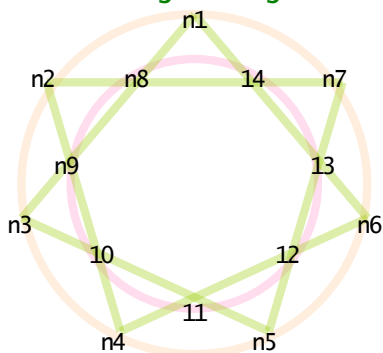


Part 2: Basic Studies of Magic Stars I: Kanji Setsuda

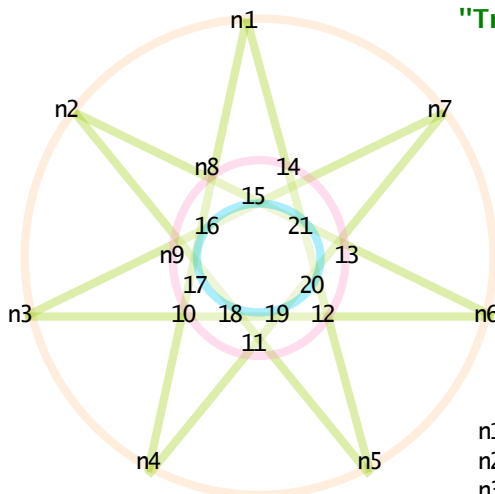
Chapter 1: Introduction: What are Magic Stars?

"Double-Ringed" Magic Star 7



$$\begin{aligned} n1+n8+n9+n3 &= K \\ n2+n9+n10+n4 &= K \\ n3+n10+n11+n5 &= K \\ n4+n11+n12+n6 &= K \\ n5+n12+n13+n7 &= K \\ n6+n13+n14+n1 &= K \\ n7+n14+n8+n2 &= K \end{aligned}$$

"Triple-Ringed" MS7



$$\begin{aligned} n1+n8+n16+n17+n10+n4 &= K2 \\ n2+n9+n17+n18+n11+n5 &= K2 \\ n3+n10+n18+n19+n12+n6 &= K2 \\ n4+n11+n19+n20+n13+n7 &= K2 \\ n5+n12+n20+n21+n14+n1 &= K2 \\ n6+n13+n21+n15+n8+n2 &= K2 \\ n7+n14+n15+n16+n9+n3 &= K2 \end{aligned}$$

[Basic Forms]

You may probably know well about 'Magic Stars', but let me explain a little about them in my own way and report about the result of my recent study here.

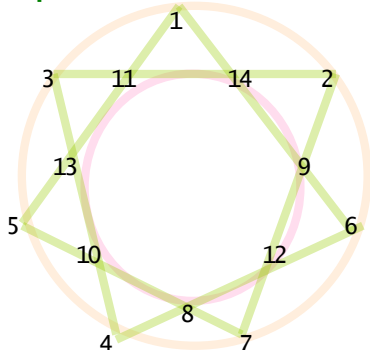
What do the magic stars look like? I want you to give your kind look at the representative figures above and below about (1) "Double-Ringed" Magic Star of Order 7 and (2) "Triple-Ringed" Magic Star of Order 7.

The former type of MS7 is made up of the 2×7 natural numbers $\{1, 2, 3, 4, 5, \dots, 12, 13, 14\}$, while the latter one is made up of the 3×7 natural numbers $\{1, 2, 3, 4, 5, \dots, 19, 20, 21\}$, each number of which must be used strictly once and neither duplication nor lack of any one is allowed.

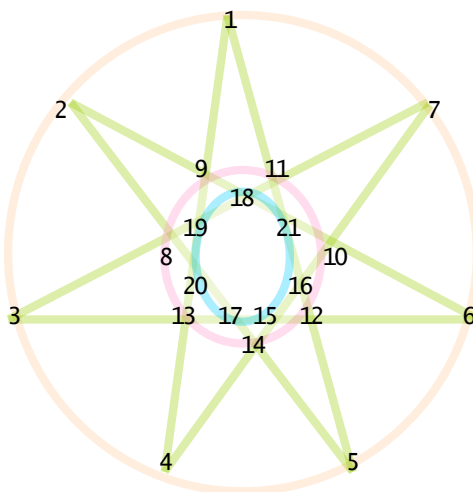
(1) Every 4 numbers on each of 7 diagonals must add up to the characteristic Magic Constant 30 for the Double-Ringed type of Magic Stars 7.

(2) Every 6 numbers on each diagonal must add up to its own Magic Constant 66 for the Triple-Ringed MS7.

* Sample Solutions *



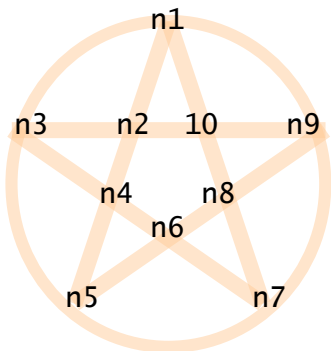
$$\begin{aligned} 1+11+13+5 &= 30; & 3+13+10+4 &= 30; \\ 5+10+8+7 &= 30; & 4+8+12+6 &= 30; \\ 7+12+9+2 &= 30; & 6+9+14+1 &= 30; \\ 2+14+11+3 &= 30; \end{aligned}$$



$$\begin{aligned} 1+9+19+20+13+4 &= 66; \\ 2+8+20+17+14+5 &= 66; \\ 3+13+17+15+12+6 &= 66; \\ 4+14+15+16+10+7 &= 66; \\ 5+12+16+21+11+1 &= 66; \\ 6+10+21+18+9+2 &= 66; \\ 7+11+18+19+8+3 &= 66; \end{aligned}$$

Let's study here for a while about the Double-Ringed Magic Stars of various orders. Please read the next list of various Definition Diagrams as follows, will you? As you see, they all have the same structure in common, so simple and impressive.

*** Basic Forms and Definitions for "Double-Ringed" Magic Stars ***

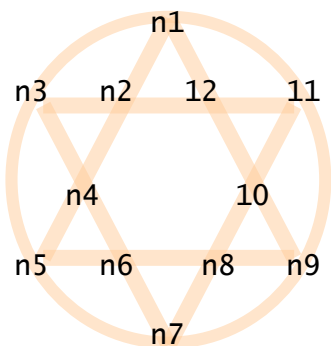


*** Basic Definition for Order 5 ***

$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & 11 \\ n3+n4+n6+n7 &= C & \dots & 12 \\ n5+n6+n8+n9 &= C & \dots & 13 \\ n7+n8+n10+n1 &= C & \dots & 14 \\ n9+n10+n2+n3 &= C & \dots & 15 \end{aligned}$$

$$\begin{aligned} 2*(n1+n2+n3+\dots+n9+n10) &= 5*C \\ 2*55 &= 5*C \end{aligned}$$

Therefore $C=22 \dots r0$

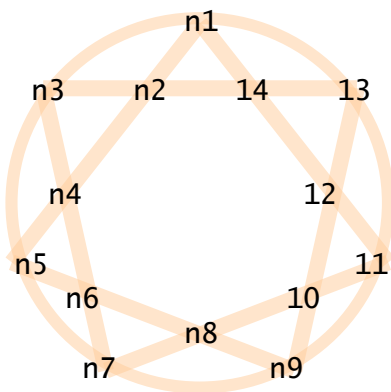


*** Basic Definition for Order 6 ***

$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & 11 \\ n3+n4+n6+n7 &= C & \dots & 12 \\ n5+n6+n8+n9 &= C & \dots & 13 \\ n7+n8+n10+n11 &= C & \dots & 14 \\ n9+n10+n12+n1 &= C & \dots & 15 \\ n11+n12+n2+n3 &= C & \dots & 16 \end{aligned}$$

$$\begin{aligned} 2*(n1+n2+n3+\dots+n11+n12) &= 6*C \\ 2*78 &= 6*C \end{aligned}$$

Therefore $C=26 \dots r0$

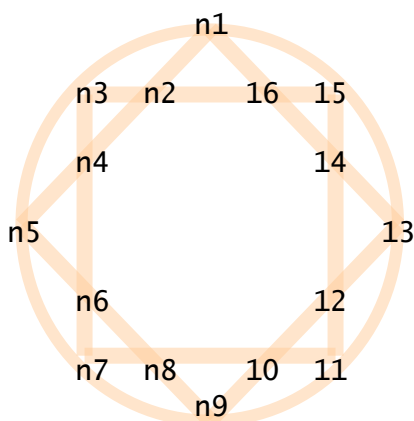


*** Basic Definition for Order 7 ***

$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & 11 \\ n3+n4+n6+n7 &= C & \dots & 12 \\ n5+n6+n8+n9 &= C & \dots & 13 \\ n7+n8+n10+n11 &= C & \dots & 14 \\ n9+n10+n12+n13 &= C & \dots & 15 \\ n11+n12+n14+n1 &= C & \dots & 16 \\ n13+n14+n2+n3 &= C & \dots & 17 \end{aligned}$$

$$\begin{aligned} 2*(n1+n2+n3+\dots+n13+n14) &= 7*C \\ 2*105 &= 7*C \end{aligned}$$

Therefore $C=30 \dots r0$



*** Basic Definition for Order 8 ***

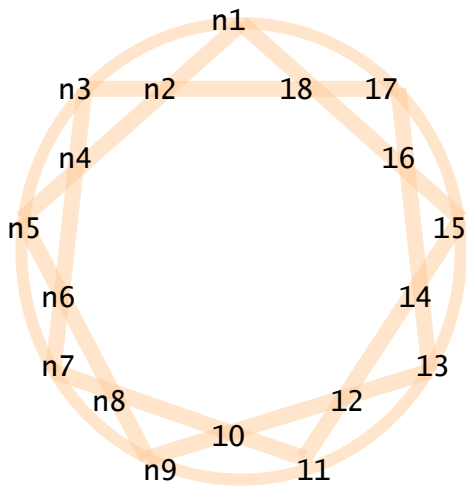
$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & 11 \\ n3+n4+n6+n7 &= C & \dots & 12 \\ n5+n6+n8+n9 &= C & \dots & 13 \\ n7+n8+n10+n11 &= C & \dots & 14 \\ n9+n10+n12+n13 &= C & \dots & 15 \\ n11+n12+n14+n15 &= C & \dots & 16 \\ n13+n14+n16+n1 &= C & \dots & 17 \\ n15+n16+n2+n3 &= C & \dots & 18 \end{aligned}$$

$$\begin{aligned} 2*(n1+n2+n3+\dots+n15+n16) &= 8*C \\ 2*136 &= 8*C \end{aligned}$$

Therefore $C=34 \dots r0$

Since all numbers look like making a picture of any brightest star, we used to call those objects as 'Magic Stars'.

However, they will look quite different in higher orders.



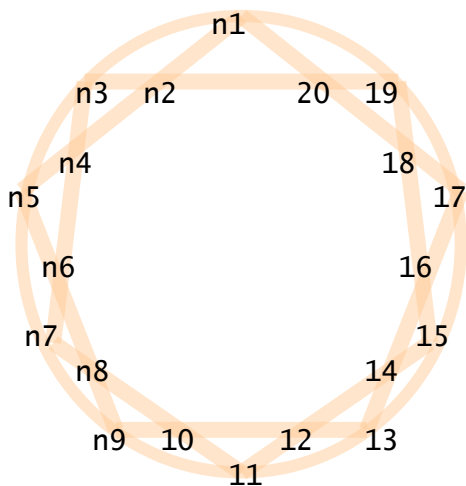
* Basic Definition for **Order 9** *

$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & \text{11} \\ n3+n4+n6+n7 &= C & \dots & \text{12} \\ n5+n6+n8+n9 &= C & \dots & \text{13} \\ n7+n8+n10+n11 &= C & \dots & \text{14} \\ n9+n10+n12+n13 &= C & \dots & \text{15} \\ n11+n12+n14+n15 &= C & \dots & \text{16} \\ n13+n14+n16+n17 &= C & \dots & \text{17} \\ n15+n16+n18+n1 &= C & \dots & \text{18} \\ n17+n18+n2+n3 &= C & \dots & \text{19} \end{aligned}$$

$$2*(n1+n2+n3+\dots+n17+n18)=9*C$$

$$2*171=9*C$$

Therefore $C=38 \dots r0$



* Basic Definition for **Order 10** *

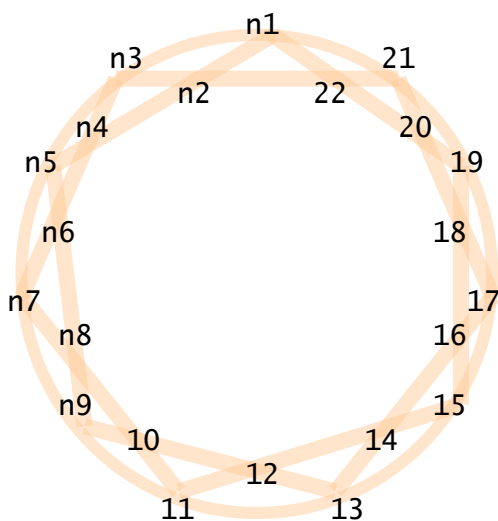
$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & \text{11} \\ n3+n4+n6+n7 &= C & \dots & \text{12} \\ n5+n6+n8+n9 &= C & \dots & \text{13} \\ n7+n8+n10+n11 &= C & \dots & \text{14} \\ n9+n10+n12+n13 &= C & \dots & \text{15} \\ n11+n12+n14+n15 &= C & \dots & \text{16} \\ n13+n14+n16+n17 &= C & \dots & \text{17} \\ n15+n16+n18+n19 &= C & \dots & \text{18} \\ n17+n18+n20+n1 &= C & \dots & \text{19} \\ n19+n20+n2+n3 &= C & \dots & \text{110} \end{aligned}$$

$$2*(n1+n2+n3+\dots+n19+n20)=10*C$$

$$2*210=10*C$$

Therefore $C=42 \dots r0$

They indeed look like pictures of a noble bracelet or a royal crown in higher orders rather than a brightest star, don't they? They are getting closer to the outer circle of a double frame. The higher order, the closer.



* Basic Definition for **Order 11** *

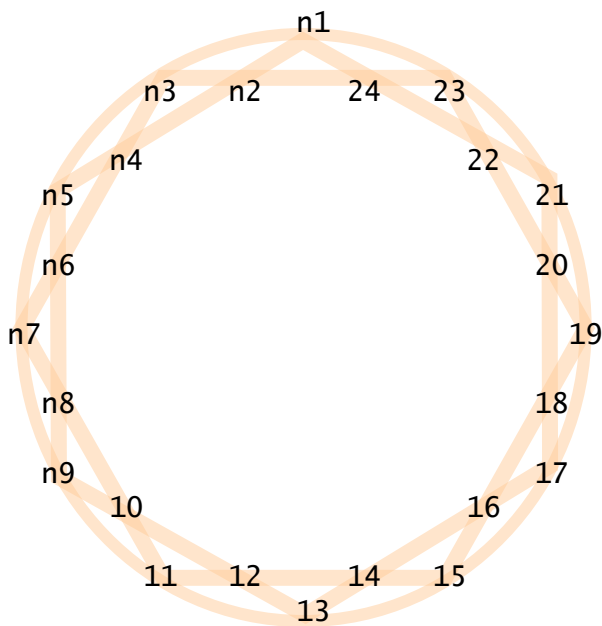
$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & \text{11} \\ n3+n4+n6+n7 &= C & \dots & \text{12} \\ n5+n6+n8+n9 &= C & \dots & \text{13} \\ n7+n8+n10+n11 &= C & \dots & \text{14} \\ n9+n10+n12+n13 &= C & \dots & \text{15} \\ n11+n12+n14+n15 &= C & \dots & \text{16} \\ n13+n14+n16+n17 &= C & \dots & \text{17} \\ n15+n16+n18+n19 &= C & \dots & \text{18} \\ n17+n18+n20+n21 &= C & \dots & \text{19} \\ n19+n20+n22+n1 &= C & \dots & \text{110} \\ n21+n22+n2+n3 &= C & \dots & \text{111} \end{aligned}$$

$$2*(n1+n2+n3+\dots+n21+n22)=11*C$$

$$2*253=11*C$$

Therefore $C=46 \dots r0$

'Double rings' and 'net structure' are the essential properties of magic stars, as you see. The outer ring only consists of variables named with 'odd' numbers, while the inner ring consists of 'even' variables. Both rings share the two common numbers among four in each diagonal, making the 'net structure' among them.



Basic Definition for Order 12

$$\begin{aligned}
 n1+n2+n4+n5 &= C & \dots & \text{r1} \\
 n3+n4+n6+n7 &= C & \dots & \text{r2} \\
 n5+n6+n8+n9 &= C & \dots & \text{r3} \\
 n7+n8+n10+n11 &= C & \dots & \text{r4} \\
 n9+n10+n12+n13 &= C & \dots & \text{r5} \\
 n11+n12+n14+n15 &= C & \dots & \text{r6} \\
 n13+n14+n16+n17 &= C & \dots & \text{r7} \\
 n15+n16+n18+n19 &= C & \dots & \text{r8} \\
 n17+n18+n20+n21 &= C & \dots & \text{r9} \\
 n19+n20+n22+n23 &= C & \dots & \text{r10} \\
 n21+n22+n24+n1 &= C & \dots & \text{r11} \\
 n23+n24+n2+n3 &= C & \dots & \text{r12}
 \end{aligned}$$

$$\begin{aligned}
 2 * (n1+n2+n3+\dots+n23+n24) &= 12 * C \\
 2 * 300 &= 12 * C
 \end{aligned}$$

Therefore $C=50 \dots r0$

Chapter 2: Algebraic Study of Double-Ringed Magic Stars 5

Let's study about various magic stars precisely, order by order.

We know we cannot compose any magic stars at all in lower orders than 5, since we cannot draw any double rings with the net structure. It is for the order 5 that we seem to be able to compose the smallest type of magic stars for the first time.

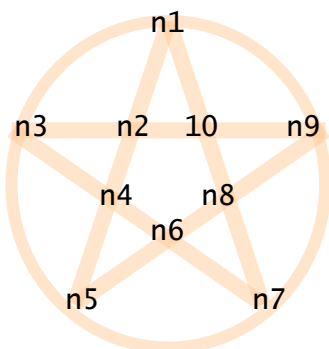
Let's make any actual Magic Stars of Order 5.

However, whenever we tried to compose any of them, we would always fail in getting the result. It seems to be impossible for us to make it up.

But why? Can we prove its impossibility? And how?

We have to make some algebraic study on our object before all.

*** Basic Form and Basic Definition for Magic Stars of Order 5 ***



*** Basic Definition ***

$$\begin{aligned}
 n1+n2+n4+n5 &= C & \dots & \text{r1} \\
 n3+n4+n6+n7 &= C & \dots & \text{r2} \\
 n5+n6+n8+n9 &= C & \dots & \text{r3} \\
 n7+n8+n10+n1 &= C & \dots & \text{r4} \\
 n9+n10+n2+n3 &= C & \dots & \text{r5}
 \end{aligned}$$

$$\begin{aligned}
 2 * (n1+n2+n3+\dots+n9+n10) &= 5 * C \\
 2 * 55 &= 5 * C
 \end{aligned}$$

Therefore $C=22 \dots r1$

Of course, we have to use each of the natural numbers {1, 2, 3, 4, 5, 6, 7, 8, 9 and 10} strictly once in a figure. We must not use any certain number twice or no

more often, and we must not neglect any number and un-use it, either.

11+14+15

$$\begin{aligned} n1+n2+n4+n5+n7+n8+n10+n1+n9+n10+n2+n3 &= 3 * C \\ (n1+n2+n10) + (n1+n2+n3+n4+n5+n7+n8+n9+n10) &= 3 * 22 \end{aligned}$$

Add n6 to both sides.

$$\begin{aligned} (n1+n2+n10) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10) &= 66+n6 \\ (n1+n2+n10) + 55 &= 66+n6 \end{aligned}$$

Therefore $n1+n2+n10=11+n6$ or $n1=11-n2+n6-n10 \dots r2$

11+12+15

$$\begin{aligned} n1+n2+n4+n5+n3+n4+n6+n7+n9+n10+n2+n3 &= 3 * C \\ (n2+n3+n4) + (n1+n2+n3+n4+n5+n6+n7+n9+n10) &= 66 \end{aligned}$$

Add n8 to both sides.

$$(n2+n3+n4) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10) = 66+n8$$

Therefore $n2+n3+n4=11+n8$ or $n3=11-n2-n4+n8 \dots r3$

11+12+13

$$\begin{aligned} n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9 &= 3 * C \\ (n4+n5+n6) + (n1+n2+n3+n4+n5+n6+n7+n8+n9) &= 66 \end{aligned}$$

Add n10 to both sides.

$$(n4+n5+n6) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10) = 66+n10$$

Therefore $n4+n5+n6=11+n10$ or $n5=11-n4-n6+n10 \dots r4$

Similarly $n6+n7+n8=11+n2$ or $n7=11+n2-n6-n8 \dots r5$

$n8+n9+n10=11+n4$ or $n9=11+n4-n8-n10 \dots r6$

11+13+15

$$\begin{aligned} n1+n2+n4+n5+n5+n6+n8+n9+n9+n10+n2+n3 &= 3 * C \\ (n2+n5+n9) + (n1+n2+n3+n4+n5+n6+n8+n9+n10) &= 66 \end{aligned}$$

Add n7 to both sides.

$$(n2+n5+n9) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10) = 66+n7$$

Therefore $n2+n5+n9=11+n7$ or $n2=11-n5+n7-n9 \dots r7$

11+12+14

$$\begin{aligned} n1+n2+n4+n5+n3+n4+n6+n7+n7+n8+n10+n1 &= 3 * C \\ (n1+n4+n7) + (n1+n2+n3+n4+n5+n6+n7+n8+n10) &= 66 \end{aligned}$$

Add n9 to both sides.

$$(n1+n4+n7) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10) = 66+n9$$

Therefore $n1+n4+n7=11+n9$ or $n4=11-n1-n7+n9 \dots r8$

Similarly $n3+n6+n9=11+n1$ or $n6=11+n1-n3-n9 \dots r9$

$n1+n5+n8=11+n3$ or $n8=11+n3-n5-n1 \dots r10$

$n3+n7+n10=11+n5$ or $n10=11-n3+n5-n7 \dots r11$

11+12+13+14+15

$$\begin{aligned} n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9+n7+n8+n10+n1-n9-n10-n2-n3 &= 3 * C \\ 2 * (n1+n4+n5+n6+n7+n8) &= 66 \end{aligned}$$

Therefore $n1+n4+n5+n6+n7+n8=33$ or $n1=33-n4-n5-n6-n7-n8 \dots r12$

-11+12+13+14+15

$$\begin{aligned} -n1-n2-n4-n5+n3+n4+n6+n7+n5+n6+n8+n9+n7+n8+n10+n1+n9+n10+n2+n3 &= 3 * C \\ 2 * (n3+n6+n7+n8+n9+n10) &= 66 \end{aligned}$$

Therefore $n3+n6+n7+n8+n9+n10=33$ or $n3=33-n6-n7-n8-n9-n10 \dots r13$

11-12+13+14+15

$$\begin{aligned} n1+n2+n4+n5-n3-n4-n6-n7+n5+n6+n8+n9+n7+n8+n10+n1+n9+n10+n2+n3 &= 3 * C \\ 2 * (n1+n2+n5+n8+n9+n10) &= 66 \end{aligned}$$

Therefore $n1+n2+n5+n8+n9+n10=33$ or $n5=33-n1-n2-n8-n9-n10 \dots r14$

11+12-13+14+15

$$n1+n2+n4+n5+n3+n4+n6+n7-n5-n6-n8-n9+n7+n8+n10+n1+n9+n10+n2+n3=3 * C$$

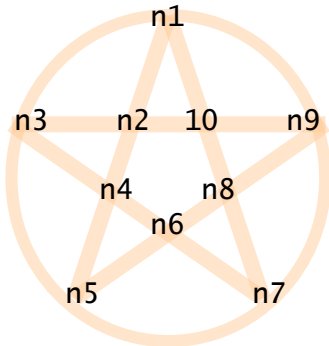
$$2*(n1+n2+n3+n4+n7+n10)=66$$

Therefore $n1+n2+n3+n4+n7+n10=33$ or $n7=33-n1-n2-n3-n4-n10 \dots r15$
 $11+12+13-14+15$

$$n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9-n7-n8-n10-n1+n9+n10+n2+n3=3*C$$

$$2*(n2+n3+n4+n5+n6+n9)=66$$

Therefore $n2+n3+n4+n5+n6+n9=33$ or $n9=33-n2-n3-n4-n5-n6 \dots r16$



* Basic Definition *

$$n1+n2+n4+n5=C \dots 11$$

$$n3+n4+n6+n7=C \dots 12$$

$$n5+n6+n8+n9=C \dots 13$$

$$n7+n8+n10+n1=C \dots 14$$

$$n9+n10+n2+n3=C \dots 15$$

* Summary: List of Relations: *

$$n1+n2+n10=11+n6 \quad \text{or} \quad n1=11-n2+n6-n10 \quad \dots r2$$

$$n2+n3+n4=11+n8 \quad \text{or} \quad n3=11-n2-n4+n8 \quad \dots r3$$

$$n4+n5+n6=11+n10 \quad \text{or} \quad n5=11-n4-n6+n10 \quad \dots r4$$

$$n6+n7+n8=11+n2 \quad \text{or} \quad n7=11+n2-n6-n8 \quad \dots r5$$

$$n8+n9+n10=11+n4 \quad \text{or} \quad n9=11+n4-n8-n10 \quad \dots r6$$

$$n2+n5+n9=11+n7 \quad \text{or} \quad n2=11-n5+n7-n9 \quad \dots r7$$

$$n1+n4+n7=11+n9 \quad \text{or} \quad n4=11-n1-n7+n9 \quad \dots r8$$

$$n3+n6+n9=11+n1 \quad \text{or} \quad n6=11+n1-n3-n9 \quad \dots r9$$

$$n1+n5+n8=11+n3 \quad \text{or} \quad n8=11+n3-n5-n1 \quad \dots r10$$

$$n3+n7+n10=11+n5 \quad \text{or} \quad n10=11-n3+n5-n7 \quad \dots r11$$

$$n1+n4+n5+n6+n7+n8=33 \quad \text{or} \quad n1=33-n4-n5-n6-n7-n8 \quad \dots r12$$

$$n3+n6+n7+n8+n9+n10=33 \quad \text{or} \quad n3=33-n6-n7-n8-n9-n10 \quad \dots r13$$

$$n1+n2+n5+n8+n9+n10=33 \quad \text{or} \quad n5=33-n1-n2-n8-n9-n10 \quad \dots r14$$

$$n1+n2+n3+n4+n7+n10=33 \quad \text{or} \quad n7=33-n1-n2-n3-n4-n10 \quad \dots r15$$

$$n2+n3+n4+n5+n6+n9=33 \quad \text{or} \quad n9=33-n2-n3-n4-n5-n6 \quad \dots r16$$

We could not find any logical contradictions among all the relations listed above.

We now have to make any 'value test' to the same relations for the next step. We want to know the reason why no solution could be found.

Let's take the value 1 for $n1$ at first and then $n2$, because both of them act as a representative position for some: $n1$ for the 'outer ring' and $n2$ for the 'inner one'.

The solution whose $n3$ takes the value 1 means the same thing to that one whose $n1=1$, because we can transform them into each other by some simple rotation and mirror reflection if necessary. Any solution whose $n5$, $n7$ or $n9=1$ means the same to the one whose $n1=1$ for the same reason.

The solution whose $n4$, $n6$, $n8$ or $n10$ takes the value 1 also means the same thing to that one whose $n2=1$, because we can transform them into each other by some simple rotation and mirror reflection without changing anything essential.

** Value Tests **

If $n1=1$ then $\{n2+n10-n6=10; n4+n7-n9=10; n5+n8-n3=10; n3+n6+n9=12;\}$
 Therefore $(n2+n10-n6)+(n4+n7-n9)+(n5+n8-n3)+(n3+n6+n9)=42;$
 $n2+n4+n5+n7+n8+n10=42;$

If $n2=1$ then $\{n1+n10-n6=10; n3+n4-n8=10; n5+n9-n7=10; n6+n7+n8=12;\}$
 Therefore $n1+n10-n6+n3+n4-n8+n5+n9-n7+n6+n7+n8=42;$

$$n1+n3+n4+n5+n9+n10=42;$$

** Impossible Magic Star of Order 5 **
 ** Possible Combinations of Values: **

 * When n1=1: *
 $n2+n4+n5+n7+n8+n10=42; n3+n6+n9=10;$
 $n2+n10-n6=10; n4+n7-n9=10; n5+n8-n3=10;$

 [1] (2,6,7,8,9,10); (3,4,5);
 {(6,7,3), (6,8,4), (6,9,5), (7,8,5), ()}
 [2] (3,5,7,8,9,10); (2,4,6);
 {(3,9,2), (5,7,2), (5,9,4), (7,9,6), ()}
 [3] (4,5,6,8,9,10); (2,3,7);
 {(4,8,2), (4,9,3), (5,8,3), (8,9,7), ()}

 * When n2=1: *
 $n1+n3+n4+n5+n9+n10=42; n6+n7+n8=12;$
 $n1+n10-n6=10; n3+n4-n8=10; n5+n9-n7=10;$

 [1] (2,6,7,8,9,10); (3,4,5);
 {(6,7,3), (6,8,4), (6,9,5), (7,8,5), ()}
 [2] (3,5,7,8,9,10); (2,4,6);
 {(3,9,2), (5,7,2), (5,9,4), (7,9,6), ()}
 [3] (4,5,6,8,9,10); (2,3,7);
 {(4,8,2), (4,9,3), (5,8,3), (8,9,7), ()}

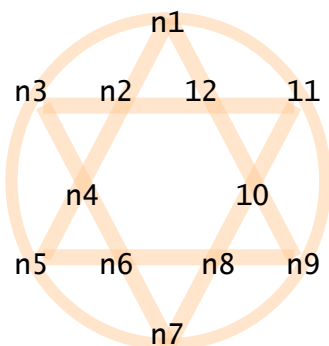
We could not really satisfy all the equations above with the serial natural numbers {1, 2, 3, 4, 5, ..., 9 and 10}, each one of which must be used strictly once.

We could not avoid repeating usage of any certain numbers for some equations.

We could not really make any solution whose n1=1. That means we could no longer make any one whose n3, n5, n7 or n9=1. The fact we could not make any one whose n2=1 also means we could no longer make any one whose n4, n6, n8 or n10=1.

As a consequence we now know we cannot practically make any magic star of order 5 only with the limited usage of natural number elements.

Chapter 3: Algebraic Study of Double-Ringed Magic Stars 6



* Basic Definition for Order 6 *

$n1+n2+n4+n5=C \quad \dots \quad 11$
 $n3+n4+n6+n7=C \quad \dots \quad 12$
 $n5+n6+n8+n9=C \quad \dots \quad 13$
 $n7+n8+n10+n11=C \quad \dots \quad 14$
 $n9+n10+n12+n1=C \quad \dots \quad 15$
 $n11+n12+n2+n3=C \quad \dots \quad 16$

 $2*(n1+n2+n3+\dots+n11+n12)=6*C$
 $2*78=6*C$

Therefore $C=26 \quad \dots \quad r0$

Let's make any real Magic Stars of order 6 next.

How many solutions can we compose for Magic Stars in all?

```

/** Compose Magic Stars of Order 6 **
/** File: 'MStar6.c' Dictated by Kanji Setsuda **
/** on Nov.25, 2003 and Nov.16, 2005; **
/** Recompiled and listed on Mar.26, 2015 **
/** with MacOSX 10.10.2 & Xcode 6.2; **
//
#include <stdio.h>
//
short cnt, cnt2;
short LSM;
short nm[13], uflg[13];
short anm[5][13];
//
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void stp09(void), stp10(void), stp11(void), stp12(void);
void ansprint(void), pr4ans(void);
//
/** Main Program *
int main(){
short n;
printf("\n");
printf("*** List of Standard Magic Stars of Order 6 **\n");
for(n=0;n<13;n++){nm[n]=0; uflg[n]=0;}
LSM=26;
cnt=0; cnt2=0;
stp01();      /** Begin the Calculations *
printf("\n");
printf(" [Count = %d] OK!\n",cnt);
printf("\n");
printf("*** Recompiled and Listed by Kanji Setsuda on\n");
printf("   Mar.26, 2015 with MacOSX 10.10.2 & Xcode 6.2 **\n");
printf("\n");
return 0;
}
/** Begin the Calculations *
/** Set N1 *
void stp01(){
short a;
for(a=1;a<8;a++){
if(uflg[a]==0){
nm[1]=a; uflg[a]=1;
stp02();
uflg[a]=0;
}}
}
/** Set N5(>n1) *
void stp02(){
short a;
for(a=nm[1]+1;a<13;a++){
if(uflg[a]==0){
nm[5]=a; uflg[a]=1;
stp03();
uflg[a]=0;
}}
}
/** Set N2 *
void stp03(){
short a;
for(a=12;a>0;a--){
if(uflg[a]==0){
nm[2]=a; uflg[a]=1;
stp04();
uflg[a]=0;
}}
}

```



```

}
/** Set n4=30-n1-n2-n5 *
void stp04(){
    short a;
    a=LSM-nm[1]-nm[2]-nm[5];
    if((0<a)&&(a<13)){
        if(uflg[a]==0){
            nm[4]=a; uflg[a]=1;
            stp05();
            uflg[a]=0;
        }
    }
}
/** Set N3(>n1) *
void stp05(){
    short a;
    for(a=nm[1]+1;a<13;a++){
        if(uflg[a]==0){
            nm[3]=a; uflg[a]=1;
            stp06();
            uflg[a]=0;
        }
    }
}
/** Set N7(>n1) *
void stp06(){
    short a;
    for(a=nm[1]+1;a<13;a++){
        if(uflg[a]==0){
            nm[7]=a; uflg[a]=1;
            stp07();
            uflg[a]=0;
        }
    }
}
/** Set n6=30-n3-n4-n7 *
void stp07(){
    short a;
    a=LSM-nm[3]-nm[4]-nm[7];
    if((0<a)&&(a<13)){
        if(uflg[a]==0){
            nm[6]=a; uflg[a]=1;
            stp08();
            uflg[a]=0;
        }
    }
}
/** Set N9(>n5) *
void stp08(){
    short a;
    for(a=nm[5]+1;a<13;a++){
        if(uflg[a]==0){
            nm[9]=a; uflg[a]=1;
            stp09();
            uflg[a]=0;
        }
    }
}
/** Set n8=30-n5-n6-n9 *
void stp09(){
    short a;
    a=LSM-nm[5]-nm[6]-nm[9];
    if((0<a)&&(a<13)){
        if(uflg[a]==0){
            nm[8]=a; uflg[a]=1;
            stp10();
            uflg[a]=0;
        }
    }
}
/** Set N11(>n1) *

```

```

void stp10(){
short a;
for(a=nm[1]+1;a<13;a++){
if(uflg[a]==0){
nm[11]=a; uflg[a]=1;
stp11();
uflg[a]=0;
}}
}
/** Set n10=30-n7-n8-n11 *
void stp11(){
short a;
a=LSM-nm[7]-nm[8]-nm[11];
if((0<a)&&(a<13)){
if(uflg[a]==0){
nm[10]=a; uflg[a]=1;
stp12();
uflg[a]=0;
}}
}
/** Set n12=30-n9-n10-n1 *
void stp12(){
short a,b;
a=LSM-nm[9]-nm[10]-nm[1];
b=LSM-nm[11]-nm[2]-nm[3];
if((a==b)&&(0<a)&&(a<13)){
if(uflg[a]==0){
nm[12]=a; uflg[a]=1;
ansprint();
uflg[a]=0;
}}
}
}
/** Print The Answers *
void ansprint(){
short n;
cnt++;
anm[cnt2][0]=cnt; anm[cnt2+1][0]=0;
for(n=1;n<13;n++){anm[cnt2][n]=nm[n]; anm[cnt2+1][n]=0;}
cnt2++;
if(cnt2==4){pr4ans(); cnt2=0;}
}
/** Print 4 Answers *
void pr4ans(){
short m;
for(m=0;m<4;m++){
printf("[%2d]",anm[m][0]);
if(m<3){printf(" ");}
}
printf("\n");
for(m=0;m<4;m++){
printf("%9d",anm[m][1]);
if(m<3){printf(" ");}
}
printf("\n");
printf(" / \ \ / \ \ / \ \ / \ \ \n");
for(m=0;m<4;m++){
printf("%3d--%2d--%2d--%2d",anm[m][3],anm[m][2],anm[m][12],anm[m][11]);
if(m<3){printf(" ");}
}
printf("\n");
printf(" \ \ / \ \ / \ \ / \ \ / \ \ / \ \ \n");
for(m=0;m<4;m++){
printf("%5d%8d",anm[m][4],anm[m][10]);
if(m<3){printf(" ");}
}
}

```

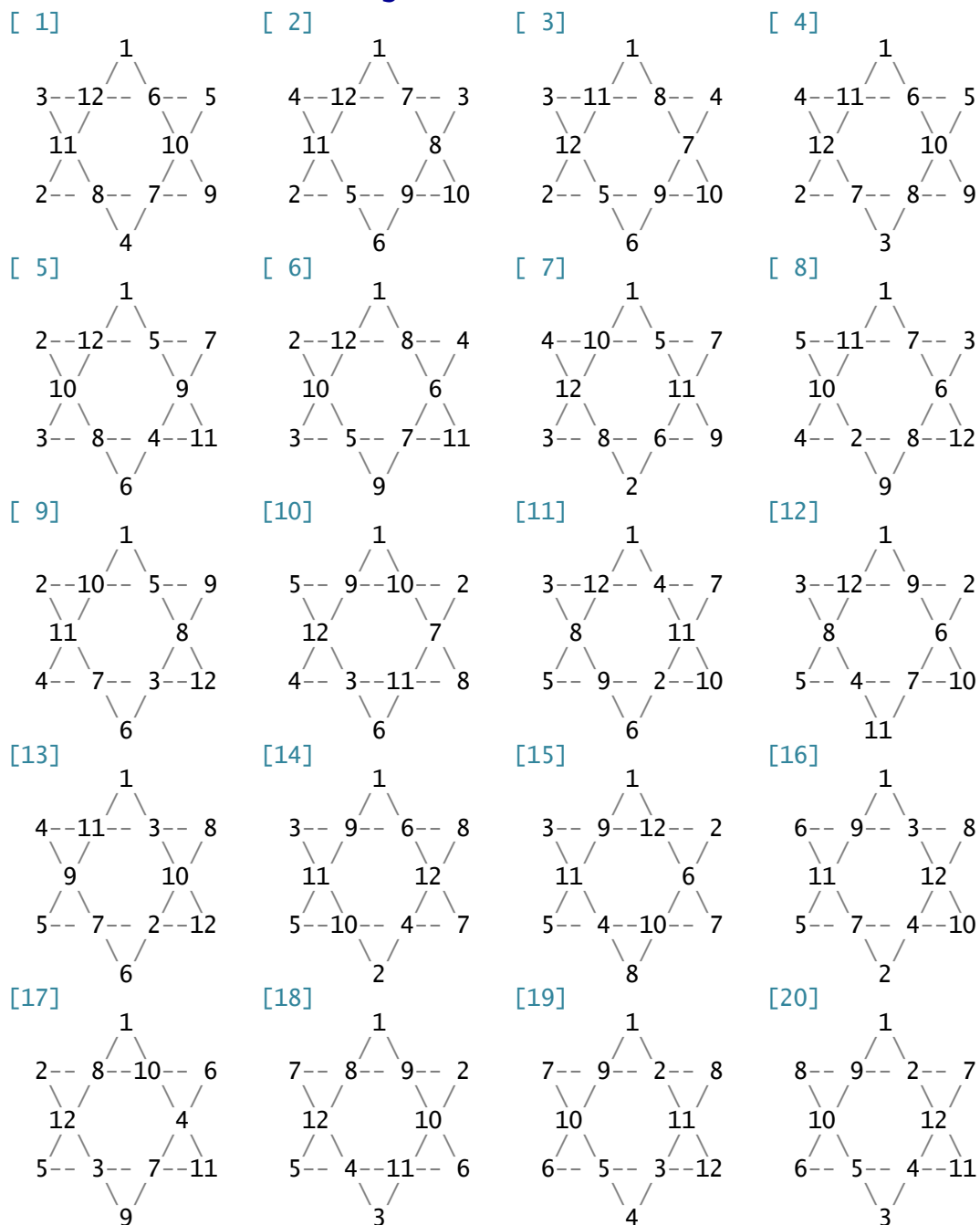
```

printf("\n");
printf(" / \ \ / \ \ / \ \ / \ \ / \ \ / \ \ / \ \ \n");
for(m=0;m<4;m++){
  printf("%3d--%2d--%2d--%2d",anm[m][5],anm[m][6],anm[m][8],anm[m][9]);
  if(m<3){printf(" ");}
}
printf("\n");
printf(" \ \ / \ \ / \ \ / \ \ \n");
for(m=0;m<4;m++){
  printf("%9d",anm[m][7]);
  if(m<3){printf(" ");}
}
printf("\n");
}
}

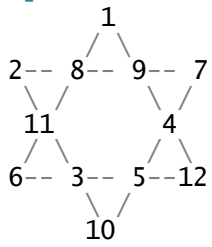
```

The next list shows the result of my recent computation by that program above. It contains all the 80 standard solutions of Magic Stars of order 6.

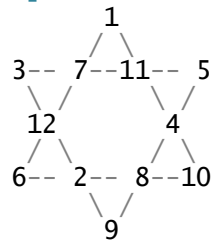
**** List of Standard Magic Stars of Order 6 ****



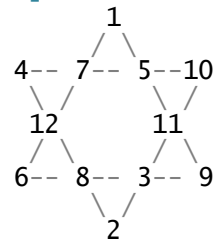
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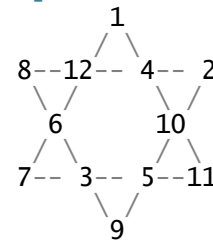
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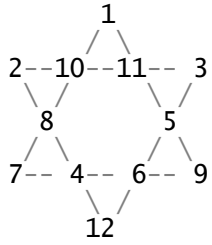
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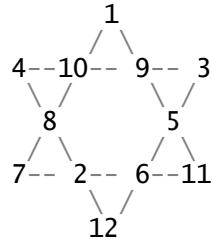
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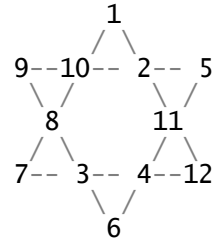
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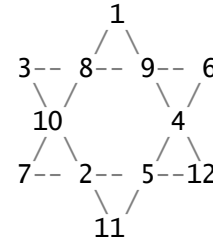
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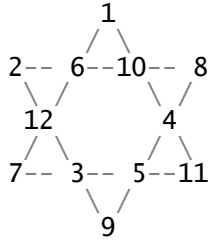
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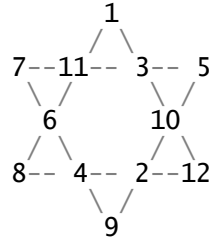
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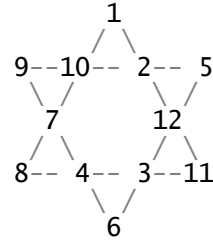
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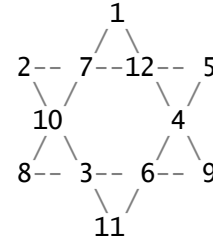
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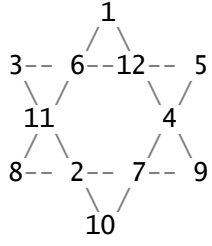
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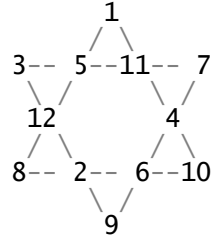
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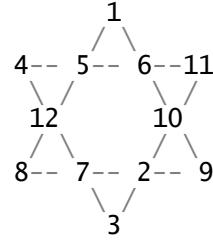
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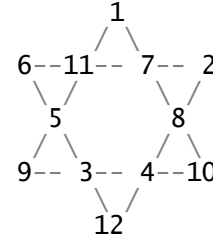
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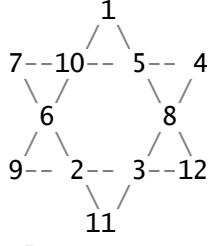
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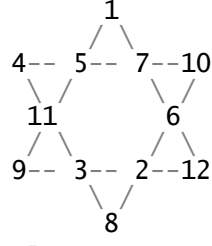
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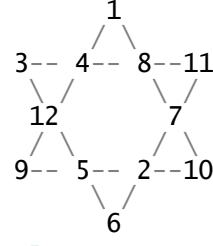
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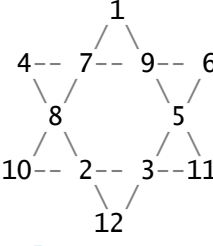
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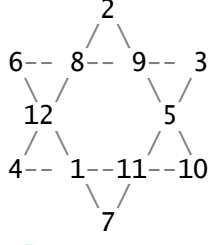
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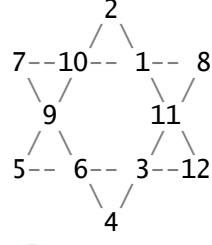
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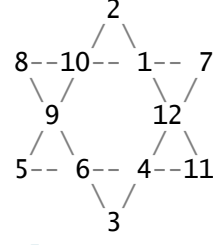
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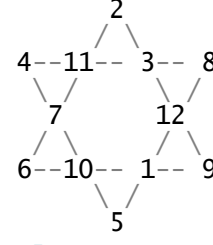
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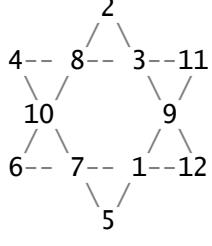
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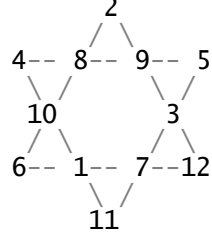
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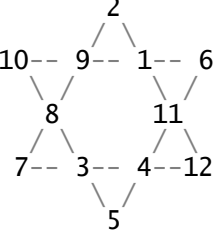
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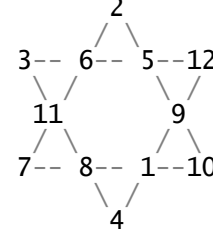
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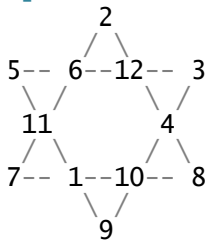
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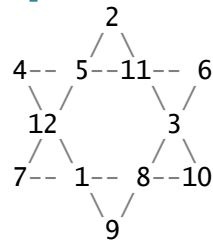
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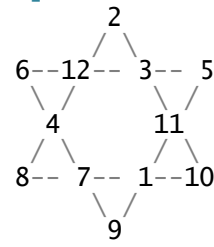
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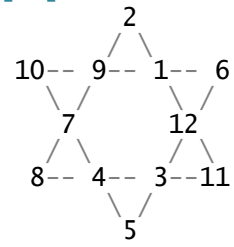
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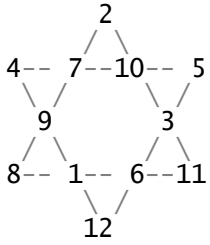
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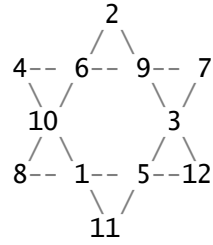
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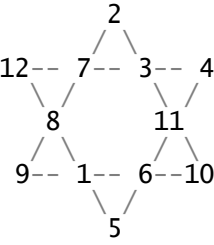
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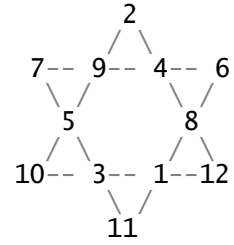
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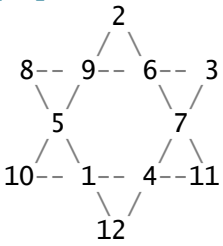
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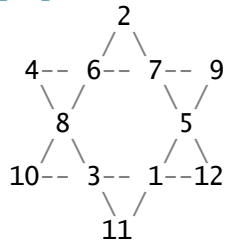
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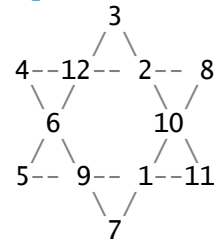
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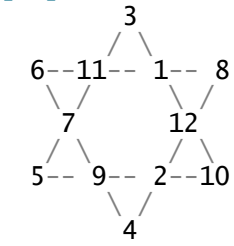
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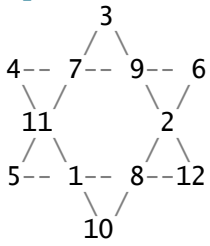
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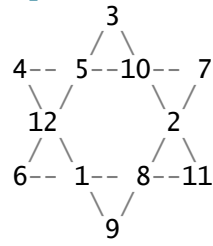
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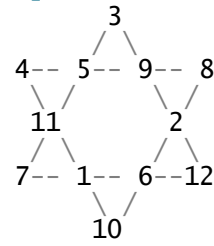
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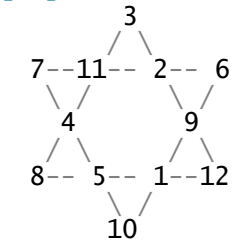
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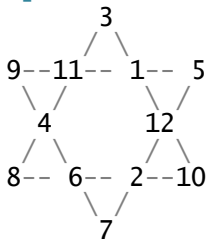
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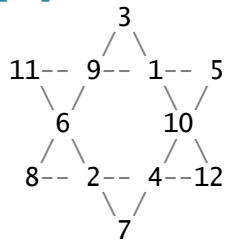
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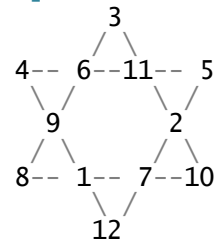
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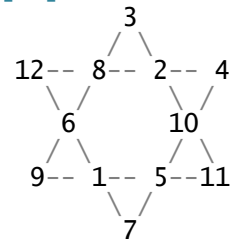
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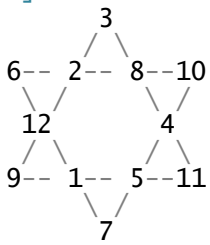
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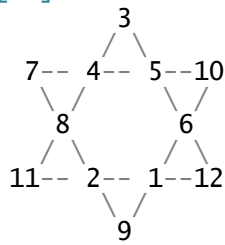
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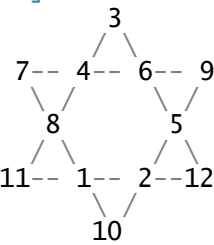
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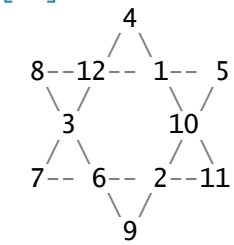
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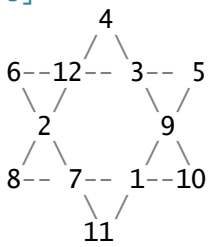
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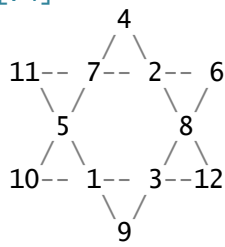
[72]



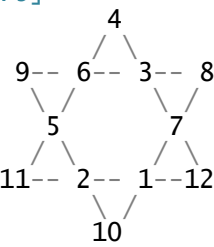
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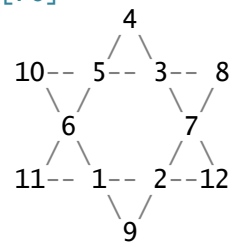
[74]



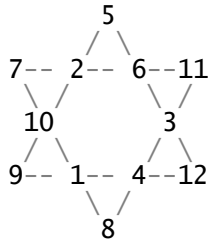
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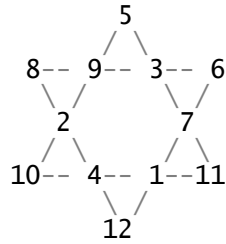
[76]



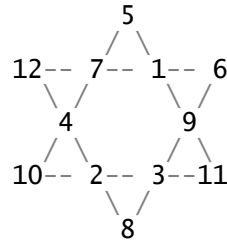
[77]



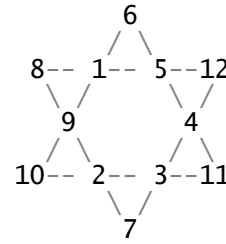
[78]



[79]



[80]



[Count = 80] OK!

** Recompiled and Listed by Kanji Setsuda on Mar.26, 2015 with MacOSX 10.10.2 & Xcode 6.2 **

As you see, we could get the 40 solutions whose $n_1=1$ and the 40 others. In the latter half any one of $\{n_2, n_4, n_6, n_8 \text{ or } n_{10}\}$ takes the value 1. It means the former 40 belong to the 'outer ring type' and the latter 40 belong to the 'inner ring type'.

It is only in the order 6 that we could have the same count of solutions for both of those two types. It seems to imply their possibility of transforming into each other.

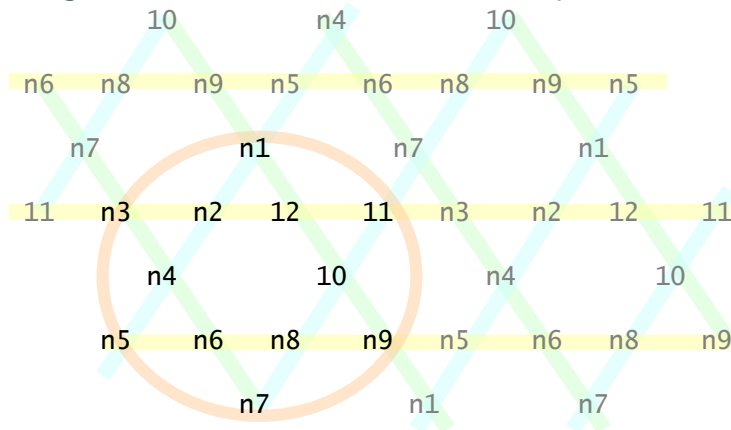
Let's **extend the space** for magic star here by duplicating several lines in the outer space, just in the same way as we often made for various Pandiagonal Magic Squares.

I have to report I could do that job successfully as shown below, and could really read another 3 different solutions out of the **Extended Space**.

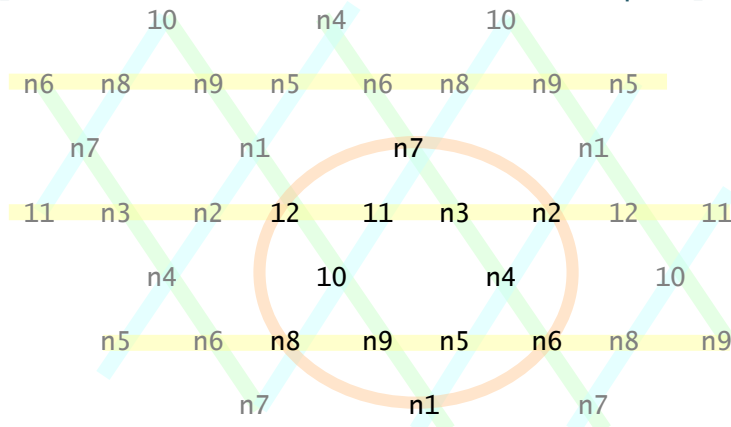
It is the fact that they were already written in the Space, where we can just find and read them out to have those 4 transformations including the original itself.

* Extended Space for Magic Star of Order 6

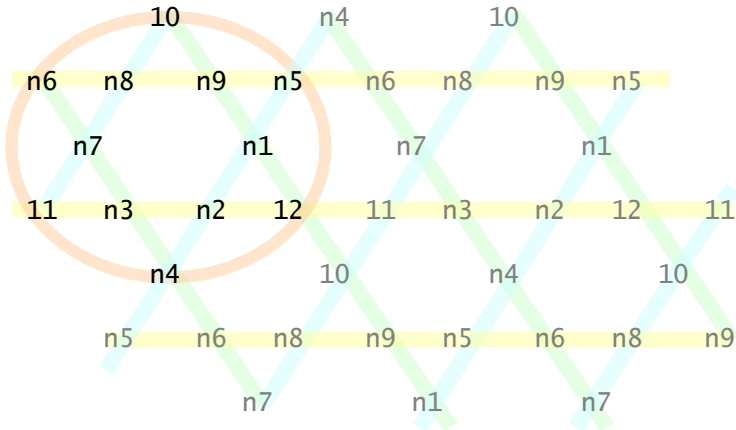
[Original Form and the Extended Space]



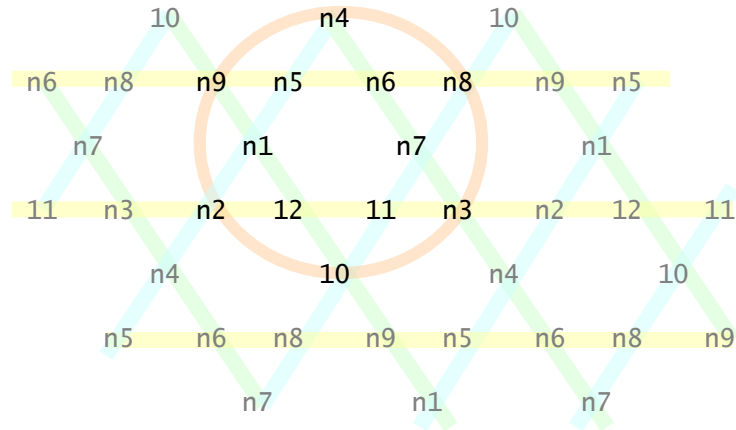
[2nd Form taken out of the Extended Space]



[3rd Form taken out]

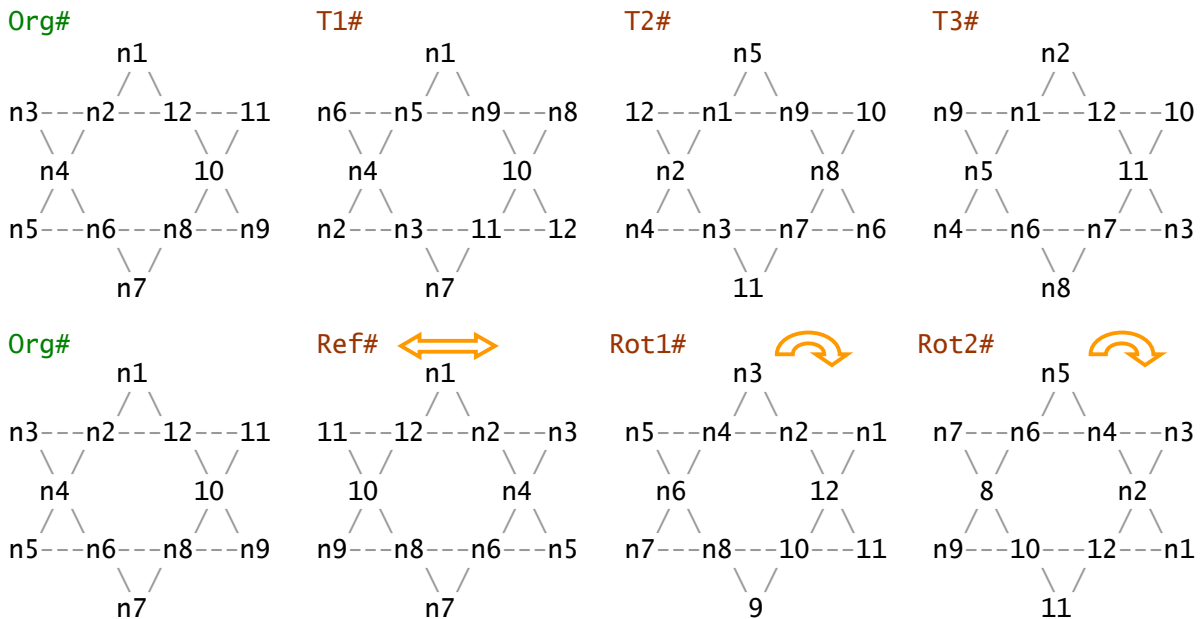


[4th Form taken out]



Let's take those 4 solutions in a group and assume some transformation system for them. Take your careful look at the next conceptual diagrams below.

**** Concept of 6 Transformations ****



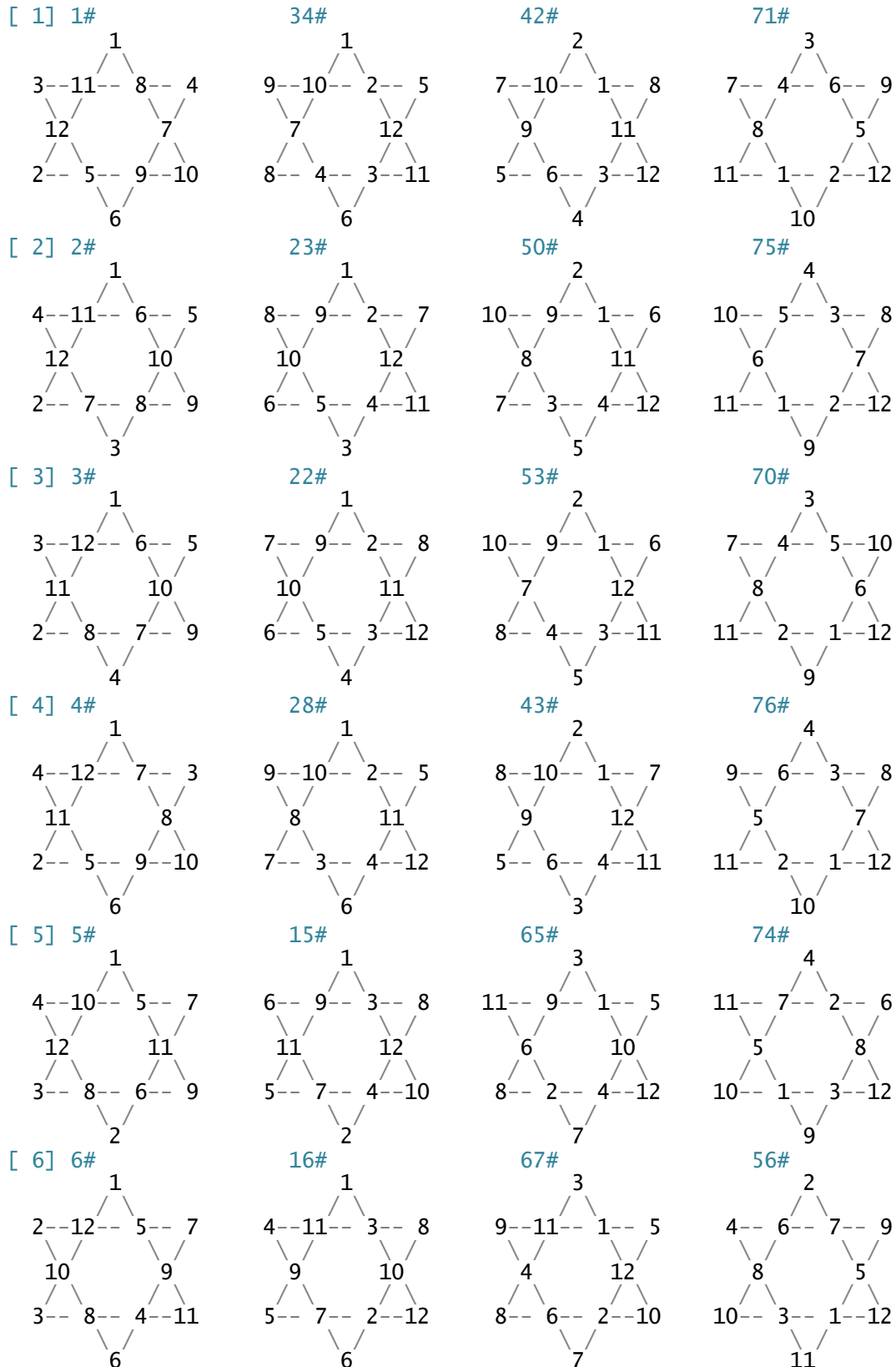
If you have both the 20 fundamental solutions and those transformations above, you can make each fundamental into three different solutions automatically. If you also assume another set of transformations such as 'mirror reflection' or 'rotation by 60 degrees', you can easily make the smart list of 80 Standard Solutions at last.

The next list shows the result of my recent classification. It gives you all the Standard Solutions by the 20 groups of four transformations. In order to get the 20 Fundamental Solutions I took the List-forming Inequality Conditions as follows:

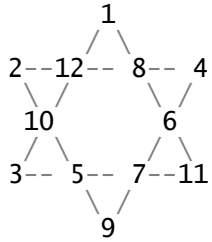
$n_1=1$; $n_1 < n_3$; $n_1 < n_5$; $n_1 < n_7$; $n_1 < n_9$; $n_1 < n_{11}$; $n_2 > n_5$; and $n_4 < n_{10}$;

**** List of Standard Solutions: Magic Stars of Order 6 ****

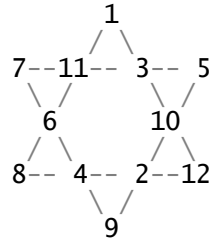
**** The Fundamental 20 and 3 Transformations for Each ****



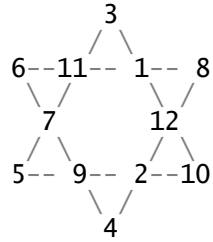
[7] 7#



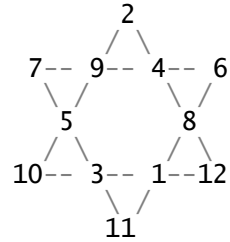
35#



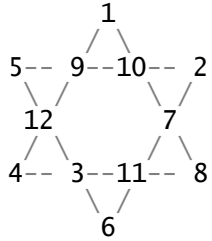
60#



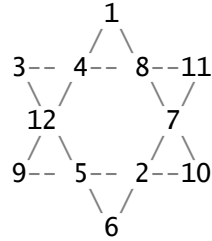
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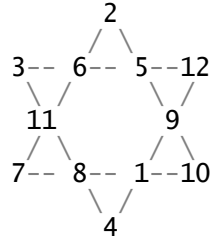
[8] 8#



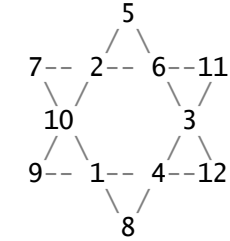
36#



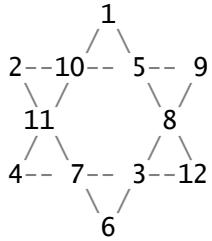
48#



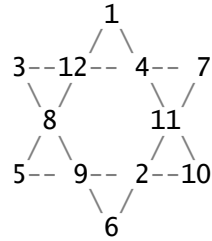
77#



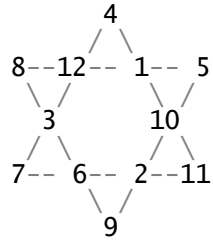
[9] 9#



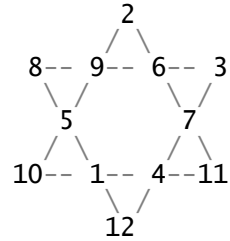
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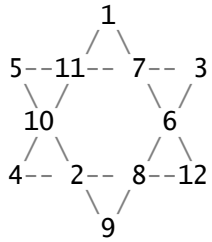
72#



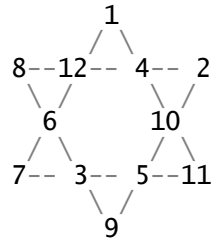
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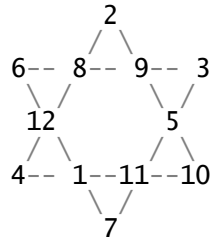
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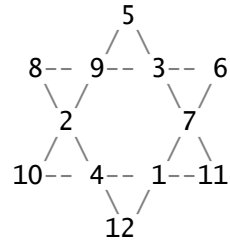
29#



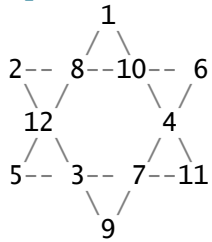
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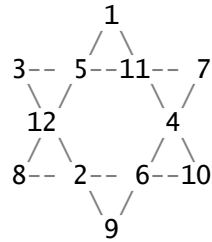
79#



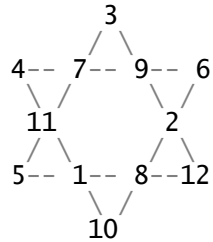
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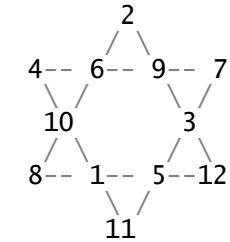
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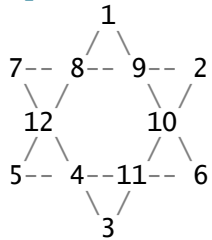
59#



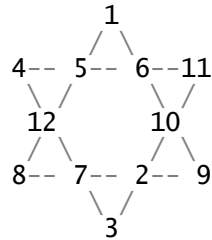
51#



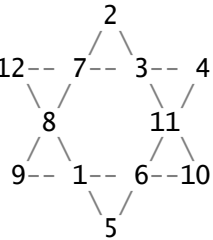
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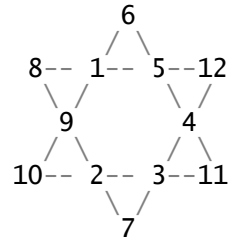
31#



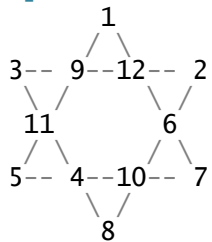
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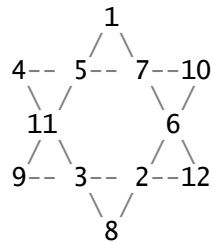
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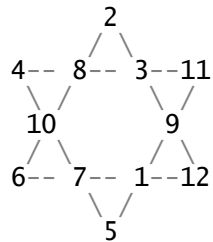
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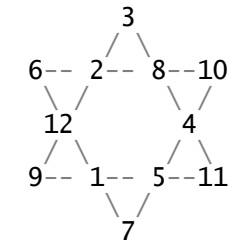
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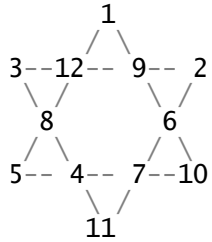
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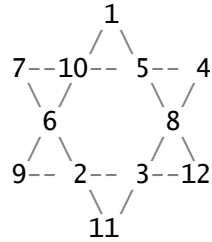
68#



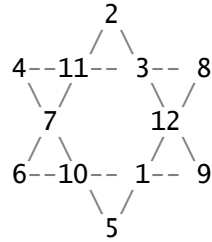
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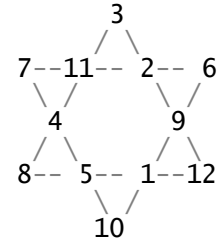
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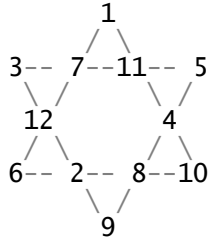
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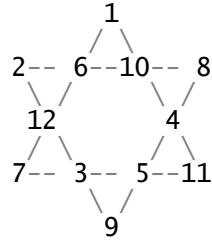
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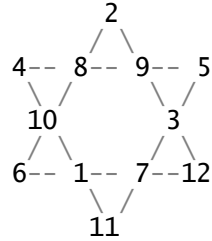
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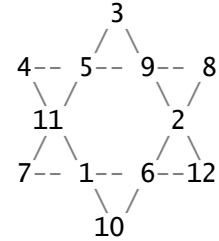
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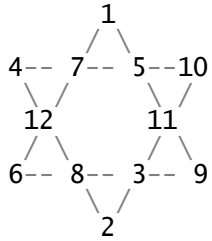
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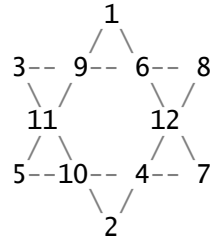
63#



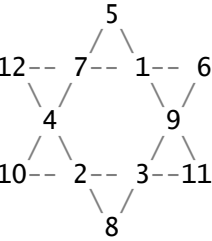
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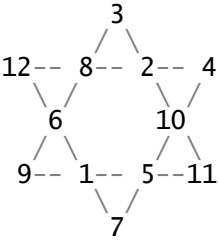
13#



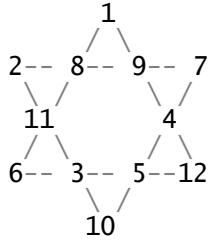
78#



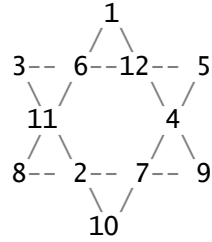
69#



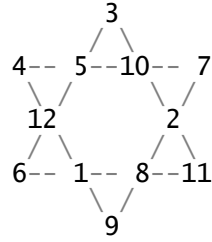
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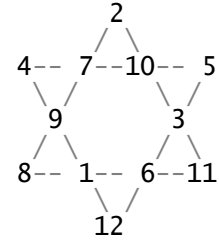
32#



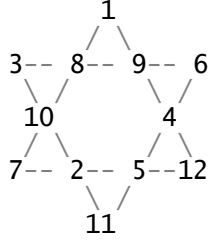
62#



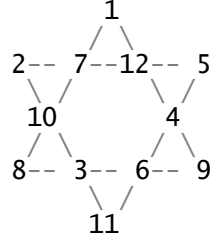
52#



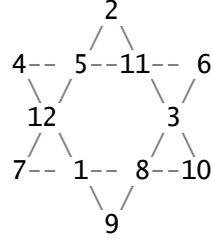
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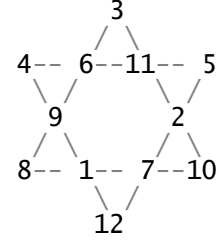
33#



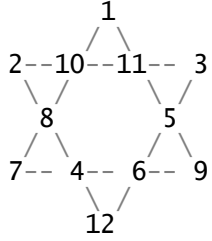
47#



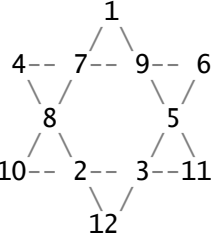
64#



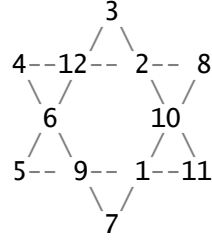
[19]26#



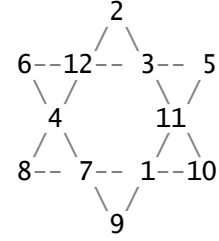
40#



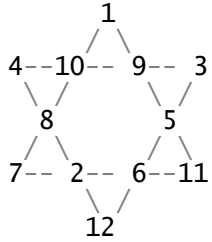
61#



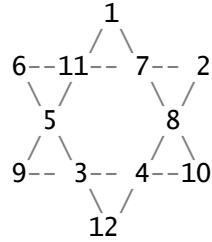
54#



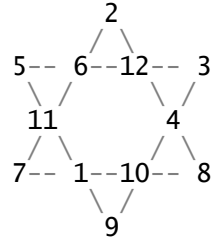
[20]27#



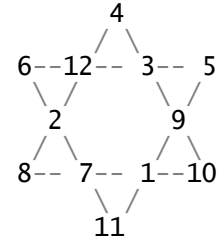
39#



49#



73#



[Count = 20 x 4 = 80]

* Duplication Monitor of Solutions *

?: 0

1: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

21: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

41: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

61: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

[OK!]

** Recompiled and Listed by Kanji Setsuda on

Mar.26, 2015 with MacOSX 10.10.2 & Xcode 6.2 **

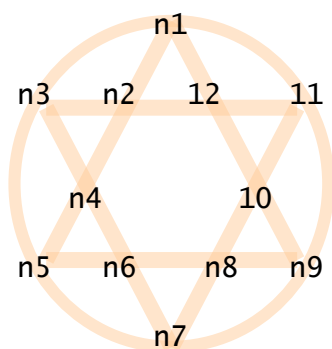
I tried to examine each solution by consulting it with the old solution list and getting the old solution number. I put it on the new list as N#.

I also made the monitor list of correspondence between the old solutions and new.

The beautiful result list above means our success. We could really reconstruct the whole set of 80 standard solutions of magic stars of order 6, using both the 20 fundamental solutions and the new transformation system. How wonderful it is!

But, what makes it possible? What conditions makes this miracle realized?

It is the time now to have some algebraic study for magic star of order 6.



* Basic Definition for **Order 6** *

$$n1+n2+n4+n5=C \quad \dots \quad 11$$

$$n3+n4+n6+n7=C \quad \dots \quad 12$$

$$n5+n6+n8+n9=C \quad \dots \quad 13$$

$$n7+n8+n10+n11=C \quad \dots \quad 14$$

$$n9+n10+n12+n1=C \quad \dots \quad 15$$

$$n11+n12+n2+n3=C \quad \dots \quad 16$$

$$2*(n1+n2+n3+\dots+n11+n12)=6*C$$

$$2*78=6*C$$

$$\text{Therefore } C=26 \quad \dots \quad r0$$

11+13+15

$$n1+n2+n4+n5+n5+n6+n8+n9+n9+n10+n12+n1=3*26$$

$$2*(n1+n5+n9)+(n2+n4+n6+n8+n10+n12)=78 \quad \dots \quad r1$$

12+14+16

$$n3+n4+n6+n7+n7+n8+n10+n11+n11+n12+n2+n3=3*26$$

$$2*(n3+n7+n11)+(n2+n4+n6+n8+n10+n12)=78 \quad \dots \quad r2$$

r1-r2

$$n1+n5+n9=n3+n7+n11 \quad \dots \quad r3$$

11+12+13

$$n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9=3*26$$

$$(n4+n5+n6)+n1+n2+n3+n4+n5+n6+n7+n8+n9=78$$

Add (n10+n11+n12) to both sides

$$(n4+n5+n6)+n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12=78+n10+n11+n12$$

Therefore n4+n5+n6=n10+n11+n12 \quad \dots \quad r4

12+13+14

$$n3+n4+n6+n7+n5+n6+n8+n9+n7+n8+n10+n11=3*26$$

$$(n6+n7+n8)+n3+n4+n5+n6+n7+n8+n9+n10+n11=78$$

Add (n1+n2+n12) to both sides

$$(n6+n7+n8)+n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12=78+n1+n2+n12$$

Therefore n6+n7+n8=n1+n2+n12 \quad \dots \quad r5

13+14+15 and add (n2+n3+n4) to both sides

Similarly $n_8+n_9+n_{10}=n_2+n_3+n_4 \quad \dots r_6$

$r_1+r_2+r_4+r_5$

$$n_1+n_2+n_4+n_5+n_3+n_4+n_6+n_7+n_7+n_8+n_{10}+n_{11}+n_9+n_{10}+n_{12}+n_1=4 \cdot C$$

$$(n_1+n_4+n_7+n_{10})+(n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104$$

Therefore $n_1+n_4+n_7+n_{10}=26 \quad \dots r_7$

$r_2+r_3+r_5+r_6$

$$n_3+n_4+n_6+n_7+n_5+n_6+n_8+n_9+n_9+n_{10}+n_{12}+n_1+n_{11}+n_{12}+n_2+n_3=4 \cdot C$$

$$(n_3+n_6+n_9+n_{12})+(n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104$$

Therefore $n_3+n_6+n_9+n_{12}=26 \quad \dots r_8$

$r_1+r_3+r_4+r_6$

$$n_1+n_2+n_4+n_5+n_5+n_6+n_8+n_9+n_7+n_8+n_{10}+n_{11}+n_{11}+n_{12}+n_2+n_3=4 \cdot C$$

$$(n_2+n_5+n_8+n_{11})+(n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104$$

Therefore $n_2+n_5+n_8+n_{11}=26 \quad \dots r_9$

$r_2+r_3+r_4+r_5$

$$n_1+n_3+n_4+n_5+n_6+n_6+n_7+n_7+n_8+n_8+n_9+n_9+n_{10}+n_{10}+n_{11}+n_{12}=4 \cdot C$$

$$(n_6+n_7+n_8+n_9+n_{10})+(n_1+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104$$

Add n_2 to both sides

$$(n_6+n_7+n_8+n_9+n_{10})+(n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104+n_2$$

Therefore $n_2=n_6+n_7+n_8+n_9+n_{10}-26 \quad \dots r_{10}$

$r_3+r_4+r_5+r_6$

$$n_5+n_6+n_8+n_9+n_7+n_8+n_{10}+n_{11}+n_9+n_{10}+n_{12}+n_1+n_{11}+n_{12}+n_2+n_3=4 \cdot C$$

$$(n_8+n_9+n_{10}+n_{11}+n_{12})+(n_1+n_2+n_3+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104$$

Add n_4 to both sides

$$(n_8+n_9+n_{10}+n_{11}+n_{12})+(n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104+n_4$$

Therefore $n_4=n_8+n_9+n_{10}+n_{11}+n_{12}-26 \quad \dots r_{11}$

Similarly $n_6=n_1+n_2+n_{10}+n_{11}+n_{12}-26 \quad \dots r_{12}$

$$n_8=n_1+n_2+n_3+n_4+n_{12}-26 \quad \dots r_{13}$$

$$n_{10}=n_2+n_3+n_4+n_5+n_6-26 \quad \dots r_{14}$$

$$n_{12}=n_4+n_5+n_6+n_7+n_8-26 \quad \dots r_{15}$$

$r_2+r_3+r_4+r_6$

$$n_3+n_4+n_6+n_7+n_5+n_6+n_8+n_9+n_7+n_8+n_{10}+n_{11}+n_{11}+n_{12}+n_2+n_3=4 \cdot C$$

$$(n_3+n_6+n_7+n_8+n_{11})+(n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104$$

Add n_1 to both sides

$$(n_3+n_6+n_7+n_8+n_{11})+(n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104+n_1$$

Therefore $n_1=n_3+n_6+n_7+n_8+n_{11}-26 \quad \dots r_{16}$

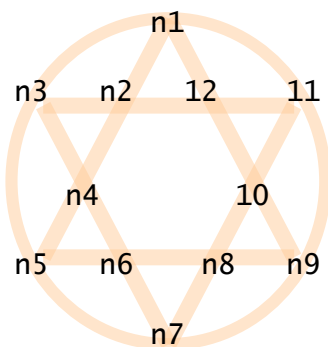
$r_1+r_3+r_4+r_5$

$$n_1+n_2+n_4+n_5+n_5+n_6+n_8+n_9+n_7+n_8+n_{10}+n_{11}+n_9+n_{10}+n_{12}+n_1=4 \cdot C$$

$$(n_1+n_5+n_8+n_9+n_{10})+(n_1+n_2+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104$$

Add n_3 to both sides

$$(n_1+n_5+n_8+n_9+n_{10})+(n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12})=104+n_3$$



* Basic Definition for Order 6 *

$$n_1+n_2+n_4+n_5=C \quad \dots r_1$$

$$n_3+n_4+n_6+n_7=C \quad \dots r_2$$

$$n_5+n_6+n_8+n_9=C \quad \dots r_3$$

$$n_7+n_8+n_{10}+n_{11}=C \quad \dots r_4$$

$$n_9+n_{10}+n_{12}+n_1=C \quad \dots r_5$$

$$n_{11}+n_{12}+n_2+n_3=C \quad \dots r_6$$

Therefore $n_3=n_1+n_5+n_8+n_9+n_{10}-26 \quad \dots r_{17}$

Similarly $n_5=n_3+n_7+n_{10}+n_{11}+n_{12}-26 \quad \dots r_{18}$

$$n_7=n_1+n_2+n_5+n_9+n_{12}-26 \quad \dots r_{19}$$

$$n_9=n_2+n_3+n_4+n_7+n_{11}-26 \quad \dots r_{20}$$

$$n_{11}=n_1+n_4+n_5+n_6+n_9-26$$

$$\dots r_{21}$$

**** Summary: List of Relations ****

$n_1+n_5+n_9=n_3+n_7+n_{11}$	$\dots r_3$
$n_4+n_5+n_6=n_{10}+n_{11}+n_{12}$	$\dots r_4$
$n_6+n_7+n_8=n_1+n_2+n_{12}$	$\dots r_5$
$n_8+n_9+n_{10}=n_2+n_3+n_4$	$\dots r_6$
$n_1+n_4+n_7+n_{10}=26$	$\dots r_7$
$n_3+n_6+n_9+n_{12}=26$	$\dots r_8$
$n_2+n_5+n_8+n_{11}=26$	$\dots r_9$
$n_2=n_6+n_7+n_8+n_9+n_{10}-26$	$\dots r_{10}$
$n_4=n_8+n_9+n_{10}+n_{11}+n_{12}-26$	$\dots r_{11}$
$n_6=n_1+n_2+n_{10}+n_{11}+n_{12}-26$	$\dots r_{12}$
$n_8=n_1+n_2+n_3+n_4+n_{12}-26$	$\dots r_{13}$
$n_{10}=n_2+n_3+n_4+n_5+n_6-26$	$\dots r_{14}$
$n_{12}=n_4+n_5+n_6+n_7+n_8-26$	$\dots r_{15}$
$n_1=n_3+n_6+n_7+n_8+n_{11}-26$	$\dots r_{16}$
$n_3=n_1+n_5+n_8+n_9+n_{10}-26$	$\dots r_{17}$
$n_5=n_3+n_7+n_{10}+n_{11}+n_{12}-26$	$\dots r_{18}$
$n_7=n_1+n_2+n_5+n_9+n_{12}-26$	$\dots r_{19}$
$n_9=n_2+n_3+n_4+n_7+n_{11}-26$	$\dots r_{20}$
$n_{11}=n_1+n_4+n_5+n_6+n_9-26$	$\dots r_{21}$

Yes. All of them really make it possible. We are lucky we could find that miracle.

It is only for the order 6 that we could draw the Extended Space and find that beautiful transformation system. Nothing such as the Extended Space could be drawn for the other orders than 6, and no beautiful transformation system such as of order 6 could be found anywhere else, either. This miraculous phenomenon is realized only for magic stars of order 6, as well as the pandiagonal magic squares of order 4 or 8.

Study of magic stars of higher orders might get rather boring, I am afraid.

Chapter 4: Algebraic Study of Double Ringed Magic Stars 7

Let's study about Magic Stars of order 7 next.

Two more numerical elements and one more equation are added to the case of order 6. How do these new comers change the structure?

*** Algebra for Magic Star of Order 7 ***

Calculations:

$$11+13+15+17$$

$$n_1+n_2+n_4+n_5+n_5+n_6+n_8+n_9+n_9+n_{10}+n_{12}+n_{13}+n_{13}+n_{14}+n_2+n_3=4 \cdot C$$

$$(n_2+n_5+n_9+n_{13})+(n_1+n_2+n_3+n_4+n_5+n_6+n_8+n_9+n_{10}+n_{12}+n_{13}+n_{14})=120$$

Add n_7+n_{11} to both sides.

$$(n_2+n_5+n_9+n_{13})+(n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12}+n_{13}+n_{14})=120+n_7+n_{11}$$

$$(n_2+n_5+n_9+n_{13})+ \quad 105 \quad =120+n_7+n_{11}$$

Therefore

$$n_2=15-n_5+n_7-n_9+n_{11}-n_{13}$$

$$\dots r_1$$

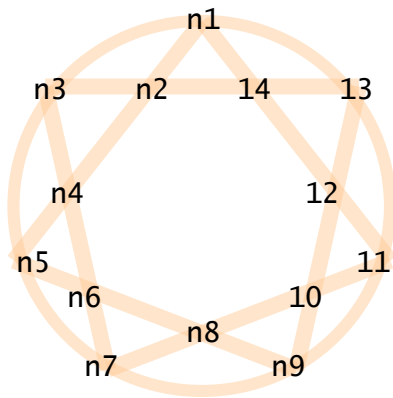
$$11+12+14+16$$

$$n_1+n_2+n_4+n_5+n_3+n_4+n_6+n_7+n_7+n_8+n_{10}+n_{11}+n_{11}+n_{12}+n_{14}+n_1=4 \cdot C$$

$$(n_1+n_4+n_7+n_{11})+(n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_{10}+n_{11}+n_{12}+n_{14})=120$$

Add n_9+n_{13} to both sides.

$$(n_1+n_4+n_7+n_{11})+(n_1+n_2+n_3+n_4+n_5+n_6+n_7+n_8+n_9+n_{10}+n_{11}+n_{12}+n_{13}+n_{14})=120+n_9+n_{13}$$



- * Basic Definition *
- $n1+n2+n4+n5=C \quad \dots \quad r1$
 - $n3+n4+n6+n7=C \quad \dots \quad r2$
 - $n5+n6+n8+n9=C \quad \dots \quad r3$
 - $n7+n8+n10+n11=C \quad \dots \quad r4$
 - $n9+n10+n12+n13=C \quad \dots \quad r5$
 - $n11+n12+n14+n1=C \quad \dots \quad r6$
 - $n13+n14+n2+n3=C \quad \dots \quad r7$

$$2*(n1+n2+n3+\dots+n13+n14)=7*C$$

$$2*105=7*C$$

Therefore $C=30 \quad \dots \quad r0$

Therefore

$$n4=15-n1-n7+n9-n11+n13 \quad \dots \quad r2$$

Similarly

$$n6=15+n1-n3-n9+n11-n13 \quad \dots \quad r3$$

$$n8=15-n1+n3-n5-n11+n13 \quad \dots \quad r4$$

$$n10=15+n1-n3+n5-n7-n13 \quad \dots \quad r5$$

$$n12=15-n1+n3-n5+n7-n9 \quad \dots \quad r6$$

$$n14=15-n3+n5-n7+n9-n11 \quad \dots \quad r7$$

$r1+r2+r5+r6$

$$n1+n2+n4+n5+n3+n4+n6+n7+n9+n10+n12+n13+n11+n12+n14+n1=4*C$$

$$(n1+n4+n12)+(n1+n2+n3+n4+n5+n6+n7+n9+n10+n11+n12+n13+n14)=120$$

Add $n8$ to both sides

$$(n1+n4+n12)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12+n13+n14)=120+n8$$

$$n1+n4+n12+105=120+n8$$

Therefore $n1=15-n4+n8-n12 \quad \dots \quad r8$

$r2+r3+r6+r7$

$$n3+n4+n6+n7+n5+n6+n8+n9+n11+n12+n14+n1+n13+n14+n2+n3=4*C$$

$$(n3+n6+n14)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n11+n12+n13+n14)=120$$

Add $n10$ to both sides

$$(n3+n6+n14)+(n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12+n13+n14)=120+n10$$

$$n3+n6+n14+105=120+n10$$

Therefore $n3=15-n6+n10-n14 \quad \dots \quad r9$

Similarly $n5=15-n2+n12-n8 \quad \dots \quad r10$

$$n7=15-n4+n14-n10 \quad \dots \quad r11$$

$$n9=15+n2-n6-n12 \quad \dots \quad r12$$

$$n11=15+n4-n8-n14 \quad \dots \quad r13$$

$$n13=15-n2+n6-n10 \quad \dots \quad r14$$

We now know how the outer 'ring' of $\{n1, n3, n5, n7, n9, n11$ and $n13\}$ relates to the inner one of $\{n2, n4, n6, n8, n10, n12$ and $n14\}$ by those relational equations: $r1 \sim r14$. However, they do not always mean their total exchangeability with each other.

The next list shows the result of my new calculation. It contains all the 72 standard solutions of magic star of order 7.

**** List of Standard Magic Stars of Order 7 ****

1#		2#		3#		4#	
	1		1		1		1
3	14	8	5	3	14	7	6
		13				4	14
13		12		13		9	3
			9		10		
2	10	6		2	7	5	8
			11				
	4	11			11		
		7		5	8	6	10
						3	8

<p>5#</p> <p>1</p> <p>4 14 10 2</p> <p>12 13</p> <p>3 9 8 6</p> <p>5 11 7</p>	<p>6#</p> <p>1</p> <p>4 14 5 7</p> <p>12 11</p> <p>3 8 2 13</p> <p>6 9 10</p>	<p>7#</p> <p>1</p> <p>4 14 7 5</p> <p>12 13</p> <p>3 6 2 9</p> <p>8 11 10</p>	<p>8#</p> <p>1</p> <p>6 14 8 2</p> <p>12 11</p> <p>3 5 4 10</p> <p>7 9 13</p>
<p>9#</p> <p>1</p> <p>6 13 8 3</p> <p>12 7</p> <p>4 10 9 14</p> <p>2 5 11</p>	<p>10#</p> <p>1</p> <p>9 13 5 3</p> <p>12 10</p> <p>4 7 6 14</p> <p>2 8 11</p>	<p>11#</p> <p>1</p> <p>3 11 14 2</p> <p>13 9</p> <p>5 10 12 6</p> <p>4 8 7</p>	<p>12#</p> <p>1</p> <p>3 11 14 2</p> <p>13 7</p> <p>5 10 12 8</p> <p>4 6 9</p>
<p>13#</p> <p>1</p> <p>7 14 6 3</p> <p>10 12</p> <p>5 4 2 11</p> <p>9 8 13</p>	<p>14#</p> <p>1</p> <p>4 12 9 5</p> <p>10 6</p> <p>7 13 11 14</p> <p>3 2 8</p>	<p>15#</p> <p>1</p> <p>11 12 2 5</p> <p>10 13</p> <p>7 6 4 14</p> <p>3 9 8</p>	<p>16#</p> <p>1</p> <p>2 13 11 4</p> <p>9 8</p> <p>7 14 12 10</p> <p>5 3 6</p>
<p>17#</p> <p>1</p> <p>5 13 8 4</p> <p>9 11</p> <p>7 14 12 10</p> <p>2 6 3</p>	<p>18#</p> <p>1</p> <p>5 9 13 3</p> <p>12 6</p> <p>8 11 14 10</p> <p>2 4 7</p>	<p>19#</p> <p>1</p> <p>2 10 14 4</p> <p>11 6</p> <p>8 12 13 9</p> <p>5 3 7</p>	<p>20#</p> <p>1</p> <p>4 11 9 6</p> <p>10 7</p> <p>8 14 12 13</p> <p>2 3 5</p>
<p>21#</p> <p>1</p> <p>3 12 5 10</p> <p>9 11</p> <p>8 14 7 13</p> <p>4 6 2</p>	<p>22#</p> <p>1</p> <p>9 14 5 2</p> <p>7 13</p> <p>8 4 3 11</p> <p>6 10 12</p>	<p>23#</p> <p>1</p> <p>5 7 12 6</p> <p>13 3</p> <p>9 8 10 14</p> <p>4 2 11</p>	<p>24#</p> <p>1</p> <p>5 8 14 3</p> <p>12 4</p> <p>9 6 10 11</p> <p>7 2 13</p>
<p>25#</p> <p>1</p> <p>6 12 2 10</p> <p>8 13</p> <p>9 11 4 14</p> <p>5 7 3</p>	<p>26#</p> <p>1</p> <p>4 14 5 7</p> <p>6 11</p> <p>9 8 2 13</p> <p>12 3 10</p>	<p>27#</p> <p>1</p> <p>4 7 5 14</p> <p>12 11</p> <p>10 8 2 13</p> <p>6 9 3</p>	<p>28#</p> <p>1</p> <p>13 7 4 6</p> <p>12 11</p> <p>10 3 5 14</p> <p>2 9 8</p>
<p>29#</p> <p>1</p> <p>13 8 3 6</p> <p>11 12</p> <p>10 4 5 14</p> <p>2 9 7</p>	<p>30#</p> <p>1</p> <p>12 11 2 5</p> <p>8 13</p> <p>10 4 3 14</p> <p>6 7 9</p>	<p>31#</p> <p>1</p> <p>4 5 12 9</p> <p>13 3</p> <p>11 7 8 14</p> <p>6 2 10</p>	<p>32#</p> <p>1</p> <p>14 10 4 2</p> <p>8 13</p> <p>11 3 6 12</p> <p>5 7 9</p>

33#	1	34#	1	35#	1	36#	1
7 4	9 10	7 11	2 10	4 6	8 12	4 9	5 12
13	6	6	13	10	7	7	10
12	2 3 14	12	9 3 14	13	5 2 14	13	8 2 14
	5		5		3		3
8	11	8	4	11	9	11	6
37#	1	38#	1	39#	2	40#	2
7 10	4 9	8 10	3 9	5 13	4 8	7 9	10 4
6	11	6	12	12	10	14	12
13	12 8 14	13	11 7 14	3	7 1 14	5	1 3 6
	3		4		9		13
5	2	5	2	6	11	8	11
41#	2	42#	2	43#	2	44#	2
8 9	10 3	9 10	3 8	8 13	5 4	3 11	10 6
14	12	12	14	9	11	9	5
5	1 4 6	6	4 1 11	6	3 1 12	8	14 13
	13		13		7		1
7	11	5	7	10	14	4	7
45#	2	46#	2	47#	2	48#	2
12 11	1 6	11 5	6 8	4 11	12 3	5 13	4 8
9	14	14	12	8	6	6	10
8	5 3 13	9	1 3 10	9	13 10	9	7 1 14
	10		13		1		3
4	7	4	7	5	7	12	11
49#	2	50#	2	51#	2	52#	2
6 7	13 4	8 3	10 9	13 8	4 5	8 12	1 9
11	3	14	5	9	10	5	14
10	5 9 12	11	1 4 13	11	1 3 14	11	10 13
	1		6		6		6
8	14	7	12	7	12	7	3
53#	2	54#	2	55#	3	56#	4
8 6	11 5	10 3	8 9	5 9	12 4	13 10	1 6
10	4	12	6	11	2	9	14
12	3 7 13	13	1 4 14	7	8 10 13	7	3 11
	1		5		1		12
9	14	7	11	6	14	5	8
57#	4	58#	4	59#	4	60#	5
7 2	12 9	8 2	11 9	8 14	1 7	8 1	9 12
14	3	14	3	2	12	14	3
10	1 5 11	10	1 5 12	10	11 5 13	10	2 4 13
	6		6		3		7
8	13	7	13	9	6	6	11

<p>61#</p> <pre> 5 9 3 7 11 12 4 10 14 1 2 6 8 13 </pre>	<p>62#</p> <pre> 5 7 11 4 10 14 6 1 2 13 12 </pre>	<p>63#</p> <pre> 5 6 13 2 10 14 8 3 1 14 11 </pre>	<p>64#</p> <pre> 5 7 9 3 11 4 8 12 12 14 6 1 2 13 10 </pre>
<p>65#</p> <pre> 5 7 10 4 9 3 8 12 13 6 2 1 14 11 </pre>	<p>66#</p> <pre> 5 8 1 9 12 11 2 13 14 4 6 3 7 10 </pre>	<p>67#</p> <pre> 5 12 3 4 11 9 7 13 14 1 2 6 8 10 </pre>	<p>68#</p> <pre> 6 8 2 9 11 12 1 10 14 3 5 4 7 13 </pre>
<p>69#</p> <pre> 6 11 9 3 7 5 8 10 13 2 1 4 12 14 </pre>	<p>70#</p> <pre> 6 8 9 3 10 4 7 11 14 5 1 2 13 12 </pre>	<p>71#</p> <pre> 8 11 7 3 9 5 6 10 13 2 1 4 12 14 </pre>	<p>72#</p> <pre> 8 10 4 7 9 6 2 12 13 3 5 1 11 14 </pre>

All the solutions 39#~72# have the value '1' in any position of the inner ring {n2, n4, n6, n8, n10, n12 and n14} and '1' cannot be found anywhere in the outer ring.

Therefore, you can rearrange and make the list smart for both types: the solutions whose n1=1 and the ones whose n2=1 as follows.

**** Smart List of Standard Magic Stars of Order 7 ****

[Type n1=1]

<p>1#</p> <pre> 1 3 14 8 5 13 12 2 9 10 6 11 4 7 </pre>	<p>2#</p> <pre> 1 3 14 7 6 13 12 2 10 9 4 11 5 8 </pre>	<p>3#</p> <pre> 1 4 14 9 3 13 12 2 8 7 5 11 6 10 </pre>	<p>4#</p> <pre> 1 5 14 7 4 13 12 2 10 9 6 11 3 8 </pre>
<p>5#</p> <pre> 1 4 14 10 2 12 13 3 6 9 8 11 5 7 </pre>	<p>6#</p> <pre> 1 4 14 5 7 12 11 3 13 8 2 9 6 10 </pre>	<p>7#</p> <pre> 1 4 14 7 5 12 13 3 9 6 2 11 8 10 </pre>	<p>8#</p> <pre> 1 6 14 8 2 12 11 3 10 5 4 9 7 13 </pre>
<p>9#</p> <pre> 1 6 13 8 3 12 7 4 14 10 9 5 2 11 </pre>	<p>10#</p> <pre> 1 9 13 5 3 12 10 4 14 7 6 8 2 11 </pre>	<p>11#</p> <pre> 1 3 11 14 2 13 9 5 6 10 12 8 4 7 </pre>	<p>12#</p> <pre> 1 3 11 14 2 13 7 5 8 10 12 6 4 9 </pre>

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10	12	10	6	10	13	9	8
5	11	7	14	7	14	7	10
4	2	13	11	6	4	14	12
8		2	8	9		3	6
9	13	3	3	8		5	
17#	1	18#	1	19#	1	20#	1
5 13	8 4	5 9	13 3	2 10	14 4	4 11	9 6
9	11	12	6	11	6	10	7
7	10	8	10	8	9	8	13
14	12	11	14	12	13	14	12
6		4		3		3	
2	3	2	7	5	7	2	5
21#	1	22#	1	23#	1	24#	1
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9	11	7	13	13	3	12	4
8	13	8	11	9	14	9	11
14	7	4	3	8	10	6	10
6		6		2		2	
4	2	10	12	4	11	7	13
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8	13	6	11	12	11	12	11
9	14	9	13	10	13	10	14
11	4	8	2	8	2	3	5
7		3		9		9	
5	3	12	10	6	3	2	8
29#	1	30#	1	31#	1	32#	1
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11	12	8	13	13	3	8	13
10	14	10	14	11	14	11	12
4	5	4	3	7	8	3	6
9		7		2		7	
2	7	6	9	6	10	5	9
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13	6	6	13	10	7	7	10
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2	3	9	3	5	2	8	2
5		5		3		3	
8	11	8	4	11	9	11	6
37#	1	38#	1				
7 10	4 9	8 10	3 9				
6	11	6	12				
13	14	13	14				
12	8	11	7				
3		4					
5	2	5	2				

[Type n2=1]

39#	40#	41#	42#
14	14	14	14
12 1 7 10	10 1 8 11	12 1 8 9	11 1 6 12
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11 4 8	12 4 7	10 4 7	9 4 5
43#	44#	45#	46#
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3 2	3 4	3 2	3 4
12 6 7 9	12 7 13 2	12 9 13 6	12 10 11 5
10 4 8	9 6 5	7 4 5	8 6 2
47#	48#	49#	50#
13	13	11	13
14 1 6 9	14 1 8 7	14 1 9 6	7 1 12 10
4 3	4 3	5 2	5 3
12 2 7 8	12 2 9 6	13 4 12 8	11 4 8 2
10 5 11	10 5 11	7 3 10	14 6 9
51#	52#	53#	54#
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6 7 2	14 4 11	12 3 9	11 3 9
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6 3	7 5	8 2	9 3
12 4 10 2	10 3 13 2	7 11 12 4	13 8 10 14
13 5 9	6 9 8	5 9 3	2 4 5
59#	60#	61#	62#
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9 2	9 2	9 4	10 2
12 3 7 6	12 3 7 14	6 7 13 2	13 9 12 8
13 4 11	5 4 11	3 12 5	7 3 5
63#	64#	65#	66#
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8 1 9 12	2 1 14 13	7 1 14 8	7 1 14 8
11 2	11 3	12 2	12 9
13 4 6 14	12 9 10 7	13 5 11 10	13 5 11 3
7 3 10	8 5 10 4	6 3 9	6 10 2

<p>67#</p> <pre> 7 5 1 14 10 13 6 9 12 3 8 11 4 2</pre>	<p>68#</p> <pre> 2 9 1 12 8 14 5 13 11 11 4 10 6 7 3</pre>	<p>69#</p> <pre> 4 9 1 13 7 14 3 11 10 5 12 6 8 2</pre>	<p>70#</p> <pre> 7 11 1 13 5 14 4 8 12 6 3 10 2 9</pre>
<p>71#</p> <pre> 7 5 1 13 11 14 4 8 12 6 9 10 2 3</pre>	<p>72#</p> <pre> 8 5 1 13 11 14 3 7 12 6 9 10 2 4</pre>		

[Count(Tn1+Tn2) = 38 + 34 = 72] OK!

Now I have nothing good to report of any smart classification and transformation system. I have not yet discovered what should be the fundamental solutions of magic stars of order 7 and how they could be transformed so as to reconstruct the complete set of standard solutions. It is still one of the problems to be solved in the future.

(The original was written in Japanese first on Aug.12, 2000;
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 Revised on July 3, 2008 working with MacOSX and Xcode 2.2;
 Recompiled on Mar.26, 2015 with MacOSX 10.10.2 & Xcode 6.2;)

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 English_Page_Index: <http://kanjisetuda.la.coocan.jp/pages/EnglishP1.html>