

## Chapter 6: Fundamental Studies of Multi-Dimensional Extra-Cubic Magic Forms and their Decompositions

### Section 2: Study of 4-Dimensional Extra-Cubic Magic Forms of Order 2 and their Decompositions

#### Section 2-2: Transformations and 'Fundamental Solutions' of Pan-Magic Squares 4 x 4: Kanji Setsuda

#1. In the former section we studied about 4-dimensional extra-cubic magic form of order 2, and finally obtained three 'initial' solutions of (1) self-complementary type and of (2) pan-diagonal (or complete or 'composite and complete') type of magic squares of order 4, by decomposing the original and converting its higher dimension down to lower.

\*\*\*\* Four Dimensional Extra-Cubic Forms  $2^4$  \*\*\*\*

\*\* With 2 Types of Decomposed Magic Squares  $4^2$  \*\*

\*\* Under the Conditions:  $n1 < n16, n1 < n5, n1 < n9, n2 > n3, n2 > n9, n3 > n5$  \*\*

0/n1	/n2	0/SC	/PM
n1---n5	n9---13	n1 n2 n9 10	n1 n2 10 n9
n2---n6	10---14	n3 n4 11 12	n3 n4 12 11
n3- -n7	11- -15	n5 n6 13 14	n7 n8 16 15
n4---n8	12---16	n7 n8 15 16	n5 n6 14 13

[Basic Positions] [Self-Complementary] [Pan-Magic]

\*\* Three Reperesentative Solutions: \*\*

1/n1	/n2	1/SC	/PD
1---12	8---13	1 15 8 10	1 15 10 8
15--- 6	10--- 3	14 4 11 5	14 4 5 11
14- - 7	11- - 2	12 6 13 3	7 9 16 2
4--- 9	5---16	7 9 2 16	12 6 3 13

2/n1	/n2	2/SC	/PD
1--- 8	12---13	1 15 12 6	1 15 6 12
15---10	6--- 3	14 4 7 9	14 4 9 7
14- -11	7- - 2	8 10 13 3	11 5 16 2
4--- 5	9---16	11 5 2 16	8 10 3 13

3/n1	/n2	3/SC	/PD
1--- 8	14---11	1 15 14 4	1 15 4 14
15---10	4--- 5	12 6 7 9	12 6 9 7
12- -13	7- - 2	8 10 11 5	13 3 16 2
6--- 3	9---16	13 3 2 16	8 10 5 11

[Count = 3]

Each solution on the right most is transformed from each Self-Complementary one in the center. It is made into Pan-diagonal (or 'Complete' or 'C&C') type by exchanging the whole lines of both 3rd and 4th rows and columns.

#2. But are these three solutions really 'fundamental' to all pan-diagonal magic squares of order 4?

We have to verify that by demonstrating any clear evidences such as 'transformation

methods' of making these three into 48.

I know there are various ways of making them, but now let me introduce you 3 typical transformation systems making all 48 pan-diagonal magic squares 4x4 out of F3.

The goal objects we want to get to and F3 on the start line are listed full as below.

```
** 'Composite & Complete' Magic Squares 4x4 **
```

```
*** Print All 48 Solutions ***
```

```

      1/G1      2/G2      3/G1      4/G3      5/G2      6/G3
1 15 10 8   1 15 6 12   1 15 10 8   1 15 4 14   1 15 6 12   1 15 4 14
14 4 5 11  14 4 9 7   12 6 3 13  12 6 9 7   8 10 3 13   8 10 5 11
7 9 16 2   11 5 16 2   7 9 16 2   13 3 16 2   11 5 16 2   13 3 16 2
12 6 3 13  8 10 3 13  14 4 5 11  8 10 5 11  14 4 9 7   12 6 9 7

      7/G2      8/G3      9/G1      10/G3     11/G1     12/G2
1 14 11 8   1 14 4 15   1 14 7 12   1 14 4 15   1 12 7 14   1 12 6 15
12 7 2 13  12 7 9 6   8 11 2 13   8 11 5 10   8 13 2 11   8 13 3 10
6 9 16 3   13 2 16 3   10 5 16 3   13 2 16 3   10 3 16 5   11 2 16 5
15 4 5 10  8 11 5 10  15 4 9 6   12 7 9 6   15 6 9 4   14 7 9 4

      13/G1     14/G2     15/G1     16/G3     17/G2     18/G3
2 16 9 7   2 16 5 11   2 16 9 7   2 16 3 13   2 16 5 11   2 16 3 13
13 3 6 12  13 3 10 8   11 5 4 14   11 5 10 8   7 9 4 14   7 9 6 12
8 10 15 1   12 6 15 1   8 10 15 1   14 4 15 1   12 6 15 1   14 4 15 1
11 5 4 14  7 9 4 14   13 3 6 12   7 9 6 12   13 3 10 8   11 5 10 8

      19/G2     20/G3     21/G1     22/G3     23/G1     24/G2
2 13 12 7   2 13 3 16   2 13 8 11   2 13 3 16   2 11 8 13   2 11 5 16
11 8 1 14  11 8 10 5   7 12 1 14   7 12 6 9   7 14 1 12   7 14 4 9
5 10 15 4   14 1 15 4   9 6 15 4   14 1 15 4   9 4 15 6   12 1 15 6
16 3 6 9   7 12 6 9   16 3 10 5   11 8 10 5   16 5 10 3   13 8 10 3

      25/G2     26/G1     27/G2     28/G3     29/G1     30/G3
3 16 9 6   3 16 5 10   3 16 9 6   3 16 2 13   3 16 5 10   3 16 2 13
13 2 7 12  13 2 11 8   10 5 4 15   10 5 11 8   6 9 4 15   6 9 7 12
8 11 14 1   12 7 14 1   8 11 14 1   15 4 14 1   12 7 14 1   15 4 14 1
10 5 4 15  6 9 4 15   13 2 7 12   6 9 7 12   13 2 11 8   10 5 11 8

      31/G1     32/G3     33/G2     34/G3     35/G2     36/G1
3 13 12 6   3 13 2 16   3 13 8 10   3 13 2 16   4 15 10 5   4 15 6 9
10 8 1 15  10 8 11 5   6 12 1 15   6 12 7 9   14 1 8 11   14 1 12 7
5 11 14 4   15 1 14 4   9 7 14 4   15 1 14 4   7 12 13 2   11 8 13 2
16 2 7 9   6 12 7 9   16 2 11 5   10 8 11 5   9 6 3 16   5 10 3 16

      37/G2     38/G3     39/G1     40/G3     41/G1     42/G3
4 15 10 5   4 15 1 14   4 15 6 9   4 15 1 14   4 14 11 5   4 14 1 15
9 6 3 16   9 6 12 7   5 10 3 16   5 10 8 11   9 7 2 16   9 7 12 6
7 12 13 2   16 3 13 2   11 8 13 2   16 3 13 2   6 12 13 3   16 2 13 3
14 1 8 11   5 10 8 11   14 1 12 7   9 6 12 7   15 1 8 10   5 11 8 10

      43/G2     44/G3     45/G1     46/G2     47/G1     48/G2
4 14 7 9   4 14 1 15   5 16 3 10   5 16 2 11   6 15 4 9   6 15 1 12
5 11 2 16  5 11 8 10   4 9 6 15   4 9 7 14   3 10 5 16   3 10 8 13
10 8 13 3   16 2 13 3   14 7 12 1   15 6 12 1   13 8 11 2   16 5 11 2
15 1 12 6   9 7 12 6   11 2 13 8   10 3 13 8   12 1 14 7   9 4 14 7

```

('/Gn' on the right shoulder means the group each solution belongs to.)

```
[Count = 48]
```

```
** 'Composite & Complete' Magic Squares 4x4 **
```

```
*** Print All Fundamental Solutions ***
```

		1/			2/			3/			
1	15	10	8	1	15	6	12	1	15	4	14
14	4	5	11	14	4	9	7	12	6	9	7
7	9	16	2	11	5	16	2	13	3	16	2
12	6	3	13	8	10	3	13	8	10	5	11

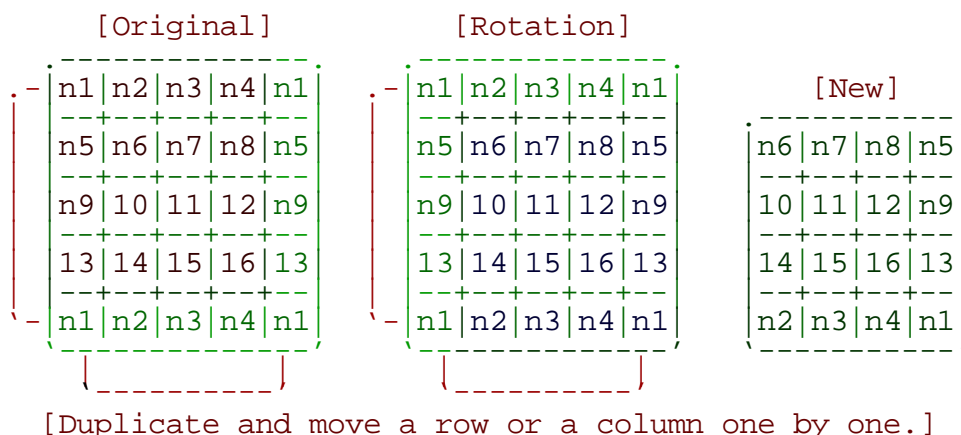
[Count = 3]

We have to make these three fundamental solutions into all those 48 goal objects above.

- (1) Classify all those 48 into three groups each of which contains 16 solutions including the fundamental one. Compare every solution to the F3, and find any similar ones whose rows and columns consist of the same 4 members and collect them into the same group.
- (2) Analyse all in the same group and try to imagine and invent anything like a 'transformation method' making all from each fundamental.
- (3) Any transformation method you make must be simple and common enough to apply to every group.

The first two transformation systems were originally invented by making the unique property of pan-diagonal magic squares fully expressed.

In any pan-diagonal magic square, as you know, you can move each row or each column on any edge far to the opposite side and put it beyond the frame. You can do that in any direction as you like, and repeat it as often as you want.

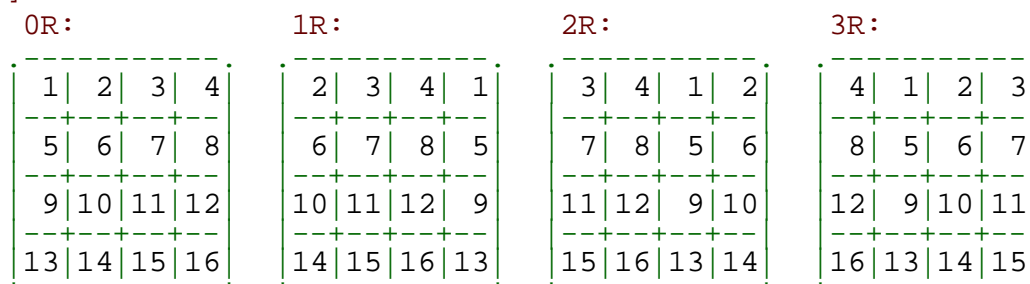


You can get a new pan-magic square by this kind of 'rotation.' Take a look at the figure above, and watch every row or every column of the new square having the same four numbers with the original, and all pan-diagonals also consisting of the same members.

#3. Let's take these rotations for our transformation system as follows:

**\*\* Concept Diagram: Transformation System 1: \*\***

[0]



4R:

5	6	7	8
9	10	11	12
13	14	15	16
1	2	3	4

5R:

6	7	8	5
10	11	12	9
14	15	16	13
2	3	4	1

6R:

7	8	5	6
11	12	9	10
15	16	13	14
3	4	1	2

7R:

8	5	6	7
12	9	10	11
16	13	14	15
4	1	2	3

8R:

9	10	11	12
13	14	15	16
1	2	3	4
5	6	7	8

9R:

10	11	12	9
14	15	16	13
2	3	4	1
6	7	8	5

10R:

11	12	9	10
15	16	13	14
3	4	1	2
7	8	5	6

11R:

12	9	10	11
16	13	14	15
4	1	2	3
8	5	6	7

12R:

13	14	15	16
1	2	3	4
5	6	7	8
9	10	11	12

13R:

14	15	16	13
2	3	4	1
6	7	8	5
10	11	12	9

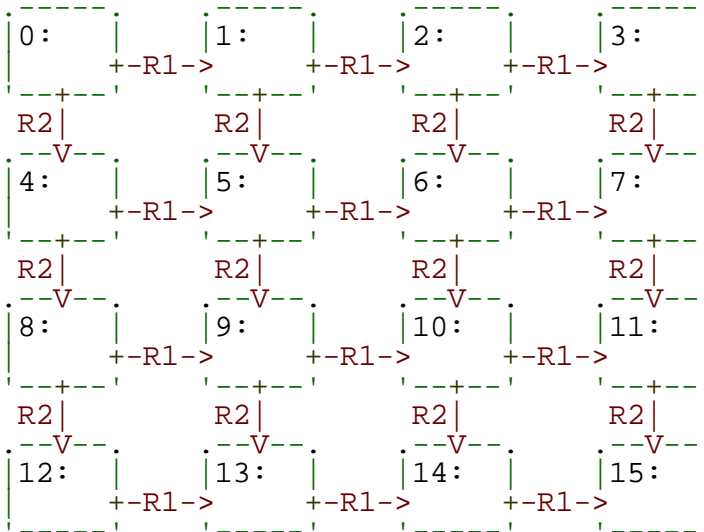
14R:

15	16	13	14
3	4	1	2
7	8	5	6
11	12	9	10

15R:

16	13	14	15
4	1	2	3
8	5	6	7
12	9	10	11

### \*\* Process Transition Diagram \*\*



/\*\*/

/\* Rotation Type 1 and 2 \*/

void rotat1(void){

short n;

```

    d[1]=c[2];    d[2]=c[3];    d[3]=c[4];    d[4]=c[1];
    d[5]=c[6];    d[6]=c[7];    d[7]=c[8];    d[8]=c[5];
    d[9]=c[10];   d[10]=c[11];   d[11]=c[12];  d[12]=c[9];
    d[13]=c[14];  d[14]=c[15];  d[15]=c[16];  d[16]=c[13];
    for(n=1; n<17; n++){ c[n]=d[n]; }

```

}

```

/**/
void rotat2(void) {
    short n;
    d[1]=c[5];    d[2]=c[6];    d[3]=c[7];    d[4]=c[8];
    d[5]=c[9];    d[6]=c[10];   d[7]=c[11];   d[8]=c[12];
    d[9]=c[13];   d[10]=c[14];  d[11]=c[15];  d[12]=c[16];
    d[13]=c[1];   d[14]=c[2];   d[15]=c[3];   d[16]=c[4];
    for(n=1;n<17;n++){c[n]=d[n];}
}
/**/
/* normalisation */
void normals(void) {
    short n;
    for(n=1;n<17;n++){d[n]=c[n];}
    if(c[4]>c[13]){
        d[1]=c[1];    d[2]=c[5];    d[3]=c[9];    d[4]=c[13];
        d[5]=c[2];    d[6]=c[6];    d[7]=c[10];   d[8]=c[14];
        d[9]=c[3];    d[10]=c[7];   d[11]=c[11];  d[12]=c[15];
        d[13]=c[4];   d[14]=c[8];   d[15]=c[12];  d[16]=c[16];
        for(n=1;n<17;n++){c[n]=d[n];}}
    if(c[1]>c[13]){
        d[1]=c[13];   d[2]=c[14];   d[3]=c[15];   d[4]=c[16];
        d[5]=c[9];    d[6]=c[10];   d[7]=c[11];   d[8]=c[12];
        d[9]=c[5];    d[10]=c[6];   d[11]=c[7];   d[12]=c[8];
        d[13]=c[1];   d[14]=c[2];   d[15]=c[3];   d[16]=c[4];
        for(n=1;n<17;n++){c[n]=d[n];}}
    if(c[1]>c[16]){
        d[1]=c[16];   d[2]=c[12];   d[3]=c[8];    d[4]=c[4];
        d[5]=c[15];   d[6]=c[11];   d[7]=c[7];    d[8]=c[3];
        d[9]=c[14];   d[10]=c[10];  d[11]=c[6];   d[12]=c[2];
        d[13]=c[13];  d[14]=c[9];   d[15]=c[5];   d[16]=c[1];
        for(n=1;n<17;n++){c[n]=d[n];}}
    if(c[1]>c[4]){
        d[1]=c[4];    d[2]=c[3];    d[3]=c[2];    d[4]=c[1];
        d[5]=c[8];    d[6]=c[7];    d[7]=c[6];    d[8]=c[5];
        d[9]=c[12];   d[10]=c[11];  d[11]=c[10];  d[12]=c[9];
        d[13]=c[16];  d[14]=c[15];  d[15]=c[14];  d[16]=c[13];
        for(n=1;n<17;n++){c[n]=d[n];}}
}
/**/

```

We only use two types of rotations that do the same thing in two directions: (1) left-right, and (2) up-down. I dictated the 'transformation rules' as shown above for our computer program, and drew the 'process transition diagram' illustrating how to use them.

Let's apply this new system to each 'representative' solution and transform it into other 15 solutions.

The next list gives you the first result of our experiment. It is kept 'pre-normalised'.

```

** Pan-Diagonal (or 'Complete' or 'C&C') Magic Square 4x4 **
** Transform Each Fundamental into 15 Solutions (Pre-normalised) **

```

[1]

0T:	1/	1T:	2T:	3T:
<pre>-----  1 15 10  8  -----+-----+----- 14  4  5 11  -----+-----+-----  7  9 16  2  -----+-----+----- 12  6  3 13  -----</pre>		<pre>----- 15 10  8  1  -----+-----+-----  4  5 11 14  -----+-----+-----  9 16  2  7  -----+-----+-----  6  3 13 12  -----</pre>	<pre>----- 10  8  1 15  -----+-----+-----  5 11 14  4  -----+-----+----- 16  2  7  9  -----+-----+-----  3 13 12  6  -----</pre>	<pre>-----  8  1 15 10  -----+-----+----- 11 14  4  5  -----+-----+-----  2  7  9 16  -----+-----+----- 13 12  6  3  -----</pre>
4T:		5T:	6T:	7T:
<pre>----- 14  4  5 11  -----+-----+-----  7  9 16  2  -----+-----+----- 12  6  3 13  -----+-----+-----  1 15 10  8  -----</pre>		<pre>-----  4  5 11 14  -----+-----+-----  9 16  2  7  -----+-----+-----  6  3 13 12  -----+-----+----- 15 10  8  1  -----</pre>	<pre>-----  5 11 14  4  -----+-----+----- 16  2  7  9  -----+-----+-----  3 13 12  6  -----+-----+----- 10  8  1 15  -----</pre>	<pre>----- 11 14  4  5  -----+-----+-----  2  7  9 16  -----+-----+----- 13 12  6  3  -----+-----+-----  8  1 15 10  -----</pre>
8T:		9T:	10T:	11T:
<pre>-----  7  9 16  2  -----+-----+----- 12  6  3 13  -----+-----+-----  1 15 10  8  -----+-----+----- 14  4  5 11  -----</pre>		<pre>-----  9 16  2  7  -----+-----+-----  6  3 13 12  -----+-----+----- 15 10  8  1  -----+-----+-----  4  5 11 14  -----</pre>	<pre>----- 16  2  7  9  -----+-----+-----  3 13 12  6  -----+-----+----- 10  8  1 15  -----+-----+-----  5 11 14  4  -----</pre>	<pre>-----  2  7  9 16  -----+-----+----- 13 12  6  3  -----+-----+-----  8  1 15 10  -----+-----+----- 11 14  4  5  -----</pre>
12T:		13T:	14T:	15T:
<pre>----- 12  6  3 13  -----+-----+-----  1 15 10  8  -----+-----+----- 14  4  5 11  -----+-----+-----  7  9 16  2  -----</pre>		<pre>-----  6  3 13 12  -----+-----+----- 15 10  8  1  -----+-----+-----  4  5 11 14  -----+-----+-----  9 16  2  7  -----</pre>	<pre>-----  3 13 12  6  -----+-----+----- 10  8  1 15  -----+-----+-----  5 11 14  4  -----+-----+----- 16  2  7  9  -----</pre>	<pre>----- 13 12  6  3  -----+-----+-----  8  1 15 10  -----+-----+----- 11 14  4  5  -----+-----+-----  2  7  9 16  -----</pre>

But which solution could be identified with each result? Could you really find the same answer in the solution list we already know?

Could you understand we have to rewrite each result so smart as to meet with the list standard under the conditions:  $n1 < n16$ ;  $n1 < n4$ ;  $n1 < n13$ ;  $n2 > n5$ ?

The next list shows all the results 'normalised' appropriately.

**\*\*\* Pan-Diagonal Magic Squares 4x4: \*\*\***

\* Transform Each Fundamental into 15 Solutions (Normalised) \*

\*\* each 'n/' shows the old number of the solution list \*\*

[1]

0T:	1/	1T:	9/	2T:	26/	3T:	29/
<pre>-----  1 15 10  8  -----+-----+----- 14  4  5 11  -----+-----+-----  7  9 16  2  -----+-----+----- 12  6  3 13  -----</pre>		<pre>-----  1 14  7 12  -----+-----+-----  8 11  2 13  -----+-----+----- 10  5 16  3  -----+-----+----- 15  4  9  6  -----</pre>		<pre>-----  3 16  5 10  -----+-----+----- 13  2 11  8  -----+-----+----- 12  7 14  1  -----+-----+-----  6  9  4 15  -----</pre>		<pre>-----  3 16  5 10  -----+-----+-----  6  9  4 15  -----+-----+----- 12  7 14  1  -----+-----+----- 13  2 11  8  -----</pre>	

4T: 3/	5T: 11/	6T: 41/	7T: 45/
<pre> 1 15 10  8  --+-+--+ 12  6  3 13  --+-+--+  7  9 16  2  --+-+--+ 14  4  5 11  --+-+--+ </pre>	<pre> 1 12  7 14  --+-+--+  8 13  2 11  --+-+--+ 10  3 16  5  --+-+--+ 15  6  9  4  --+-+--+ </pre>	<pre> 4 14 11  5  --+-+--+  9  7  2 16  --+-+--+  6 12 13  3  --+-+--+ 15  1  8 10  --+-+--+ </pre>	<pre> 5 16  3 10  --+-+--+  4  9  6 15  --+-+--+ 14  7 12  1  --+-+--+ 11  2 13  8  --+-+--+ </pre>

8T: 13/	9T: 39/	10T: 36/	11T: 21/
<pre> 2 16  9  7  --+-+--+ 13  3  6 12  --+-+--+  8 10 15  1  --+-+--+ 11  5  4 14  --+-+--+ </pre>	<pre> 4 15  6  9  --+-+--+  5 10  3 16  --+-+--+ 11  8 13  2  --+-+--+ 14  1 12  7  --+-+--+ </pre>	<pre> 4 15  6  9  --+-+--+ 14  1 12  7  --+-+--+ 11  8 13  2  --+-+--+  5 10  3 16  --+-+--+ </pre>	<pre> 2 13  8 11  --+-+--+  7 12  1 14  --+-+--+  9  6 15  4  --+-+--+ 16  3 10  5  --+-+--+ </pre>

12T: 15/	13T: 47/	14T: 31/	15T: 23/
<pre> 2 16  9  7  --+-+--+ 11  5  4 14  --+-+--+  8 10 15  1  --+-+--+ 13  3  6 12  --+-+--+ </pre>	<pre> 6 15  4  9  --+-+--+  3 10  5 16  --+-+--+ 13  8 11  2  --+-+--+ 12  1 14  7  --+-+--+ </pre>	<pre> 3 13 12  6  --+-+--+ 10  8  1 15  --+-+--+  5 11 14  4  --+-+--+ 16  2  7  9  --+-+--+ </pre>	<pre> 2 11  8 13  --+-+--+  7 14  1 12  --+-+--+  9  4 15  6  --+-+--+ 16  5 10  3  --+-+--+ </pre>

[2]

0T: 2/	1T: 7/	2T: 25/	3T: 27/
<pre> 1 15  6 12  --+-+--+ 14  4  9  7  --+-+--+ 11  5 16  2  --+-+--+  8 10  3 13  --+-+--+ </pre>	<pre> 1 14 11  8  --+-+--+ 12  7  2 13  --+-+--+  6  9 16  3  --+-+--+ 15  4  5 10  --+-+--+ </pre>	<pre> 3 16  9  6  --+-+--+ 13  2  7 12  --+-+--+  8 11 14  1  --+-+--+ 10  5  4 15  --+-+--+ </pre>	<pre> 3 16  9  6  --+-+--+ 10  5  4 15  --+-+--+  8 11 14  1  --+-+--+ 13  2  7 12  --+-+--+ </pre>

4T: 5/	5T: 12/	6T: 43/	7T: 48/
<pre> 1 15  6 12  --+-+--+  8 10  3 13  --+-+--+ 11  5 16  2  --+-+--+ 14  4  9  7  --+-+--+ </pre>	<pre> 1 12  6 15  --+-+--+  8 13  3 10  --+-+--+ 11  2 16  5  --+-+--+ 14  7  9  4  --+-+--+ </pre>	<pre> 4 14  7  9  --+-+--+  5 11  2 16  --+-+--+ 10  8 13  3  --+-+--+ 15  1 12  6  --+-+--+ </pre>	<pre> 6 15  1 12  --+-+--+  3 10  8 13  --+-+--+ 16  5 11  2  --+-+--+  9  4 14  7  --+-+--+ </pre>

8T: 14/	9T: 37/	10T: 35/	11T: 19/
<pre> 2 16  5 11  --+-+--+ 13  3 10  8  --+-+--+ 12  6 15  1  --+-+--+  7  9  4 14  --+-+--+ </pre>	<pre> 4 15 10  5  --+-+--+  9  6  3 16  --+-+--+  7 12 13  2  --+-+--+ 14  1  8 11  --+-+--+ </pre>	<pre> 4 15 10  5  --+-+--+ 14  1  8 11  --+-+--+  7 12 13  2  --+-+--+  9  6  3 16  --+-+--+ </pre>	<pre> 2 13 12  7  --+-+--+ 11  8  1 14  --+-+--+  5 10 15  4  --+-+--+ 16  3  6  9  --+-+--+ </pre>

12T: 17/	13T: 46/	14T: 33/	15T: 24/
$\begin{array}{ c c c c } \hline 2 16  5 11  \\ \hline 7  9  4 14  \\ \hline 12  6 15  1  \\ \hline 13  3 10  8  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 5 16  2 11  \\ \hline 4  9  7 14  \\ \hline 15  6 12  1  \\ \hline 10  3 13  8  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 13  8 10  \\ \hline 6 12  1 15  \\ \hline 9  7 14  4  \\ \hline 16  2 11  5  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 11  5 16  \\ \hline 7 14  4  9  \\ \hline 12  1 15  6  \\ \hline 13  8 10  3  \\ \hline \end{array}$

[3]

0T: 4/	1T: 8/	2T: 42/	3T: 38/
$\begin{array}{ c c c c } \hline 1 15  4 14  \\ \hline 12  6  9  7  \\ \hline 13  3 16  2  \\ \hline 8 10  5 11  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 1 14  4 15  \\ \hline 12  7  9  6  \\ \hline 13  2 16  3  \\ \hline 8 11  5 10  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 14  1 15  \\ \hline 9  7 12  6  \\ \hline 16  2 13  3  \\ \hline 5 11  8 10  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 15  1 14  \\ \hline 9  6 12  7  \\ \hline 16  3 13  2  \\ \hline 5 10  8 11  \\ \hline \end{array}$

4T: 6/	5T: 10/	6T: 44/	7T: 40/
$\begin{array}{ c c c c } \hline 1 15  4 14  \\ \hline 8 10  5 11  \\ \hline 13  3 16  2  \\ \hline 12  6  9  7  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 1 14  4 15  \\ \hline 8 11  5 10  \\ \hline 13  2 16  3  \\ \hline 12  7  9  6  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 14  1 15  \\ \hline 5 11  8 10  \\ \hline 16  2 13  3  \\ \hline 9  7 12  6  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 15  1 14  \\ \hline 5 10  8 11  \\ \hline 16  3 13  2  \\ \hline 9  6 12  7  \\ \hline \end{array}$

8T: 16/	9T: 28/	10T: 32/	11T: 20/
$\begin{array}{ c c c c } \hline 2 16  3 13  \\ \hline 11  5 10  8  \\ \hline 14  4 15  1  \\ \hline 7  9  6 12  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 16  2 13  \\ \hline 10  5 11  8  \\ \hline 15  4 14  1  \\ \hline 6  9  7 12  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 13  2 16  \\ \hline 10  8 11  5  \\ \hline 15  1 14  4  \\ \hline 6 12  7  9  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 13  3 16  \\ \hline 11  8 10  5  \\ \hline 14  1 15  4  \\ \hline 7 12  6  9  \\ \hline \end{array}$

12T: 18/	13T: 30/	14T: 34/	15T: 22/
$\begin{array}{ c c c c } \hline 2 16  3 13  \\ \hline 7  9  6 12  \\ \hline 14  4 15  1  \\ \hline 11  5 10  8  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 16  2 13  \\ \hline 6  9  7 12  \\ \hline 15  4 14  1  \\ \hline 10  5 11  8  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 13  2 16  \\ \hline 6 12  7  9  \\ \hline 15  1 14  4  \\ \hline 10  8 11  5  \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 13  3 16  \\ \hline 7 12  6  9  \\ \hline 14  1 15  4  \\ \hline 11  8 10  5  \\ \hline \end{array}$

[Count = 3 x 16]

I checked how often each old number('n/') was consulted. I found nothing wrong.

1:1, 2:1, 3:1, 4:1, 5:1, 6:1, 7:1, 8:1,  
9:1, 10:1, 11:1, 12:1, 13:1, 14:1, 15:1, 16:1,  
17:1, 18:1, 19:1, 20:1, 21:1, 22:1, 23:1, 24:1,  
25:1, 26:1, 27:1, 28:1, 29:1, 30:1, 31:1, 32:1,  
33:1, 34:1, 35:1, 36:1, 37:1, 38:1, 39:1, 40:1,  
41:1, 42:1, 43:1, 44:1, 45:1, 46:1, 47:1, 48:1

All Right!



#4. The next way I like to introduce you uses the 'Extended Form'. If you accept such a useful form as shown below, you can easily get a new square simply by moving the frame 4x4 itself in any direction as you like, instead of moving any row or any column. This form stands for the 'Extended Space.'

If you move just the frame 4x4 in the wide extended space, you can take out various pan-magic squares.

\*\*\* Pan-Diagonal Magic Square 4x4 \*\*\*

\*\*\* Basic Form in Extended Space \*\*\*

14 15 16 13 14 15 16 13 14 15	<b>Pan-Diagonals:</b>													
<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-right: 1px dashed black; padding: 2px 5px;">2 3 4</td> <td style="border-right: 1px dashed black; padding: 2px 5px;">1   2   3   4</td> <td style="padding: 2px 5px;">1 2 3</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 5px;">6 7 8</td> <td style="border-right: 1px dashed black; padding: 2px 5px;">5   6   7   8</td> <td style="padding: 2px 5px;">5 6 7</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 5px;">10 11 12</td> <td style="border-right: 1px dashed black; padding: 2px 5px;">9   10   11   12</td> <td style="padding: 2px 5px;">9 10 11</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 5px;">14 15 16</td> <td style="border-right: 1px dashed black; padding: 2px 5px;">13   14   15   16</td> <td style="padding: 2px 5px;">13 14 15</td> </tr> </table>	2 3 4	1   2   3   4	1 2 3	6 7 8	5   6   7   8	5 6 7	10 11 12	9   10   11   12	9 10 11	14 15 16	13   14   15   16	13 14 15	$n1+ n6+n11+n16=K \dots (1)$ $n1+ n8+n11+n14=K \dots (2)$ $n2+ n7+n12+n13=K \dots (3)$ $n2+ n5+n12+n15=K \dots (4)$ $n3+ n8+ n9+n14=K \dots (5)$ $n3+ n6+ n9+n16=K \dots (6)$ $n4+ n5+n10+n15=K \dots (7)$ $n4+ n7+n10+n13=K \dots (8)$	
2 3 4	1   2   3   4	1 2 3												
6 7 8	5   6   7   8	5 6 7												
10 11 12	9   10   11   12	9 10 11												
14 15 16	13   14   15   16	13 14 15												
2 3 4 1 2 3 4 1 2 3														

\*\*\* Move the Square Frame 4x4 and Take Fifteen \*\*\*

\*\*\* Different Patterns out of the Extended Space \*\*\*

[1]

<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="border-right: 1px dashed black; padding: 2px 5px;">1   15   10   8</td> <td style="padding: 2px 5px;">1   15 10</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 5px;">14   4   5   11</td> <td style="padding: 2px 5px;">14   4 5</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 5px;">7   9   16   2</td> <td style="padding: 2px 5px;">7   9 16</td> </tr> <tr> <td style="border-right: 1px dashed black; padding: 2px 5px;">12   6   3   13</td> <td style="padding: 2px 5px;">12   6 3</td> </tr> </table>	1   15   10   8	1   15 10	14   4   5   11	14   4 5	7   9   16   2	7   9 16	12   6   3   13	12   6 3	<table style="border-collapse: collapse; width: 100%;"> <tr> <td style="padding: 2px 5px;">14 4 5 11</td> <td style="padding: 2px 5px;">14 4 5</td> </tr> <tr> <td style="padding: 2px 5px;">7 9 16 2</td> <td style="padding: 2px 5px;">7 9 16</td> </tr> </table>	14 4 5 11	14 4 5	7 9 16 2	7 9 16
1   15   10   8	1   15 10												
14   4   5   11	14   4 5												
7   9   16   2	7   9 16												
12   6   3   13	12   6 3												
14 4 5 11	14 4 5												
7 9 16 2	7 9 16												

	1/	2/	3/	4/
1 15 10 8	15 10 8 1	10 8 1 15	8 1 15 10	
14 4 5 11	4 5 11 14	5 11 14 4	11 14 4 5	
7 9 16 2	9 16 2 7	16 2 7 9	2 7 9 16	
12 6 3 13	6 3 13 12	3 13 12 6	13 12 6 3	
	5/	6/	7/	8/
14 4 5 11	4 5 11 14	5 11 14 4	11 14 4 5	
7 9 16 2	9 16 2 7	16 2 7 9	2 7 9 16	
12 6 3 13	6 3 13 12	3 13 12 6	13 12 6 3	
1 15 10 8	15 10 8 1	10 8 1 15	8 1 15 10	
	9/	10/	11/	12/
7 9 16 2	9 16 2 7	16 2 7 9	2 7 9 16	
12 6 3 13	6 3 13 12	3 13 12 6	13 12 6 3	
1 15 10 8	15 10 8 1	10 8 1 15	8 1 15 10	
14 4 5 11	4 5 11 14	5 11 14 4	11 14 4 5	

	13/		14/		15/		16/								
12	6	3	13	6	3	13	12	3	13	12	6	13	12	6	3
1	15	10	8	15	10	8	1	10	8	1	15	8	1	15	10
14	4	5	11	4	5	11	14	5	11	14	4	11	14	4	5
7	9	16	2	9	16	2	7	16	2	7	9	2	7	9	16

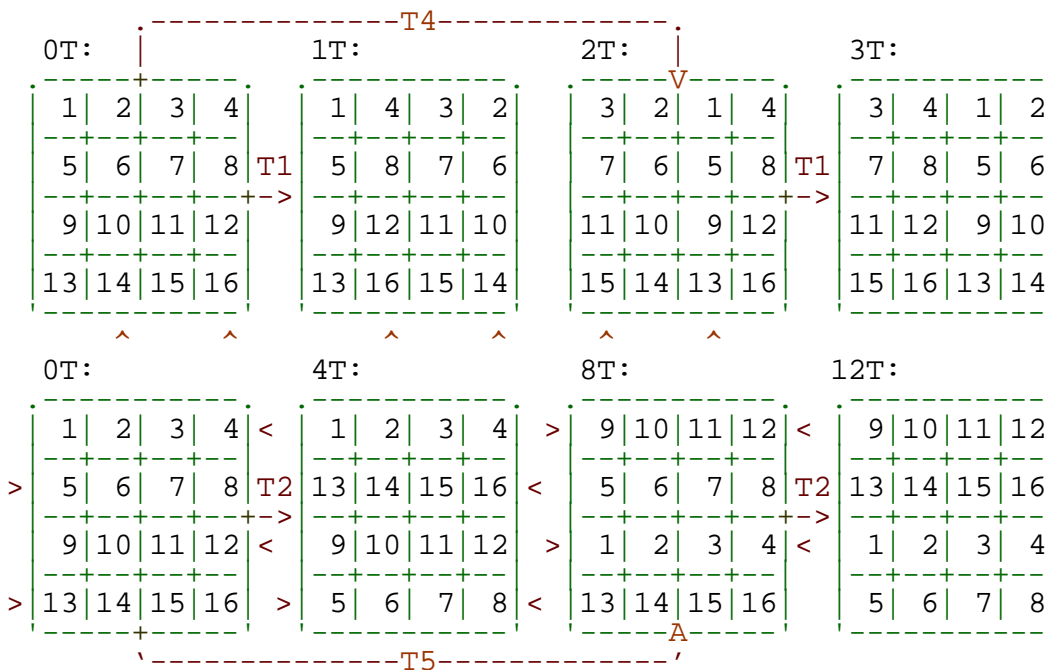
This result is just the same with the one prenormalised of Rotation Method mentioned above. You can expect to do the same job for the result and to go on in the same way. You can rewrite any result smart with n1=1 on the left top. Reflect it in various ways or rotate it in 180 degrees clockwise for your normalisation.

And you can finally get to the same reconstruction list, though I skip showing it again.

#5. The third system I like to show you uses the so-called 'Exchange-a-half' transformation methods to make F3 into 48 solutions.

We had to invent 2x2 types of transformation method in all.

**\*\* 2x2 Transformation Methods: \*\***



Type 1 exchanges 2nd column with 4th one. Type 2 exchanges 2nd row with 4th. Type 4 exchanges 1st column with 3rd one. Type 5 exchanges 1st row with 3rd. We don't use Type 3 and 6 because we don't have to.

/\*\*/

**/\* Transformation Rules: Type 1~6 \*/**

/\*\*/

**void trnsf1(void) {**

**short** n;

```

    d[1]=c[1];      d[2]=c[4];      d[3]=c[3];      d[4]=c[2];
    d[5]=c[5];      d[6]=c[8];      d[7]=c[7];      d[8]=c[6];
    d[9]=c[9];      d[10]=c[12];     d[11]=c[11];     d[12]=c[10];
    d[13]=c[13];    d[14]=c[16];     d[15]=c[15];     d[16]=c[14];
  
```

```

    for(n=1; n<17; n++) { c[n]=d[n]; }
  
```

**}**

/\*\*/

```

void trnsf2(void){
  short n;
  d[1]=c[1];    d[2]=c[2];    d[3]=c[3];    d[4]=c[4];
  d[5]=c[13];   d[6]=c[14];   d[7]=c[15];   d[8]=c[16];
  d[9]=c[9];    d[10]=c[10];   d[11]=c[11];  d[12]=c[12];
  d[13]=c[5];   d[14]=c[6];    d[15]=c[7];   d[16]=c[8];
  for(n=1;n<17;n++){ c[n]=d[n]; }
}
/**/
/* Type 3 = T1 + T2 */
void trnsf3(void){
  short n;
  d[1]=c[1];    d[2]=c[4];    d[3]=c[3];    d[4]=c[2];
  d[5]=c[13];   d[6]=c[16];   d[7]=c[15];   d[8]=c[14];
  d[9]=c[9];    d[10]=c[12];  d[11]=c[11];  d[12]=c[10];
  d[13]=c[5];   d[14]=c[8];    d[15]=c[7];   d[16]=c[6];
  for(n=1;n<17;n++){ c[n]=d[n]; }
}
/**/
void trnsf4(void){
  short n;
  d[1]=c[3];    d[2]=c[2];    d[3]=c[1];    d[4]=c[4];
  d[5]=c[7];    d[6]=c[6];    d[7]=c[5];    d[8]=c[8];
  d[9]=c[11];   d[10]=c[10];  d[11]=c[9];   d[12]=c[12];
  d[13]=c[15];  d[14]=c[14];  d[15]=c[13];  d[16]=c[16];
  for(n=1;n<17;n++){ c[n]=d[n]; }
}
/**/
void trnsf5(void){
  short n;
  d[1]=c[9];    d[2]=c[10];   d[3]=c[11];   d[4]=c[12];
  d[5]=c[5];    d[6]=c[6];    d[7]=c[7];    d[8]=c[8];
  d[9]=c[1];    d[10]=c[2];   d[11]=c[3];   d[12]=c[4];
  d[13]=c[13];  d[14]=c[14];  d[15]=c[15];  d[16]=c[16];
  for(n=1;n<17;n++){ c[n]=d[n]; }
}
/**/
/* Type 6 = T4 + T5 */
void trnsf6(void){
  short n;
  d[1]=c[11];   d[2]=c[10];   d[3]=c[9];    d[4]=c[12];
  d[5]=c[7];    d[6]=c[6];    d[7]=c[5];    d[8]=c[8];
  d[9]=c[3];    d[10]=c[2];   d[11]=c[1];   d[12]=c[4];
  d[13]=c[15];  d[14]=c[14];  d[15]=c[13];  d[16]=c[16];
  for(n=1;n<17;n++){ c[n]=d[n]; }
}
/**/
/* normalisation */
void normal s(void){
  short n;
  for(n=1;n<17;n++){ d[n]=c[n]; }
  if(c[4]>c[13]){

```

```

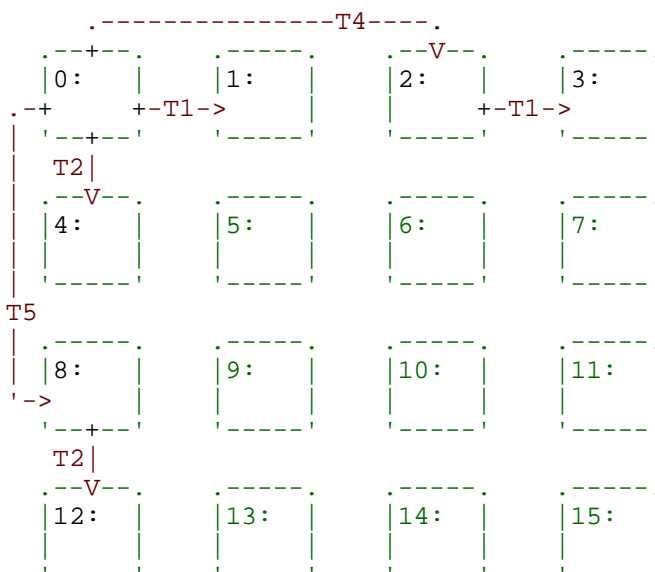
d[1]=c[1];    d[2]=c[5];    d[3]=c[9];    d[4]=c[13];
d[5]=c[2];    d[6]=c[6];    d[7]=c[10];   d[8]=c[14];
d[9]=c[3];    d[10]=c[7];   d[11]=c[11];  d[12]=c[15];
d[13]=c[4];   d[14]=c[8];   d[15]=c[12];  d[16]=c[16];
for(n=1;n<17;n++){c[n]=d[n];}
if(c[1]>c[13]){
d[1]=c[13];   d[2]=c[14];   d[3]=c[15];   d[4]=c[16];
d[5]=c[9];    d[6]=c[10];   d[7]=c[11];   d[8]=c[12];
d[9]=c[5];    d[10]=c[6];   d[11]=c[7];   d[12]=c[8];
d[13]=c[1];   d[14]=c[2];   d[15]=c[3];   d[16]=c[4];
for(n=1;n<17;n++){c[n]=d[n];}
if(c[1]>c[16]){
d[1]=c[16];   d[2]=c[12];   d[3]=c[8];    d[4]=c[4];
d[5]=c[15];   d[6]=c[11];   d[7]=c[7];    d[8]=c[3];
d[9]=c[14];   d[10]=c[10];  d[11]=c[6];   d[12]=c[2];
d[13]=c[13];  d[14]=c[9];   d[15]=c[5];   d[16]=c[1];
for(n=1;n<17;n++){c[n]=d[n];}
if(c[1]>c[4]){
d[1]=c[4];    d[2]=c[3];    d[3]=c[2];    d[4]=c[1];
d[5]=c[8];    d[6]=c[7];    d[7]=c[6];    d[8]=c[5];
d[9]=c[12];   d[10]=c[11]; d[11]=c[10];  d[12]=c[9];
d[13]=c[16];  d[14]=c[15]; d[15]=c[14];  d[16]=c[13];
for(n=1;n<17;n++){c[n]=d[n];}
}
/**/

```

The next diagrams for 'Process Transition' will tell you how to use them. There are two possible ways for Step 2. Either will do the same thing. Choose one as you like. Normalise your answers in the end, if necessary.

**\*\* Process Transition Diagrams \*\***

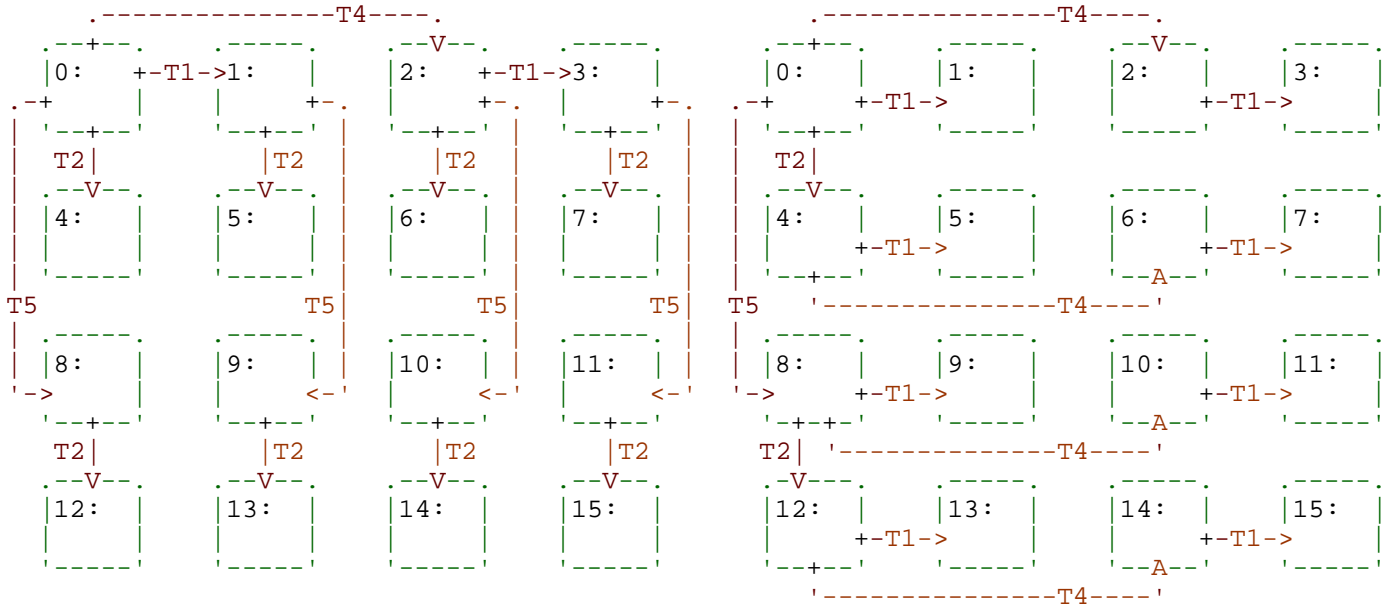
[Step 1:]



Please choose one you like out of two ways illustrated below for the Step 2, and make No.5, 6, 7, 9, 10, 11, 13, 14 and 15 solutions.

[Step 2/]

[\Step 2]



**\*\*\* Pan-Diagonal Magic Squares 4x4: \*\*\***

**\*\* Transform Each Fundamental into 15 Solutions \*\***

**\*\*\* Basic Diagram Illustrating its Concept \*\*\***

[0]

0T:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1T:

1	2	3	4
13	14	15	16
9	10	11	12
5	6	7	8

2T:

9	10	11	12
5	6	7	8
1	2	3	4
13	14	15	16

3T:

9	10	11	12
13	14	15	16
1	2	3	4
5	6	7	8

4T:

1	4	3	2
5	8	7	6
9	12	11	10
13	16	15	14

5T:

1	4	3	2
13	16	15	14
9	12	11	10
5	8	7	6

6T:

9	12	11	10
5	8	7	6
1	4	3	2
13	16	15	14

7T:

9	12	11	10
13	16	15	14
1	4	3	2
5	8	7	6

8T:

3	2	1	4
7	6	5	8
11	10	9	12
15	14	13	16

9T:

3	2	1	4
15	14	13	16
11	10	9	12
7	6	5	8

10T:

11	10	9	12
7	6	5	8
3	2	1	4
15	14	13	16

11T:

11	10	9	12
15	14	13	16
3	2	1	4
7	6	5	8

12T:

3	4	1	2
7	8	5	6
11	12	9	10
15	16	13	14

13T:

3	4	1	2
15	16	13	14
11	12	9	10
7	8	5	6

14T:

11	12	9	10
7	8	5	6
3	4	1	2
15	16	13	14

15T:

11	12	9	10
15	16	13	14
3	4	1	2
7	8	5	6

\*\*\* Print the Original and Transformed Solutions \*\*\*

[1]

0T: 1/	1T: 9/	2T: 29/	3T: 26/
$\begin{array}{ c c c c } \hline 1 & 15 & 10 & 8 \\ \hline 14 & 4 & 5 & 11 \\ \hline 7 & 9 & 16 & 2 \\ \hline 12 & 6 & 3 & 13 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 1 & 14 & 7 & 12 \\ \hline 8 & 11 & 2 & 13 \\ \hline 10 & 5 & 16 & 3 \\ \hline 15 & 4 & 9 & 6 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 & 16 & 5 & 10 \\ \hline 6 & 9 & 4 & 15 \\ \hline 12 & 7 & 14 & 1 \\ \hline 13 & 2 & 11 & 8 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 & 16 & 5 & 10 \\ \hline 13 & 2 & 11 & 8 \\ \hline 12 & 7 & 14 & 1 \\ \hline 6 & 9 & 4 & 15 \\ \hline \end{array}$
4T: 3/	5T: 11/	6T: 45/	7T: 41/
$\begin{array}{ c c c c } \hline 1 & 15 & 10 & 8 \\ \hline 12 & 6 & 3 & 13 \\ \hline 7 & 9 & 16 & 2 \\ \hline 14 & 4 & 5 & 11 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 1 & 12 & 7 & 14 \\ \hline 8 & 13 & 2 & 11 \\ \hline 10 & 3 & 16 & 5 \\ \hline 15 & 6 & 9 & 4 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 5 & 16 & 3 & 10 \\ \hline 4 & 9 & 6 & 15 \\ \hline 14 & 7 & 12 & 1 \\ \hline 11 & 2 & 13 & 8 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 & 14 & 11 & 5 \\ \hline 9 & 7 & 2 & 16 \\ \hline 6 & 12 & 13 & 3 \\ \hline 15 & 1 & 8 & 10 \\ \hline \end{array}$
8T: 15/	9T: 47/	10T: 23/	11T: 31/
$\begin{array}{ c c c c } \hline 2 & 16 & 9 & 7 \\ \hline 11 & 5 & 4 & 14 \\ \hline 8 & 10 & 15 & 1 \\ \hline 13 & 3 & 6 & 12 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 6 & 15 & 4 & 9 \\ \hline 3 & 10 & 5 & 16 \\ \hline 13 & 8 & 11 & 2 \\ \hline 12 & 1 & 14 & 7 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 & 11 & 8 & 13 \\ \hline 7 & 14 & 1 & 12 \\ \hline 9 & 4 & 15 & 6 \\ \hline 16 & 5 & 10 & 3 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 & 13 & 12 & 6 \\ \hline 10 & 8 & 1 & 15 \\ \hline 5 & 11 & 14 & 4 \\ \hline 16 & 2 & 7 & 9 \\ \hline \end{array}$
12T: 13/	13T: 39/	14T: 21/	15T: 36/
$\begin{array}{ c c c c } \hline 2 & 16 & 9 & 7 \\ \hline 13 & 3 & 6 & 12 \\ \hline 8 & 10 & 15 & 1 \\ \hline 11 & 5 & 4 & 14 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 & 15 & 6 & 9 \\ \hline 5 & 10 & 3 & 16 \\ \hline 11 & 8 & 13 & 2 \\ \hline 14 & 1 & 12 & 7 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 & 13 & 8 & 11 \\ \hline 7 & 12 & 1 & 14 \\ \hline 9 & 6 & 15 & 4 \\ \hline 16 & 3 & 10 & 5 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 & 15 & 6 & 9 \\ \hline 14 & 1 & 12 & 7 \\ \hline 11 & 8 & 13 & 2 \\ \hline 5 & 10 & 3 & 16 \\ \hline \end{array}$

[2]

0T: 2/	1T: 7/	2T: 27/	3T: 25/
$\begin{array}{ c c c c } \hline 1 & 15 & 6 & 12 \\ \hline 14 & 4 & 9 & 7 \\ \hline 11 & 5 & 16 & 2 \\ \hline 8 & 10 & 3 & 13 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 1 & 14 & 11 & 8 \\ \hline 12 & 7 & 2 & 13 \\ \hline 6 & 9 & 16 & 3 \\ \hline 15 & 4 & 5 & 10 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 & 16 & 9 & 6 \\ \hline 10 & 5 & 4 & 15 \\ \hline 8 & 11 & 14 & 1 \\ \hline 13 & 2 & 7 & 12 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 & 16 & 9 & 6 \\ \hline 13 & 2 & 7 & 12 \\ \hline 8 & 11 & 14 & 1 \\ \hline 10 & 5 & 4 & 15 \\ \hline \end{array}$
4T: 5/	5T: 12/	6T: 48/	7T: 43/
$\begin{array}{ c c c c } \hline 1 & 15 & 6 & 12 \\ \hline 8 & 10 & 3 & 13 \\ \hline 11 & 5 & 16 & 2 \\ \hline 14 & 4 & 9 & 7 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 1 & 12 & 6 & 15 \\ \hline 8 & 13 & 3 & 10 \\ \hline 11 & 2 & 16 & 5 \\ \hline 14 & 7 & 9 & 4 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 6 & 15 & 1 & 12 \\ \hline 3 & 10 & 8 & 13 \\ \hline 16 & 5 & 11 & 2 \\ \hline 9 & 4 & 14 & 7 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 & 14 & 7 & 9 \\ \hline 5 & 11 & 2 & 16 \\ \hline 10 & 8 & 13 & 3 \\ \hline 15 & 1 & 12 & 6 \\ \hline \end{array}$

8T: 17/	9T: 46/	10T: 24/	11T: 33/																																																
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[Every answer is normalised so as to put n1 on the left top.]  
 (Each 'n/' above stands for the number of the old solutions list.)

[Count = 3 x 16]

I checked if any repetition or drop-off could be found among the results.

```

1:1,    2:1,    3:1,    4:1,    5:1,    6:1,    7:1,    8:1,
9:1,   10:1,   11:1,   12:1,   13:1,   14:1,   15:1,   16:1,
17:1,  18:1,  19:1,  20:1,  21:1,  22:1,  23:1,  24:1,
25:1,  26:1,  27:1,  28:1,  29:1,  30:1,  31:1,  32:1,
33:1,  34:1,  35:1,  36:1,  37:1,  38:1,  39:1,  40:1,
41:1,  42:1,  43:1,  44:1,  45:1,  46:1,  47:1,  48:1

```

**Nothing Wrong!**

As a result you could make all 48 solutions of panmagic squares 4x4 out of the 3 initial solutions including themselves, and they are just the same with the ones in the old solution list we got before.

We can call these three solutions the real 'fundamental' for all, can't we?

We could demonstrate that by our powerful transformation systems, couldn't we?

But why did we examine so many systems of transformation as three?

Because we wanted to know the inner structure of panmagic squares well by watching it from various view points.

And we also wanted know all 48 solutions belong to the noble 'family tree' whose origin was thought to be our 4-dimensional extra-cubic form of order 2.

In order to prove that we had to use the 'Exchange-a-half' methods of transformation applied in our System 3. The fact that any similar pair solution has the same eight 'DNA' numbers in common indicates their same common origin, I think. It is natural to think these two are brothers and sisters.

We could discover all 48 solutions belong to the same noble family, couldn't we?

**#6.** Could we think of 'the most fundamental' solution for all? Say, the only one origin! This intellectual imagination was born in my mind during the study of 4-dimensional magic form of order 2. There prove to be essentially only one solution for that object. It is because in the 4-dimensional world we can only consider there is one meaning for the fact that the four numbers {n2, n4, n5, n13} next to n1 must take their values only out of {15, 14, 12, 8}. Permutation does not mean anything at all right here. Three different answers could make any sense only in the 2-dimensional world.

My imagination grew wilder and wider. Doesn't each fundamental solution have any memory of its parent? Or doesn't it have any common notable property with its brother and sister? Can't we find any beautiful structural relations among those F3?

**\*\*\* Print All 3 Fundamental Solutions \*\*\***

1/	2/	3/
<pre> 1   15   6   12   --+-+--+-+-- 14   4   9   7   --+-+--+-+-- 11   5   16   2   --+-+--+-+-- 8   10   3   13   </pre>	<pre> 1   15   10   8   --+-+--+-+-- 14   4   5   11   --+-+--+-+-- 7   9   16   2   --+-+--+-+-- 12   6   3   13   </pre>	<pre> 1   15   4   14   --+-+--+-+-- 12   6   9   7   --+-+--+-+-- 13   3   16   2   --+-+--+-+-- 8   10   5   11   </pre>



\*\*\* Find Any Similar Pairs to Each Other \*\*\*

```
[ 1]      1/      <-- Unmatched -->      2/
  1 15  6 12    * *  6 12    * * 10  8    1 15 10  8
 14  4  9  7    * *  9  7    * *  5 11    14  4  5 11
11  5 16  2    11  5  * *    7  9  * *    7  9 16  2
 8 10  3 13    8 10  * *    12  6  * *    12  6  3 13

[ 2]      1/      <-- Unmatched -->      3/
  1 15  6 12    * *  6 12    * *  4 14    1 15  4 14
 14  4  9  7    14  4  * *    12  6  * *    12  6  9  7
11  5 16  2    11  5  * *    13  3  * *    13  3 16  2
 8 10  3 13    * *  3 13    * *  5 11    8 10  5 11

[ 3]      2/      <-- Unmatched -->      1/
  1 15 10  8    * * 10  8    * *  6 12    1 15  6 12
 14  4  5 11    * *  5 11    * *  9  7    14  4  9  7
 7  9 16  2    7  9  * *    11  5  * *    11  5 16  2
12  6  3 13    12  6  * *    8 10  * *    8 10  3 13

[ 4]      3/      <-- Unmatched -->      1/
  1 15  4 14    * *  4 14    * *  6 12    1 15  6 12
12  6  9  7    12  6  * *    14  4  * *    14  4  9  7
13  3 16  2    13  3  * *    11  5  * *    11  5 16  2
 8 10  5 11    * *  5 11    * *  3 13    8 10  3 13
```

Let's take any two similar patterns and compare them precisely. We want to find something like 'transformation rules' between any couples of initial three.

```
/**/
/* Transformation Procedures: Type 1 & 2 */
void trnsf1(void){
    d[1]=c[1];    d[2]=c[2];    d[3]=c[14];    d[4]=c[13];
    d[5]=c[5];    d[6]=c[6];    d[7]=c[10];    d[8]=c[9];
    d[9]=c[8];    d[10]=c[7];    d[11]=c[11];    d[12]=c[12];
    d[13]=c[4];    d[14]=c[3];    d[15]=c[15];    d[16]=c[16];
}
/**/

void trnsf2(void){
    d[1]=c[1];    d[2]=c[2];    d[3]=c[6];    d[4]=c[5];
    d[5]=c[4];    d[6]=c[3];    d[7]=c[7];    d[8]=c[8];
    d[9]=c[16];    d[10]=c[15];    d[11]=c[11];    d[12]=c[12];
    d[13]=c[13];    d[14]=c[14];    d[15]=c[10];    d[16]=c[9];
}
/**/
```

Between two pairs (No.1, No.2), (No.1, No.3) I found such impressive correspondences as illustrated above. They are simple enough to be called 'transformation rules.'

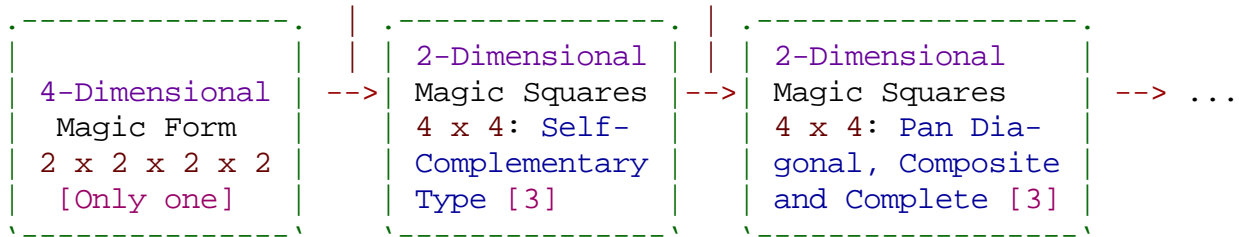
We could easily transform No.1 into both No.2 and No.3, or make No.2 into No.1, and then make No.3 of No.1.

I should say we can transform any one into the other two. Each one of those three initial solutions could be nominated for the 'representative' of all. They are brothers and sisters of the same parent!

We can say that we could find the noble 'family tree' they belong to, can't we?

**\*\*\* The Family Tree of Magic Squares 4 x 4 \*\*\***

[Down Converter]      [Type Converter]



We thus discovered the beautiful structural relations among those different types of objects shown above, I say, the only 'family tree.'

It makes me happy.

**(Revised on February 17, 2003 by Kanji Setsuda)**

**(The original article was written on October 21, 2001)**

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