

Part 2: Basic Studies of Magic Stars IV: Kanji Setsuda

Chapter 10: Advanced Study of Some Double-Ringed Magic Stars

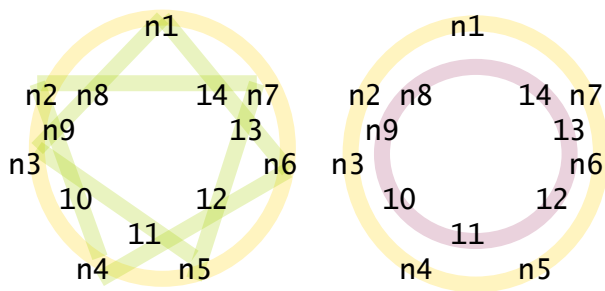
We have never tried anything new for the case of those 'Double-Ringed' magic stars in the former chapters and we haven't yet known what solutions we could have just when we add those limitations to the values of variables taken on every rings.

Let's make such a similar experiment for some cases of double-ringed magic stars.

But, as a result we had to know we could not find any special solutions of those magic stars of the orders 6, 8 and 10. We must start our study from the order 7.

Section 1: Special Type of Double-Ringed Magic Stars of Order 7

* Basic Diagrams and Additional Conditions: *



Add the next limitations to the values of 7 variables on the outer ring:

$$\{n1, n2, n3, n4, n5, n6, n7\} = \{1, 2, 3, 4, 5, 6, 7\}$$

To the 7 ones on the inner ring:

$$\{n8, n9, n10, n11, n12, n13, n14\} = \{8, 9, 10, 11, 12, 13, 14\}$$

And add the new ones for another type:

$$\{n1, n2, n3, n4, n5, n6, n7\} = \{8, 9, 10, 11, 12, 13, 14\}$$

$$\{n8, n9, n10, n11, n12, n13, n14\} = \{1, 2, 3, 4, 5, 6, 7\}$$

What solutions could we make?

As a result we could have only four special solutions in all as listed below.

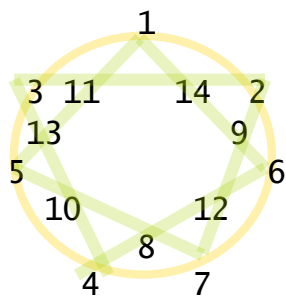
Will you watch the decomposed patterns by /D7i listed aside? They are beautiful, aren't they? I would say they demonstrate the examples of 'Latin Type' of the 7 variables on each ring, especially in their low layers of decomposed patterns.

We could have got no solutions of that kind for the orders 6, 8 and 10.

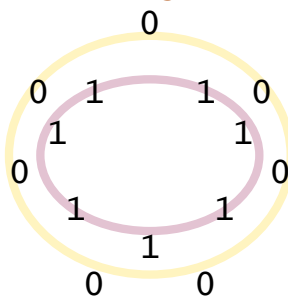
* Solution List: "Latin" Type of Double-Ringed Magic Stars of Order 7 *

[Type: n1=1]

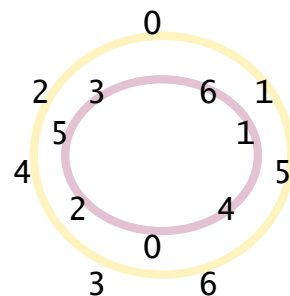
[1]



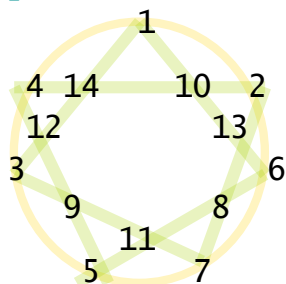
/D7i High



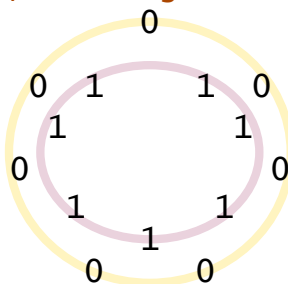
Low



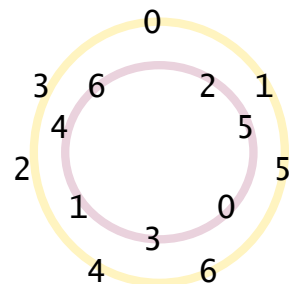
[2]



/D7i High



Low



[Type: n8=1]

[3]

				/D7i	High		Low							
	14				1		6							
11	1	5	13	1	0	0	1	3	0	4	5			
	3		2		0		0		2		1			
12		6	4	7	9	1	0	0	1	4		5	6	1
	10		8				1	0	1		2	3	0	

[4]

				/D7i	High		Low							
	13				1		5							
14	1	6	9	1	0	0	1	6	0	5	1			
	4		3		0		0		3		2			
12		2	5	7	8	1	0	0	1	4		1	6	0
	10		11				1	0	1		2	4	3	

[Total Count = 4] OK!

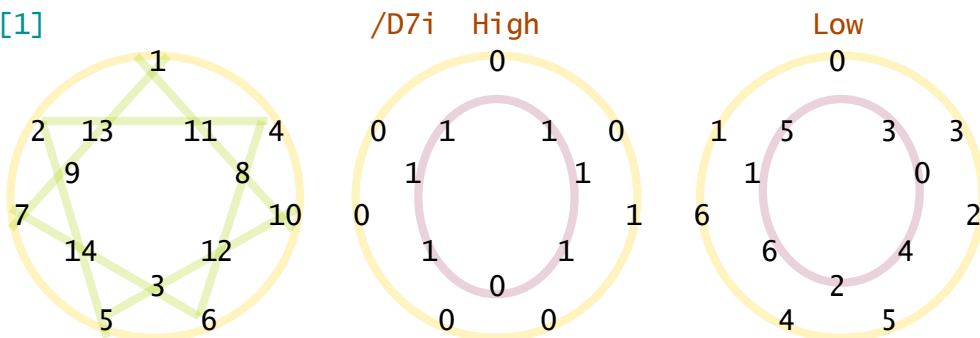
You should see if every 7 numbers on each of both outer ring and inner one consist of {0, 1, 2, 3, 4, 5, 6} in the lower layer of the decomposed patterns. Neither any number appears twice or more often, nor any number disappears at all there.

The next list reports about the solutions of 'pure Latin type' of DMS7 only in the lower layer of the decomposed patterns. They do never care about the higher pattern of the decomposed. We don't mind whatever patterns would come to the high layer.

Finally we could have 14 solutions of that type in all. They contain those four solutions of our former type.

**** Lower 'Latin' Type of Double-Ringed Magic Stars of Order 7 ****

[1]



[2]

				/D7i	High		Low					
	1				0		0					
3	11	14	2	0	1	1	0	2	3	6	1	
	13		9		1		1		5		1	
5		10	8	12	6	0	1	0	4	2	4	5
	4		7				1	0		3	0	6

[3]					/D7i	High				Low	
	1				0	0				0	
4	14	10	2	0	1	1	0	3	6	2	1
	12		13		1		1		4		5
3		9	8	6	0	1	1	0	2	1	0
		11				1				3	
	5		7		0	0			4		6

[4]					/D7i	High				Low	
	1				0	0				0	
5	8	14	3	0	1	1	0	4	0	6	2
	12		4		1		0		4		3
9		6	10	11	1	0	1	1	5		2
		7	2			0				1	
			13		0	1			6		5

[5]					/D7i	High				Low	
	1				0	0				0	
7	14	6	3	0	1	0	0	6	6	5	2
	10		12		1		1		2		4
5		4	2	11	0	0	0	1	4	3	1
		8				1				0	
		9	13		1	1			1		5

[6]					/D7i	High				Low	
	1				0	0				0	
7	5	14	4	0	0	1	0	6	4	6	3
	11		6		1		0		3		5
13		2	8	9	1	0	1	1	5	1	0
		3				0				2	
		10	12		1	1			2		4

[7]					/D7i	High				Low	
	2				0	0				1	
5	13	4	8	0	1	0	1	4	5	3	0
	12		10		1		1		4		2
3		7	1	14	0	0	0	1	2	6	0
		9				1				1	
		6	11		0	1			5		3

[8]					/D7i	High				Low	
	2				0	0				1	
6	7	13	4	0	0	1	0	5	6	5	3
	11		3		1		0		3		2
10		5	9	12	1	0	1	1	2	4	1
		1				0				0	
		8	14		1	1			0		6

[9]				/D7i	High		Low
	2				0		1
8	9	10	3	1	1	1	0
5	14		12	0	1	1	0
	1		4	0	0	0	0
	7	13			1		
		11		0	1		6
							5
							3

[10]				/D7i	High		Low
	2				0		1
8	13	5	4	1	1	0	0
6	9		11	0	1	1	1
	3		1	0	0	0	5
	10	7			0		
		14		1	1		2
							6
							6

[11]				/D7i	High		Low
	4				0		3
8	2	11	9	1	0	1	1
10	14		3	1	1	0	0
	1		12	1	0	0	1
	6	5			0	0	2
	7	13		0	0	1	0
					0	1	6
							5

[12]				/D7i	High		Low
	5				0		4
9	3	7	11	1	0	0	1
10	12		4	1	1	0	0
	1		14	1	0	0	1
	6	2			0	0	2
	8	13		1	0	0	0
					1	1	0
							5

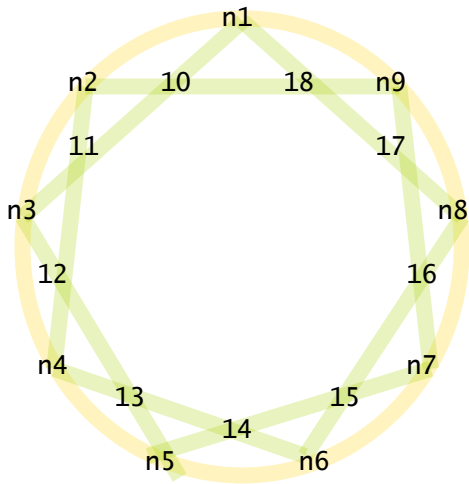
[13]				/D7i	High		Low
	8				1		0
10	4	7	9	1	0	0	1
12	6		2	1	0	0	0
	3		13	1	0	0	1
	1	5			0	0	4
	11	1			0	0	2
		14		1	1		3
							6

[14]				/D7i	High		Low
	8				1		0
11	7	3	9	1	0	0	1
10	5		6	1	0	0	1
	2		13	1	0	0	1
	1				0	0	2
	4	1			0	0	1
					0	0	3
	12	14		1	1		4
							6

[Total Count = 14] OK!

Section 2: Special Type of Double-Ringed Magic Stars of Order 9

Let's make such a similar experiment for Double-ringed Magic Stars 9, and add those limitations below to the values of variables taken on every ring.



* **Basic Conditions:** *

$$\begin{aligned}
 n1+n10+n11+n3 &= C & \dots & sm1 \\
 n2+n11+n12+n4 &= C & \dots & sm2 \\
 n3+n12+n13+n5 &= C & \dots & sm3 \\
 n4+n13+n14+n6 &= C & \dots & sm4 \\
 n5+n14+n15+n7 &= C & \dots & sm5 \\
 n6+n15+n16+n8 &= C & \dots & sm6 \\
 n7+n16+n17+n9 &= C & \dots & sm7 \\
 n8+n17+n18+n1 &= C & \dots & sm8 \\
 n9+n18+n10+n2 &= C & \dots & sm9
 \end{aligned}$$

$$\begin{aligned}
 sm1+sm2+sm3+sm4+ \dots +sm8+sm9 \\
 2*(n1+n2+n3+ \dots +n17+n18) &= 9*C \\
 2*(1+2+3+4+ \dots +16+17+18) &= 9*C \\
 342 &= 9*C
 \end{aligned}$$

Therefore C=38

* **Additional Conditions:**

[Type 1] $\{n1, n2, n3, n4, n5, n6, n7, n8, n9\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\{n10, n11, n12, n13, n14, n15, n16, n17, n18\} = \{10, 11, 12, 13, 14, 15, 16, 17, 18\}$

[Type 2] $\{n1, n2, n3, n4, n5, n6, n7, n8, n9\} = \{10, 11, 12, 13, 14, 15, 16, 17, 18\}$

$\{n10, n11, n12, n13, n14, n15, n16, n17, n18\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

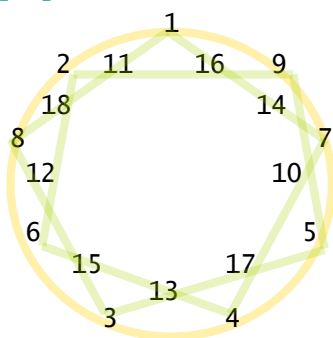
What solutions could we make?

As a result, we could have only 48 special solutions in all as listed below.

** List of "Latin" Type of Double-Ringed Magic Stars of Order 9 **

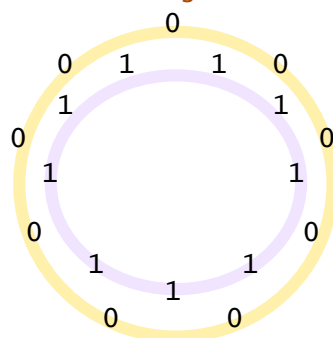
[Type n1=1]

[1]

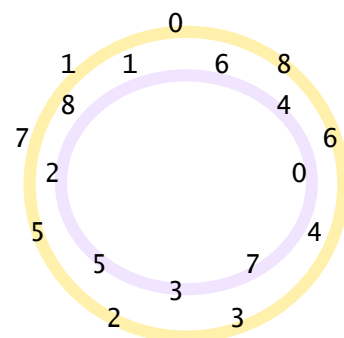


/D9i

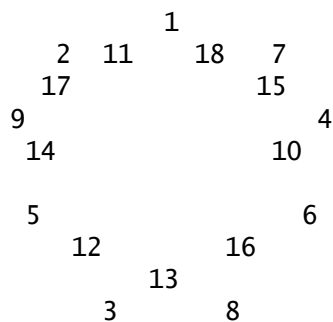
High



Low

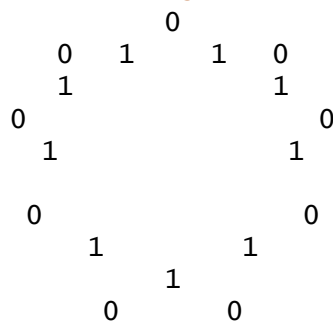


[2]

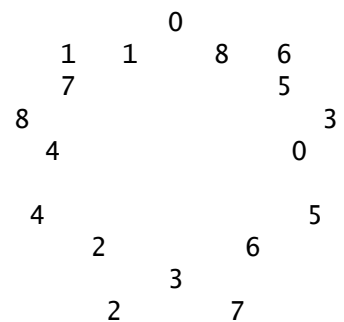


/D9i

High



Low



[3]		/D9i	High		Low
	1		0		0
3	18	0	1	0	2
12		1		1	2
7		0		0	6
14		1		1	4
9		0		0	8
13		1		1	3
4		0		0	3

[4]		/D9i	High		Low
	1		0		0
3	10	0	1	0	2
18		1		1	8
9		0		0	8
12		1		1	2
5		0		0	4
15		1		1	5
2		0		0	1

[5]		/D9i	High		Low
	1		0		0
3	12	0	1	0	2
16		1		1	6
9		0		0	8
13		1		1	3
6		0		0	5
14		1		1	4
2		0		0	1

[6]		/D9i	High		Low
	1		0		0
3	12	0	1	0	2
16		1		1	6
9		0		0	8
13		1		1	3
6		0		0	5
11		1		1	1
5		0		0	4

[7]		/D9i	High		Low
	1		0		0
4	11	0	1	0	3
18		1		1	8
8		0		0	7
13		1		1	3
3		0		0	2
12		1		1	2
5		0		0	4

[8]		/D9i	High		Low
	1		0		0
4	15	0	1	0	3
14		1		1	4
8	12	0	1	0	5
17	7	1		1	2
	16				6
	9	0		0	7
	10	1		1	0
3		0		0	2
11	13	1	1	1	1
	5				3
2	18	0	1	0	8
	6	0	0		1
					5

[9]		/D9i	High		Low
	1		0		0
4	11	0	1	0	3
17		1		1	7
9	15	0	1	0	1
14	8	1		1	5
	16				7
	6	0		0	8
	12	1		1	4
3		0		0	2
10	13	1	1	1	0
	2				3
5	18	0	1	0	8
	7	0	0		4
					6

[10]		/D9i	High		Low
	1		0		0
5	14	0	1	0	4
16		1		1	4
7	11	0	1	0	6
15	8	1		1	5
	17				6
	9	0		0	8
	10	1		1	0
2		0		0	1
12	13	1	1	1	2
	3				3
4	18	0	1	0	8
	6	0	0		3
					5

[11]		/D9i	High		Low
	1		0		0
5	10	0	1	0	4
18		1		1	8
9	15	0	1	0	0
13	8	1		1	5
	16				6
	6	0		0	8
	11	1		1	3
2		0		0	1
12	14	1	1	1	2
	3				4
4	17	0	1	0	7
	7	0	0		3
					6

[12]		/D9i	High		Low
	1		0		0
5	10	0	1	0	4
18		1		1	8
9	17	0	1	0	0
11	6	1		1	7
	12				8
	8	0		0	8
	13	1		1	1
4		0		0	3
16	14	1	1	1	6
	7				3
2	15	0	1	0	5
	3	0	0		1
					2

[13]	/D9i	High	Low
1	0	0	0
5 12 14 7	0 1 1 0	4 2 4 6	
16 17	1 1 1	6 7	
9 6	0 0	8 5	
15 11	1 1	5 1	
2 3	0 0	1 2	
10 13	1 1	0 3	
18 8	0 0	8 7	
4		3	

[14]	/D9i	High	Low
1	0	0	0
5 12 13 8	0 1 1 0	4 2 3 7	
16 18	1 1 1	6 8	
9 6	0 0	8 5	
14 10	1 1	4 0	
3 2	0 0	2 1	
11 15	1 1	1 5	
17 7	0 0	7 6	
4		3	

[15]	/D9i	High	Low
1	0	0	0
5 16 14 3	0 1 1 0	4 6 4 2	
12 15	1 1 1	2 5	
9 8	0 0	8 7	
17 13	1 1	7 3	
4 7	0 0	3 6	
10 11	1 1	0 1	
18 6	0 0	8 5	
2		1	

[16]	/D9i	High	Low
1	0	0	0
5 16 11 6	0 1 1 0	4 6 1 5	
12 18	1 1 1	2 8	
9 8	0 0	8 7	
14 10	1 1	4 0	
7 4	0 0	6 3	
13 17	1 1	3 7	
15 3	0 0	5 2	
2		1	

[17]	/D9i	High	Low
1	0	0	0
6 15 14 3	0 1 1 0	5 5 4 2	
18 16	1 1 1	8 6	
4 7	0 0	3 6	
12 10	1 1	2 0	
2 9	0 0	1 8	
17 13	1 1	7 3	
11 8	0 0	4 7	

[18]		/D9i	High	Low
	1		0	0
6	17	13	2	5
12		15		7
8		9		3
16		14		1
				5
4		7		0
	11	10		7
	18	5		6
3				3

[19]		/D9i	High	Low
	1		0	0
6	11	14	7	5
17		18		1
9		5		7
13		10		8
				3
2		3		1
	12	15		2
	16	8		6
4				3

[20]		/D9i	High	Low
	1		0	0
7	10	16	5	6
18		13		8
9		8		8
11		14		1
				8
2		6		1
	15	12		5
	17	4		7
3				2

[21]		/D9i	High	Low
	1		0	0
7	12	15	4	6
16		17		6
9		5		8
13		14		3
				8
2		3		1
	10	11		0
	18	8		5
6				8

[22]		/D9i	High	Low
	1		0	0
8	13	14	3	7
15		18		5
9		5		8
11		10		1
				8
4		7		3
	16	17		6
	12	6		2
2				1

[28]				/D9i	High		Low
	12	1	14		1		4
	5		9	16	0	0	0
	18			4	1	0	1
	8			11	0	0	2
				3	1	0	0
	13			15	0	0	1
	2			7	1	0	0
				6	0	0	1
	10			17	1	1	0
							5
							6
							7

[29]				/D9i	High		Low
	11	1	13		1		3
	6		9	17	0	0	1
	18			4	1	0	0
	7			12	0	0	1
				2	1	0	0
	14			15	0	0	1
	3			8	1	0	0
				5	0	0	1
	10			16	1	1	0
							4
							6

[30]				/D9i	High		Low
	10	1	14		1		4
	6		9	18	0	0	1
	17			4	1	0	0
	7			11	0	0	1
				3	1	0	0
	15			13	0	0	1
	2			8	1	0	0
				5	0	0	1
	12			16	1	1	0
							4
							6

[31]				/D9i	High		Low
	12	1	14		1		4
	6		9	16	0	0	1
	17			5	1	0	0
	7			10	0	0	1
				2	1	0	0
	13			15	0	0	1
	3			8	1	0	0
				4	0	0	1
	11			18	1	1	0
							4
							8

.....

[46]				/D9i	High		Low
	17	1	13		1		3
	9		8	12	0	0	1
	15			6	1	0	0
	2			11	0	0	1
				4	1	0	0
	10			16	0	0	1
	7			5	1	0	0
				3	0	0	1
	14			18	1	1	0
							4
							8

[47]						/D9i	High					Low			
		16					1					6			
	14	1	5	18		1	0	0	1		4	0	4	8	
	9			6		0			0		8			5	
12				11		1			1	2					1
	2			4		0			0	1				3	
13				10		1			1	3					0
	7		8			0		0			6		7		
		3					0					2			
	17		15			1	1				7		5		

[48]						/D9i	High					Low			
		10					1					0			
	12	1	8	17		1	0	0	1		2	0	7	7	
	9			4		0			0		8			3	
18				16		1			1	8					6
	3			2		0			0	2				1	
14				15		1			1	4					5
	6		7			0		0			5		6		
		5					0					4			
	11		13			1	1				1		3		

[Total Count = 48] OK!

Section 3: Special Type of Double-Ringed Magic Stars of Order 11

Let's go on making another experiment for Double-Ringed Magic Stars 11.

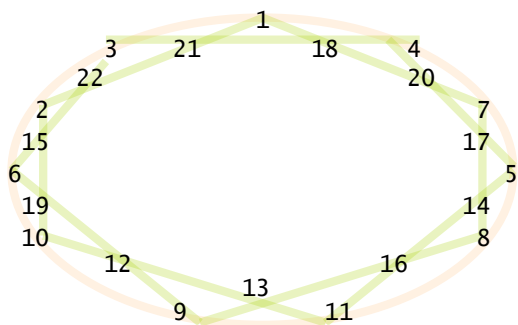
[Type 1] {n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,n11}={1,2,3,4,5,6,7,8,9,10,11}
 {n12,n13,n14,n15,n16,n17,n18,n19,n20,n21,n22}={12,13,14,15,16,17,18,19,20,21,22}

[Type 2] {n1,n2,n3,n4,n5,n6,n7,n8,n9,n10,n11}={12,13,14,15,16,17,18,19,20,21,22}
 {n12,n13,n14,n15,n16,n17,n18,n19,n20,n21,n22}={1,2,3,4,5,6,7,8,9,10,11}

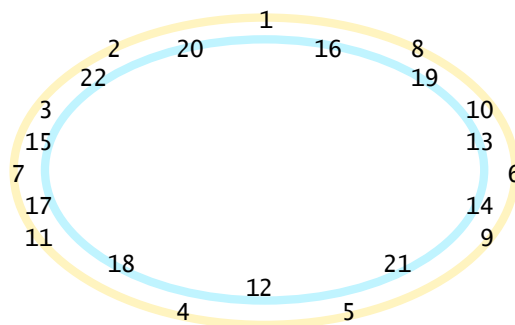
** List of "Latin" Type of Double-Ringed Magic Stars of Order 11 **

[Type n1=1]

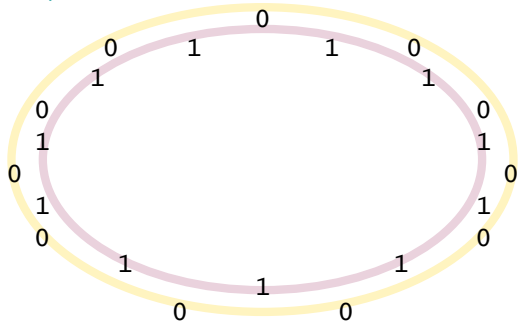
1#



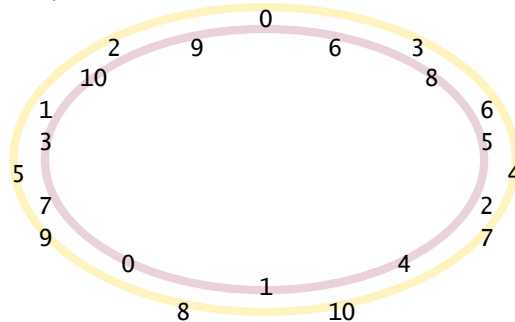
3#

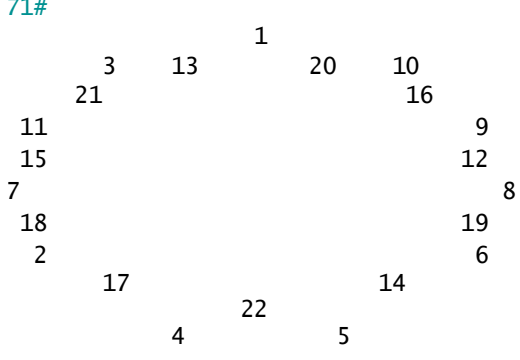
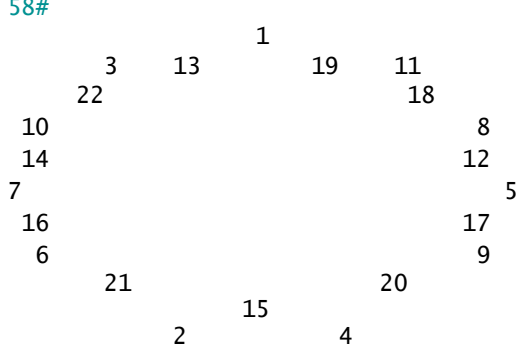
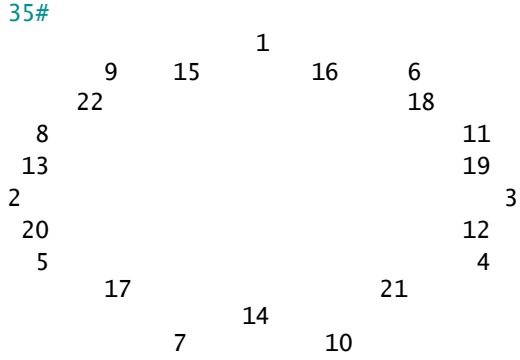
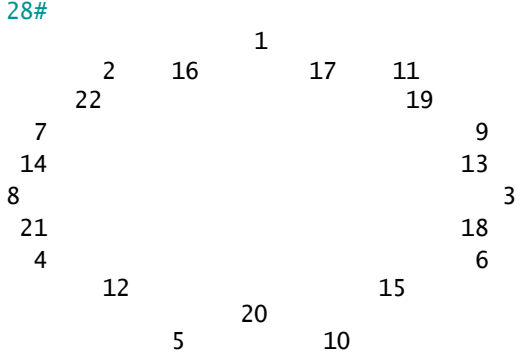
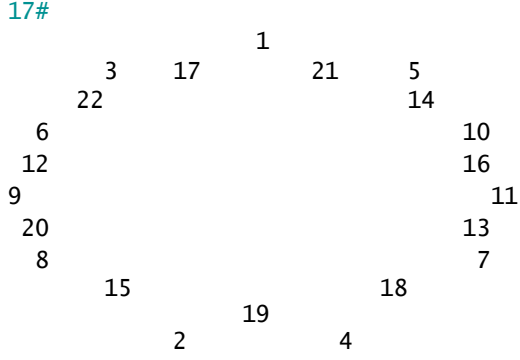
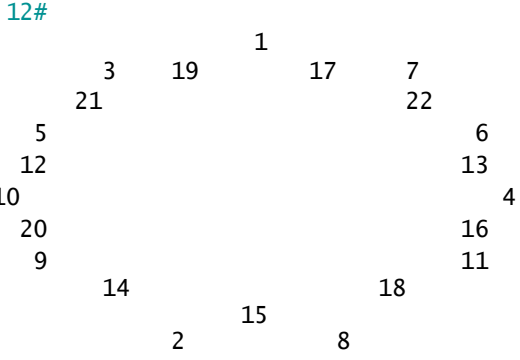
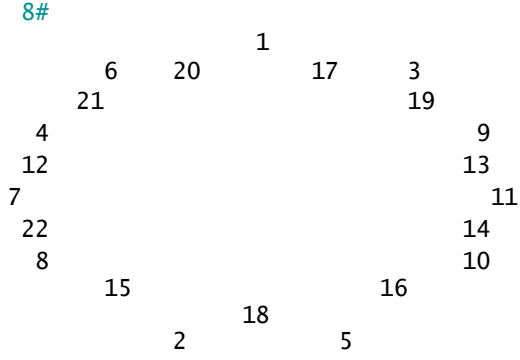
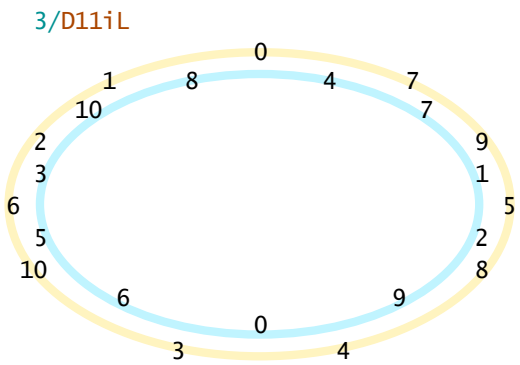
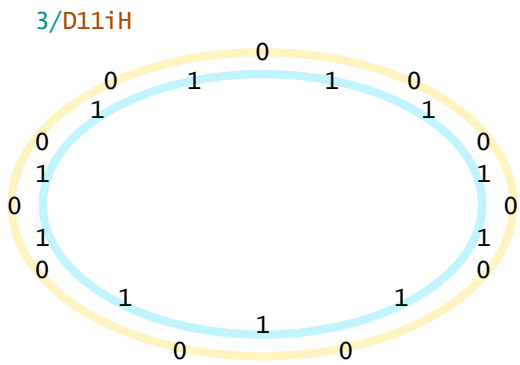


1/D11iH

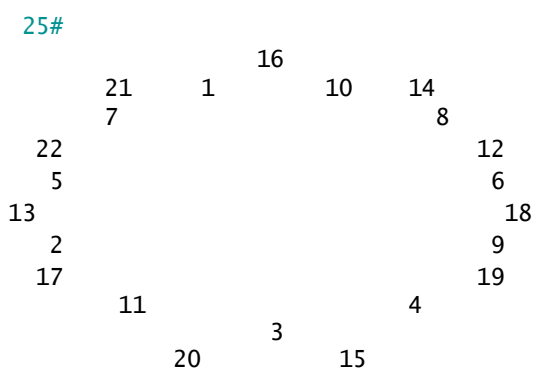
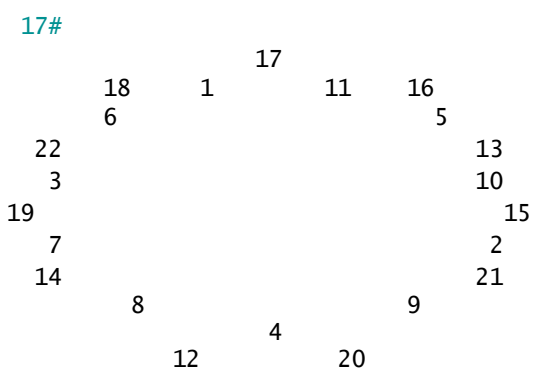
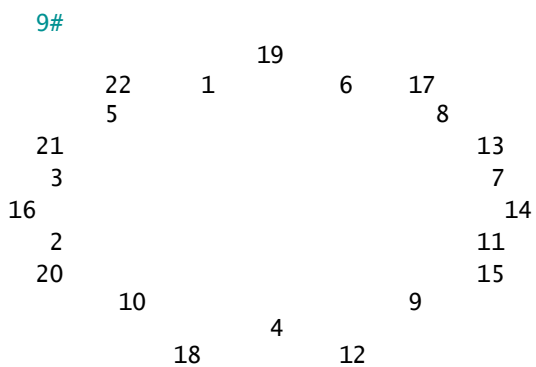
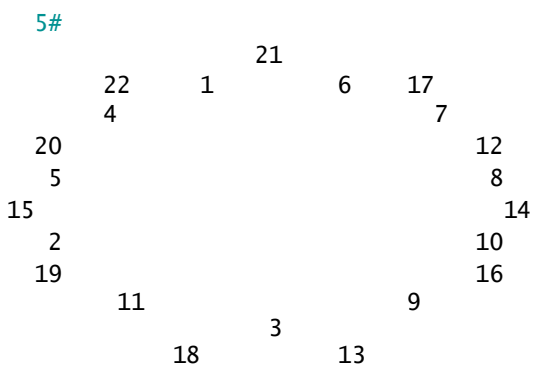
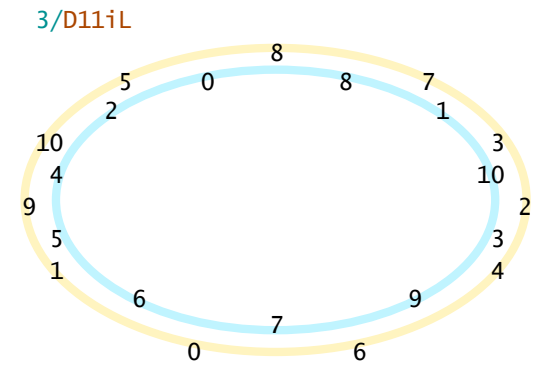
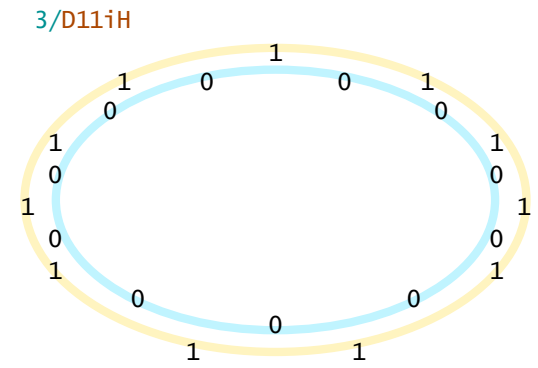
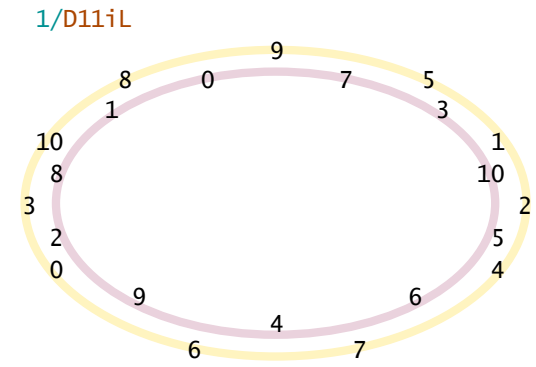
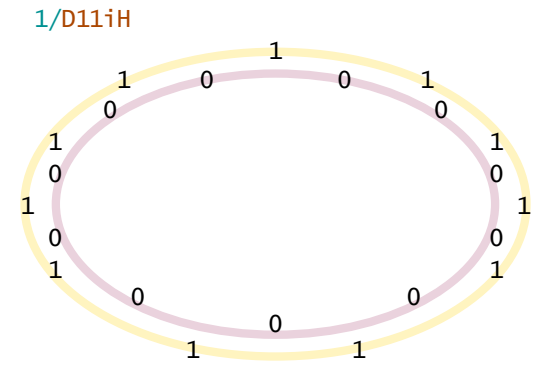
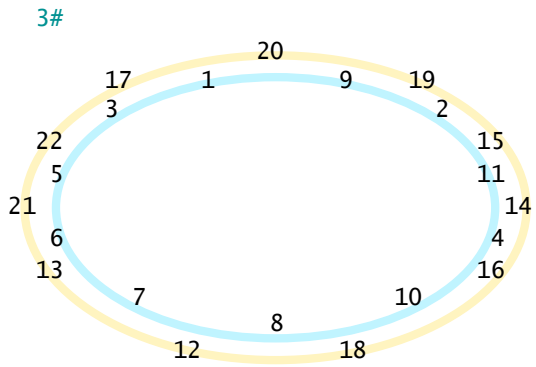
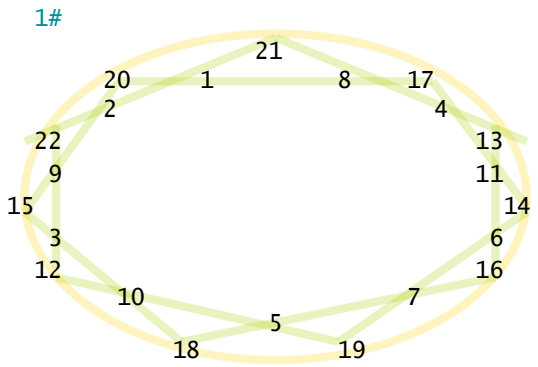


1/D11iL

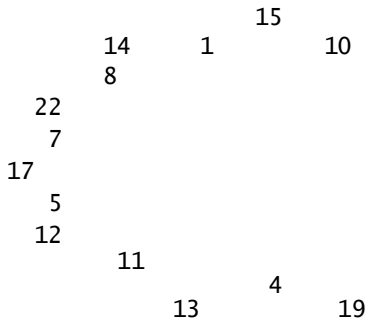




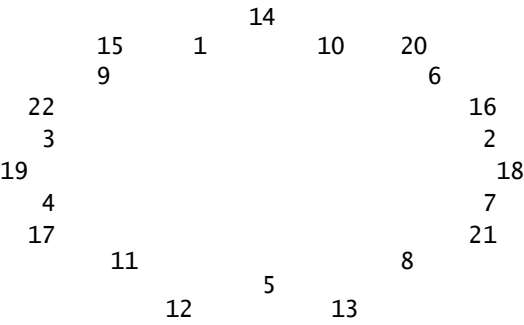
[Type n12=1]



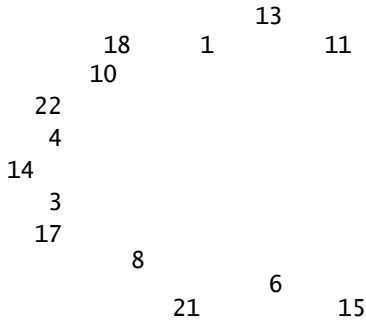
31#



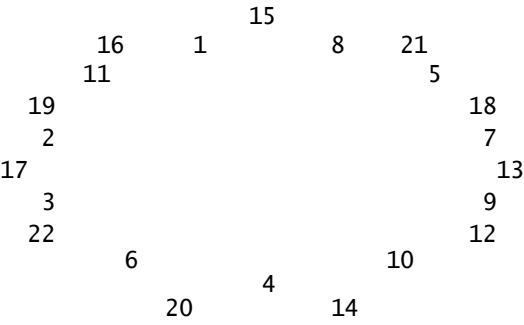
42#



57#



75#



[Count = 84 + 84 = 168] OK!

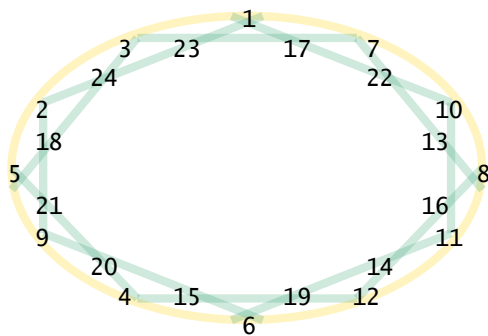
It is quite a relief that there are not too many solutions of this type, and there are the same counts of solutions for both Type $n1=1$ and Type $n12=1$. It might indicate any high symmetry exists between them, don't you think? We are now ready to study the inner structure of solution set and any transformational method among them.

Section 4: Special Type of Double-Ringed Magic Stars of Order 12

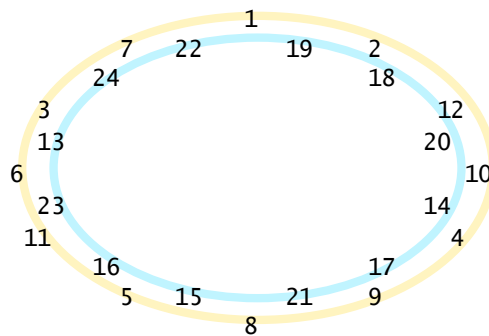
** "Latin" Type of Double-Ringed Magic Stars of Order 12 **

[Type $n1=1$]

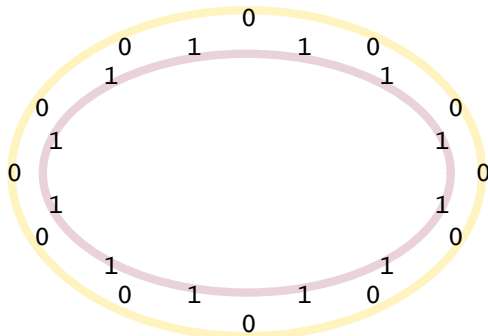
1#



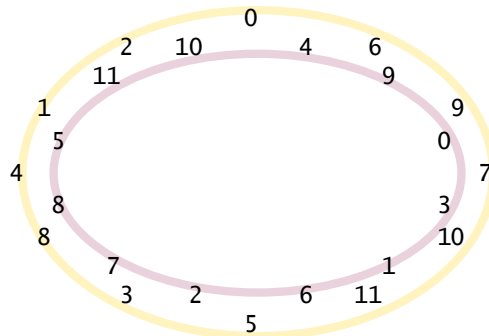
12#



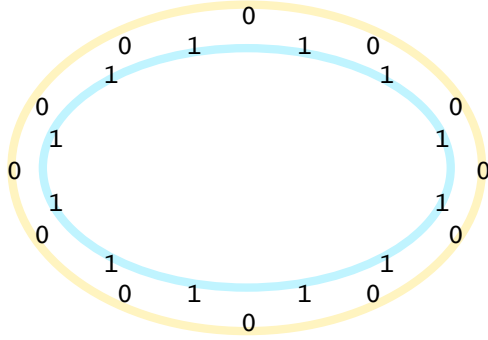
1/D12iH



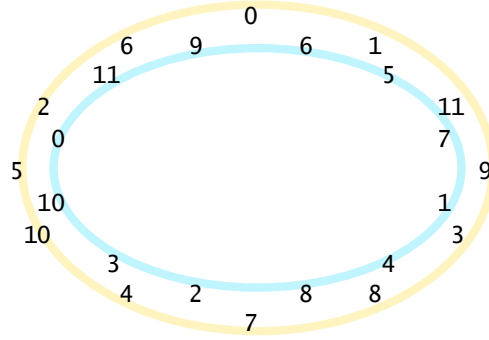
1/D12iL



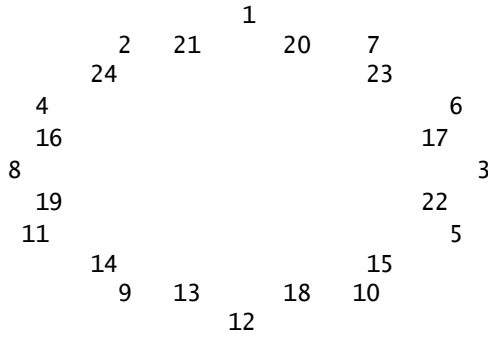
12/D12iH



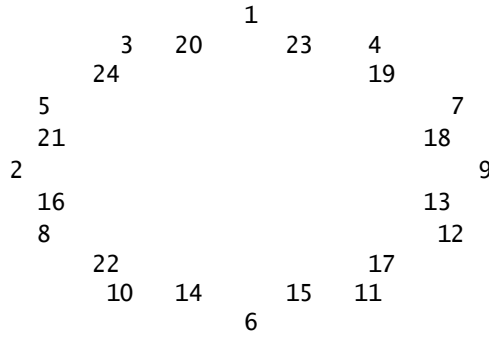
12/D12iL



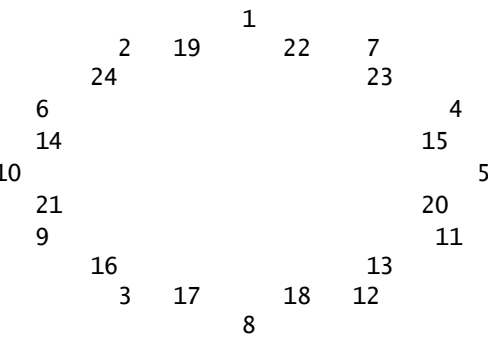
22#



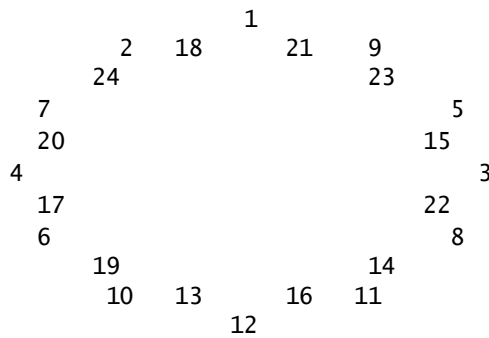
31#



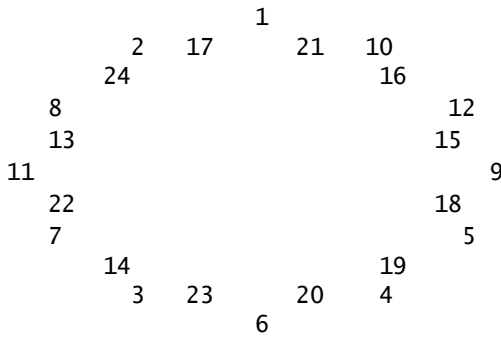
45#



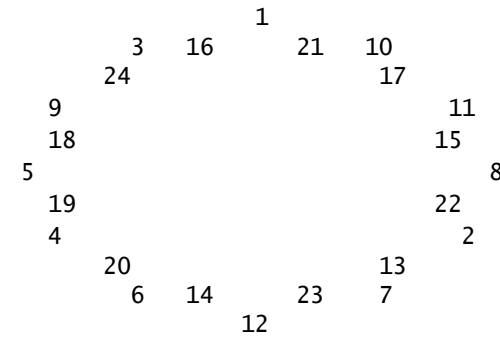
73#



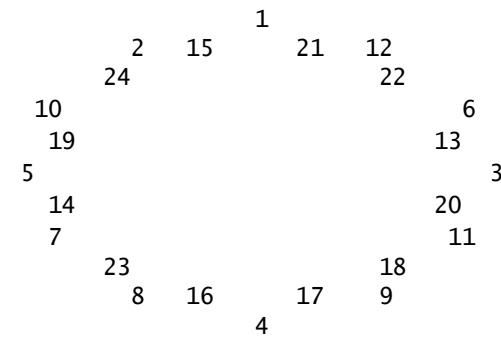
101#



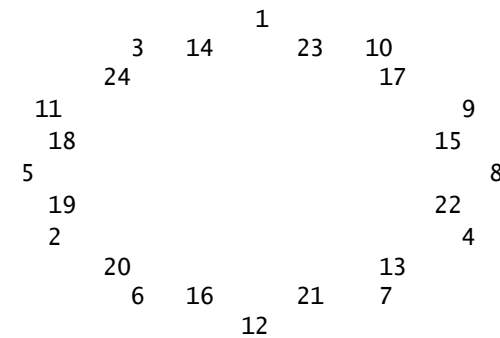
136#



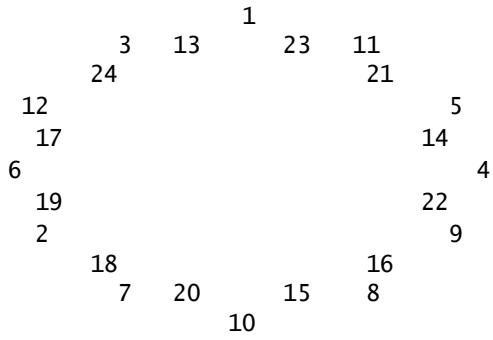
167#



218#

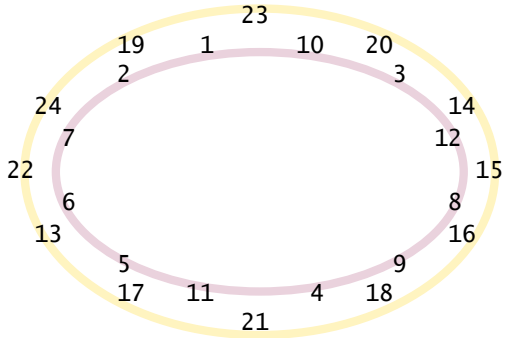


259#

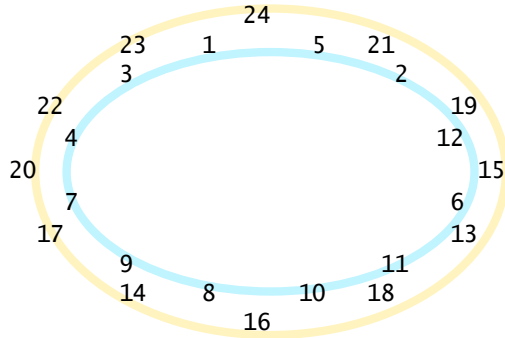


[Type n2=1]

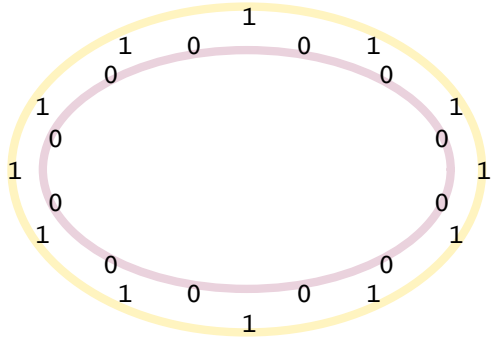
1#



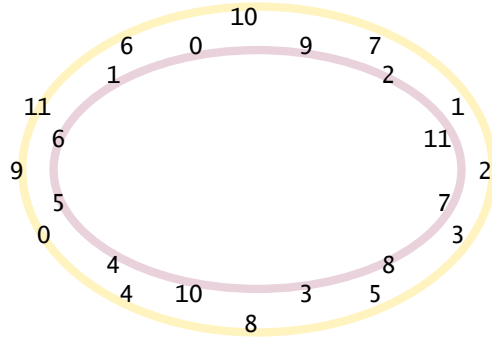
12#



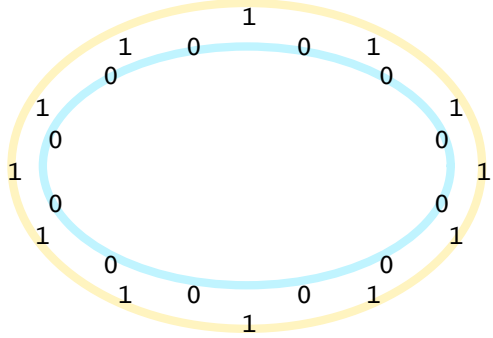
1/D12iH



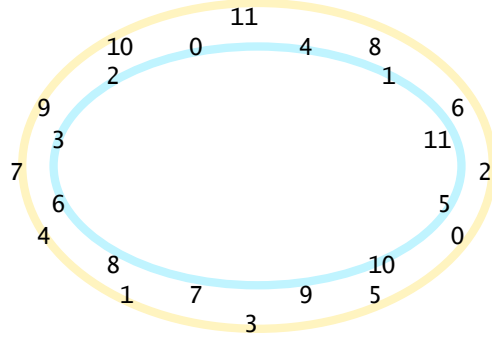
1/D12iL



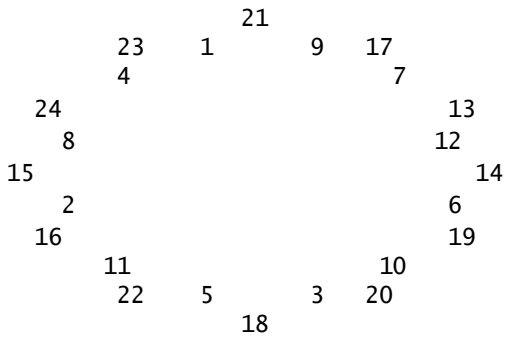
12/D12iH



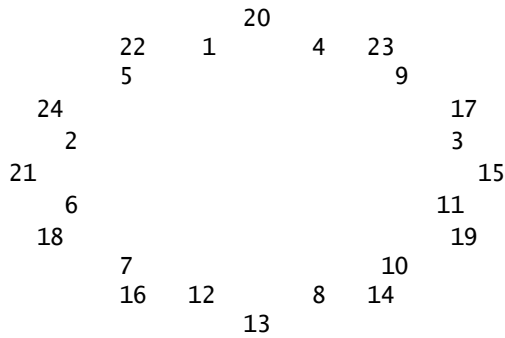
12/D12iL

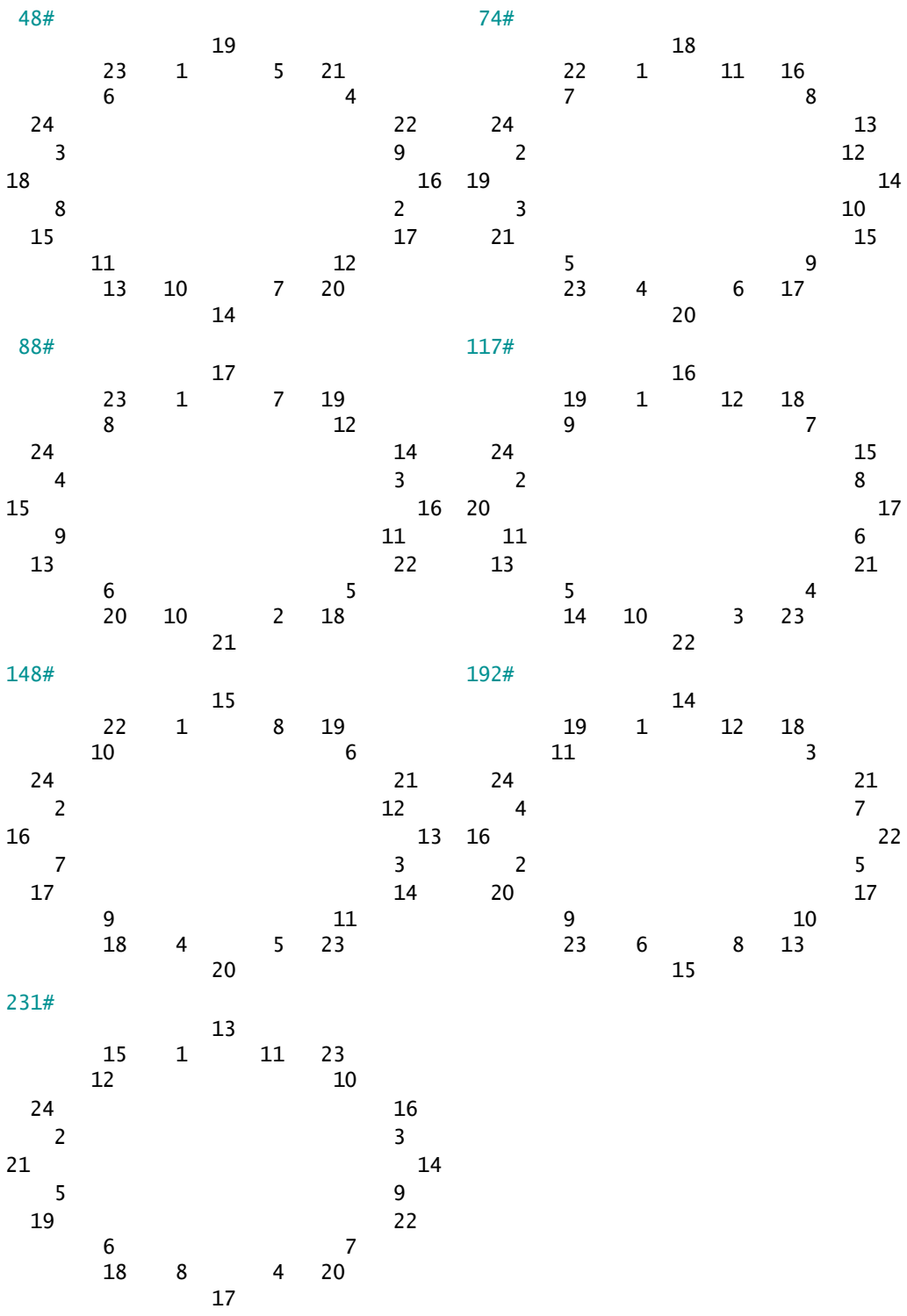


18#



29#





[Count = 340 + 326 = 666] OK!

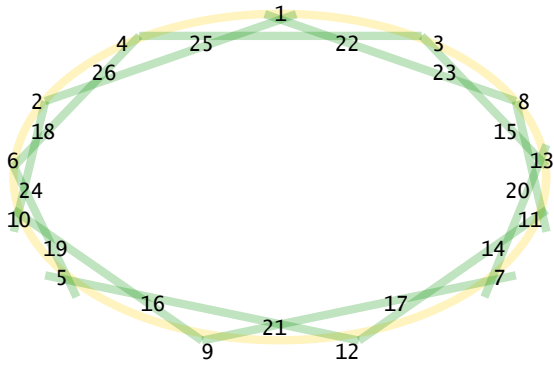
You may naturally think you would have no such solutions for the order 12, since I reported that we could not make any solutions of this type for the orders 6, 8 and 10. But, it proved to be there are 666 solutions of this type for the order 12. However, the solution count of Type $n_1=1$ is different from the one of Type $n_2=1$. It might indicate any symmetry breaks between them.

Section 5: Special Type of Double-Ringed Magic Stars of Order 13

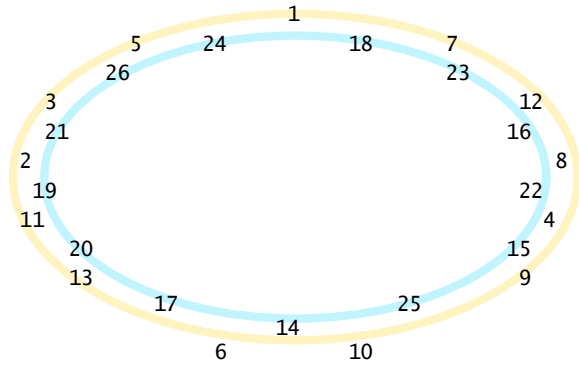
** "Latin" Type of Double-Ringed Magic Stars of Order 13 **

[Type n1=1]

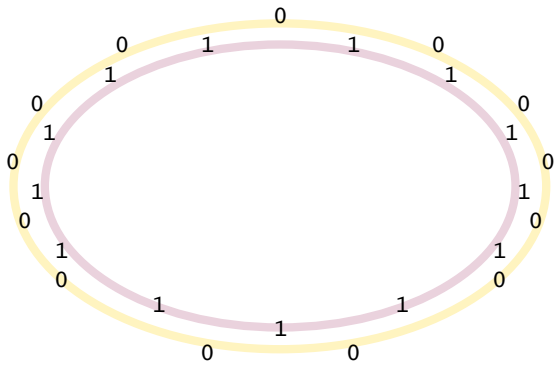
1#



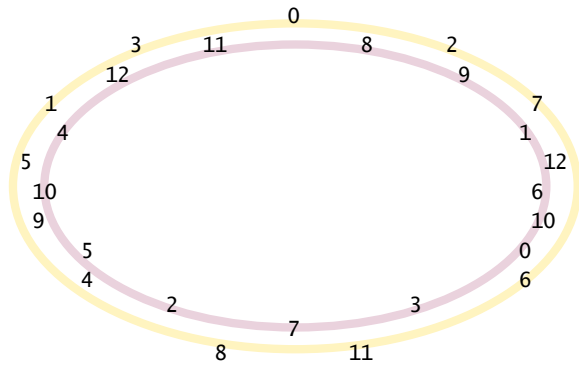
9#



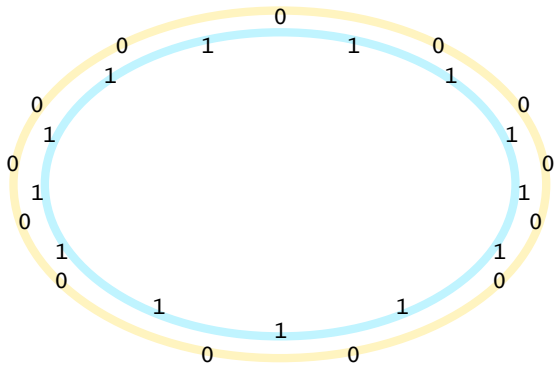
1/D13iH



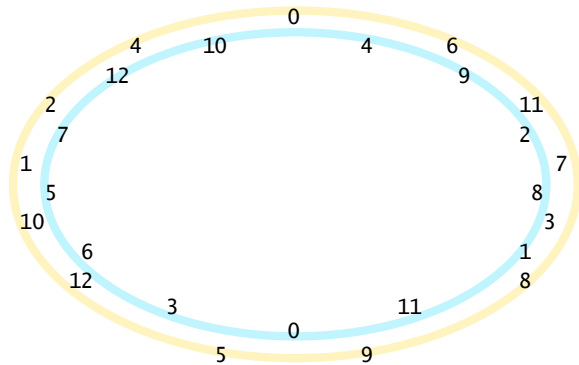
1/D13iL



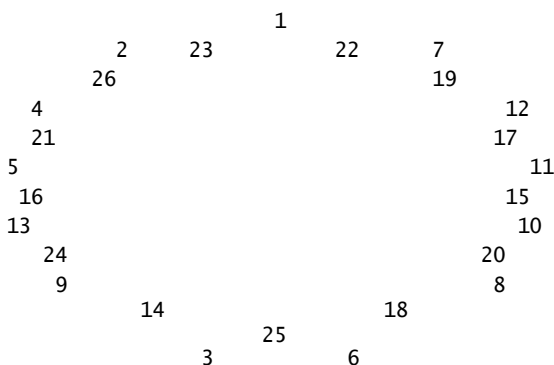
9/D13iH



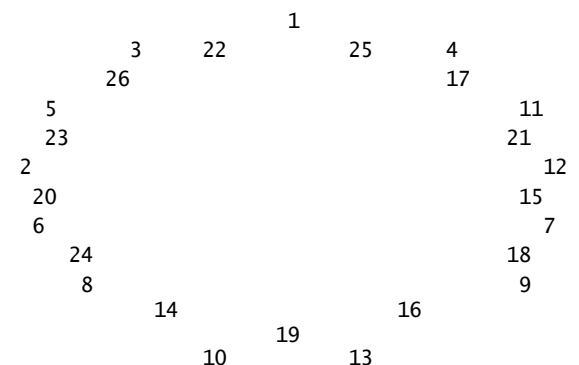
9/D13iL



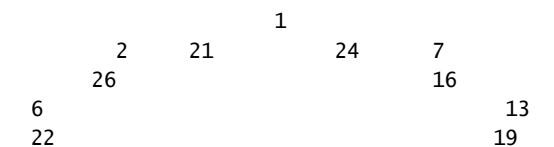
14#



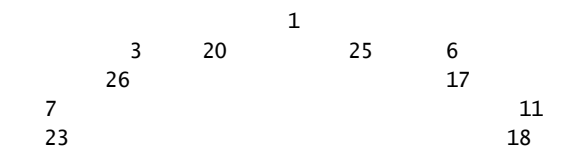
43#



71#



106#



4				12	2			13
18				17	19			15
8				5	5			10
23				14	24			14
9				11	9			12
	20			25		21		22
		3	15	10		4	16	8

150#

191#

			1					
	2	19		21	12			
	26				22			
8				10				7
17				14		9		15
9				6		23		10
18				25		16		19
11				5		6		13
24				16		24		14
3				7		12		11
	15			20		20		22
		4	23	13				

240#

311#

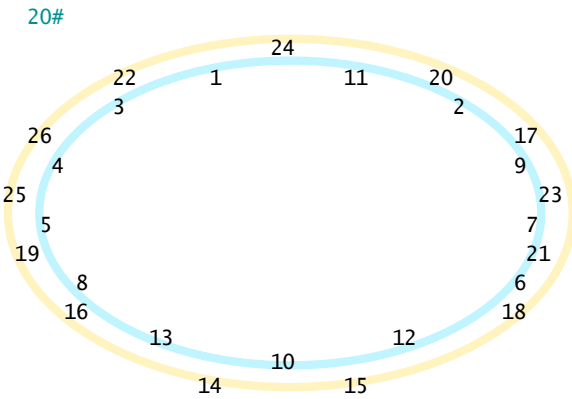
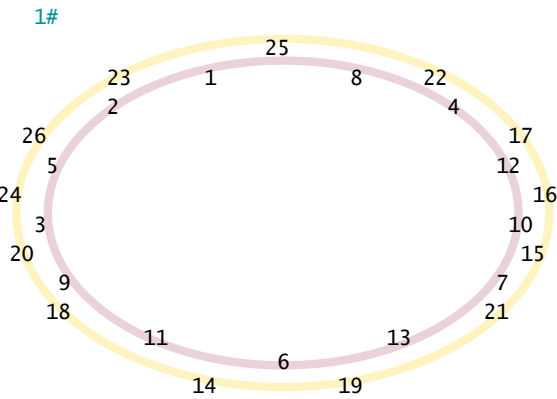
			1					
	2	17		24	11			
	26				20			
10				9				12
22				15		11		22
4				8		21		4
19				23		14		17
3				7		8		3
25				18		25		24
6				5		13		9
	14			16		15		20
		12	21	13				

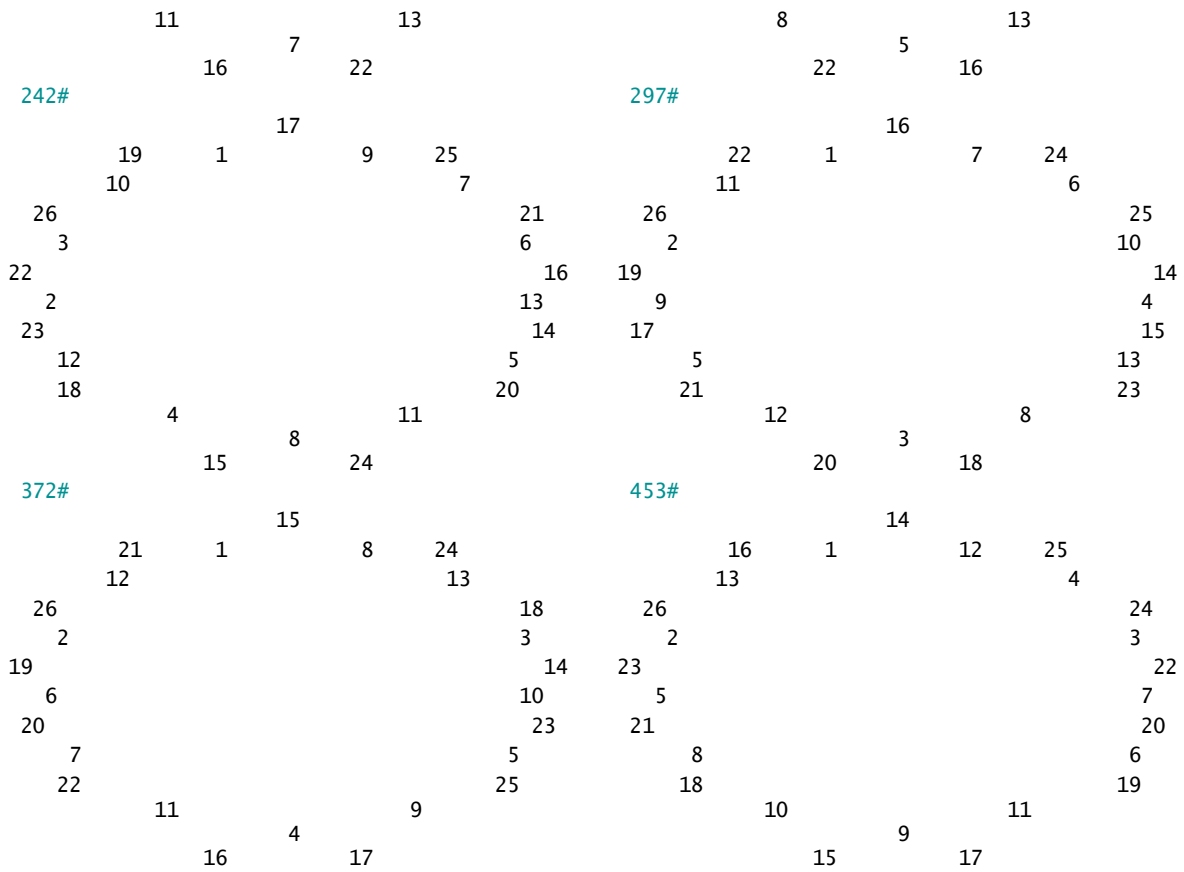
363#

449#

			1					
	2	15		24	13			
	26				22			
12				7				11
20				14		13		20
6				5		16		5
19				23		9		19
3				10		15		4
25				17		10		22
4				9		24		8
	18			16		6		
		8	21	11		18		21
						2	23	7

[Type n2=1]





[Count = 544 + 544 = 1088] OK!

Table of Total Solution Counts of Double-ringed Magic Stars

Order	'Latin' Type of DMS	'Standard' Type of DMS
6	0	80
7	4	72
8	0	112
9	48	3014
10	0	10882
11	168	53528
12	666	396930
13	1088	2434692

Our special 'Latin Type' of Double-ringed Magic Stars are supposed to be very rare and precious jewels, especially in higher orders.

(Written by Kanji Setsuda on February 13, 2007
and July 8, 2008; Working with MacOSX and Xcode2.2)

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