

## Part 3: "New Algebraic Study of Magic Squares"

### Chapter 5: Outlines of Magic Squares of Order 7: Kanji Setsuda

In this chapter we are going to study only about three types of magic squares of order 7: Self-Complementary, Pan-Diagonal and Simultaneous of both S-C and P-D.

I intend to present you a simple guide-book that explains only about the outlines of typical magic squares of order 7 here.

#### Section 1: Self-Complementary Magic Squares of Order 7

##### #1. Definition of the Self-Complementary Type

First of all, let's study about the Self-Complementary Magic Squares of order 7. See the diagram below and the simultaneous equations we assume. Complementary Pairs of 50 are all located symmetrically with respect to  $n_{25}$  (Geometric center of our object).

** Basic Form **	** List of Basic Conditions: C=175 **																																																																																											
<table style="width: 100%; border-collapse: collapse;"> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">1</td><td style="border-right: 1px dashed black; padding: 2px 5px;">2</td><td style="border-right: 1px dashed black; padding: 2px 5px;">3</td><td style="border-right: 1px dashed black; padding: 2px 5px;">4</td><td style="border-right: 1px dashed black; padding: 2px 5px;">5</td><td style="border-right: 1px dashed black; padding: 2px 5px;">6</td><td style="padding: 2px 5px;">7</td></tr> <tr><td colspan="7" style="border-top: 1px dashed black; border-bottom: 1px dashed black;"></td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">8</td><td style="border-right: 1px dashed black; padding: 2px 5px;">9</td><td style="border-right: 1px dashed black; padding: 2px 5px;">10</td><td style="border-right: 1px dashed black; padding: 2px 5px;">11</td><td style="border-right: 1px dashed black; padding: 2px 5px;">12</td><td style="border-right: 1px dashed black; padding: 2px 5px;">13</td><td style="padding: 2px 5px;">14</td></tr> <tr><td colspan="7" style="border-top: 1px dashed black; border-bottom: 1px dashed black;"></td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">15</td><td style="border-right: 1px dashed black; padding: 2px 5px;">16</td><td style="border-right: 1px dashed black; padding: 2px 5px;">17</td><td style="border-right: 1px dashed black; padding: 2px 5px;">18</td><td style="border-right: 1px dashed black; padding: 2px 5px;">19</td><td style="border-right: 1px dashed black; padding: 2px 5px;">20</td><td style="padding: 2px 5px;">21</td></tr> <tr><td colspan="7" style="border-top: 1px dashed black; border-bottom: 1px dashed black;"></td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">22</td><td style="border-right: 1px dashed black; padding: 2px 5px;">23</td><td style="border-right: 1px dashed black; padding: 2px 5px;">24</td><td style="border-right: 1px dashed black; padding: 2px 5px;">25</td><td style="border-right: 1px dashed black; padding: 2px 5px;">26</td><td style="border-right: 1px dashed black; padding: 2px 5px;">27</td><td style="padding: 2px 5px;">28</td></tr> <tr><td colspan="7" style="border-top: 1px dashed black; border-bottom: 1px dashed black;"></td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">29</td><td style="border-right: 1px dashed black; padding: 2px 5px;">30</td><td style="border-right: 1px dashed black; padding: 2px 5px;">31</td><td style="border-right: 1px dashed black; padding: 2px 5px;">32</td><td style="border-right: 1px dashed black; padding: 2px 5px;">33</td><td style="border-right: 1px dashed black; padding: 2px 5px;">34</td><td style="padding: 2px 5px;">35</td></tr> <tr><td colspan="7" style="border-top: 1px dashed black; border-bottom: 1px dashed black;"></td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">36</td><td style="border-right: 1px dashed black; padding: 2px 5px;">37</td><td style="border-right: 1px dashed black; padding: 2px 5px;">38</td><td style="border-right: 1px dashed black; padding: 2px 5px;">39</td><td style="border-right: 1px dashed black; padding: 2px 5px;">40</td><td style="border-right: 1px dashed black; padding: 2px 5px;">41</td><td style="padding: 2px 5px;">42</td></tr> <tr><td colspan="7" style="border-top: 1px dashed black; border-bottom: 1px dashed black;"></td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">43</td><td style="border-right: 1px dashed black; padding: 2px 5px;">44</td><td style="border-right: 1px dashed black; padding: 2px 5px;">45</td><td style="border-right: 1px dashed black; padding: 2px 5px;">46</td><td style="border-right: 1px dashed black; padding: 2px 5px;">47</td><td style="border-right: 1px dashed black; padding: 2px 5px;">48</td><td style="padding: 2px 5px;">49</td></tr> </table>	1	2	3	4	5	6	7								8	9	10	11	12	13	14								15	16	17	18	19	20	21								22	23	24	25	26	27	28								29	30	31	32	33	34	35								36	37	38	39	40	41	42								43	44	45	46	47	48	49	$n_1+n_2+n_3+n_4+n_5+n_6+n_7=C$ ... rw1; $n_8+n_9+n_{10}+n_{11}+n_{12}+n_{13}+n_{14}=C$ ... rw2; $n_{15}+n_{16}+n_{17}+n_{18}+n_{19}+n_{20}+n_{21}=C$ ... rw3; $n_{22}+n_{23}+n_{24}+n_{25}+n_{26}+n_{27}+n_{28}=C$ ... rw4; $n_{29}+n_{30}+n_{31}+n_{32}+n_{33}+n_{34}+n_{35}=C$ ... rw5; $n_{36}+n_{37}+n_{38}+n_{39}+n_{40}+n_{41}+n_{42}=C$ ... rw6; $n_{43}+n_{44}+n_{45}+n_{46}+n_{47}+n_{48}+n_{49}=C$ ... rw7; $n_1+n_8+n_{15}+n_{22}+n_{29}+n_{36}+n_{43}=C$ ... c11; $n_2+n_9+n_{16}+n_{23}+n_{30}+n_{37}+n_{44}=C$ ... c12; $n_3+n_{10}+n_{17}+n_{24}+n_{31}+n_{38}+n_{45}=C$ ... c13; $n_4+n_{11}+n_{18}+n_{25}+n_{32}+n_{39}+n_{46}=C$ ... c14; $n_5+n_{12}+n_{19}+n_{26}+n_{33}+n_{40}+n_{47}=C$ ... c15; $n_6+n_{13}+n_{20}+n_{27}+n_{34}+n_{41}+n_{48}=C$ ... c16; $n_7+n_{14}+n_{21}+n_{28}+n_{35}+n_{42}+n_{49}=C$ ... c17; $n_1+n_9+n_{17}+n_{25}+n_{33}+n_{41}+n_{49}=C$ ... pd1; $n_7+n_{13}+n_{19}+n_{25}+n_{31}+n_{37}+n_{43}=C$ ... pb7;
1	2	3	4	5	6	7																																																																																						
8	9	10	11	12	13	14																																																																																						
15	16	17	18	19	20	21																																																																																						
22	23	24	25	26	27	28																																																																																						
29	30	31	32	33	34	35																																																																																						
36	37	38	39	40	41	42																																																																																						
43	44	45	46	47	48	49																																																																																						

##### \*\* Conditions for the Symmetric Complementary Pairs: CC=50 \*\*

$$\begin{aligned}
 n_1+n_{49}=n_2+n_{48}=n_3+n_{47}=n_4+n_{46}=n_5+n_{45}=n_6+n_{44}=n_7+n_{43}= \\
 n_8+n_{42}=n_9+n_{41}=n_{10}+n_{40}=n_{11}+n_{39}=n_{12}+n_{38}=n_{13}+n_{37}=n_{14}+n_{36}= \\
 n_{15}+n_{35}=n_{16}+n_{34}=n_{17}+n_{33}=n_{18}+n_{32}=n_{19}+n_{31}=n_{20}+n_{30}=n_{21}+n_{29}= \\
 n_{22}+n_{28}=n_{23}+n_{27}=n_{24}+n_{26}=n_{25}+n_{25}=CC \dots scc
 \end{aligned}$$

To the multi-equations of Symmetric Complementary Pairs we can expect:

$$\begin{aligned}
 n_{49}=C-n_1; n_{48}=C-n_2; n_{47}=C-n_3; n_{46}=C-n_4; n_{45}=C-n_5; n_{44}=C-n_6; n_{43}=C-n_7; \\
 n_{42}=C-n_8; n_{41}=C-n_9; \dots n_{27}=C-n_{23}; n_{26}=C-n_{24}; \text{ and } n_{25}=C-n_{25}=25
 \end{aligned}$$

If you put any temporary value to  $n_1$ , then  $n_{49}$  is defined according to  $C-n_1$  at the same time. If you define  $n_2$ , then  $n_{48}$  is defined automatically. ... Therefore we must really deal with about half of all entries.

##### #2. New Notation for those Complementary Pairs

We know there are too many solutions for us to calculate and count up through.

Though we have to deal with 49 numbers, we have only 17 equations in all.

Under these conditions we can only draw the rough outline of our object. We can only get some guide-lines to know its structure.

But when you adopt the next new notation for the Complementary Pairs, you may probably have a new approach to the structural study of this type.

$$\underline{n}_1=C-n_1=n_{49}; \underline{n}_2=C-n_2=n_{48}; \underline{n}_3=C-n_3=n_{47}; \underline{n}_4=C-n_4=n_{46}; \underline{n}_5=C-n_5=n_{45};$$

$n_6=C-n_6=n_{44}$ ;  $n_7=C-n_7=n_{43}$ ;  $n_8=C-n_8=n_{42}$ ;  $n_9=C-n_9=n_{41}$ ;  $n_{10}=C-n_{10}=n_{40}$ ; ...;  
 $n_{23}=C-n_{23}=n_{27}$ ;  $n_{24}=C-n_{24}=n_{26}$ ; and  $n_{25}=C-n_{25}=n_{25}=25$

```

** New Basic Form **      ** List of Basic Conditions: C=175 **
-----
| n1 | n2 | n3 | n4 | n5 | n6 | n7 |   n1+n2+n3+n4+n5+n6+n7=C   ... rw1;
|---+---+---+---+---+---+
| n8 | n9 | 10 | 11 | 12 | 13 | 14 |   n8+n9+n10+n11+n12+n13+n14=C  ... rw2;
|---+---+---+---+---+---+
| n8 | n9 | 10 | 11 | 12 | 13 | 14 |   n15+n16+n17+n18+n19+n20+n21=C  ... rw3;
|---+---+---+---+---+---+
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |   n22+n23+n24+n25+n24+n23+n22=C  ... rw4;
|---+---+---+---+---+---+
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |   n21+n20+n19+n18+n17+n16+n15=C  ... rw5;
|---+---+---+---+---+---+
| 22 | 23 | 24 | 25 | 24 | 23 | 22 |   n14+n13+n12+n11+n10+n9+n8=C   ... rw6;
|---+---+---+---+---+---+
| 22 | 23 | 24 | 25 | 24 | 23 | 22 |   n7+n6+n5+n4+n3+n2+n1=C       ... rw7;
|---+---+---+---+---+---+
| 21 | 20 | 19 | 18 | 17 | 16 | 15 |   n1+n8+n15+n22+n21+n14+n7=C   ... c11;
|---+---+---+---+---+---+
| 21 | 20 | 19 | 18 | 17 | 16 | 15 |   n2+n9+n16+n23+n20+n13+n6=C   ... c12;
|---+---+---+---+---+---+
| 14 | 13 | 12 | 11 | 10 | n9 | n8 |   n3+n10+n17+n24+n19+n12+n5=C  ... c13;
|---+---+---+---+---+---+
| 14 | 13 | 12 | 11 | 10 | n9 | n8 |   n4+n11+n18+n25+n18+n11+n4=C  ... c14;
|---+---+---+---+---+---+
| n7 | n6 | n5 | n4 | n3 | n2 | n1 |   n5+n12+n19+n24+n17+n10+n3=C  ... c15;
|---+---+---+---+---+---+
| n7 | n6 | n5 | n4 | n3 | n2 | n1 |   n6+n13+n20+n23+n16+n9+n2=C   ... c16;
|---+---+---+---+---+---+
| n7 | n6 | n5 | n4 | n3 | n2 | n1 |   n7+n14+n21+n22+n15+n8+n1=C   ... c17;
|---+---+---+---+---+---+
| n7 | n6 | n5 | n4 | n3 | n2 | n1 |   n1+n9+n17+n25+n17+n9+n1=C   ... pd1;
|---+---+---+---+---+---+
| n7 | n6 | n5 | n4 | n3 | n2 | n1 |   n7+n13+n19+n25+n19+n13+n7=C   ... pb7;
|---+---+---+---+---+---+

```

```

** Conditions for the Symmetric Complementary Pairs: CC=50 **
n1+n1=n2+n2=n3+n3=n4+n4=n5+n5=n6+n6=n7+n7=
n8+n8=n9+n9=n10+n10=n11+n11=n12+n12=n13+n13=n14+n14=
n15+n15=n16+n16=n17+n17=n18+n18=n19+n19=n20+n20=n21+n21=
n22+n22=n23+n23=n24+n24=n25+n25=CC ... scc

```

You only have to draw the next diagrams as below to examine the contents of any row, column, or primary diagonal. You will be able to see which rows or columns you can exchange with each other, without any calculations. Even if you exchange such a couple of rows and columns as below, you can still have your Self-Complementary MS77 unspoiled, while you will make a different solution from it.

```

** New Basic Form **      ** Results of Row/Column Exchanges **
-----
| n1 | n2 | n3 | n4 | n5 | n6 | n7 |   | n1 | n6 | n3 | n4 | n5 | n2 | n7 |
|---+---+---+---+---+---+   |---+---+---+---+---+---+
| n8 | n9 | 10 | 11 | 12 | 13 | 14 |   | n8 | 13 | 10 | 11 | 12 | n9 | 14 |
|---+---+---+---+---+---+   |---+---+---+---+---+---+
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | * | 21 | 20 | 19 | 18 | 17 | 16 | 15 | * | 15 | 20 | 17 | 18 | 19 | 16 | 21 |
|---+---+---+---+---+---+   |---+---+---+---+---+---+
| 22 | 23 | 24 | 25 | 24 | 23 | 22 |   | 22 | 23 | 24 | 25 | 24 | 23 | 22 |   | 22 | 23 | 24 | 25 | 24 | 23 | 22 |
|---+---+---+---+---+---+   |---+---+---+---+---+---+
| 21 | 20 | 19 | 18 | 17 | 16 | 15 | * | 15 | 16 | 17 | 18 | 19 | 20 | 21 | * | 21 | 16 | 19 | 18 | 17 | 20 | 15 |
|---+---+---+---+---+---+   |---+---+---+---+---+---+
| 14 | 13 | 12 | 11 | 10 | n9 | n8 |   | 14 | 13 | 12 | 11 | 10 | n9 | n8 |   | 14 | n9 | 12 | 11 | 10 | 13 | n8 |
|---+---+---+---+---+---+   |---+---+---+---+---+---+
| n7 | n6 | n5 | n4 | n3 | n2 | n1 |   | n7 | n6 | n5 | n4 | n3 | n2 | n1 |   | n7 | n2 | n5 | n4 | n3 | n6 | n1 |
|---+---+---+---+---+---+   |---+---+---+---+---+---+

```

This implies that you can make 16 different transformations in all for each solution of the fundamental type. ( $2 \times 2 \times 2 \times 2 = 16$  including the original)

And in the case when you want to have the set of fundamental solutions, you have to take such the selecting inequality conditions as follows, as you can easily guess.

$n_1=1$ ;  $n_2 > n_8$ ;  $n_2 > n_6$ ;  $n_3 < n_5$ ;  $n_8 > n_{36}$ ; and  $n_{15} < n_{29}$ ;

Let me show you the compact lists of fundamental solutions of our object here.

**\*\* Self-Complementary Magic Squares of Order 7: Fundamental Solutions with  $n_1=1$  \*\***

1/ 1 48 3 46 5 44 28 43 11 26 37 36 12 10 8 16 35 32 31 23 30 41 29 33 25 17 21 9 20 27 19 18 15 34 42 40 38 14 13 24 39 7 22 6 45 4 47 2 49	1298448/ 1 48 3 46 5 44 28 43 11 27 37 36 12 9 8 18 33 34 21 31 30 40 35 24 25 26 15 10 20 19 29 16 17 32 42 41 38 14 13 23 39 7 22 6 45 4 47 2 49	2752583/ 1 48 3 46 5 44 28 43 10 30 37 34 12 9 8 26 27 18 31 36 29 39 33 35 25 15 17 11 21 14 19 32 23 24 42 41 38 16 13 20 40 7 22 6 45 4 47 2 49	3834145/ 1 48 3 46 5 44 28 43 9 29 37 36 11 10 8 16 35 32 31 26 27 38 33 30 25 20 17 12 23 24 19 18 15 34 42 40 39 14 13 21 41 7 22 6 45 4 47 2 49
5165688/ 1 48 3 46 5 44 28 43 10 29 38 35 11 9 8 24 30 19 33 34 27 37 32 36 25 14 18 13 23 16 17 31 20 26 42 41 39 15 12 21 40 7 22 6 45 4 47 2 49	6172186/ 1 48 3 46 5 44 28 43 10 27 38 37 11 9 8 17 34 31 29 30 26 36 35 32 25 18 15 14 24 20 21 19 16 33 42 41 39 13 12 23 40 7 22 6 45 4 47 2 49	7276495/ 1 48 3 46 5 44 28 43 9 27 38 37 11 10 8 19 33 29 32 30 24 35 34 36 25 14 16 15 26 20 18 21 17 31 42 40 39 13 12 23 41 7 22 6 45 4 47 2 49	8067454/ 1 48 3 46 5 44 28 43 10 27 38 37 11 9 8 17 36 29 31 30 24 34 35 32 25 18 15 16 26 20 19 21 14 33 42 41 39 13 12 23 40 7 22 6 45 4 47 2 49
9124439/ 1 48 3 46 5 44 28 43 10 29 38 35 11 9 8 16 37 31 36 24 23 33 30 32 25 18 20 17 27 26 14 19 13 34 42 41 39 15 12 21 40 7 22 6 45 4 47 2 49	9860391/ 1 48 3 46 5 44 28 43 9 27 38 37 11 10 8 24 34 35 33 20 21 32 19 36 25 14 31 18 29 30 17 15 16 26 42 40 39 13 12 23 41 7 22 6 45 4 47 2 49	10699608/ 1 48 3 46 5 44 28 43 10 27 38 37 11 9 8 18 34 35 33 26 21 31 30 36 25 14 20 19 29 24 17 15 16 32 42 41 39 13 12 23 40 7 22 6 45 4 47 2 49	11417766/ 1 48 3 46 5 44 28 43 9 27 38 37 11 10 8 24 36 21 34 33 19 30 32 35 25 15 18 20 31 17 16 29 14 26 42 40 39 13 12 23 41 7 22 6 45 4 47 2 49
12259890/ 1 48 3 46 5 44 28 43 10 27 38 37 11 9 8 18 36 35 33 26 19 29 30 34 25 16 20 21 31 24 17 15 14 32 42 41 39 13 12 23 40 7 22 6 45 4 47 2 49	12973650/ 1 48 3 46 5 44 28 43 10 29 38 35 11 9 8 26 37 31 36 20 17 27 16 32 25 18 34 23 33 30 14 19 13 24 42 41 39 15 12 21 40 7 22 6 45 4 47 2 49	13553321/ 1 48 3 46 5 44 28 43 10 27 38 37 11 9 8 21 33 35 32 30 16 26 31 36 25 14 19 24 34 20 18 15 17 29 42 41 39 13 12 23 40 7 22 6 45 4 47 2 49	14230504/ 1 48 3 46 5 44 28 43 10 27 38 37 11 9 8 34 35 33 19 32 14 24 20 21 25 29 30 26 36 18 31 17 15 16 42 41 39 13 12 23 40 7 22 6 45 4 47 2 49
14749321/ 1 48 3 46 5 44 28 43 10 29 38 35 11 9 8 24 31 33 34 32 13 23 30 36 25 14 20 27 37 18 16 17 19 26 42 41 39 15 12 21 40 7 22 6 45 4 47 2 49	15037003/ 1 48 3 46 5 44 28 43 10 30 35 36 12 9 8 27 34 26 32 37 11 21 33 31 25 19 17 29 39 13 18 24 16 23 42 41 38 14 15 20 40 7 22 6 45 4 47 2 49	15170081/ 1 48 3 46 5 44 28 43 11 27 37 36 12 9 8 26 33 34 29 35 10 20 31 32 25 18 19 30 40 15 21 16 17 24 42 41 38 14 13 23 39 7 22 6 45 4 47 2 49	15342180/ 1 48 3 46 5 44 28 43 11 26 37 36 12 10 9 23 35 30 29 17 32 42 16 31 25 19 34 8 18 33 21 20 15 27 41 40 38 14 13 24 39 7 22 6 45 4 47 2 49

.....

**\*\* Self-Complementary Magic Squares of Order 7: Fundamental Solutions with n4=1 \*\***

1/ 2 46 34 1 44 45 3 42 11 17 29 36 12 28 9 20 35 27 18 26 40 43 31 37 25 13 19 7 10 24 32 23 15 30 41 22 38 14 21 33 39 8 47 5 6 49 16 4 48	131425/ 2 46 34 1 44 45 3 42 10 18 37 27 12 29 9 33 28 15 20 31 39 43 24 36 25 14 26 7 11 19 30 35 22 17 41 21 38 23 13 32 40 8 47 5 6 49 16 4 48	247579/ 2 46 34 1 44 45 3 42 10 17 29 36 11 30 9 26 37 19 18 28 38 43 27 35 25 15 23 7 12 22 32 31 13 24 41 20 39 14 21 33 40 8 47 5 6 49 16 4 48	463841/ 2 46 34 1 44 45 3 42 10 14 38 29 11 31 9 27 33 24 15 30 37 43 28 32 25 18 22 7 13 20 35 26 17 23 41 19 39 21 12 36 40 8 47 5 6 49 16 4 48
655928/ 2 46 34 1 44 45 3 42 10 15 38 27 11 32 9 20 37 24 21 28 36 43 33 31 25 19 17 7 14 22 29 26 13 30 41 18 39 23 12 35 40 8 47 5 6 49 16 4 48	948454/ 2 46 34 1 44 45 3 42 10 14 38 27 11 33 9 28 37 18 19 29 35 43 26 30 25 20 24 7 15 21 31 32 13 22 41 17 39 23 12 36 40 8 47 5 6 49 16 4 48	1207374/ 2 46 34 1 44 45 3 42 10 14 37 26 11 35 9 27 31 21 22 32 33 43 30 38 25 12 20 7 17 18 28 29 19 23 41 15 39 24 13 36 40 8 47 5 6 49 16 4 48	1368906/ 2 46 34 1 44 45 3 42 10 13 35 28 11 36 9 26 33 23 21 31 32 43 30 38 25 12 20 7 18 19 29 27 17 24 41 14 39 22 15 37 40 8 47 5 6 49 16 4 48

1598454/	1823644/	2113109/	2380888/
2 46 34 1 44 45 3	2 46 34 1 44 45 3	2 46 34 1 44 45 3	2 46 34 1 44 45 3
42 10 12 36 27 11 37	42 10 13 33 28 11 38	42 10 13 36 23 12 39	42 11 13 36 21 12 40
9 26 33 28 18 30 31	9 26 36 27 15 32 30	9 33 35 20 18 31 29	9 27 32 24 20 35 28
43 29 35 25 15 21 7	43 31 29 25 21 19 7	43 24 28 25 22 26 7	43 33 31 25 19 17 7
19 20 32 22 17 24 41	20 18 35 23 14 24 41	21 19 32 30 15 17 41	22 15 30 26 18 23 41
13 39 23 14 38 40 8	12 39 22 17 37 40 8	11 38 27 14 37 40 8	10 38 29 14 37 39 8
47 5 6 49 16 4 48	47 5 6 49 16 4 48	47 5 6 49 16 4 48	47 5 6 49 16 4 48
2707286/	3071058/	6836309/	11080870/
2 46 34 1 44 45 3	2 46 34 1 44 45 3	2 46 34 1 44 45 3	2 46 34 1 44 45 3
42 11 13 36 20 12 41	41 11 17 37 31 12 26	40 11 18 37 33 12 24	39 10 19 37 35 12 23
10 27 35 18 26 31 28	8 18 35 29 22 23 40	8 23 35 19 21 28 41	8 24 36 21 17 28 41
43 29 33 25 17 21 7	43 30 36 25 14 20 7	43 30 36 25 14 20 7	43 30 32 25 18 20 7
22 19 24 32 15 23 40	10 27 28 21 15 32 42	9 22 29 31 15 27 42	9 22 33 29 14 26 42
9 38 30 14 37 39 8	24 38 19 13 33 39 9	26 38 17 13 32 39 10	27 38 15 13 31 40 11
47 5 6 49 16 4 48	47 5 6 49 16 4 48	47 5 6 49 16 4 48	47 5 6 49 16 4 48
. . . . .			

**\*\* Self-Complementary Magic Squares 7x7: Fundamental Solutions with /D7i \*\***

1/ /D7i	7994/ /D7i
1 48 3 46 5 44 28 0606063 0523416	1 48 3 46 5 44 28 0606063 0523416
43 11 26 37 36 12 10 6135511 0341042	43 12 26 37 36 11 10 6135511 0441032
8 16 35 32 31 23 30 1244434 0163211	8 17 31 34 23 32 30 1244344 0225131
41 29 33 25 17 21 9 5443221 5043261	41 35 29 25 21 15 9 5443221 5603601
20 27 19 18 15 34 42 2322245 5543056	20 18 27 16 19 33 42 2232245 5351446
40 38 14 13 24 39 7 5511350 4265236	40 39 14 13 24 38 7 5511350 4365226
22 6 45 4 47 2 49 3060606 0523416	22 6 45 4 47 2 49 3060606 0523416
17069/ /D7i	25852/ /D7i
1 48 3 46 5 44 28 0606063 0523416	1 48 3 46 5 44 28 0606063 0523416
43 13 24 38 36 11 10 6135511 0522032	43 14 24 38 35 11 10 6135411 0622632
8 18 33 31 21 34 30 1244244 0342651	8 16 37 31 32 21 30 1254424 0112361
41 35 27 25 23 15 9 5433321 5653101	41 23 33 25 17 27 9 5343231 5143251
20 16 29 19 17 32 42 2242245 5104236	20 29 18 19 13 34 42 2422145 5034556
40 39 14 12 26 37 7 5511350 4364416	40 39 15 12 26 36 7 5521350 4304406
22 6 45 4 47 2 49 3060606 0523416	22 6 45 4 47 2 49 3060606 0523416
34027/ /D7i	42190/ /D7i
1 48 3 46 5 44 28 0606063 0523416	1 48 3 46 5 44 28 0606063 0523416
43 15 21 38 37 11 10 6225511 0062132	43 16 21 38 36 11 10 6225511 0162032
8 32 33 27 14 31 30 1443144 0345621	8 15 37 27 26 32 30 1253344 0015431
41 16 24 25 26 34 9 5233341 5123451	41 33 31 25 19 17 9 5443221 5423421
20 19 36 23 17 18 42 2253225 5401236	20 18 24 23 13 35 42 2233145 5321566
40 39 13 12 29 35 7 5511440 4354066	40 39 14 12 29 34 7 5511440 4364056
22 6 45 4 47 2 49 3060606 0523416	22 6 45 4 47 2 49 3060606 0523416
51192/ /D7i	59136/ /D7i
1 48 3 46 5 44 28 0606063 0523416	1 48 3 46 5 44 28 0606063 0523416
43 17 19 38 37 11 10 6225511 0242132	43 18 19 38 36 11 10 6225511 0342032
8 24 34 35 21 23 30 1344234 0256611	8 16 37 35 26 23 30 1254334 0116411
41 14 32 25 18 36 9 5143251 5633301	41 21 33 25 17 29 9 5243241 5643201
20 27 29 15 16 26 42 2342235 5500146	20 27 24 15 13 34 42 2332145 5520556
40 39 13 12 31 33 7 5511440 4354246	40 39 14 12 31 32 7 5511440 4364236
22 6 45 4 47 2 49 3060606 0523416	22 6 45 4 47 2 49 3060606 0523416
.....	

## Section 2: Pan-diagonal Magic Squares of Order 7

### #1. Definitions of 'Extended Space' and the Pan-diagonals

The next type we study about is so called "Pan-diagonal" Magic Squares of Order 7.

#### \*\* Basic Form & List of Basic Equations \*\*

```

47 48 49 43 44 45 46 47 48 49 43 44 45 46
 5  6  7 | 1| 2| 3| 4| 5| 6| 7 | 1  2  3  4
 12 13 14 | 8| 9|10|11|12|13|14 | 8  9 10 11
 19 20 21 |15|16|17|18|19|20|21 |15 16 17 18
 26 27 28 |22|23|24|25|26|27|28 |22 23 24 25
 33 34 35 |29|30|31|32|33|34|35 |29 30 31 32
 40 41 42 |36|37|38|39|40|41|42 |36 37 38 39
 47 48 49 |43|44|45|46|47|48|49 |43 44 45 46
 5  6  7  1  2  3  4  5  6  7  1  2  3  4
                [Extended Space]

```

#### \*\* Basic Conditions: C=175 \*\*

```

n1+n2+n3+n4+n5+n6+n7=C      ... rw1; | n1+n8+n15+n22+n29+n36+n43=C      ... c1;
n8+n9+n10+n11+n12+n13+n14=C ... rw2; | n2+n9+n16+n23+n30+n37+n44=C      ... c2;
n15+n16+n17+n18+n19+n20+n21=C ... rw3; | n3+n10+n17+n24+n31+n38+n45=C      ... c3;
n22+n23+n24+n25+n26+n27+n28=C ... rw4; | n4+n11+n18+n25+n32+n39+n46=C      ... c4;
n29+n30+n31+n32+n33+n34+n35=C ... rw5; | n5+n12+n19+n26+n33+n40+n47=C      ... c5;
n36+n37+n38+n39+n40+n41+n42=C ... rw6; | n6+n13+n20+n27+n34+n41+n48=C      ... c6;
n43+n44+n45+n46+n47+n48+n49=C ... rw7; | n7+n14+n21+n28+n35+n42+n49=C      ... c7;

```

#### \*\* Conditions for the Pan-diagonals: C=175 \*\*

```

n1+n9+n17+n25+n33+n41+n49=C      ... pd1; | n1+n14+n20+n26+n32+n38+n44=C      ... pb1;
n2+n10+n18+n26+n34+n42+n43=C      ... pd2; | n2+n8+n21+n27+n33+n39+n45=C      ... pb2;
n3+n11+n19+n27+n35+n36+n44=C      ... pd3; | n3+n9+n15+n28+n34+n40+n46=C      ... pb3;
n4+n12+n20+n28+n29+n37+n45=C      ... pd4; | n4+n10+n16+n22+n35+n41+n47=C      ... pb4;
n5+n13+n21+n22+n30+n38+n46=C      ... pd5; | n5+n11+n17+n23+n29+n42+n48=C      ... pb5;
n6+n14+n15+n23+n31+n39+n47=C      ... pd6; | n6+n12+n18+n24+n30+n36+n49=C      ... pb6;
n7+n8+n16+n24+n32+n40+n48=C      ... pd7; | n7+n13+n19+n25+n31+n37+n43=C      ... pb7;

```

The Basic Form for 'Extended Space' is invented to make you easily find all pan-diagonals in straight lines in place of those notorious "broken wrapping" lines.

When you deal with 49 numbers, you only have 14+14 equations in all. You will surely have comparatively fewer solutions this time, although you will really take more time to calculate and find the correct solutions. Even if you use your super personal computer, you will surely have to wait and wait for a long time.

### #2. Algebraic Study of this Type

We must study the structure of this type in advance by some 'Algebraic' calculations. We must know the secret keys to improve our clever calculation.

How about looking for the various combinations of 6 numbers which should add up to the same sums?

[Sums of 6 Numbers Combined into a Triangle on the Corners]

$$\begin{array}{l}
 n1 \ n2 \ n3 \ n4 \ n5 \ n6 \ n7 \\
 n8 \ n9 \ 10 \ 11 \ 12 \ 13 \ 14 \\
 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \\
 22 \ 23 \ 24 \ 25 \ 26 \ 27 \ 28 \\
 29 \ 30 \ 31 \ 32 \ 33 \ 34 \ 35 \\
 36 \ 37 \ 38 \ 39 \ 40 \ 41 \ 42 \\
 43 \ 44 \ 45 \ 46 \ 47 \ 48 \ 49
 \end{array}
 \quad
 \begin{array}{l}
 n1+ n2+ n3+ n8+ n9+n15 = P \dots(1) \\
 n5+ n6+ n7+n13+n14+n21 = Q \dots(2) \\
 n29+n36+n37+n43+n44+n45 = R \dots(3) \\
 n35+n41+n42+n47+n48+n49 = S \dots(4)
 \end{array}$$

$P = S; Q = R;$  are expected.

rw1+rw2+pb3+pd5

$$\begin{array}{l}
 n1+ n2+ n3+ n4+ n5+ n6+ n7 = K \dots rw1 \\
 n8+ n9+n10+n11+n12+n13+n14 = K \dots rw2 \\
 n3+ n9+n15+n28+n34+n40+n46 = K \dots pb3 \\
 +) n5+n13+n21+n22+n30+n38+n46 = K \dots pd5
 \end{array}$$

$$\begin{array}{l}
 n1+n2+n3+n3+n4+n5+n5+n6+n7+n8+n9+n9+n10+n11+n12+n13+n13+n14 \\
 +n15+n21+n22+n28+n30+n34+n38+n40+n46+n46 = 4K \\
 (n1+n2+n3+n8+n9+n15)+(n5+n6+n7+n13+n14+n21)+(n4+n22+n28+n46)+ \\
 (n3+n5+n9+n10+n11+n12+n13+n30+n34+n38+n40+n46) = 4K \\
 \text{Put } (n4+n22+n28+n46) = W \\
 P+Q+W+(n3+n5+n9+n10+n11+n12+n13+n30+n34+n38+n40+n46) = 4K \dots(30)
 \end{array}$$

c16+c17+pb4+pd5

$$\begin{array}{l}
 n6+n13+n20+n27+n34+n41+n48 = K \dots c16 \\
 n7+n14+n21+n28+n35+n42+n49 = K \dots c17 \\
 n4+n10+n16+n22+n35+n41+n47 = K \dots pb4 \\
 +) n5+n13+n21+n22+n30+n38+n46 = K \dots pd5
 \end{array}$$

$$\begin{array}{l}
 n4+n5+n6+n7+n10+n13+n13+n14+n16+n20+n21+n21+n22+n22+n27+n28 \\
 +n30+n34+n35+n35+n38+n41+n41+n42+n46+n47+n48+n49 = 4K \\
 (n5+n6+n7+n13+n14+n21)+(n35+n41+n42+n47+n48+n49)+(n4+n22+n28+n46)+ \\
 (n10+n13+n16+n20+n21+n22+n27+n30+n34+n35+n38+n41) = 4K \\
 Q+S+W+(n10+n13+n16+n20+n21+n22+n27+n30+n34+n35+n38+n41) = 4K \dots(31)
 \end{array}$$

(30)-(31)

$$\begin{array}{l}
 P-S = (n16+n20+n21+n22+n27+n35+n41)-(n3+n5+n9+n11+n12+n40+n46) \\
 P+(n3+n5+n9+n11+n12+n40+n46) = S+(n16+n20+n21+n22+n27+n35+n41) \dots(32)
 \end{array}$$

This conclusion means that if you want  $P = S$ , then you must also accept  $(n3+n5+n9+n11+n12+n40+n46) = (n16+n20+n21+n22+n27+n35+n41)$

In the same way

$$\begin{array}{l}
 n1+n8+n15+n22+n29+n36+n43 = K \dots c11 \\
 n2+n9+n16+n23+n30+n37+n44 = K \dots c12 \\
 n3+n9+n15+n28+n34+n40+n46 = K \dots pb3 \\
 +) n4+n12+n20+n28+n29+n37+n45 = K \dots pd4
 \end{array}$$

$$\begin{array}{l}
 (n1+n2+n3+n8+n9+n15)+(n29+n36+n37+n43+n44+n45)+(n4+n22+n28+n46)+ \\
 (n9+n12+n15+n16+n20+n23+n28+n29+n30+n34+n37+n40) = 4K \\
 P+R+W+(n9+n12+n15+n16+n20+n23+n28+n29+n30+n34+n37+n40) = 4K \dots(33)
 \end{array}$$

$$\begin{array}{l}
 n36+n37+n38+n39+n40+n41+n42 = K \dots rw6 \\
 n43+n44+n45+n46+n47+n48+n49 = K \dots rw7 \\
 n4+n12+n20+n28+n29+n37+n45 = K \dots pd4 \\
 +) n4+n10+n16+n22+n35+n41+n47 = K \dots pb4
 \end{array}$$

$$\begin{array}{l}
 (n29+n36+n37+n43+n44+n45)+(n35+n41+n42+n47+n48+n49)+(n4+n22+n28+n46)+ \\
 (n4+n10+n12+n16+n20+n37+n38+n39+n40+n41+n45+n47) = 4K \\
 R+S+W+(n4+n10+n12+n16+n20+n37+n38+n39+n40+n41+n45+n47) = 4K \dots(34)
 \end{array}$$

(33)-(34)

$$P+(n9+n15+n23+n28+n29+n30+n34) = S+(n4+n10+n38+n39+n41+n45+n47) \dots(35)$$

(32)+(35)

$$P+n3+n5+n9+n11+n12+n40+n46 = S+n16+n20+n21+n22+n27+n35+n41 \dots (32)$$

$$+ ) P+n9+n15+n23+n28+n29+n30+n34 = S+n4+n10+n38+n39+n41+n45+n47 \dots (35)$$

---

$$2P+(n3+n9+n15+n28+n34+n40+n46)+(n5+n9+n11+n12+n23+n29+n30) \\ = 2S+(n4+n10+n16+n22+n35+n41+n47)+(n20+n21+n27+n38+n39+n41+n45)$$

But  $n3+n9+n15+n28+n34+n40+n46 = K \dots (20)$

$$n4+n10+n16+n22+n35+n41+n47 = K \dots (22)$$

$$2P+K+(n5+n9+n11+n12+n23+n29+n30) = 2S+K+(n20+n21+n27+n38+n39+n41+n45)$$

Therefore

$$2P+(n5+n9+n11+n12+n23+n29+n30) = 2S+(n20+n21+n27+n38+n39+n41+n45) \dots (36)$$

If you want  $P = S$ , then you must accept

$$(n5+n9+n11+n12+n23+n29+n30) = (n20+n21+n27+n38+n39+n41+n45)$$

Add  $(n2+n8+n17+n33+n42+n48)$  to both sides of (36) and calculate.

$$2P+\{(n5+n11+n17+n23+n29+n42+n48)+(n2+n8+n9+n12+n30+n33)\} \\ = 2S+\{(n2+n8+n21+n27+n33+n39+n45)+(n17+n20+n38+n41+n42+n48)\}$$

But  $n5+n11+n17+n23+n29+n42+n48 = K \dots (24)$

$$n2+n8+n21+n27+n33+n39+n45 = K \dots (18)$$

$$2P+K+(n2+n8+n9+n12+n30+n33) = 2S+K+(n17+n20+n38+n41+n42+n48)$$

Therefore

$$2P+(n2+n8+n9+n12+n30+n33) = 2S+(n17+n20+n38+n41+n42+n48) \dots (37)$$

This conclusion is a little bit shorter than (36)

Add  $(n1+n3+n15+n35+n47+n49)$  to both sides.

$$2P+(n1+n2+n3+n8+n9+n15)+(n12+n30+n33+n35+n47+n49) \\ = 2S+(n35+n41+n42+n47+n48+n49)+(n1+n3+n15+n17+n20+n38) \\ 3P+(n12+n30+n33+n35+n47+n49) = 3S+(n1+n3+n15+n17+n20+n38) \dots (38)$$

- $n1+ n2+ n3+ n8+ n9+n15 = n35+n41+n42+n47+n48+n49 \dots (T6.1)$
- $n5+ n6+ n7+n13+n14+n21 = n29+n36+n37+n43+n44+n45 \dots (T6.2)$
- $n4+n10+n11+n16+n17+n18 = n26+n27+n28+n33+n34+n40 \dots (T6.3)$
- $n4+n11+n12+n18+n19+n20 = n22+n23+n24+n30+n31+n38 \dots (T6.4)$
- $n2+ n8+ n9+n12+n30+n33 = n17+n20+n38+n41+n42+n48 \dots (T6.5)$
- $n6+n10+n13+n14+n31+n34 = n16+n19+n36+n37+n40+n44 \dots (T6.6)$
- $n1+ n3+n15+n17+n20+n38 = n12+n30+n33+n35+n47+n49 \dots (T6.7)$
- $n5+ n7+n16+n19+n21+n40 = n10+n29+n31+n34+n43+n45 \dots (T6.8)$

### [6 Entries Combined into a Triangle on the Corner]

$n1$	$n2$	$n3$	$n4$	$n5$	$n6$	$n7$	$n1+ n2+ n3+ n8+ n9+n15 = P \dots (1)$
$n8$	$n9$	10	11	12	13	14	$n5+ n6+ n7+n13+n14+n21 = Q \dots (2)$
15	16	17	18	19	20	21	$n29+n36+n37+n43+n44+n45 = R \dots (3)$
22	23	24	25	26	27	28	$n35+n41+n42+n47+n48+n49 = S \dots (4)$
29	30	31	32	33	34	35	
36	37	38	39	40	41	42	If you want $P = S$ , then you must
43	44	45	46	47	48	49	accept $Q = R$ , (T6.5), and (T6.6).

### [Other 6 Entries Combined]

$n1$	$n2$	$n3$	$n4$	$n5$	$n6$	$n7$	$n1$	$n2$	$n3$	$n4$	$n5$	$n6$	$n7$
$n8$	$n9$	10	11	12	13	14	$n8$	$n9$	10	11	12	13	14
15	16	17	18	19	20	21	15	16	17	18	19	20	21
22	23	24	25	26	27	28	22	23	24	25	26	27	28
29	30	31	32	33	34	35	29	30	31	32	33	34	35
36	37	38	39	40	41	42	36	37	38	39	40	41	42
43	44	45	46	47	48	49	43	44	45	46	47	48	49

$$n2+n8+n9+n12+n30+n33 = n1+n3+n15+n17+n20+n38 = \\ n17+n20+n38+n41+n42+n48 \quad n12+n30+n33+n35+n47+n49$$

Why don't you take these conclusions into your program coding positively and



replace some independent variables for some subordinate ones?

But it is still such a hard work for you to calculate solutions of order 7 that you should know it is practically impossible to count them up to the last. When I once tried to do it, I had to wait and wait for a long time every time. I got almost sick.

Let me show you some sample sets of fundamental solutions of Pan-diagonal Magic Squares 7x7 with the Decomposition layers by the number system of 7-th increment.

### #3. Sample Sets of the Fundamental Solutions of Pan-diagonal MS77

\* Pan-Diagonal Magic Squares of Order 7: Fundamental Solutions: Set #1 \*

														1/ /D7i																2/ /D7i																																																																																															
1	48	39	30	28	19	10	0654321	0531642	1	48	39	23	35	19	10	0653421	0531642	18	9	7	47	38	29	27	2106543	3164205	18	9	7	47	38	22	34	2106534	3164205	35	26	17	8	6	46	37	4321065	6420531	28	33	17	8	6	46	37	3421065	6420531	45	36	34	25	16	14	5	6543210	2053164	45	36	27	32	16	14	5	6534210	2053164	13	4	44	42	33	24	15	1065432	5316420	13	4	44	42	26	31	15	1065342	5316420	23	21	12	3	43	41	32	3210654	1642053	30	21	12	3	43	41	25	4210653	1642053	40	31	22	20	11	2	49	5432106	4205316	40	24	29	20	11	2	49	5342106	4205316
														3/ /D7i																4/ /D7i																																																																																															
1	48	39	30	21	26	10	0654231	0531642	1	48	39	16	35	26	10	0652431	0531642	25	9	7	47	38	29	20	3106542	3164205	25	9	7	47	38	15	34	3106524	3164205	35	19	24	8	6	46	37	4231065	6420531	21	33	24	8	6	46	37	2431065	6420531	45	36	34	18	23	14	5	6542310	2053164	45	36	20	32	23	14	5	6524310	2053164	13	4	44	42	33	17	22	1065423	5316420	13	4	44	42	19	31	22	1065243	5316420	16	28	12	3	43	41	32	2310654	1642053	30	28	12	3	43	41	18	4310652	1642053	40	31	15	27	11	2	49	5423106	4205316	40	17	29	27	11	2	49	5243106	4205316
														5/ /D7i																6/ /D7i																																																																																															
1	48	39	23	21	33	10	0653241	0531642	1	48	39	16	28	33	10	0652341	0531642	32	9	7	47	38	22	20	4106532	3164205	32	9	7	47	38	15	27	4106523	3164205	28	19	31	8	6	46	37	3241065	6420531	21	26	31	8	6	46	37	2341065	6420531	45	36	27	18	30	14	5	6532410	2053164	45	36	20	25	30	14	5	6523410	2053164	13	4	44	42	26	17	29	1065324	5316420	13	4	44	42	19	24	29	1065234	5316420	16	35	12	3	43	41	25	2410653	1642053	23	35	12	3	43	41	18	3410652	1642053	40	24	15	34	11	2	49	5324106	4205316	40	17	22	34	11	2	49	5234106	4205316
. . . . .																																																																																																																													

\* Pan-Diagonal Magic Squares of Order 7: Fundamental Solutions: Set #2 \*

														1/ /D7i																2/ /D7i																																																																																															
1	45	40	35	23	18	13	0654321	0246135	1	45	40	28	30	18	13	0653421	0246135	21	9	4	48	36	31	26	2106543	6135024	21	9	4	48	36	24	33	2106534	6135024	34	22	17	12	7	44	39	4321065	5024613	27	29	17	12	7	44	39	3421065	5024613	47	42	30	25	20	8	3	6543210	4613502	47	42	23	32	20	8	3	6534210	4613502	11	6	43	38	33	28	16	1065432	3502461	11	6	43	38	26	35	16	1065342	3502461	24	19	14	2	46	41	29	3210654	2461350	31	19	14	2	46	41	22	4210653	2461350	37	32	27	15	10	5	49	5432106	1350246	37	25	34	15	10	5	49	5342106	1350246
														3/ /D7i																4/ /D7i																																																																																															
1	45	40	35	16	25	13	0654231	0246135	1	45	40	21	30	25	13	0652431	0246135	28	9	4	48	36	31	19	3106542	6135024	28	9	4	48	36	17	33	3106524	6135024	34	15	24	12	7	44	39	4231065	5024613	20	29	24	12	7	44	39	2431065	5024613	47	42	30	18	27	8	3	6542310	4613502	47	42	16	32	27	8	3	6524310	4613502	11	6	43	38	33	21	23	1065423	3502461	11	6	43	38	19	35	23	1065243	3502461	17	26	14	2	46	41	29	2310654	2461350	31	26	14	2	46	41	15	4310652	2461350	37	32	20	22	10	5	49	5423106	1350246	37	18	34	22	10	5	49	5243106	1350246



														5/ /D7i																6/ /D7i	
1	45	40	28	16	32	13	0653241	0246135	1	45	40	21	23	32	13	0652341	0246135														
35	9	4	48	36	24	19	4106532	6135024	35	9	4	48	36	17	26	4106523	6135024														
27	15	31	12	7	44	39	3241065	5024613	20	22	31	12	7	44	39	2341065	5024613														
47	42	23	18	34	8	3	6532410	4613502	47	42	16	25	34	8	3	6523410	4613502														
11	6	43	38	26	21	30	1065324	3502461	11	6	43	38	19	28	30	1065234	3502461														
17	33	14	2	46	41	22	2410653	2461350	24	33	14	2	46	41	15	3410652	2461350														
37	25	20	29	10	5	49	5324106	1350246	37	18	27	29	10	5	49	5234106	1350246														

.....

\* Pan-Diagonal Magic Squares of Order 7: Fundamental Solutions: Set #3 \*

														1/ /D7i																2/ /D7i	
1	49	41	33	25	17	9	0654321	0654321	1	49	41	26	32	17	9	0653421	0654321														
40	32	24	16	8	7	48	5432106	4321065	40	25	31	16	8	7	48	5342106	4321065														
23	15	14	6	47	39	31	3210654	1065432	30	15	14	6	47	39	24	4210653	1065432														
13	5	46	38	30	22	21	1065432	5432106	13	5	46	38	23	29	21	1065342	5432106														
45	37	29	28	20	12	4	6543210	2106543	45	37	22	35	20	12	4	6534210	2106543														
35	27	19	11	3	44	36	4321065	6543210	28	34	19	11	3	44	36	3421065	6543210														
18	10	2	43	42	34	26	2106543	3210654	18	10	2	43	42	27	33	2106534	3210654														

  

														3/ /D7i																4/ /D7i	
1	49	41	33	18	24	9	0654231	0654321	1	49	41	19	32	24	9	0652431	0654321														
40	32	17	23	8	7	48	5423106	4321065	40	18	31	23	8	7	48	5243106	4321065														
16	22	14	6	47	39	31	2310654	1065432	30	22	14	6	47	39	17	4310652	1065432														
13	5	46	38	30	15	28	1065423	5432106	13	5	46	38	16	29	28	1065243	5432106														
45	37	29	21	27	12	4	6542310	2106543	45	37	15	35	27	12	4	6524310	2106543														
35	20	26	11	3	44	36	4231065	6543210	21	34	26	11	3	44	36	2431065	6543210														
25	10	2	43	42	34	19	3106542	3210654	25	10	2	43	42	20	33	3106524	3210654														

  

														5/ /D7i																6/ /D7i	
1	49	41	26	18	31	9	0653241	0654321	1	49	41	19	25	31	9	0652341	0654321														
40	25	17	30	8	7	48	5324106	4321065	40	18	24	30	8	7	48	5234106	4321065														
16	29	14	6	47	39	24	2410653	1065432	23	29	14	6	47	39	17	3410652	1065432														
13	5	46	38	23	15	35	1065324	5432106	13	5	46	38	16	22	35	1065234	5432106														
45	37	22	21	34	12	4	6532410	2106543	45	37	15	28	34	12	4	6523410	2106543														
28	20	33	11	3	44	36	3241065	6543210	21	27	33	11	3	44	36	2341065	6543210														
32	10	2	43	42	27	19	4106532	3210654	32	10	2	43	42	20	26	4106523	3210654														

.....

## Section 3: Simultaneous Magic Squares of Order 7: Both Self-Complementary and Pan-Diagonal

### #1. Definition of Simultaneous Magic Squares 7x7: Both S-C and P-D Type

The next goal we are trying to build is the Simultaneous type of magic squares of order 7: both Self-complementary and Pan-diagonal.

All the conditions we mentioned above must be used here and be made active.

#### \*\* Basic Form & List of Basic Equations \*\*

47	48	49	43	44	45	46	47	48	49	43	44	45	46
5	6	7	1	2	3	4	5	6	7	1	2	3	4
12	13	14	8	9	10	11	12	13	14	8	9	10	11
19	20	21	15	16	17	18	19	20	21	15	16	17	18
26	27	28	22	23	24	25	26	27	28	22	23	24	25
33	34	35	29	30	31	32	33	34	35	29	30	31	32
40	41	42	36	37	38	39	40	41	42	36	37	38	39
47	48	49	43	44	45	46	47	48	49	43	44	45	46
5	6	7	1	2	3	4	5	6	7	1	2	3	4

#### \*\* Basic Conditions for the Rows and Columns: C=175 \*\*

$n1+n2+n3+n4+n5+n6+n7=C$	... rw1;	$n1+n8+n15+n22+n29+n36+n43=C$	... c11;
$n8+n9+n10+n11+n12+n13+n14=C$	... rw2;	$n2+n9+n16+n23+n30+n37+n44=C$	... c12;
$n15+n16+n17+n18+n19+n20+n21=C$	... rw3;	$n3+n10+n17+n24+n31+n38+n45=C$	... c13;
$n22+n23+n24+n25+n26+n27+n28=C$	... rw4;	$n4+n11+n18+n25+n32+n39+n46=C$	... c14;
$n29+n30+n31+n32+n33+n34+n35=C$	... rw5;	$n5+n12+n19+n26+n33+n40+n47=C$	... c15;
$n36+n37+n38+n39+n40+n41+n42=C$	... rw6;	$n6+n13+n20+n27+n34+n41+n48=C$	... c16;
$n43+n44+n45+n46+n47+n48+n49=C$	... rw7;	$n7+n14+n21+n28+n35+n42+n49=C$	... c17;

#### \*\* Conditions for the Pan-diagonals: C=175 \*\*

$n1+n9+n17+n25+n33+n41+n49=C$	... pd1;	$n1+n14+n20+n26+n32+n38+n44=C$	... pb1;
$n2+n10+n18+n26+n34+n42+n43=C$	... pd2;	$n2+n8+n21+n27+n33+n39+n45=C$	... pb2;
$n3+n11+n19+n27+n35+n36+n44=C$	... pd3;	$n3+n9+n15+n28+n34+n40+n46=C$	... pb3;
$n4+n12+n20+n28+n29+n37+n45=C$	... pd4;	$n4+n10+n16+n22+n35+n41+n47=C$	... pb4;
$n5+n13+n21+n22+n30+n38+n46=C$	... pd5;	$n5+n11+n17+n23+n29+n42+n48=C$	... pb5;
$n6+n14+n15+n23+n31+n39+n47=C$	... pd6;	$n6+n12+n18+n24+n30+n36+n49=C$	... pb6;
$n7+n8+n16+n24+n32+n40+n48=C$	... pd7;	$n7+n13+n19+n25+n31+n37+n43=C$	... pb7;

#### \*\* Conditions for the Symmetric Complementary Pairs: CC=50 \*\*

$n1+n49=n2+n48=n3+n47=n4+n46=n5+n45=n6+n44=n7+n43=$
$n8+n42=n9+n41=n10+n40=n11+n39=n12+n38=n13+n37=n14+n36=$
$n15+n35=n16+n34=n17+n33=n18+n32=n19+n31=n20+n30=n21+n29=$
$n22+n28=n23+n27=n24+n26=n25+n25=CC \dots scc$

The last set of equations appears again to define Complementary Pairs of CC. Place all those pairs symmetrically with respect to n25(Geometric center of our object).

This makes our object a Self-complementary type as well as a Pan-diagonal one.

### #2. Powerful Combinations of Those Properties

For the Simultaneous type you can combine the properties of Pan-diagonal with those of Self-complementary. For instance, various sums of 6 numbers combined

could be more clearly defined like this:

[6 Entries Combined in a Triangle on each Corner]

n1	n2	n3	n4	n5	n6	n7	n1+	n2+	n3+	n8+	n9+n15	= P ... (1)
n8	n9	10	11	12	13	14	n5+	n6+	n7+n13+n14+n21	= Q ... (2)		
15	16	17	18	19	20	21	n29+n36+n37+n43+n44+n45	= R ... (3)				
22	23	24	25	26	27	28	n35+n41+n42+n47+n48+n49	= S ... (4)				
29	30	31	32	33	34	35						
36	37	38	39	40	41	42						
43	44	45	46	47	48	49						

If you want  $P = S$ , then you must accept  $Q = R$ , (T6.5), and (T6.6).

$n1+n49=50$ ;  $n2+n48=50$ ;  $n3+n47=50$ ;  $n8+n42=50$ ;  $n9+n41=50$ ;  $n15+n35=50$ ;  
 $P+S=(n1+n49)+(n2+n48)+(n3+n47)+(n8+n42)+(n9+n41)+(n15+n35)= 50 \times 6 = 300$ ;  
 If  $P=S$ , then  $P=S=300/2=150$

$n1+ n2+ n3+ n8+ n9+n15 = n35+n41+n42+n47+n48+n49 = 150$ ;  
 In the same way  $Q=R=150$   
 $n5+ n6+ n7+n13+n14+n21 = n29+n36+n37+n43+n44+n45 = 150$ ;  
 $n2+ n8+ n9+n12+n30+n33 = n17+n20+n38+n41+n42+n48 = 150$ ;  
 $n1+ n3+n15+n17+n20+n38 = n12+n30+n33+n35+n47+n49 = 150$ ;

You may well take  $(P=S)\&(Q=R)$  positively into your program coding. These new conditions will help you a lot. Because they are the guide-lines we have expected. And that would neither spoil your object, nor lose any solutions you should have found.

That would also save your time to calculate and count them up far. This may be the best relief to you.

**\*\* Simultaneous Magic Squares 7x7: Both Self-Complementary and Pan-Diagonal \*\***

[Compact List of Standard Solutions: with Check-sum Errors, normalized by  $n1<n49$ ,  $n1<n7<n43$ ]

1# R C\P/p							135# R C\P/p							314# R C\P/p						
1	48	10	46	33	14	23 0 0\0/0	1	48	8	41	36	19	22 0 0\0/0	1	48	9	35	39	24	19 0 0\0/0
47	5	12	43	6	28	34 0 0\0/0	47	6	20	39	7	27	29 0 0\0/0	47	7	29	46	8	16	22 0 0\0/0
39	8	41	24	30	15	18 0 0\0/0	40	13	46	15	32	12	17 0 0\0/0	38	5	33	6	36	27	30 0 0\0/0
13	21	31	25	19	29	37 0 0\0/0	5	16	26	25	24	34	45 0 0\0/0	10	32	37	25	13	18	40 0 0\0/0
32	35	20	26	9	42	11 0 0\0/0	33	38	18	35	4	37	10 0 0\0/0	20	23	14	44	17	45	12 0 0\0/0
16	22	44	7	38	45	3 0 0\0/0	21	23	43	11	30	44	3 0 0\0/0	28	34	42	4	21	43	3 0 0\0/0
27	36	17	4	40	2	49 0 0\0/0	28	31	14	9	42	2	49 0 0\0/0	31	26	11	15	41	2	49 0 0\0/0
441# R C\P/p							553# R C\P/p							666# R C\P/p						
1	48	10	41	46	12	17 0 0\0/0	1	48	10	36	46	18	16 0 0\0/0	1	48	5	37	44	16	24 0 0\0/0
47	8	13	45	7	27	28 0 0\0/0	47	9	31	33	11	21	23 0 0\0/0	47	10	23	36	8	21	30 0 0\0/0
36	19	34	29	11	26	20 0 0\0/0	35	5	37	6	38	28	26 0 0\0/0	39	12	46	18	28	17	15 0 0\0/0
6	15	32	25	18	35	44 0 0\0/0	7	30	42	25	8	20	43 0 0\0/0	7	9	31	25	19	41	43 0 0\0/0
30	24	39	21	16	31	14 0 0\0/0	24	22	12	44	13	45	15 0 0\0/0	35	33	22	32	4	38	11 0 0\0/0
22	23	43	5	37	42	3 0 0\0/0	27	29	39	17	19	41	3 0 0\0/0	20	29	42	14	27	40	3 0 0\0/0
33	38	4	9	40	2	49 0 0\0/0	34	32	4	14	40	2	49 0 0\0/0	26	34	6	13	45	2	49 0 0\0/0
788# R C\P/p							895# R C\P/p							1001# R C\P/p						
1	48	7	41	37	17	24 0 0\0/0	1	48	7	44	45	16	14 0 0\0/0	1	48	5	43	44	16	18 0 0\0/0
47	11	12	46	10	31	18 0 0\0/0	47	12	18	40	13	22	23 0 0\0/0	47	13	17	42	4	21	31 0 0\0/0
36	20	45	22	8	21	23 0 0\0/0	35	8	46	11	21	24	30 0 0\0/0	36	9	39	22	26	23	20 0 0\0/0
6	15	16	25	34	35	44 0 0\0/0	9	19	33	25	17	31	41 0 0\0/0	10	15	38	25	12	35	40 0 0\0/0
27	29	42	28	5	30	14 0 0\0/0	20	26	29	39	4	42	15 0 0\0/0	30	27	24	28	11	41	14 0 0\0/0
32	19	40	4	38	39	3 0 0\0/0	27	28	37	10	32	38	3 0 0\0/0	19	29	46	8	33	37	3 0 0\0/0
26	33	13	9	43	2	49 0 0\0/0	36	34	5	6	43	2	49 0 0\0/0	32	34	6	7	45	2	49 0 0\0/0
1091# R C\P/p							1201# R C\P/p							1283# R C\P/p						
1	48	5	43	40	16	22 0 0\0/0	1	48	5	46	43	11	21 0 0\0/0	1	48	7	42	46	22	9 0 0\0/0
47	14	11	42	6	23	32 0 0\0/0	47	15	14	44	8	30	17 0 0\0/0	47	16	14	45	6	17	30 0 0\0/0
35	12	46	26	20	19	17 0 0\0/0	34	10	37	23	12	31	28 0 0\0/0	31	12	40	18	21	27	26 0 0\0/0
13	9	29	25	21	41	37 0 0\0/0	9	24	32	25	18	26	41 0 0\0/0	11	15	37	25	13	35	39 0 0\0/0
33	31	30	24	4	38	15 0 0\0/0	22	19	38	27	13	40	16 0 0\0/0	24	23	29	32	10	38	19 0 0\0/0
18	27	44	8	39	36	3 0 0\0/0	33	20	42	6	36	35	3 0 0\0/0	20	33	44	5	36	34	3 0 0\0/0
28	34	10	7	45	2	49 0 0\0/0	29	39	7	4	45	2	49 0 0\0/0	41	28	4	8	43	2	49 0 0\0/0

<p>1356#</p> <p>1 48 5 46 35 16 24 47 17 12 44 9 36 10 32 13 43 19 11 28 29 8 27 20 25 30 23 42 21 22 39 31 7 37 18 40 14 41 6 38 33 3 26 34 15 4 45 2 49</p> <p>1640#</p> <p>1 48 9 45 37 12 23 47 21 19 22 6 46 14 24 7 39 20 34 33 18 8 40 35 25 15 10 42 32 17 16 30 11 43 26 36 4 44 28 31 29 3 27 38 13 5 41 2 49</p> <p>1874#</p> <p>1 48 5 34 43 29 15 47 26 19 46 11 12 14 23 6 40 9 18 42 37 20 28 33 25 17 22 30 13 8 32 41 10 44 27 36 38 39 4 31 24 3 35 21 7 16 45 2 49</p> <p>2082#</p> <p>1 48 6 38 41 33 8 47 30 11 40 5 28 14 18 23 46 15 13 34 26 7 19 21 25 29 31 43 24 16 37 35 4 27 32 36 22 45 10 39 20 3 42 17 9 12 44 2 49</p> <p>2203#</p> <p>1 48 5 26 41 37 17 47 34 11 44 7 14 18 15 22 46 21 8 40 23 20 12 19 25 31 38 30 27 10 42 29 4 28 35 32 36 43 6 39 16 3 33 13 9 24 45 2 49</p> <p>2289#</p> <p>1 48 5 19 40 42 20 47 38 28 27 4 16 15 11 14 44 9 37 43 17 18 26 29 25 21 24 32 33 7 13 41 6 36 39 35 34 46 23 22 12 3 30 8 10 31 45 2 49</p> <p>2673#</p> <p>1 48 9 37 47 12 21 46 7 23 40 8 16 35 39 6 33 22 30 26 19 14 18 45 25 5 32 36 31 24 20 28 17 44 11 15 34 42 10 27 43 4 29 38 3 13 41 2 49</p> <p>3221#</p> <p>1 48 8 44 45 12 17 46 11 20 35 10 22 31 36 3 41 16 32 24 23 13 21 43 25 7 29 37 27 26 18 34 9 47 14 19 28 40 15 30 39 4 33 38 5 6 42 2 49</p>	<p>1429#</p> <p>1 48 5 43 46 8 24 47 18 12 41 6 28 23 31 10 34 33 11 35 21 14 20 37 25 13 30 36 29 15 39 17 16 40 19 27 22 44 9 38 32 3 26 42 4 7 45 2 49</p> <p>1688#</p> <p>1 48 6 46 45 18 11 47 22 17 36 8 29 16 26 7 40 13 23 35 31 9 30 38 25 12 20 41 19 15 27 37 10 43 24 34 21 42 14 33 28 3 39 32 5 4 44 2 49</p> <p>1930#</p> <p>1 48 5 20 43 34 24 47 27 38 29 4 17 13 22 10 35 6 42 41 19 11 32 36 25 14 18 39 31 9 8 44 15 40 28 37 33 46 21 12 23 3 26 16 7 30 45 2 49</p> <p>2112#</p> <p>1 48 5 15 42 40 24 47 31 20 41 4 11 21 18 28 43 23 14 37 12 16 6 17 25 33 44 34 38 13 36 27 7 22 32 29 39 46 9 30 19 3 26 10 8 35 45 2 49</p> <p>2229#</p> <p>1 48 5 16 39 42 24 47 35 20 44 7 10 12 14 21 41 17 13 46 23 22 19 18 25 32 31 28 27 4 37 33 9 29 36 38 40 43 6 30 15 3 26 8 11 34 45 2 49</p> <p>2292#</p> <p>1 48 6 29 33 40 18 47 39 16 36 5 19 13 9 8 43 12 30 46 27 26 35 28 25 22 15 24 23 4 20 38 7 42 41 37 31 45 14 34 11 3 32 10 17 21 44 2 49</p> <p>2848#</p> <p>1 48 11 26 47 23 19 46 8 29 41 6 7 38 36 10 30 18 37 28 16 15 17 45 25 5 33 35 34 22 13 32 20 40 14 12 43 44 9 21 42 4 31 27 3 24 39 2 49</p> <p>3344#</p> <p>1 48 8 23 41 30 24 46 12 37 31 5 16 28 35 14 40 7 47 21 11 6 18 33 25 17 32 44 39 29 3 43 10 36 15 22 34 45 19 13 38 4 26 20 9 27 42 2 49</p>	<p>1508#</p> <p>1 48 7 18 41 39 21 47 19 36 40 4 6 23 28 12 35 5 42 33 20 13 24 34 25 16 26 37 30 17 8 45 15 38 22 27 44 46 10 14 31 3 29 11 9 32 43 2 49</p> <p>1763#</p> <p>1 48 7 40 45 14 20 47 23 18 34 11 38 4 24 15 37 19 9 42 29 6 33 28 25 22 17 44 21 8 41 31 13 35 26 46 12 39 16 32 27 3 30 36 5 10 43 2 49</p> <p>1996#</p> <p>1 48 6 46 40 11 23 47 28 9 32 7 37 15 20 5 38 29 17 42 24 19 34 36 25 14 16 31 26 8 33 21 12 45 30 35 13 43 18 41 22 3 27 39 10 4 44 2 49</p> <p>2144#</p> <p>1 48 7 41 44 26 8 47 32 17 23 5 31 20 15 10 46 13 34 36 21 11 28 38 25 12 22 39 29 14 16 37 4 40 35 30 19 45 27 33 18 3 42 24 6 9 43 2 49</p> <p>2246#</p> <p>1 48 5 26 44 39 12 47 36 15 41 4 10 22 13 17 42 18 19 43 23 21 16 30 25 20 34 29 27 7 31 32 8 33 37 28 40 46 9 35 14 3 38 11 6 24 45 2 49</p> <p>2295#</p> <p>1 48 10 43 38 13 22 46 3 17 45 6 24 34 42 9 35 23 29 18 19 11 20 36 25 14 30 39 31 32 21 27 15 41 8 16 26 44 5 33 47 4 28 37 12 7 40 2 49</p>	<p>1566#</p> <p>1 48 7 41 45 22 11 47 20 18 40 15 21 14 27 8 46 6 17 34 37 12 26 31 25 19 24 38 13 16 33 44 4 42 23 36 29 35 10 32 30 3 39 28 5 9 43 2 49</p> <p>1815#</p> <p>1 48 7 37 45 17 20 47 24 8 35 9 14 38 23 6 46 31 21 32 16 28 10 39 25 11 40 22 34 18 29 19 4 44 27 12 36 41 15 42 26 3 30 33 5 13 43 2 49</p> <p>2048#</p> <p>1 48 7 34 46 22 17 47 29 10 31 6 14 38 18 8 45 30 26 35 13 27 11 41 25 9 39 23 37 15 24 20 5 42 32 12 36 44 19 40 21 3 33 28 4 16 43 2 49</p> <p>2173#</p> <p>1 48 6 34 43 31 12 47 33 14 41 5 27 8 15 20 37 18 10 46 29 11 28 26 25 24 22 39 21 4 40 32 13 30 35 42 23 45 9 36 17 3 38 19 7 16 44 2 49</p> <p>2273#</p> <p>1 48 7 19 44 38 18 47 37 28 24 9 14 16 10 11 45 8 35 46 20 21 27 33 25 17 23 29 30 4 15 42 5 39 40 34 36 41 26 22 13 3 32 12 6 31 43 2 49</p> <p>2548#</p> <p>1 48 8 30 43 23 22 46 6 26 47 5 10 35 41 13 34 18 31 21 17 11 12 36 25 14 38 39 33 29 19 32 16 37 9 15 40 45 3 24 44 4 28 27 7 20 42 2 49</p> <p>3129#</p> <p>1 48 12 34 47 14 19 46 10 28 41 6 29 15 33 13 18 20 23 42 26 5 39 43 25 7 11 45 24 8 27 30 32 37 17 35 21 44 9 22 40 4 31 36 3 16 38 2 49</p> <p>3454#</p> <p>1 48 6 45 34 19 22 46 13 17 47 12 29 11 36 8 43 9 18 26 35 10 30 23 25 27 20 40 15 24 32 41 7 42 14 39 21 38 3 33 37 4 28 31 16 5 44 2 49</p> <p>3541#</p> <p>1 48 6 42 45 11 22 46 14 12 47 13 20 23 35 9 40 24 7 31 29 17 16 32 25 18 34 33 21 19 43 26 10 41 15 27 30 37 3 38 36 4 28 39 5 8 44 2 49</p>
---	---	---	---

. . . . .

[Simple List 2 of Standard Solutions normalized by  $n1 < n49$ ,  $n1 < n7 < n43$ ]

1#	2295#	4902#	7116#
1 48 10 46 33 14 23	1 48 10 43 38 13 22	1 48 11 43 44 12 16	1 48 13 33 40 22 18
47 5 12 43 6 28 34	46 3 17 45 6 24 34	45 3 20 41 10 27 29	44 3 24 46 15 7 36
39 8 41 24 30 15 18	42 9 35 23 29 18 19	42 14 37 17 24 19 22	41 5 39 8 34 21 27
13 21 31 25 19 29 37	11 20 36 25 14 30 39	4 18 35 25 15 32 46	20 19 38 25 12 31 30
32 35 20 26 9 42 11	31 32 21 27 15 41 8	28 31 26 33 13 36 8	23 29 16 42 11 45 9
16 22 44 7 38 45 3	16 26 44 5 33 47 4	21 23 40 9 30 47 5	14 43 35 4 26 47 6
27 36 17 4 40 2 49	28 37 12 7 40 2 49	34 38 6 7 39 2 49	32 28 10 17 37 2 49
9577#	11836#	14186#	16452#
1 48 11 46 31 14 24	1 48 12 31 45 14 24	1 48 12 44 28 18 24	1 48 13 34 43 14 22
43 3 22 41 8 37 21	42 3 37 40 16 22 15	41 3 14 46 11 23 37	40 3 24 44 11 12 41
44 10 35 12 34 17 23	44 9 27 4 29 32 30	45 8 43 15 34 10 20	45 8 29 19 33 23 18
5 32 30 25 20 18 45	7 33 39 25 11 17 43	19 17 29 25 21 33 31	20 15 46 25 4 35 30
27 33 16 38 15 40 6	20 18 21 46 23 41 6	30 40 16 35 7 42 5	32 27 17 31 21 42 5
29 13 42 9 28 47 7	35 28 34 10 13 47 8	13 27 39 4 36 47 9	9 38 39 6 26 47 10
26 36 19 4 39 2 49	26 36 5 19 38 2 49	26 32 22 6 38 2 49	28 36 7 16 37 2 49
18788#	20853#	23086#	25100#
1 48 15 37 46 18 10	1 48 14 42 43 6 21	1 48 15 36 44 8 23	1 48 16 42 35 9 24
39 3 28 41 12 19 33	38 3 24 45 20 28 17	37 3 26 29 10 32 38	36 3 20 28 12 43 33
44 14 29 7 34 23 24	46 9 27 10 16 32 35	46 20 31 22 34 17 5	46 23 39 19 32 10 6
8 20 45 25 5 30 42	13 31 39 25 11 19 37	7 11 41 25 9 39 43	5 13 29 25 21 37 45
26 27 16 43 21 36 6	15 18 34 40 23 41 4	45 33 16 28 19 30 4	44 40 18 31 11 27 4
17 31 38 9 22 47 11	33 22 30 5 26 47 12	12 18 40 21 24 47 13	17 7 38 22 30 47 14
40 32 4 13 35 2 49	29 44 7 8 36 2 49	27 42 6 14 35 2 49	26 41 15 8 34 2 49
27252#	29261#	31340#	33324#
1 48 17 42 32 19 16	1 48 18 26 43 22 17	1 48 19 44 45 7 11	1 48 20 45 44 9 8
35 3 11 45 10 28 43	34 3 30 38 11 14 45	33 3 16 40 26 22 35	32 3 13 43 16 33 35
46 24 37 20 27 9 12	46 21 27 13 40 19 9	46 12 37 9 18 23 30	46 26 27 19 14 22 21
14 6 29 25 21 44 36	15 8 44 25 6 42 35	21 14 42 25 8 36 29	10 12 39 25 11 38 40
38 41 23 30 13 26 4	41 31 10 37 23 29 4	20 27 32 41 13 38 4	29 28 36 31 23 24 4
7 22 40 5 39 47 15	5 36 39 12 20 47 16	15 28 24 10 34 47 17	15 17 34 7 37 47 18
34 31 18 8 33 2 49	33 28 7 24 32 2 49	39 43 5 6 31 2 49	42 41 6 5 30 2 49
35327#	37262#	39148#	40981#
1 48 21 36 40 13 16	1 48 22 34 45 7 18	1 48 24 30 18 31 23	1 48 24 41 44 7 10
31 3 20 43 5 38 35	30 3 21 42 13 40 26	29 3 12 46 16 36 33	28 3 11 38 23 35 37
46 33 11 28 23 26 8	46 35 11 27 9 33 14	45 40 39 13 22 7 9	46 32 31 20 8 21 17
6 18 41 25 9 32 44	6 19 38 25 12 31 44	15 8 6 25 44 42 35	14 5 34 25 16 45 36
42 24 27 22 39 17 4	36 17 41 23 39 15 4	41 43 28 37 11 10 5	33 29 42 30 19 18 4
15 12 45 7 30 47 19	24 10 37 8 29 47 20	17 14 34 4 38 47 21	13 15 27 12 39 47 22
34 37 10 14 29 2 49	32 43 5 16 28 2 49	27 19 32 20 26 2 49	40 43 6 9 26 2 49
42828#	44443#	46507#	48052#
1 48 28 8 46 30 14	1 48 27 8 44 29 18	1 48 28 32 45 14 7	1 48 29 30 38 5 24
27 3 37 45 21 9 33	26 3 41 33 20 14 38	24 3 16 42 19 30 41	23 3 13 44 19 34 39
43 35 11 6 24 38 18	45 35 19 4 37 28 7	46 39 17 21 12 27 13	46 32 7 35 9 36 10
19 16 40 25 10 34 31	16 10 39 25 11 40 34	15 6 40 25 10 44 35	28 17 42 25 8 33 22
32 12 26 44 39 15 7	43 22 13 46 31 15 5	37 23 38 29 33 11 4	40 14 41 15 43 18 4
17 41 29 5 13 47 23	12 36 30 17 9 47 24	9 20 31 8 34 47 26	11 16 31 6 37 47 27
36 20 4 42 22 2 49	32 21 6 42 23 2 49	43 36 5 18 22 2 49	26 45 12 20 21 2 49
49641#	51202#	52648#	54248#
1 48 30 27 43 16 10	1 48 31 43 28 14 10	1 48 32 7 44 22 21	1 48 33 34 44 7 8
22 3 24 42 17 38 29	21 3 6 37 34 42 32	20 3 38 33 26 15 40	19 3 20 37 27 40 29
46 44 5 18 13 35 14	46 33 41 5 11 15 24	46 34 11 9 31 36 8	46 35 9 14 11 38 22
9 19 39 25 11 31 41	23 12 20 25 30 38 27	27 13 45 25 5 37 23	18 24 45 25 5 26 32
36 15 37 32 45 6 4	26 35 39 45 9 17 4	42 14 19 41 39 16 4	28 12 39 36 41 15 4
21 12 33 8 26 47 28	18 8 16 13 44 47 29	10 35 24 17 12 47 30	21 10 23 13 30 47 31
40 34 7 23 20 2 49	40 36 22 7 19 2 49	29 28 6 43 18 2 49	42 43 6 16 17 2 49
55625#	57033#	58293#	59572#
1 48 34 13 38 20 21	1 48 35 24 44 12 11	1 48 36 21 44 12 13	1 48 37 18 41 7 23
18 3 22 45 24 19 44	17 3 31 18 30 42 34	16 3 24 39 40 23 30	15 3 20 28 38 26 45
46 40 7 23 15 36 8	46 41 21 5 28 27 7	46 32 15 5 7 42 28	46 34 21 19 14 33 8
33 9 39 25 11 41 17	13 14 40 25 10 36 37	33 19 41 25 9 31 17	39 6 40 25 10 44 11

```

42 14 35 27 43 10 4 43 23 22 45 29 9 4 22 8 43 45 35 18 4 42 17 36 31 29 16 4
6 31 26 5 28 47 32 16 8 20 32 19 47 33 20 27 10 11 26 47 34 5 24 12 22 30 47 35
29 30 12 37 16 2 49 39 38 6 26 15 2 49 37 38 6 29 14 2 49 27 43 9 32 13 2 49
60744# 61961# 62983# 63962#
1 48 38 21 42 9 16 1 48 39 17 30 18 22 1 48 40 11 35 24 16 1 48 41 23 36 6 20
14 3 23 26 31 35 43 13 3 15 36 44 26 38 12 3 33 32 28 30 37 11 3 24 28 37 40 32
46 40 13 18 20 33 5 46 40 31 8 7 27 16 46 43 5 6 31 36 8 46 33 7 12 19 42 16
28 11 44 25 6 39 22 41 5 21 25 29 45 9 27 21 41 25 9 29 23 35 29 45 25 5 21 15
45 17 30 32 37 10 4 34 23 43 42 19 10 4 42 14 19 44 45 7 4 34 8 31 38 43 17 4
7 15 19 24 27 47 36 12 24 6 14 35 47 37 13 20 22 18 17 47 38 18 10 13 22 26 47 39
34 41 8 29 12 2 49 28 32 20 33 11 2 49 34 26 15 39 10 2 49 30 44 14 27 9 2 49
64862# 65681# 66414# 67106#
1 48 42 26 32 12 14 1 48 43 20 31 15 17 1 48 44 20 29 16 17 1 48 45 12 26 20 23
10 3 22 20 37 39 44 9 3 18 26 38 36 45 8 3 18 27 38 36 45 7 3 22 33 40 34 36
46 33 21 5 34 27 9 46 42 21 11 22 27 6 46 41 19 10 24 28 7 46 44 13 8 18 35 11
35 19 43 25 7 31 15 37 10 34 25 16 40 13 39 13 35 25 15 37 11 41 19 29 25 21 31 9
41 23 16 45 29 17 4 44 23 28 39 29 8 4 43 22 26 40 31 9 4 39 15 32 42 37 6 4
6 11 13 30 28 47 40 5 14 12 24 32 47 41 5 14 12 23 32 47 42 14 16 10 17 28 47 43
36 38 18 24 8 2 49 33 35 19 30 7 2 49 33 34 21 30 6 2 49 27 30 24 38 5 2 49
67673# 68188# 68591# 68921#
1 48 43 29 26 18 10 1 48 43 22 17 21 23 1 48 41 23 26 15 21 1 48 43 21 26 14 22
6 5 16 30 35 41 42 5 6 16 36 32 38 42 4 11 18 36 38 37 31 3 9 19 30 42 40 32
47 39 17 4 28 27 13 47 37 10 11 30 31 9 45 34 7 8 17 44 20 46 38 13 6 17 39 16
36 19 38 25 12 31 14 46 24 35 25 15 26 4 47 28 40 25 10 22 3 45 23 35 25 15 27 5
37 23 22 46 33 11 3 41 19 20 39 40 13 3 30 6 33 42 43 16 5 34 11 33 44 37 12 4
8 9 15 20 34 45 44 8 12 18 14 34 44 45 19 13 12 14 32 39 46 18 10 8 20 31 41 47
40 32 24 21 7 2 49 27 29 33 28 7 2 49 29 35 24 27 9 2 49 28 36 24 29 7 2 49
69202# 71470# 73745# 76394#
1 47 10 46 37 12 22 1 47 12 45 40 7 23 1 47 12 36 37 19 23 1 47 15 30 45 18 19
48 5 16 44 14 30 18 46 2 16 37 14 28 32 45 2 29 40 4 22 33 44 2 34 37 11 14 33
39 9 43 15 17 21 31 42 9 44 21 24 15 20 43 9 30 15 44 18 16 41 7 28 10 42 26 21
8 27 24 25 26 23 42 11 17 31 25 19 33 39 8 26 39 25 11 24 42 12 27 46 25 4 23 38
19 29 33 35 7 41 11 30 35 26 29 6 41 8 34 32 6 35 20 41 7 29 24 8 40 22 43 9
32 20 36 6 34 45 2 18 22 36 13 34 48 4 17 28 46 10 21 48 5 17 36 39 13 16 48 6
28 38 13 4 40 3 49 27 43 10 5 38 3 49 27 31 13 14 38 3 49 31 32 5 20 35 3 49
. . . . .

```

\*\* Calculated and Listed by Kanji Setsuda on  
Feb. 7, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 \*\*

How many solutions could we find? The computation result was very surprising.

We have got too many solutions to count them up through, taking too much time.

Watch the sample lists of Standard Solutions above and below. Simultaneous type of squares 7x7 prove to be no longer rare, precious jewels at all.

When Prof. Mutsumi Suzuki discovered all of order 5, he predicted that there must exist any Simultaneous magic squares: both Self-complementary and Pan-diagonal of orders 7, 9, 11, 13, ...

I really found many examples of those 'miraculous' solutions of order 7 and 9 for Prof. M. Suzuki.

\*\* Simultaneous Magic Squares 7x7: Self-Complementary and Pan-Diagonal: \*\*

\*\* Sample Solutions with n1,n9,n17,n25,n33,n41,n49={1,9,17,25,33,41,49} \*\*

```

1#|R|C\P/p| 132#|R|C\P/p| 260#|R|C\P/p|
1 48 26 18 34 36 12|0|0\0/0| 1 47 22 4 43 38 20|0|0\0/0| 1 46 24 13 36 32 23|0|0\0/0|
47 9 21 45 7 11 35|0|0\0/0| 48 9 35 40 5 6 32|0|0\0/0| 48 9 21 43 3 6 45|0|0\0/0|
19 13 17 20 40 44 22|0|0\0/0| 23 24 17 21 37 42 11|0|0\0/0| 22 16 17 35 38 39 8|0|0\0/0|
27 46 42 25 8 4 23|0|0\0/0| 16 31 36 25 14 19 34|0|0\0/0| 30 31 40 25 10 19 20|0|0\0/0|
28 6 10 30 33 37 31|0|0\0/0| 39 8 13 29 33 26 27|0|0\0/0| 42 11 12 15 33 34 28|0|0\0/0|
15 39 43 5 29 41 3|0|0\0/0| 18 44 45 10 15 41 2|0|0\0/0| 5 44 47 7 29 41 2|0|0\0/0|
38 14 16 32 24 2 49|0|0\0/0| 30 12 7 46 28 3 49|0|0\0/0| 27 18 14 37 26 4 49|0|0\0/0|

```



365# R C\P/p										463# R C\P/p										571# R C\P/p												
1	45	28	14	47	29	11	0	0	0	0	1	44	27	37	40	19	7	0	0	0	0	1	43	23	5	40	44	19	0	0	0	0
48	9	32	26	7	16	37	0	0	0	0	48	9	3	46	5	28	36	0	0	0	0	48	9	39	42	3	12	22	0	0	0	0
19	20	17	23	44	42	10	0	0	0	0	21	24	17	42	12	39	20	0	0	0	0	26	35	17	16	32	36	13	0	0	0	0
15	38	46	25	4	12	35	0	0	0	0	18	34	35	25	15	16	32	0	0	0	0	4	30	21	25	29	20	46	0	0	0	0
40	8	6	27	33	30	31	0	0	0	0	30	11	38	8	33	26	29	0	0	0	0	37	14	18	34	33	15	24	0	0	0	0
13	34	43	24	18	41	2	0	0	0	0	14	22	45	4	47	41	2	0	0	0	0	28	38	47	8	11	41	2	0	0	0	0
39	21	3	36	22	5	49	0	0	0	0	43	31	10	13	23	6	49	0	0	0	0	31	6	10	45	27	7	49	0	0	0	0
663# R C\P/p										762# R C\P/p										888# R C\P/p												
1	42	31	11	32	44	14	0	0	0	0	1	40	28	20	39	42	5	0	0	0	0	1	39	24	43	45	8	15	0	0	0	0
47	9	27	40	4	5	43	0	0	0	0	48	9	29	44	4	3	38	0	0	0	0	48	9	13	34	3	40	28	0	0	0	0
20	21	17	22	48	35	12	0	0	0	0	24	19	17	15	43	34	23	0	0	0	0	29	27	17	44	12	32	14	0	0	0	0
26	37	34	25	16	13	24	0	0	0	0	18	36	37	25	13	14	32	0	0	0	0	4	30	31	25	19	20	46	0	0	0	0
38	15	2	28	33	29	30	0	0	0	0	27	16	7	35	33	31	26	0	0	0	0	36	18	38	6	33	23	21	0	0	0	0
7	45	46	10	23	41	3	0	0	0	0	12	47	46	6	21	41	2	0	0	0	0	22	10	47	16	37	41	2	0	0	0	0
36	6	18	39	19	8	49	0	0	0	0	45	8	11	30	22	10	49	0	0	0	0	35	42	5	7	26	11	49	0	0	0	0
994# R C\P/p										1110# R C\P/p										1214# R C\P/p												
1	38	28	11	40	34	23	0	0	0	0	1	37	28	39	40	18	12	0	0	0	0	1	36	24	44	47	12	11	0	0	0	0
48	9	20	47	4	5	42	0	0	0	0	48	9	8	43	3	35	29	0	0	0	0	48	9	10	46	5	20	37	0	0	0	0
26	36	17	43	15	32	6	0	0	0	0	27	36	17	45	4	30	16	0	0	0	0	32	21	17	43	8	31	23	0	0	0	0
21	13	19	25	31	37	29	0	0	0	0	6	26	19	25	31	24	44	0	0	0	0	15	22	34	25	16	28	35	0	0	0	0
44	18	35	7	33	14	24	0	0	0	0	34	20	46	5	33	14	23	0	0	0	0	27	19	42	7	33	29	18	0	0	0	0
8	45	46	3	30	41	2	0	0	0	0	21	15	47	7	42	41	2	0	0	0	0	13	30	45	4	40	41	2	0	0	0	0
27	16	10	39	22	12	49	0	0	0	0	38	32	10	11	22	13	49	0	0	0	0	39	38	3	6	26	14	49	0	0	0	0
1315# R C\P/p										1404# R C\P/p										1491# R C\P/p												
1	35	14	38	46	18	23	0	0	0	0	1	34	22	44	42	12	20	0	0	0	0	1	32	37	40	47	4	14	0	0	0	0
48	9	37	44	3	8	26	0	0	0	0	48	9	23	45	11	32	7	0	0	0	0	48	9	15	22	16	21	44	0	0	0	0
43	5	17	19	34	28	29	0	0	0	0	36	21	17	26	3	35	37	0	0	0	0	23	8	17	43	26	38	20	0	0	0	0
11	30	40	25	10	20	39	0	0	0	0	4	40	19	25	31	10	46	0	0	0	0	31	39	45	25	5	11	19	0	0	0	0
21	22	16	31	33	45	7	0	0	0	0	13	15	47	24	33	29	14	0	0	0	0	30	12	24	7	33	42	27	0	0	0	0
24	42	47	6	13	41	2	0	0	0	0	43	18	39	5	27	41	2	0	0	0	0	6	29	34	28	35	41	2	0	0	0	0
27	32	4	12	36	15	49	0	0	0	0	30	38	8	6	28	16	49	0	0	0	0	36	46	3	10	13	18	49	0	0	0	0
1552# R C\P/p										1626# R C\P/p										1696# R C\P/p												
1	31	34	38	37	28	6	0	0	0	0	1	30	40	3	42	46	13	0	0	0	0	1	29	36	22	39	43	5	0	0	0	0
48	9	20	47	15	26	10	0	0	0	0	48	9	38	35	14	5	26	0	0	0	0	48	9	38	42	15	4	19	0	0	0	0
27	29	17	18	5	36	43	0	0	0	0	22	44	17	19	21	34	18	0	0	0	0	27	24	17	3	30	34	40	0	0	0	0
8	46	11	25	39	4	42	0	0	0	0	11	27	7	25	43	23	39	0	0	0	0	13	44	18	25	32	6	37	0	0	0	0
7	14	45	32	33	21	23	0	0	0	0	32	16	29	31	33	6	28	0	0	0	0	10	16	20	47	33	26	23	0	0	0	0
40	24	35	3	30	41	2	0	0	0	0	24	45	36	15	12	41	2	0	0	0	0	31	46	35	8	12	41	2	0	0	0	0
44	22	13	12	16	19	49	0	0	0	0	37	4	8	47	10	20	49	0	0	0	0	45	7	11	28	14	21	49	0	0	0	0
1761# R C\P/p										1833# R C\P/p										1913# R C\P/p												
1	28	27	36	39	29	15	0	0	0	0	1	27	36	34	47	26	4	0	0	0	0	1	26	36	44	38	19	11	0	0	0	0
48	9	40	42	12	18	6	0	0	0	0	48	9	31	39	20	18	10	0	0	0	0	48	9	23	37	16	35	7	0	0	0	0
37	19	17	5	24	30	43	0	0	0	0	29	28	17	12	7	37	45	0	0	0	0	30	29	17	22	5	32	40	0	0	0	0
3	46	16	25	34	4	47	0	0	0	0	6	42	15	25	35	8	44	0	0	0	0	4	47	8	25	42	3	46	0	0	0	0
7	20	26	45	33	31	13	0	0	0	0	5	13	43	38	33	22	21	0	0	0	0	10	18	45	28	33	21	20	0	0	0	0
44	32	38	8	10	41	2	0	0	0	0	40	32	30	11	19	41	2	0	0	0	0	43	15	34	13	27	41	2	0	0	0	0
35	21	11	14	23	22	49	0	0	0	0	46	24	3	16	14	23	49	0	0	0	0	39	31	12	6	14	24	49	0	0	0	0
1969# R C\P/p										2025# R C\P/p										2080# R C\P/p												
1	24	36	38	46	20	10	0	0	0	0	1	23	38	39	47	22	5	0	0	0	0	1	22	35	26	43	37	11	0	0	0	0
48	9	31	37	21	22	7	0	0	0	0	48	9	32	35	21	20	10	0	0	0	0	47	9	34	46	12	6	21	0	0	0	0
32	27	17	16	3	35	45	0	0	0	0	31	26	17	13	8	34	46	0	0	0	0	36	40	17	19	8	23	32	0	0	0	0
6	42	11	25	39	8	44	0	0	0	0	6	43	14	25	36	7	44	0	0	0	0	5	20	2	25	48	30	45	0	0	0	0
5	15	47	34	33	23	18	0	0	0	0	4	16	42	37	33	24	19	0	0	0	0	18	27	42	31	33	10	14	0	0	0	0
43	28	29	13	19	41	2	0	0	0	0	40	30	29	15	18	41	2	0	0	0	0	29	44	38	4	16	41	3	0	0	0	0
40	30	4	12	14	26	49	0	0	0	0	45	28	3	11	12	27	49	0	0	0	0	39	13	7	24	15	28	49	0	0	0	0
2122# R C\P/p										2162# R C\P/p										2204# R C\P/p												
1	21	40	35	24	46	8	0	0	0	0	1	20	32	36	43	24	19	0	0	0	0	1	19	30	42	45	24	14	0	0	0	0
48	9	30	45	7	13	23	0	0	0	0	48	9	27	42	6	15	28	0	0	0	0	48	9	27	47	6	4	34	0	0	0	0
31	28	17	11	34	18	36	0	0	0	0	40	39	17	37	5	16	21	0	0	0	0	43	28	17	32	11	15	29	0	0	0	0
12	44	3	25	47	6	38	0	0																								



2243# R C\P/p	2277# R C\P/p	2306# R C\P/p
1 18 30 34 40 28 24 0 0\0/0	1 16 39 42 47 7 23 0 0\0/0	1 15 46 38 40 30 5 0 0\0/0
48 9 42 46 15 3 12 0 0\0/0	48 9 44 4 12 30 28 0 0\0/0	48 9 43 16 13 22 24 0 0\0/0
44 21 17 13 14 23 43 0 0\0/0	37 19 17 29 40 18 15 0 0\0/0	31 27 17 11 42 18 29 0 0\0/0
11 31 5 25 45 19 39 0 0\0/0	5 36 24 25 26 14 45 0 0\0/0	3 44 14 25 36 6 47 0 0\0/0
7 27 36 37 33 29 6 0 0\0/0	35 32 10 21 33 31 13 0 0\0/0	21 32 8 39 33 23 19 0 0\0/0
38 47 35 4 8 41 2 0 0\0/0	22 20 38 46 6 41 2 0 0\0/0	26 28 37 34 7 41 2 0 0\0/0
26 22 10 16 20 32 49 0 0\0/0	27 43 3 8 11 34 49 0 0\0/0	45 20 10 12 4 35 49 0 0\0/0
2319# R C\P/p	2341# R C\P/p	2351# R C\P/p
1 14 43 40 28 38 11 0 0\0/0	1 13 44 45 42 20 10 0 0\0/0	1 12 45 44 47 18 8 0 0\0/0
48 9 31 34 4 5 44 0 0\0/0	48 9 34 19 11 7 47 0 0\0/0	48 9 43 10 14 22 29 0 0\0/0
35 21 17 23 47 8 24 0 0\0/0	35 12 17 27 46 14 24 0 0\0/0	35 23 17 19 39 16 26 0 0\0/0
20 32 13 25 37 18 30 0 0\0/0	22 32 29 25 21 18 28 0 0\0/0	4 37 20 25 30 13 46 0 0\0/0
26 42 3 27 33 29 15 0 0\0/0	26 36 4 23 33 38 15 0 0\0/0	24 34 11 31 33 27 15 0 0\0/0
6 45 46 16 19 41 2 0 0\0/0	3 43 39 31 16 41 2 0 0\0/0	21 28 36 40 7 41 2 0 0\0/0
39 12 22 10 7 36 49 0 0\0/0	40 30 8 5 6 37 49 0 0\0/0	42 32 3 6 5 38 49 0 0\0/0
2360# R C\P/p	2369# R C\P/p	2378# R C\P/p
1 11 44 45 47 4 23 0 0\0/0	1 10 46 35 44 18 21 0 0\0/0	1 8 37 45 44 19 21 0 0\0/0
48 9 35 12 19 10 42 0 0\0/0	47 9 48 19 28 8 16 0 0\0/0	47 9 40 34 14 4 27 0 0\0/0
37 7 17 36 34 20 24 0 0\0/0	37 14 17 11 26 27 43 0 0\0/0	48 22 17 24 18 11 35 0 0\0/0
28 32 29 25 21 18 22 0 0\0/0	20 45 12 25 38 5 30 0 0\0/0	12 20 7 25 43 30 38 0 0\0/0
26 30 16 14 33 43 13 0 0\0/0	7 23 24 39 33 36 13 0 0\0/0	15 39 32 26 33 28 2 0 0\0/0
8 40 31 38 15 41 2 0 0\0/0	34 42 22 31 2 41 3 0 0\0/0	23 46 36 16 10 41 3 0 0\0/0
27 46 3 5 6 39 49 0 0\0/0	29 32 6 15 4 40 49 0 0\0/0	29 31 6 5 13 42 49 0 0\0/0
2382# R C\P/p	2389# R C\P/p	2391# R C\P/p
1 7 46 42 47 12 20 0 0\0/0	1 6 47 42 46 19 14 0 0\0/0	1 5 43 47 46 12 21 0 0\0/0
48 9 44 13 21 5 35 0 0\0/0	48 9 45 13 16 15 29 0 0\0/0	48 9 39 20 15 8 36 0 0\0/0
39 10 17 24 36 18 31 0 0\0/0	39 26 17 22 32 12 27 0 0\0/0	44 19 17 32 26 10 27 0 0\0/0
23 34 22 25 28 16 27 0 0\0/0	7 30 10 25 40 20 43 0 0\0/0	16 22 13 25 37 28 34 0 0\0/0
19 32 14 26 33 40 11 0 0\0/0	23 38 18 28 33 24 11 0 0\0/0	23 40 24 18 33 31 6 0 0\0/0
15 45 29 37 6 41 2 0 0\0/0	21 35 34 37 5 41 2 0 0\0/0	14 42 35 30 11 41 2 0 0\0/0
30 38 3 8 4 43 49 0 0\0/0	36 31 4 8 3 44 49 0 0\0/0	29 38 4 3 7 45 49 0 0\0/0
2394# R C\P/p	2528# R C\P/p	2669# R C\P/p
1 48 13 39 42 20 12 0 0\0/0	1 47 14 46 40 11 16 0 0\0/0	1 46 18 38 37 15 20 0 0\0/0
47 9 16 43 6 14 40 0 0\0/0	48 9 6 42 5 37 28 0 0\0/0	48 9 5 42 7 40 24 0 0\0/0
32 5 33 21 35 27 22 0 0\0/0	31 20 33 35 12 26 18 0 0\0/0	28 31 33 39 3 27 14 0 0\0/0
19 24 46 25 4 26 31 0 0\0/0	7 23 29 25 21 27 43 0 0\0/0	6 21 16 25 34 29 44 0 0\0/0
28 23 15 29 17 45 18 0 0\0/0	32 24 38 15 17 30 19 0 0\0/0	36 23 47 11 17 19 22 0 0\0/0
10 36 44 7 34 41 3 0 0\0/0	22 13 45 8 44 41 2 0 0\0/0	26 10 43 8 45 41 2 0 0\0/0
38 30 8 11 37 2 49 0 0\0/0	34 39 10 4 36 3 49 0 0\0/0	30 35 13 12 32 4 49 0 0\0/0
. . . . .		

\*\* Calculated and Listed by Kanji Setsuda on  
Feb. 8, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 \*\*

I haven't yet known how many solutions there exist in all. Magic squares of higher orders than 6 have too many solutions for us to count piece by piece up to the last.

It might be meaningless to know that, I myself think, unless you could divide the total count into some meaningful factors that indicate some structural necessities of our object.

### #3. About 'Complete Euler Squares' 7x7 for this Type

Give your kind look at the next list. It shows some special solutions accompanied with decomposition layers by the number system of 7-th increment for each.

This is the sample list of fundamental solutions I got when I defined the two primary diagonals strictly like these:

n1=1; n9=9; n17=17; n25=25; n33=33; n41=41; n49=49;  
{n7, n13, n19, n31, n37, n43}={7, 13, 19, 31, 37, 43}

The latter definition means each variable on the left side could take any value only of the six values listed on the right.

**\*\* Simultaneous Magic Squares 7x7: Both Self-Complementary and Pan-Diagonal \*\***

\*\* n1=1; n9=9; n17=17; n25=25; n33=33; n41=41; n49=49; \*\*

\*\* {n7, n13, n19, n31, n37, n43}={7, 13, 19, 31, 37, 43} \*\*

1/ /D7i							2/ /D7i						
1 48 39 14 44 22 7	0651630	0536106	1 30 40 45 48 4	7	0456600	0142536							
18 9 23 34 38 13 40	2134515	3115254	26 9 22 29 34 13	42	3134415	4100556							
35 29 17 5 19 46 24	4420263	6024432	44 12 17 15 19 32	36	6122245	1420430							
42 20 47 25 3 30 8	5263041	6543210	39 23 47 25 3 27	11	5363031	3143253							
26 4 31 45 33 21 15	3046422	4322460	14 18 31 35 33 38	6	1244450	6326425							
10 37 12 16 27 41 32	1512354	2141553	8 37 16 21 28 41	24	1522353	0116652							
43 28 6 36 11 2 49	6305106	0650316	43 46 2 5 10 20	49	6600126	0314256							
3/ /D7i							4/ /D7i						
1 30 48 35 42 12 7	0464510	0156646	1 44 47 23 24 29	7	0663340	0141206							
16 9 26 27 44 13 40	2133615	1145154	38 9 2 28 16 37	45	5103256	2116112							
46 22 17 3 19 32 36	6320245	3022430	11 32 17 36 31 40	8	1425451	3320240							
45 21 39 25 11 29 5	6253140	2633304	35 46 30 25 20 4	15	4643202	6313530							
14 18 31 47 33 28 4	1246430	6324463	42 10 19 14 33 18	39	5121425	6246433							
10 37 6 23 24 41 34	1503354	2151255	5 13 34 22 48 41	12	0143651	4550554							
43 38 8 15 2 20 49	6512026	0200156	43 21 26 27 3 6	49	6233006	0645256							
5/ /D7i							6/ /D7i						
1 36 34 38 29 24 13	0545431	0052025	1 39 46 21 47 8 13	0562611	0336405								
30 9 22 48 27 7 32	4136304	1105563	27 9 12 24 36 19	48	3113526	5142045							
40 4 17 3 31 35 45	5020446	4322262	28 30 17 34 7 44	15	3424062	6125610							
44 42 39 25 11 8 6	6553110	1633305	45 18 40 25 10 32	5	6253140	2343234							
5 15 19 47 33 46 10	0226461	4044432	35 6 43 16 33 20	22	4062423	6501450							
18 43 23 2 28 41 20	2630352	3011655	2 31 14 26 38 41	23	0413553	1264251							
37 26 21 12 16 14 49	5321216	1464166	37 42 3 29 4 11	49	5504016	1620336							
7/ /D7i							8/ /D7i						
1 45 40 27 47 2 13	0653601	0245415	1 45 40 35 23 18	13	0654321	0246135							
21 9 12 24 36 31 42	2113545	6142026	21 9 4 48 36 31	26	2106543	6135024							
34 30 17 28 7 44 15	4423062	5126610	34 22 17 12 7 44	39	4321065	5024613							
39 18 46 25 4 32 11	5263041	3333333	47 42 30 25 20 8	3	6543210	4613502							
35 6 43 22 33 20 16	4063422	6500451	11 6 43 38 33 28	16	1065432	3502461							
8 19 14 26 38 41 29	1213554	0464250	24 19 14 2 46 41	29	3210654	2461350							
37 48 3 23 10 5 49	5603106	1521246	37 32 27 15 10 5	49	5432106	1350246							
9/ /D7i							10/ /D7i						
1 39 28 45 47 2 13	0536601	0362415	1 30 48 38 23 22	13	0465331	0152105							
27 9 12 36 18 31 42	3115245	5140326	18 9 5 40 26 31	46	2105346	3144423							
46 30 17 34 7 26 15	6424032	3125640	44 47 17 29 7 16	15	6624022	1420610							
21 6 40 25 10 44 29	2053164	6543210	36 8 11 25 39 42	14	5113551	0033366							
35 24 43 16 33 20 4	4362420	6201453	35 34 43 21 33 3	6	4462400	6506425							
8 19 32 14 38 41 23	1241553	0436251	4 19 24 10 45 41	32	0231654	3422253							
37 48 3 5 22 11 49	5600316	1524036	37 28 27 12 2 20	49	5331026	1654156							
11/ /D7i							12/ /D7i						
1 45 46 27 35 8 13	0663411	0235605	1 39 28 10 48 30	19	0531642	0362514							
21 9 6 18 30 43 48	2102466	6153105	27 9 47 29 18 7	38	3164205	5140362							
28 36 17 34 19 38 3	3524250	6025422	46 35 17 6 37 26	8	6420531	3625140							
39 24 40 25 10 26 11	5353131	3243243	16 5 36 25 14 45	34	2053164	1403625							
47 12 31 16 33 14 22	6142413	4421460	42 24 13 44 33 15	4	5316420	6251403							
2 7 20 32 44 41 29	0024654	1653150	12 43 32 21 3 41	23	1642053	4036251							
37 42 15 23 4 5 49	5523006	1601346	31 20 2 40 22 11	49	4205316	2514036							
13/ /D7i							14/ /D7i						
1 40 38 29 45 3 19	0554602	0420224	1 42 46 27 30 10	19	0563412	0635124							
20 9 32 22 44 37 11	2143651	5130113	14 9 22 11 34 37	48	1131456	6103515							
42 26 17 2 7 46 35	5320064	6421636	38 24 17 15 43 32	6	5322640	2220035							
27 36 34 25 16 14 23	3543213	5053161	45 29 47 25 3 21	5	6463020	2043264							
15 4 43 48 33 24 8	2066431	0305420	44 18 7 35 33 26	12	6204431	1366444							
39 13 6 28 18 41 30	5103254	3556351	2 13 16 39 28 41	36	0125355	1513650							
31 47 5 21 12 10 49	4602116	2446426	31 40 20 23 4 8	49	4523016	2451306							

	15/ /D7i		16/ /D7i
1 39 46 10 48 12 19	0561612 0332544	1 39 34 22 48 12 19	0543612 0350544
27 9 35 5 36 43 20	3140562 5164005	27 9 47 5 24 43 20	3160362 5144205
28 47 17 18 13 44 8	3622161 6423510	40 35 17 6 37 32 8	5420541 4625130
16 29 24 25 26 21 34	2433324 1023465	4 29 36 25 14 21 46	0453126 3003663
42 6 37 32 33 3 22	5054403 6513420	42 18 13 44 33 15 10	5216421 6351402
30 7 14 45 15 41 23	4016253 1662051	30 7 26 45 3 41 23	4036053 1642251
31 38 2 40 4 11 49	4505016 2214336	31 38 2 28 16 11 49	4503216 2216136
	17/ /D7i		18/ /D7i
1 46 22 21 14 40 31	0632154 0306642	1 40 34 21 2 46 31	0542064 0456132
24 9 34 45 6 37 20	3146052 2152515	30 9 22 45 6 37 26	4136053 1102514
48 38 17 3 43 18 8	6520621 5222030	36 38 17 15 43 18 8	5522621 0220030
11 27 15 25 35 23 39	1323435 3503613	23 39 3 25 47 11 27	3503613 1323435
42 32 7 47 33 12 2	5406410 6364441	42 32 7 35 33 12 14	5404411 6366446
30 13 44 5 16 41 26	4160253 1514154	24 13 44 5 28 41 20	3160352 2514655
19 10 36 29 28 4 49	2154306 4200636	19 4 48 29 16 10 49	2064216 4350126
	19/ /D7i		20/ /D7i
1 45 40 4 42 12 31	0650514 0243642	1 45 40 12 18 28 31	0651234 0244362
21 9 35 11 36 43 20	2141562 6163005	21 9 27 35 36 43 4	2134560 6156003
34 47 17 18 13 44 2	4622160 5423511	34 39 17 2 13 44 26	4520163 5321514
22 23 24 25 26 27 28	3333333 0123456	30 47 8 25 42 3 20	4613502 1403625
48 6 37 32 33 3 16	6054402 5513421	24 6 37 48 33 11 16	3056412 2515431
30 7 14 39 15 41 29	4015254 1663050	46 7 14 15 23 41 29	6012354 3660150
19 38 8 46 10 5 49	2516106 4203246	19 22 32 38 10 5 49	2345106 4032246
	21/ /D7i		22/ /D7i
1 27 48 47 11 10 31	0366114 0554322	1 48 30 38 11 16 31	0645124 0512312
21 9 12 34 36 43 20	2114562 6145005	18 9 5 40 14 43 46	2105166 3144603
44 22 17 18 13 26 35	6322134 1023546	44 23 17 29 37 22 3	6324530 1120102
45 46 8 25 42 4 5	6613500 2303634	42 26 35 25 15 24 8	5343231 6463020
15 24 37 32 33 28 6	2354430 0213465	47 28 13 21 33 27 6	6312430 4656455
30 7 14 16 38 41 29	4012554 1661250	4 7 36 10 45 41 32	0051654 3602253
19 40 39 3 2 23 49	2550036 4432116	19 34 39 12 20 2 49	2451206 4534516
	23/ /D7i		24/ /D7i
1 45 28 16 42 12 31	0632514 0261642	1 45 28 24 6 40 31	0633054 0262542
21 9 47 11 24 43 20	2161362 6143205	21 9 27 47 12 43 16	2136162 6154401
46 35 17 6 37 32 2	6420540 3625131	46 39 17 2 37 20 14	6520521 3321156
10 23 36 25 14 27 40	1353135 2103654	18 35 8 25 42 15 32	2413524 3603603
48 18 13 44 33 15 4	6216420 5351403	36 30 13 48 33 11 4	5416410 0155433
30 7 26 39 3 41 29	4035054 1643250	34 7 38 3 23 41 29	4050354 5622150
19 38 8 34 22 5 49	2514306 4205046	19 10 44 26 22 5 49	2163306 4214046
	25/ /D7i		26/ /D7i
1 39 34 21 35 8 37	0542415 0356601	1 45 28 27 35 2 37	0633405 0265611
27 9 12 36 24 19 48	3115326 5140245	21 9 12 36 24 31 42	2115345 6140226
40 30 17 46 7 32 3	5426040 4123632	46 30 17 40 7 32 3	6425040 3124632
45 6 28 25 22 44 5	6033360 2563014	39 6 34 25 16 44 11	5043261 3553113
47 18 43 4 33 20 10	6260421 4303452	47 18 43 10 33 20 4	6261420 4302453
2 31 26 14 38 41 23	0431553 1246251	8 19 26 14 38 41 29	1231554 0446250
13 42 15 29 16 11 49	1524216 5600136	13 48 15 23 22 5 49	1623306 5501046
	27/ /D7i		28/ /D7i
1 47 30 27 29 4 37	0643405 0415031	1 42 36 38 11 10 37	0555115 0602321
39 9 14 22 16 43 32	5113264 3160103	18 9 5 34 20 43 46	2104266 3145503
24 12 17 42 31 44 5	3125460 2426214	44 23 17 35 31 22 3	6324430 1126202
35 48 40 25 10 2 15	4653102 6543210	48 26 29 25 21 24 2	6343230 5403621
45 6 19 8 33 38 26	6021453 2540424	47 28 19 15 33 27 6	6322430 4640455
18 7 34 28 36 41 11	2043551 3656053	4 7 30 16 45 41 32	0042654 3611253
13 46 21 23 20 3 49	1623206 5361526	13 40 39 12 14 8 49	1551116 5434606
	29/ /D7i		30/ /D7i
1 30 48 22 21 10 43	0463216 0150620	1 46 42 2 27 14 43	0650316 0361560
16 9 36 18 47 37 12	2152651 1103414	20 9 34 12 39 37 24	2141553 5154312
46 26 17 5 19 35 27	6320243 3424465	32 28 17 15 31 47 5	4322460 3620244
44 42 11 25 39 8 6	6513510 1633305	44 40 29 25 21 10 6	6543210 1403625
23 15 31 45 33 24 4	3246430 1022423	45 3 19 35 33 22 18	6024432 2246403
38 13 3 32 14 41 34	5104154 2523655	26 13 11 38 16 41 30	3115254 4532151
7 40 29 28 2 20 49	0543026 6406156	7 36 23 48 8 4 49	0536106 6015036

31/ /D7i							32/ /D7i										
1	44	32	3	38	14	43	0640516	0132260	1	42	27	44	14	4	43	0536106	0651630
34	9	46	11	28	37	10	4161351	5133612	24	9	11	40	20	37	34	3115254	2134515
30	26	17	15	31	48	8	4322461	1420250	47	5	17	29	31	28	18	6024432	4420263
21	45	27	25	23	5	29	2633304	6253140	48	38	35	25	15	12	2	6543210	5263041
42	2	19	35	33	24	20	5024432	6146425	32	22	19	21	33	45	3	4322460	3046422
40	13	22	39	4	41	16	5135052	4503351	16	13	30	10	39	41	26	2141553	1512354
7	36	12	47	18	6	49	0516206	6044356	7	46	36	6	23	8	49	0650316	6305106

  

33/ /D7i								
1	38	26	14	21	32	43	0531246	0246630
28	9	46	34	15	37	6	3164250	6135015
48	40	17	5	31	23	11	6520431	5424213
8	30	3	25	47	20	42	1403625	0123456
39	27	19	45	33	10	2	5326410	3542421
44	13	35	16	4	41	22	6142053	1561350
7	18	29	36	24	12	49	0245316	6300246

[Count = 33] OK!

In the list above each solution has its decomposition layers by 7th increment, in separate forms for higher position and lower.

Find such solutions that should contain the next special properties:

### Definitions of "Complete Euler Squares" of Order 7

- (1) Every row and every column must consist of {0, 1, 2, 3, 4, 5 and 6} using each strictly once even in both high and low layers.
- (2) Every pan-diagonal must consist of {0, 1, 2, 3, 4, 5 and 6} using each strictly once even in both high and low layers.
- (3) Suppose you combine each value of the corresponding positions high and low, and calculate the value of each variable in the next equation:  $V_n = V_h * 7 + V_l$ .

The result must always be any one of integers from 00 to 66 (N7i) appearing strictly once. Neither repetition nor drop-off of a certain integer must be found.

I would like to call them the "Complete Euler Squares" from now on.

The sum of each row, each column and each pandiagonal is always calculated in the same way as:

$$(0+1+2+3+4+5+6) \times 7^1 + (0+1+2+3+4+5+6) \times 7^0 = 168 \text{ (Decimal; equivalent to 175 in the classical style)}$$

Once you know of this property, you don't have to calculate any longer to know the magic sum. You have to see only the constant pattern of the contents.

Could you find and pick up any solutions of this kind among those above?

Yes. There are two such solutions: No. 8 and No. 12.

There are only two solutions among 33, really rare, precious jewels.

You must be surprised that 31 non-'Euler Squares' exist among 33.

In the case of order 5, Simultaneous type of magic squares are all 'Complete Euler Squares,' and nothing else. On the other hand in the case of order 7, 'non-Euler Squares' are found far more than Euler ones. It is really surprising.

What do we have to do with these 'non-Euler Squares'? How embarrassing it is!

### #4. How many 'Complete Euler Squares' 7x7 could we find?

The next job we have to do is to calculate every 'Complete Euler Squares' of order 7 for our Simultaneous MS77: both Self-complementary and Pan-diagonal and to get the complete list of standard solutions of them. But how can we do that?

Guess if we have to pick up to select them out of all solutions of Simultaneous type. But, we have not yet counted the latter through. It is terrible we imagine how long it should take a time for us to do that job?

We have to make our objects in any other direct method of composition. But how?  
 I had to invent any new effective way of compositions to do that job.  
 I wrote some articles and reported of them in some sections of Chapter 4, Part 4.  
 Let me introduce you only about the conclusions there as follows:

**\* Complete Euler Squares for Simultaneous MS77: Both S-C. & P-D. \***

<b>1/H</b>	<b>9/L</b>		<b>1#</b>	<b>1/H</b>	<b>10/L</b>		<b>2#</b>
0641352	0516234	1 48 30 14 24 39 19		0641352	0516432	1 48 30 14 26 39 17	
5206413	6234051	42 17 4 47 29 13 23		5206413	6432051	42 19 4 45 29 13 23	
1352064	4051623	12 22 41 16 7 45 32		1352064	2051643	10 22 41 16 7 47 32	
6413520	1623405	44 35 10 25 40 15 6		6413520	1643205	44 35 12 25 38 15 6	
2064135	3405162	18 5 43 34 9 28 38		2064135	3205164	18 3 43 34 9 28 40	
3520641	5162340	27 37 21 3 46 33 8		3520641	5164320	27 37 21 5 46 31 8	
4135206	2340516	31 11 26 36 20 2 49		4135206	4320516	33 11 24 36 20 2 49	
<b>2/H</b>	<b>9/L</b>		<b>3#</b>	<b>2/H</b>	<b>10/L</b>		<b>4#</b>
0621354	0516234	1 48 16 14 24 39 33		0621354	0516432	1 48 16 14 26 39 31	
5406213	6234051	42 31 4 47 15 13 23		5406213	6432051	42 33 4 45 15 13 23	
1354062	4051623	12 22 41 30 7 45 18		1354062	2051643	10 22 41 30 7 47 18	
6213540	1623405	44 21 10 25 40 29 6		6213540	1643205	44 21 12 25 38 29 6	
4062135	3405162	32 5 43 20 9 28 38		4062135	3205164	32 3 43 20 9 28 40	
3540621	5162340	27 37 35 3 46 19 8		3540621	5164320	27 37 35 5 46 17 8	
2135406	2340516	17 11 26 36 34 2 49		2135406	4320516	19 11 24 36 34 2 49	
<b>1/H</b>	<b>11/L</b>		<b>5#</b>	<b>3/H</b>	<b>9/L</b>		<b>9#</b>
0641352	0531624	1 48 32 9 28 38 19		0652341	0516234	1 48 37 21 24 32 12	
5206413	3162405	39 16 7 45 33 8 27		4106523	6234051	35 10 4 47 36 20 23	
1352064	6240531	14 24 40 15 6 46 30		2341065	4051623	19 22 34 9 7 45 39	
6413520	4053162	47 29 13 25 37 21 3		6523410	1623405	44 42 17 25 33 8 6	
2064135	5316240	20 4 44 35 10 26 36		1065234	3405162	11 5 43 41 16 28 31	
3520641	1624053	23 42 17 5 43 34 11		3410652	5162340	27 30 14 3 46 40 15	
4135206	2405316	31 12 22 41 18 2 49		5234106	2340516	38 18 26 29 13 2 49	
<b>3/H</b>	<b>10/L</b>		<b>10#</b>	<b>4/H</b>	<b>9/L</b>		<b>11#</b>
0652341	0516432	1 48 37 21 26 32 10		0612345	0516234	1 48 9 21 24 32 40	
4106523	6432051	35 12 4 45 36 20 23		4506123	6234051	35 38 4 47 8 20 23	
2341065	2051643	17 22 34 9 7 47 39		2345061	4051623	19 22 34 37 7 45 11	
6523410	1643205	44 42 19 25 31 8 6		6123450	1623405	44 14 17 25 33 36 6	
1065234	3205164	11 3 43 41 16 28 33		5061234	3405162	39 5 43 13 16 28 31	
3410652	5164320	27 30 14 5 46 38 15		3450612	5162340	27 30 42 3 46 12 15	
5234106	4320516	40 18 24 29 13 2 49		1234506	2340516	10 18 26 29 41 2 49	
<b>4/H</b>	<b>10/L</b>		<b>12#</b>	<b>3/H</b>	<b>11/L</b>		<b>13#</b>
0612345	0516432	1 48 9 21 26 32 38		0652341	0531624	1 48 39 16 28 31 12	
4506123	6432051	35 40 4 45 8 20 23		4106523	3162405	32 9 7 45 40 15 27	
2345061	2051643	17 22 34 37 7 47 11		2341065	6240531	21 24 33 8 6 46 37	
6123450	1643205	44 14 19 25 31 36 6		6523410	4053162	47 36 20 25 30 14 3	
5061234	3205164	39 3 43 13 16 28 33		1065234	5316240	13 4 44 42 17 26 29	
3450612	5164320	27 30 42 5 46 10 15		3410652	1624053	23 35 10 5 43 41 18	
1234506	4320516	12 18 24 29 41 2 49		5234106	2405316	38 19 22 34 11 2 49	
<b>5/H</b>	<b>9/L</b>		<b>17#</b>	<b>5/H</b>	<b>10/L</b>		<b>18#</b>
0654321	0516234	1 48 37 35 24 18 12		0654321	0516432	1 48 37 35 26 18 10	
2106543	6234051	21 10 4 47 36 34 23		2106543	6432051	21 12 4 45 36 34 23	
4321065	4051623	33 22 20 9 7 45 39		4321065	2051643	31 22 20 9 7 47 39	
6543210	1623405	44 42 31 25 19 8 6		6543210	1643205	44 42 33 25 17 8 6	
1065432	3405162	11 5 43 41 30 28 17		1065432	3205164	11 3 43 41 30 28 19	
3210654	5162340	27 16 14 3 46 40 29		3210654	5164320	27 16 14 5 46 38 29	
5432106	2340516	38 32 26 15 13 2 49		5432106	4320516	40 32 24 15 13 2 49	
<b>6/H</b>	<b>9/L</b>		<b>19#</b>	<b>6/H</b>	<b>10/L</b>		<b>20#</b>
0614325	0516234	1 48 9 35 24 18 40		0614325	0516432	1 48 9 35 26 18 38	
2506143	6234051	21 38 4 47 8 34 23		2506143	6432051	21 40 4 45 8 34 23	
4325061	4051623	33 22 20 37 7 45 11		4325061	2051643	31 22 20 37 7 47 11	
6143250	1623405	44 14 31 25 19 36 6		6143250	1643205	44 14 33 25 17 36 6	
5061432	3405162	39 5 43 13 30 28 17		5061432	3205164	39 3 43 13 30 28 19	
3250614	5162340	27 16 42 3 46 12 29		3250614	5164320	27 16 42 5 46 10 29	
1432506	2340516	10 32 26 15 41 2 49		1432506	4320516	12 32 24 15 41 2 49	

5/H	11/L					21#	7/H	9/L						25#													
0654321	0531624	1	48	39	30	28	17	12	0645312	0516234	1	48	30	42	24	11	19										
2106543	3162405	18	9	7	45	40	29	27	1206453	6234051	14	17	4	47	29	41	23										
4321065	6240531	35	24	19	8	6	46	37	5312064	4051623	40	22	13	16	7	45	32										
6543210	4053162	47	36	34	25	16	14	3	6453120	1623405	44	35	38	25	12	15	6										
1065432	5316240	13	4	44	42	31	26	15	2064531	3405162	18	5	43	34	37	28	10										
3210654	1624053	23	21	10	5	43	41	32	3120645	5162340	27	9	21	3	46	33	36										
5432106	2405316	38	33	22	20	11	2	49	4531206	2340516	31	39	26	8	20	2	49										
7/H	10/L					26#	8/H	9/L						27#													
0645312	0516432	1	48	30	42	26	11	17	0625314	0516234	1	48	16	42	24	11	33										
1206453	6432051	14	19	4	45	29	41	23	1406253	6234051	14	31	4	47	15	41	23										
5312064	2051643	38	22	13	16	7	47	32	5314062	4051623	40	22	13	30	7	45	18										
6453120	1643205	44	35	40	25	10	15	6	6253140	1623405	44	21	38	25	12	29	6										
2064531	3205164	18	3	43	34	37	28	12	4062531	3405162	32	5	43	20	37	28	10										
3120645	5164320	27	9	21	5	46	31	36	3140625	5162340	27	9	35	3	46	19	36										
4531206	4320516	33	39	24	8	20	2	49	2531406	2340516	17	39	26	8	34	2	49										
8/H	10/L					28#	7/H	11/L						29#													
0625314	0516432	1	48	16	42	26	11	31	0645312	0531624	1	48	32	37	28	10	19										
1406253	6432051	14	33	4	45	15	41	23	1206453	3162405	11	16	7	45	33	36	27										
5314062	2051643	38	22	13	30	7	47	18	5312064	6240531	42	24	12	15	6	46	30										
6253140	1643205	44	21	40	25	10	29	6	6453120	4053162	47	29	41	25	9	21	3										
4062531	3205164	32	3	43	20	37	28	12	2064531	5316240	20	4	44	35	38	26	8										
3140625	5164320	27	9	35	5	46	17	36	3120645	1624053	23	14	17	5	43	34	39										
2531406	4320516	19	39	24	8	34	2	49	4531206	2405316	31	40	22	13	18	2	49										
1/H	13/L	33#					1/H	15/L	37#		3/H	13/L	41#		3/H	15/L	45#										
1	47	31	14	23	39	20	1	47	32	10	28	37	20	1	47	39	17	28	30	13							
42	16	4	48	29	12	24	39	17	7	44	34	8	26	35	9	4	48	36	19	24	32	10	7	44	41	15	26
13	22	40	17	7	44	32	14	23	41	15	5	46	31	20	22	33	10	7	44	39	21	23	34	8	5	46	38
45	35	9	25	41	15	5	48	29	12	25	38	21	2	45	42	16	25	34	8	5	48	36	19	25	31	14	2
18	6	43	33	10	28	37	19	4	45	35	9	27	36	11	6	43	40	17	28	30	12	4	45	42	16	27	29
26	38	21	2	46	34	8	24	42	16	6	43	33	11	26	31	14	2	46	41	15	24	35	9	6	43	40	18
30	11	27	36	19	3	49	30	13	22	40	18	3	49	37	18	27	29	12	3	49	37	20	22	33	11	3	49
5/H	13/L	49#					5/H	15/L	53#		7/H	13/L	57#		7/H	15/L	61#										
1	47	38	35	23	18	13	1	47	39	31	28	16	13	1	47	31	42	23	11	20	1	47	32	38	28	9	20
21	9	4	48	36	33	24	18	10	7	44	41	29	26	14	16	4	48	29	40	24	11	17	7	44	34	36	26
34	22	19	10	7	44	39	35	23	20	8	5	46	38	41	22	12	17	7	44	32	42	23	13	15	5	46	31
45	42	30	25	20	8	5	48	36	33	25	17	14	2	45	35	37	25	13	15	5	48	29	40	25	10	21	2
11	6	43	40	31	28	16	12	4	45	42	30	27	15	18	6	43	33	38	28	9	19	4	45	35	37	27	8
26	17	14	2	46	41	29	24	21	9	6	43	40	32	26	10	21	2	46	34	36	24	14	16	6	43	33	39
37	32	27	15	12	3	49	37	34	22	19	11	3	49	30	39	27	8	19	3	49	30	41	22	12	18	3	49
1/H	17/L	65#					1/H	19/L	69#		1/H	21/L	73#		1/H	23/L	77#										
1	46	35	10	27	37	19	1	46	35	9	26	38	20	1	46	35	9	24	40	20	1	46	35	10	23	41	19
41	16	5	43	32	14	24	40	17	6	43	32	14	23	38	19	6	43	32	14	23	37	20	5	43	32	14	24
11	28	38	20	2	47	29	11	28	37	19	3	48	29	11	28	37	17	5	48	29	11	28	38	16	6	47	29
44	33	8	25	42	17	6	45	34	8	25	42	16	5	47	34	8	25	42	16	3	48	33	8	25	42	17	2
21	3	48	30	12	22	39	21	2	47	31	13	22	39	21	2	45	33	13	22	39	21	3	44	34	12	22	39
26	36	18	7	45	34	9	27	36	18	7	44	33	10	27	36	18	7	44	31	12	26	36	18	7	45	30	13
31	13	23	40	15	4	49	30	12	24	41	15	4	49	30	10	26	41	15	4	49	31	9	27	40	15	4	49
3/H	17/L	81#					3/H	19/L	85#		3/H	21/L	89#		3/H	23/L	93#										
1	46	42	17	27	30	12	1	46	42	16	26	31	13	1	46	42	16	24	33	13	1	46	42	17	23	34	12
34	9	5	43	39	21	24	33	10	6	43	39	21	23	31	12	6	43	39	21	23	30	13	5	43	39	21	24
18	28	31	13	2	47	36	18	28	30	12	3	48	36	18	28	30	10	5	48	36	18	28	31	9	6	47	36
44	40	15	25	35	10	6	45	41	15	25	35	9	5	47	41	15	25	35	9	3	48	40	15	25	35	10	2
14	3	48	37	19	22	32	14	2	47	38	20	22	32	14	2	45	40	20	22	32	14	3	44	41	19	22	32
26	29	11	7	45	41	16	27	29	11	7	44	40	17	27	29	11	7	44	38	19	26	29	11	7	45	37	20
38	20	23	33	8	4	49	37	19	24	34	8	4	49	37	17	26	34	8	4	49	38	16	27	33	8	4	49
5/H	17/L	97#					5/H	19/L	101#		5/H	21/L	105#		5/H	23/L	109#										
1	46	42	31	27	16	12	1	46	42	30	26	17	13	1	46	42	30	24	19	13	1	46	42	31	23	20	12
20	9	5	43	39	35	24	19	10	6	43	39	35	23	17	12	6	43	39	35	23	16	13	5	43	39	35	24
32	28	17	13	2	47	36	32	28	16	12	3	48	36	32	28	16	10	5	48	36	32	28	17	9	6	47	36
44	40	29	25	21	10	6	45	41	29	25	21	9	5	47	41	29	25	21	9	3	48	40	29	25	21	10	2
14	3	48	37	33	22	18	14	2	47	38	34	22	18	14	2	45	40	34	22	18	14	3	44	41	33	22	18
26	15	11	7	45	41	30	27	15	11	7	44	40	31	27	15	11	7	44	38	33	26	15	11	7	45	37	34
38	34	23	19	8	4	49	37	33	24	20	8	4	49	37	31	26	20	8	4	49	38	30	27	19	8	4	49



7/H 17/L 113#	7/H 19/L 117#	7/H 21/L 121#	7/H 23/L 125#
1 46 35 38 27 9 19	1 46 35 37 26 10 20	1 46 35 37 24 12 20	1 46 35 38 23 13 19
13 16 5 43 32 42 24	12 17 6 43 32 42 23	10 19 6 43 32 42 23	9 20 5 43 32 42 24
39 28 10 20 2 47 29	39 28 9 19 3 48 29	39 28 9 17 5 48 29	39 28 10 16 6 47 29
44 33 36 25 14 17 6	45 34 36 25 14 16 5	47 34 36 25 14 16 3	48 33 36 25 14 17 2
21 3 48 30 40 22 11	21 2 47 31 41 22 11	21 2 45 33 41 22 11	21 3 44 34 40 22 11
26 8 18 7 45 34 37	27 8 18 7 44 33 38	27 8 18 7 44 31 40	26 8 18 7 45 30 41
31 41 23 12 15 4 49	30 40 24 13 15 4 49	30 38 26 13 15 4 49	31 37 27 12 15 4 49
1/H 25/L 129#	1/H 27/L 133#	3/H 25/L 137#	3/H 27/L 141#
1 45 33 14 23 39 20	1 45 32 12 28 37 20	1 45 40 21 23 32 13	1 45 39 19 28 30 13
42 16 4 48 29 10 26	39 19 7 44 34 8 24	35 9 4 48 36 17 26	32 12 7 44 41 15 24
13 22 38 19 7 44 32	14 23 41 15 3 46 33	20 22 31 12 7 44 39	21 23 34 8 3 46 40
47 35 9 25 41 15 3	48 29 10 25 40 21 2	47 42 16 25 34 8 3	48 36 17 25 33 14 2
18 6 43 31 12 28 37	17 4 47 35 9 27 36	11 6 43 38 19 28 30	10 4 47 42 16 27 29
24 40 21 2 46 34 8	26 42 16 6 43 31 11	24 33 14 2 46 41 15	26 35 9 6 43 38 18
30 11 27 36 17 5 49	30 13 22 38 18 5 49	37 18 27 29 10 5 49	37 20 22 31 11 5 49
5/H 25/L 145#	5/H 27/L 149#	7/H 25/L 153#	7/H 27/L 157#
1 45 40 35 23 18 13	1 45 39 33 28 16 13	1 45 33 42 23 11 20	1 45 32 40 28 9 20
21 9 4 48 36 31 26	18 12 7 44 41 29 24	14 16 4 48 29 38 26	11 19 7 44 34 36 24
34 22 17 12 7 44 39	35 23 20 8 3 46 40	41 22 10 19 7 44 32	42 23 13 15 3 46 33
47 42 30 25 20 8 3	48 36 31 25 19 14 2	47 35 37 25 13 15 3	48 29 38 25 12 21 2
11 6 43 38 33 28 16	10 4 47 42 30 27 15	18 6 43 31 40 28 9	17 4 47 35 37 27 8
24 19 14 2 46 41 29	26 21 9 6 43 38 32	24 12 21 2 46 34 36	26 14 16 6 43 31 39
37 32 27 15 10 5 49	37 34 22 17 11 5 49	30 39 27 8 17 5 49	30 41 22 10 18 5 49
1/H 29/L 161#	9/H 1/L 193#	9/H 9/L 209#	9/H 13/L 213#
1 44 34 14 24 39 19	1 42 26 9 46 20 31	1 41 23 14 45 18 33	1 40 24 14 44 18 34
42 17 4 47 29 9 27	27 10 43 21 33 2 39	28 10 46 19 29 6 37	28 9 46 20 29 5 38
12 22 37 20 7 45 32	44 18 34 3 36 28 12	47 15 34 2 42 24 11	48 15 33 3 42 23 11
48 35 10 25 40 15 2	35 5 37 25 13 45 15	30 7 38 25 12 43 20	31 7 37 25 13 43 19
18 5 43 30 13 28 38	38 22 14 47 16 32 6	39 26 8 48 16 35 3	39 27 8 47 17 35 2
23 41 21 3 46 33 8	11 48 17 29 7 40 23	13 44 21 31 4 40 22	12 45 21 30 4 41 22
31 11 26 36 16 6 49	19 30 4 41 24 8 49	17 32 5 36 27 9 49	16 32 6 36 26 10 49
9/H 17/L 217#	9/H 25/L 233#	9/H 29/L 237#	11/H 1/L 241#
1 39 28 10 48 16 33	1 38 26 14 44 18 34	1 37 27 14 45 18 33	1 35 26 16 46 13 38
27 9 47 15 32 7 38	28 9 46 20 29 3 40	28 10 46 19 29 2 41	27 17 43 14 40 2 32
46 21 31 6 37 26 8	48 15 31 5 42 23 11	47 15 30 6 42 24 11	44 11 41 3 29 28 19
30 5 36 25 14 45 20	33 7 37 25 13 43 17	34 7 38 25 12 43 16	42 5 30 25 20 45 8
42 24 13 44 19 29 4	39 27 8 45 19 35 2	39 26 8 44 20 35 3	31 22 21 47 9 39 6
12 43 18 35 3 41 23	10 47 21 30 4 41 22	9 48 21 31 4 40 22	18 48 10 36 7 33 23
17 34 2 40 22 11 49	16 32 6 36 24 12 49	17 32 5 36 23 13 49	12 37 4 34 24 15 49
11/H 9/L 257#	11/H 13/L 261#	11/H 17/L 265#	11/H 25/L 281#
1 34 23 21 45 11 40	1 33 24 21 44 11 41	1 32 28 17 48 9 40	1 31 26 21 44 11 41
28 17 46 12 36 6 30	28 16 46 13 36 5 31	27 16 47 8 39 7 31	28 16 46 13 36 3 33
47 8 41 2 35 24 18	48 8 40 3 35 23 18	46 14 38 6 30 26 15	48 8 38 5 35 23 18
37 7 31 25 19 43 13	38 7 30 25 20 43 12	37 5 29 25 21 45 13	40 7 30 25 20 43 10
32 26 15 48 9 42 3	32 27 15 47 10 42 2	35 24 20 44 12 36 4	32 27 15 45 12 42 2
20 44 14 38 4 33 22	19 45 14 37 4 34 22	19 43 11 42 3 34 23	17 47 14 37 4 34 22
10 39 5 29 27 16 49	9 39 6 29 26 17 49	10 41 2 33 22 18 49	9 39 6 29 24 19 49
11/H 29/L 285#	13/H 1/L 289#	13/H 9/L 321#	13/H 13/L 337#
1 30 27 21 45 11 40	1 28 47 9 18 34 38	1 27 44 14 17 32 40	1 26 45 14 16 32 41
28 17 46 12 36 2 34	20 31 36 7 26 44 11	21 31 39 5 22 48 9	21 30 39 6 22 47 10
47 8 37 6 35 24 18	23 46 13 17 29 42 5	26 43 13 16 35 38 4	27 43 12 17 35 37 4
41 7 31 25 19 43 9	35 40 2 25 48 10 15	30 42 3 25 47 8 20	31 42 2 25 48 8 19
32 26 15 44 13 42 3	45 8 21 33 37 4 27	46 12 15 34 37 7 24	46 13 15 33 38 7 23
16 48 14 38 4 33 22	39 6 24 43 14 19 30	41 2 28 45 11 19 29	40 3 28 44 11 20 29
10 39 5 29 23 20 49	12 16 32 41 3 22 49	10 18 33 36 6 23 49	9 18 34 36 5 24 49
13/H 25/L 353#	13/H 29/L 369#		
1 24 47 14 16 32 41	1 23 48 14 17 32 40		
21 30 39 6 22 45 12	21 31 39 5 22 44 13		
27 43 10 19 35 37 4	26 43 9 20 35 38 4		
33 42 2 25 48 8 17	34 42 3 25 47 8 16		
46 13 15 31 40 7 23	46 12 15 30 41 7 24		
38 5 28 44 11 20 29	37 6 28 45 11 19 29		
9 18 34 36 3 26 49	10 18 33 36 2 27 49		



[Count(n1=1/A11) = 384/3456] OK!

\*\* Calculated and Listed by Kanji Setsuda on Feb. 16, 2007 \*\*

I have counted all the 3456 standard solutions of Complete Euler Squares for our Simultaneous MS77: Self-complementary and Pan-diagonal, independently in three different ways of compositions.

I have also counted the 38102400 standard solutions of Complete Euler Squares for Pan-diagonal Magic Squares of Order 7.

I am afraid they may look very many to you, but they are really rare, precious jewels, theoretically speaking. There are indeed far more 'Non-Euler Squares' of those two types. I cannot even imagine how many solutions of them there are in all.

(First Written on August 10, 2001 by Kanji Setsuda with MacOS9;  
Revised on April 3, 2007 by Kanji Setsuda with MacOSX & Xcode2.2  
Recompiled on February 8, 2015 with MacOSX 10.10.2 & Xcode 6.1.1)

---

E-Mail Address <jag12001@nifty.com>

<< Appendix: The Newest List of Program for Simultaneous MS77 >>

```
/** Simultaneous Type of Magic Squares of Order 7: **
/** Both Self-Complementary and Pan-Diagonal; **
/** File: 'MS77Sml1.c' Dictated by Kanji Setsuda **
/** in 2006 and Revised on February 7, 2015 **
/** with MacOSX 10.10.2 and Xcode 6.1.1; **
//
/** Basic Form for the 'SC&PD'MS77 **
//
//      .-----
//  5  6  7| 11 21 31 41 51 61 71| 1  2  3
//      |---+---+---+---+---+---|
// 12 13 14| 81 911011112113141| 8  9 10
//      |---+---+---+---+---+---|
// 19 20 21|1516171819202115 16 17
//      |---+---+---+---+---+---|
// 26 27 28|22123124125126127128|22 23 24
//      |---+---+---+---+---+---|
// 33 34 35|29130131132133134135|29 30 31
//      |---+---+---+---+---+---|
// 40 41 42|36137138139140141142|36 37 38
//      |---+---+---+---+---+---|
// 47 48 49|43144145146147148149|43 44 45
//      '-----'
/** Basic Conditions for the Rows and Columns: C=175; **
//  n1+n2+n3+n4+n5+n6+n7=C      ... rw1; | n1+n8+n15+n22+n29+n36+n43=C      ... cl1;
//  n8+n9+n10+n11+n12+n13+n14=C ... rw2; | n2+n9+n16+n23+n30+n37+n44=C      ... cl2;
//  n15+n16+n17+n18+n19+n20+n21=C ... rw3; | n3+n10+n17+n24+n31+n38+n45=C      ... cl3;
//  n22+n23+n24+n25+n26+n27+n28=C ... rw4; | n4+n11+n18+n25+n32+n39+n46=C      ... cl4;
//  n29+n30+n31+n32+n33+n34+n35=C ... rw5; | n5+n12+n19+n26+n33+n40+n47=C      ... cl5;
//  n36+n37+n38+n39+n40+n41+n42=C ... rw6; | n6+n13+n20+n27+n34+n41+n48=C      ... cl6;
//  n43+n44+n45+n46+n47+n48+n49=C ... rw7; | n7+n14+n21+n28+n35+n42+n49=C      ... cl7;
/** Conditions for the Padiagonals: C=175; **
//  n1+n9+n17+n25+n33+n41+n49=C      ... pd1; | n1+n14+n20+n26+n32+n38+n44=C      ... pb1;
//  n2+n10+n18+n26+n34+n42+n43=C      ... pd2; | n2+n8+n21+n27+n33+n39+n45=C      ... pb2;
//  n3+n11+n19+n27+n35+n36+n44=C      ... pd3; | n3+n9+n15+n28+n34+n40+n46=C      ... pb3;
//  n4+n12+n20+n28+n29+n37+n45=C      ... pd4; | n4+n10+n16+n22+n35+n41+n47=C      ... pb4;
//  n5+n13+n21+n22+n30+n38+n46=C      ... pd5; | n5+n11+n17+n23+n29+n42+n48=C      ... pb5;
//  n6+n14+n15+n23+n31+n39+n47=C      ... pd6; | n6+n12+n18+n24+n30+n36+n49=C      ... pb6;
//  n7+n8+n16+n24+n32+n40+n48=C      ... pd7; | n7+n13+n19+n25+n31+n37+n43=C      ... pb7;
```

```

/** Conditions for the Symmetric Complementary Pairs: CC=50; **
// n1+n49=n2+n48=n3+n47=n4+n46=n5+n45=n6+n44=n7+n43=
// n8+n42=n9+n41=n10+n40=n11+n39=n12+n38=n13+n37=n14+n36=
// n15+n35=n16+n34=n17+n33=n18+n32=n19+n31=n20+n30=n21+n29=
// n22+n28=n23+n27=n24+n26=n25+n25==CC ... scc
/** Conditions for the 2 types of Combined 6 Numbers: C6=150; **
// n1+n2+n3+n8+n9+n15=C6 ... c61; n35+n41+n42+n47+n48+n49=C6 ... c62;
// n5+n6+n7+n13+n14+n21=C6 ... c63; n29+n36+n37+n43+n44+n45=C6 ... c64;
// n6+n10+n13+n14+n31+n34=C6 ... c65; n16+n19+n36+n37+n40+n44=C6 ... c66;
/** Normalizing Inequality Conditions: n1<n49; n1<n7<n43; **
//
#include <stdio.h>
//
/** Global Variables *
long int cnt, cnt2;
long cntn[5];
short int cnt3;
short LSM, S6N, CC;
short nm[50], uflg[50];
short tnm[5][50];
short lnsn[5][15], pdsm[5][15];
short nt04[44], nt05[42], nt07[38], nt08[36], nt09[34];
short nt11[30], nt12[28], nt15[22], nt18[16];
//
/** Sub-Routiens *
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void stp09(void), stp10(void), stp11(void), stp12(void);
void stp13(void), stp14(void), stp15(void), stp16(void);
void stp17(void), stp18(void), stp19(void), stp20(void);
void stp21(void), stp22(void), stp23(void), stp24(void);
void stp25(void);
void ansprint(void), printxp(short x);
void print2d(void);
void pr3ans(short x);
void chksum(short x);
//
/** Main Program *
/** Set n25=25 *
int main(){
short n;
printf("\n");
printf("** Simultaneous Type of Magic Squares of Order 7: **\n");
printf(" ** Both Self-Complementary and Pan-Diagonal; **\n");
printf("[List of Standard Solutions normalized by n1<n49, n1<n7<n43]\n");
for(n=0;n<50;n++){nm[n]=0; uflg[n]=0;}
LSM=175; S6N=150; CC=50; cnt=0; cnt3=0;
nm[25]=25; uflg[25]=1;
stp01(); /** Begin the Calculations *
uflg[25]=0; nm[25]=0;
if(cnt3>0){printxp(cnt3);}
printf("\n");
printf(" [Count = %ld] OK!\n",cnt);
printf("\n");
printf("** Calculated and Listed by Kanji Setsuda on\n");
printf(" Feb. 7, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 **\n");
printf("\n");
return 0;
}
/** Begin the Calculations *
/** Level #1: **
/** Set N1 & n49 & N1<n49 *
void stp01(){
short a,b;
for(a=1;a<25;a++){b=CC-a;

```

```

    nm[1]=a; nm[49]=b;
    uflg[a]=1; uflg[b]=1;
    stp02();
    uflg[b]=0; uflg[a]=0;
}
}
/** Set N2 & n48 *
void stp02(){
    short a,b;
    for(a=49;a>0;a--){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){cnt2=0;
            nm[2]=a; nm[48]=b;
            uflg[a]=1; uflg[b]=1;
            stp03();
            uflg[b]=0; uflg[a]=0;}}
}
}
/** Set N8 & n42 *
void stp03(){
    short a,b;
    for(a=49;a>0;a--){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){if(nm[1]==1){cnt2=0;}}
            nm[8]=a; nm[42]=b;
            uflg[a]=1; uflg[b]=1;
            stp04();
            uflg[b]=0; uflg[a]=0;}}
}
}
/** Set N9 & n41 *
void stp04(){
    short a,b,n,m;
    m=1;
    for(n=1;n<50;n++){if(uflg[n]==0){nt04[m]=n; m++;}}
    nt04[m]=0;
    for(n=1;n<m;n++){
        a=nt04[n]; b=CC-a;
        nm[9]=a; nm[41]=b;
        uflg[a]=1; uflg[b]=1;
        stp05();
        uflg[b]=0; uflg[a]=0;
    }
}
}
/** Set N3 & n47 *
void stp05(){
    short a,b,n,m;
    m=1;
    for(n=1;n<50;n++){if(uflg[n]==0){nt05[m]=n; m++;}}
    nt05[m]=0;
    for(n=1;n<m;n++){
        a=nt05[n]; b=CC-a;
        nm[3]=a; nm[47]=b;
        uflg[a]=1; uflg[b]=1;
        stp06();
        uflg[b]=0; uflg[a]=0;
    }
}
}
/** Set n15=150-n1-n2-n3-n8-n9 & n35 *
void stp06(){
    short a,b;
    a=S6N-nm[1]-nm[2]-nm[3]-nm[8]-nm[9];
    if((0<a)&&(a<50)){
        b=S6N-nm[49]-nm[48]-nm[47]-nm[42]-nm[41];
        if(a+b==CC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[15]=a; nm[35]=b;
            }
        }
    }
}

```

```

        uflg[a]=1; uflg[b]=1;
        stp07();
        uflg[b]=0; uflg[a]=0;}
    }}
}
/** Level #2: *
/** Set N5 & n45 *
void stp07(){
    short a,b,n,m;
    m=1;
    for(n=1;n<50;n++){if(uflg[n]==0){nt07[m]=n; m++;}}
    nt07[m]=0;
    for(n=m-1;n>0;n--){
        a=nt07[n]; b=CC-a;
        nm[5]=a; nm[45]=b;
        uflg[a]=1; uflg[b]=1;
        stp08();
        uflg[b]=0; uflg[a]=0;
    }
}
/** Set N6 & n44 *
void stp08(){
    short a,b,n,m;
    m=1;
    for(n=1;n<50;n++){if(uflg[n]==0){nt08[m]=n; m++;}}
    nt08[m]=0;
    for(n=1;n<m;n++){
        a=nt08[n]; b=CC-a;
        nm[6]=a; nm[44]=b;
        uflg[a]=1; uflg[b]=1;
        stp09();
        uflg[b]=0; uflg[a]=0;
    }
}
/** Set N7 & n43 & N1<N7<n43 *
void stp09(){
    short a,b,n,m;
    m=1;
    for(n=1;n<50;n++){if(uflg[n]==0){nt09[m]=n; m++;}}
    nt09[m]=0;
    for(n=m-1;n>0;n--){
        a=nt09[n]; b=CC-a;
        if((nm[1]<a)&&(a<b)){
            nm[7]=a; nm[43]=b;
            uflg[a]=1; uflg[b]=1;
            stp10();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set n4=LSM-n1-n2-n3-n5-n6-n7 & n46 *
void stp10(){
    short a,b;
    a=LSM-nm[1]-nm[2]-nm[3]-nm[5]-nm[6]-nm[7];
    if((0<a)&&(a<50)){
        b=LSM-nm[49]-nm[48]-nm[47]-nm[45]-nm[44]-nm[43];
        if(a+b==CC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[4]=a; nm[46]=b;
                uflg[a]=1; uflg[b]=1;
                stp11();
                uflg[b]=0; uflg[a]=0;}
            }}
}
/** Set N14 & n36 *
void stp11(){

```

```

short a,b,n,m;
m=1;
for(n=1;n<50;n++){if(uflg[n]==0){nt11[m]=n; m++;}}
nt11[m]=0;
for(n=1;n<m;n++){
a=nt11[n]; b=CC-a;
nm[14]=a; nm[36]=b;
uflg[a]=1; uflg[b]=1;
stp12();
uflg[b]=0; uflg[a]=0;
}
}
/* Set N13 & n37 *
void stp12(){
short a,b,n,m;
m=1;
for(n=1;n<50;n++){if(uflg[n]==0){nt12[m]=n; m++;}}
nt12[m]=0;
for(n=1;n<m;n++){
a=nt12[n]; b=CC-a;
nm[13]=a; nm[37]=b;
uflg[a]=1; uflg[b]=1;
stp13();
uflg[b]=0; uflg[a]=0;
}
}
/* Set n21=150-n5-n6-n7-n13-n14 & n29 *
void stp13(){
short a,b;
a=S6N-nm[5]-nm[6]-nm[7]-nm[13]-nm[14];
if((0<a)&&(a<50)){
b=S6N-nm[45]-nm[44]-nm[43]-nm[37]-nm[36];
if(a+b==CC){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[21]=a; nm[29]=b;
uflg[a]=1; uflg[b]=1;
stp14();
uflg[b]=0; uflg[a]=0;}}
}}
}
/* Set n22=LSM-n1-n8-n15-n29-n36-n43 & n28 *
void stp14(){
short a,b;
a=LSM-nm[1]-nm[8]-nm[15]-nm[29]-nm[36]-nm[43];
if((0<a)&&(a<50)){
b=LSM-nm[49]-nm[42]-nm[35]-nm[21]-nm[14]-nm[7];
if(a+b==CC){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[22]=a; nm[28]=b;
uflg[a]=1; uflg[b]=1;
stp15();
uflg[b]=0; uflg[a]=0;}}
}}
}
/* Level #3: *
/* Set N16 & n34 *
void stp15(){
short a,b,n,m;
m=1;
for(n=1;n<50;n++){if(uflg[n]==0){nt15[m]=n; m++;}}
nt15[m]=0;
for(n=1;n<m;n++){
a=nt15[n]; b=CC-a;
nm[16]=a; nm[34]=b;
uflg[a]=1; uflg[b]=1;

```

```

    stp16();
    uflg[b]=0; uflg[a]=0;
}
}
/** Set n10=LSM-n4-n16-n22-n35-n41-n47 & n40 *
void stp16(){
short a,b;
a=LSM-nm[4]-nm[16]-nm[22]-nm[35]-nm[41]-nm[47];
if((0<a)&&(a<50)){
b=LSM-nm[3]-nm[9]-nm[15]-nm[28]-nm[34]-nm[46];
if(a+b==CC){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[10]=a; nm[40]=b;
uflg[a]=1; uflg[b]=1;
stp17();
uflg[b]=0; uflg[a]=0;}
}}
}
/** Set n31=150-n6-n10-n13-n14-n34 & n19 *
void stp17(){
short a,b;
a=S6N-nm[6]-nm[10]-nm[13]-nm[14]-nm[34];
if((0<a)&&(a<50)){
b=S6N-nm[44]-nm[40]-nm[37]-nm[36]-nm[16];
if(a+b==CC){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[31]=a; nm[19]=b;
uflg[a]=1; uflg[b]=1;
if(nm[7]+nm[13]+a+nm[25]+b+nm[37]+nm[43]==LSM){stp18();}
uflg[b]=0; uflg[a]=0;}
}}
}
/** Set n12 & n38 *
void stp18(){
short a,b,n,m;
m=1;
for(n=1;n<50;n++){if(uflg[n]==0){nt18[m]=n; m++;}}
nt18[m]=0;
for(n=1;n<m;n++){
a=nt18[n]; b=CC-a;
nm[12]=a; nm[38]=b;
uflg[a]=1; uflg[b]=1;
stp19();
uflg[b]=0; uflg[a]=0;
}
}
/** Set n20=LSM-n4-n12-n28-n29-n37-n45 & n30 *
void stp19(){
short a,b;
a=LSM-nm[4]-nm[12]-nm[28]-nm[29]-nm[37]-nm[45];
if((0<a)&&(a<50)){
b=LSM-nm[5]-nm[13]-nm[21]-nm[22]-nm[38]-nm[46];
if(a+b==CC){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[20]=a; nm[30]=b;
uflg[a]=1; uflg[b]=1;
stp20();
uflg[b]=0; uflg[a]=0;}
}}
}
/** Set n11=LSM-n8-n9-n10-n12-n13-n14 & n39 *
void stp20(){
short a,b;
a=LSM-nm[8]-nm[9]-nm[10]-nm[12]-nm[13]-nm[14];
if((0<a)&&(a<50)){

```

```

b=LSM-nm[42]-nm[41]-nm[40]-nm[38]-nm[37]-nm[36];
if(a+b==CC){
    if((uflg[a]==0)&&(uflg[b]==0)){
        nm[11]=a; nm[39]=b;
        uflg[a]=1; uflg[b]=1;
        stp21();
        uflg[b]=0; uflg[a]=0;}
    }}
}
/* Set n23=LSM-n2-n9-n16-n30-n37-n44 & n27 *
void stp21(){
    short a,b,c,d;
    a=LSM-nm[2]-nm[9]-nm[16]-nm[30]-nm[37]-nm[44];
    if((0<a)&&(a<50)){
        b=LSM-nm[48]-nm[41]-nm[34]-nm[20]-nm[13]-nm[6];
        if(a+b==CC){
            c=LSM-nm[6]-nm[14]-nm[15]-nm[31]-nm[39]-nm[47];
            d=LSM-nm[3]-nm[11]-nm[19]-nm[35]-nm[36]-nm[44];
            if((a==c)&&(b==d)){
                if((uflg[a]==0)&&(uflg[b]==0)){
                    nm[23]=a; nm[27]=b;
                    uflg[a]=1; uflg[b]=1;
                    stp22();
                    uflg[b]=0; uflg[a]=0;}
                }}}
    }
}
/* Set n17=LSM-n5-n11-n23-n29-n42-n48 & n33 *
void stp22(){
    short a,b;
    a=LSM-nm[5]-nm[11]-nm[23]-nm[29]-nm[42]-nm[48];
    if((0<a)&&(a<50)){
        b=LSM-nm[2]-nm[8]-nm[21]-nm[27]-nm[39]-nm[45];
        if(a+b==CC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[17]=a; nm[33]=b;
                uflg[a]=1; uflg[b]=1;
                if(nm[1]+nm[9]+a+nm[25]+b+nm[41]+nm[49]==LSM){stp23();}
                uflg[b]=0; uflg[a]=0;}
            }}
    }
}
/* Set n18=LSM-n15-n16-n17-n19-n20-n21 & n32 *
void stp23(){
    short a,b;
    a=LSM-nm[15]-nm[16]-nm[17]-nm[19]-nm[20]-nm[21];
    if((0<a)&&(a<50)){
        b=LSM-nm[35]-nm[34]-nm[33]-nm[31]-nm[30]-nm[29];
        if(a+b==CC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[18]=a; nm[32]=b;
                uflg[a]=1; uflg[b]=1;
                if(nm[4]+nm[11]+a+nm[25]+b+nm[39]+nm[46]==LSM){stp24();}
                uflg[b]=0; uflg[a]=0;}
            }}
    }
}
/* Set n24=LSM-n3-n10-n17-n31-n38-n45 & n26 *
void stp24(){
    short a,b;
    a=LSM-nm[3]-nm[10]-nm[17]-nm[31]-nm[38]-nm[45];
    if((0<a)&&(a<50)){
        b=LSM-nm[5]-nm[12]-nm[19]-nm[33]-nm[40]-nm[47];
        if(a+b==CC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[24]=a; nm[26]=b;
                uflg[a]=1; uflg[b]=1;
                if(nm[22]+nm[23]+a+nm[25]+b+nm[27]+nm[28]==LSM){stp25();}
            }}
    }
}

```



```

        uflg[b]=0; uflg[a]=0;}
    }}
}
/* Check The Pan-Diagonals *
void stp25(){
    short sm1,sm2,sm3,sm4;
    sm1=nm[6]+nm[12]+nm[18]+nm[24]+nm[30]+nm[36]+nm[49];
    sm2=nm[7]+nm[8]+nm[16]+nm[24]+nm[32]+nm[40]+nm[48];
    sm3=nm[1]+nm[14]+nm[20]+nm[26]+nm[32]+nm[38]+nm[44];
    sm4=nm[2]+nm[10]+nm[18]+nm[26]+nm[34]+nm[42]+nm[43];
    if((sm1==LSM)&&(sm2==LSM)&&(sm3==LSM)&&(sm4==LSM)){ansprint();}
}
//
/* Print The Answers *
void ansprint(){
    short n;
    cnt++; cnt2++;
    if(cnt2==1){
        cntn[cnt3]=cnt;
        for(n=1;n<50;n++){tnm[cnt3][n]=nm[n];}
        chksum(cnt3);
        cnt3++;
        //if(cnt3==2){print2d(); cnt3=0;}
        if(cnt3==3){pr3ans(cnt3); cnt3=0;}
        //if(cnt3==4){printxp(cnt3); cnt3=0;}
    }
}
//
/* Print 4 Answers *
void printxp(short x){
    short l,l7,m,n;
    for(n=0;n<x;n++){
        printf("%21ld#",cntn[n]);
        if(n+1<x){printf(" ");}
    }
    printf("\n");
    for(l=0;l<7;l++){l7=l*7;
        for(n=0;n<x;n++){
            printf(" ");
            for(m=1;m<8;m++){printf("%3d",tnm[n][l7+m]);}
            if(n+1<x){printf(" ");}
        }
        printf("\n");
    }
}
//
/* Print 2 Answers with /D7i *
void print2d(){
    short l,l7,m,n;
    printf("%22ld# /D7i H/      L/",cntn[0]);
    printf("%22ld# /D7i H/      L^\n",cntn[1]);
    for(l=0;l<7;l++){l7=l*7;
        for(n=0;n<2;n++){
            printf(" ");
            for(m=1;m<8;m++){printf("%3d",tnm[n][l7+m]);}
            printf(" ");
            for(m=1;m<8;m++){printf("%d", (tnm[n][l7+m]-1)/7);}
            printf(" ");
            for(m=1;m<8;m++){printf("%d", (tnm[n][l7+m]-1)%7);}
        }
        printf("\n");
    }
}
//
/* Print the Answers with Check Sum-Errors *

```

```

void pr3ans(short x){
short m,l,n;
for(m=0;m<x;m++){
printf("%20ld#IRIC\\P/pi",cntn[m]);
if(m+1<x){printf(" ");}}
printf("\\n");
for(l=0;l<7;l++){
for(m=0;m<x;m++){
for(n=1;n<8;n++){printf("%3d",tnm[m][l*7+n]);}
printf("%d\\%d\\%d\\%d",lnsm[m][l+1],lnsm[m][l+8],pdsm[m][l+1],pdsm[m][l+8]);
if(m+1<x){printf(" ");}
}
printf("\\n");
}
}
//
/* Check Sums of Rows, Columns and Pan-diagonals *
void chksum(short x){
lnsm[x][1]=nm[1]+nm[2]+nm[3]+nm[4]+nm[5]+nm[6]+nm[7]-LSM;
lnsm[x][2]=nm[8]+nm[9]+nm[10]+nm[11]+nm[12]+nm[13]+nm[14]-LSM;
lnsm[x][3]=nm[15]+nm[16]+nm[17]+nm[18]+nm[19]+nm[20]+nm[21]-LSM;
lnsm[x][4]=nm[22]+nm[23]+nm[24]+nm[25]+nm[26]+nm[27]+nm[28]-LSM;
lnsm[x][5]=nm[29]+nm[30]+nm[31]+nm[32]+nm[33]+nm[34]+nm[35]-LSM;
lnsm[x][6]=nm[36]+nm[37]+nm[38]+nm[39]+nm[40]+nm[41]+nm[42]-LSM;
lnsm[x][7]=nm[43]+nm[44]+nm[45]+nm[46]+nm[47]+nm[48]+nm[49]-LSM;
lnsm[x][8]=nm[1]+nm[8]+nm[15]+nm[22]+nm[29]+nm[36]+nm[43]-LSM;
lnsm[x][9]=nm[2]+nm[9]+nm[16]+nm[23]+nm[30]+nm[37]+nm[44]-LSM;
lnsm[x][10]=nm[3]+nm[10]+nm[17]+nm[24]+nm[31]+nm[38]+nm[45]-LSM;
lnsm[x][11]=nm[4]+nm[11]+nm[18]+nm[25]+nm[32]+nm[39]+nm[46]-LSM;
lnsm[x][12]=nm[5]+nm[12]+nm[19]+nm[26]+nm[33]+nm[40]+nm[47]-LSM;
lnsm[x][13]=nm[6]+nm[13]+nm[20]+nm[27]+nm[34]+nm[41]+nm[48]-LSM;
lnsm[x][14]=nm[7]+nm[14]+nm[21]+nm[28]+nm[35]+nm[42]+nm[49]-LSM;
//
pdsm[x][1]=nm[1]+nm[9]+nm[17]+nm[25]+nm[33]+nm[41]+nm[49]-LSM;
pdsm[x][2]=nm[2]+nm[10]+nm[18]+nm[26]+nm[34]+nm[42]+nm[43]-LSM;
pdsm[x][3]=nm[3]+nm[11]+nm[19]+nm[27]+nm[35]+nm[36]+nm[44]-LSM;
pdsm[x][4]=nm[4]+nm[12]+nm[20]+nm[28]+nm[29]+nm[37]+nm[45]-LSM;
pdsm[x][5]=nm[5]+nm[13]+nm[21]+nm[22]+nm[30]+nm[38]+nm[46]-LSM;
pdsm[x][6]=nm[6]+nm[14]+nm[15]+nm[23]+nm[31]+nm[39]+nm[47]-LSM;
pdsm[x][7]=nm[7]+nm[8]+nm[16]+nm[24]+nm[32]+nm[40]+nm[48]-LSM;
pdsm[x][8]=nm[1]+nm[14]+nm[20]+nm[26]+nm[32]+nm[38]+nm[44]-LSM;
pdsm[x][9]=nm[2]+nm[8]+nm[21]+nm[27]+nm[33]+nm[39]+nm[45]-LSM;
pdsm[x][10]=nm[3]+nm[9]+nm[15]+nm[28]+nm[34]+nm[40]+nm[46]-LSM;
pdsm[x][11]=nm[4]+nm[10]+nm[16]+nm[22]+nm[35]+nm[41]+nm[47]-LSM;
pdsm[x][12]=nm[5]+nm[11]+nm[17]+nm[23]+nm[29]+nm[42]+nm[48]-LSM;
pdsm[x][13]=nm[6]+nm[12]+nm[18]+nm[24]+nm[30]+nm[36]+nm[49]-LSM;
pdsm[x][14]=nm[7]+nm[13]+nm[19]+nm[25]+nm[31]+nm[37]+nm[43]-LSM;
}
//

```