

# Chapter 2. New Study of Magic Squares of Order 4: Kanji Setsuda

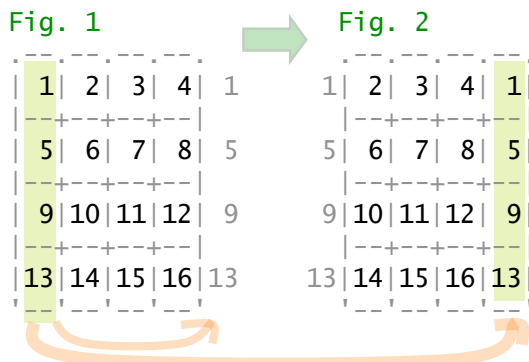
## Section 2. New Algebraic Study of Pan-Magic Squares 4x4

### #0. Classical Concept of "Pan-Magic Squares" of Order 4

Let's study about the Pan-diagonal type of Magic Squares of Order 4 here. I know there are various kinds of definition of this type, but I like to start with such a simple explanation about our classical concept for the 'Pan-magic Shift' as follows.

I want you to see the next diagrams that demonstrate how the Basic Form can be transformed into other forms of order 4. We want those two types of transformation to be always possible for making the 'Magic Squares of Order 4' with the constant sums of their rows, columns and primary diagonals every time.

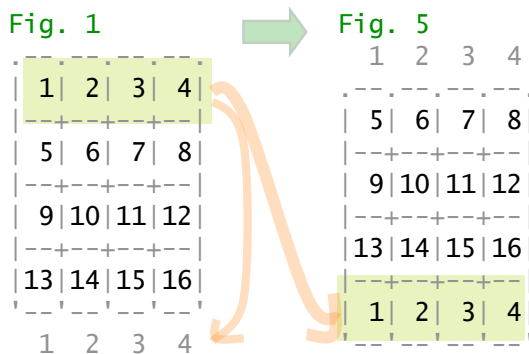
How should we define the Basic Conditions for that type at the first stage?



Diagrams for the 'Pan-magic Shifts' of Pan-magic Squares of Order 4

#### Type 1:

- (1) Select the column at the left side.
- (2) Move it with the 4 numbers beyond the wall to the right end.
- (3) Put it there for a new column.



#### Type 2:

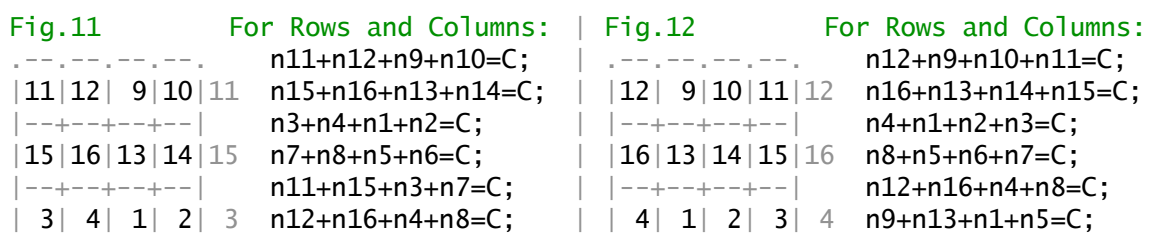
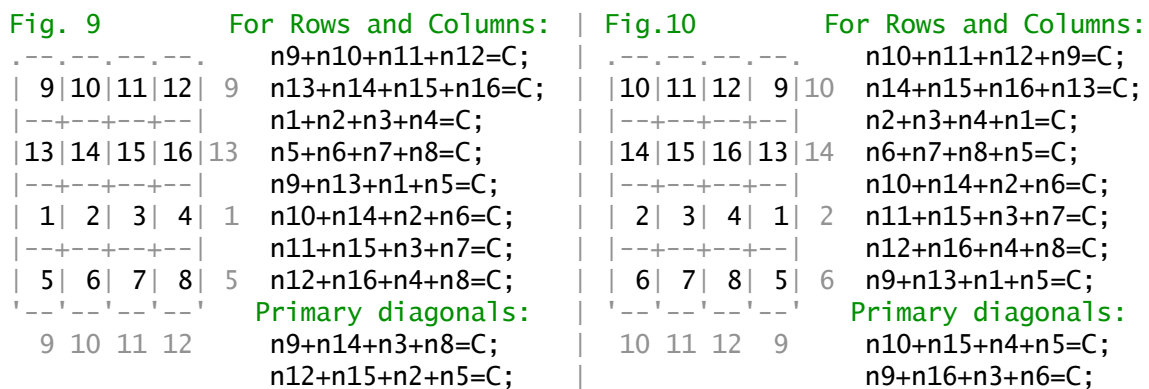
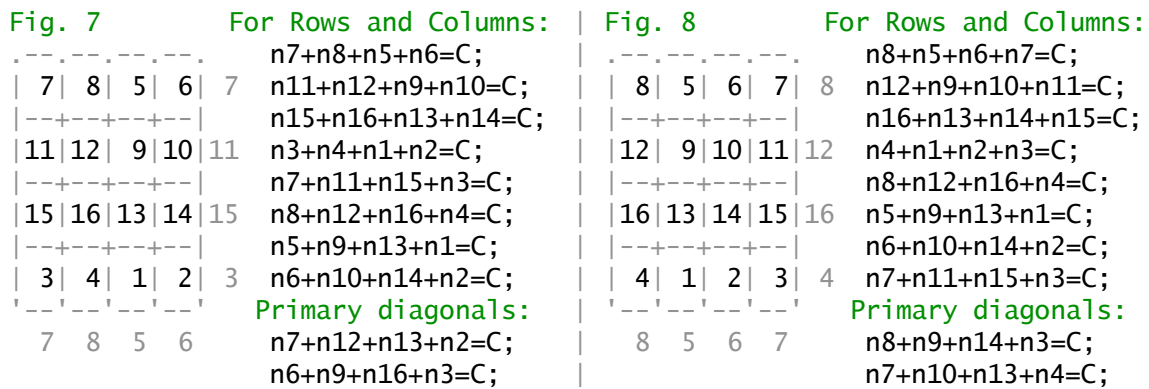
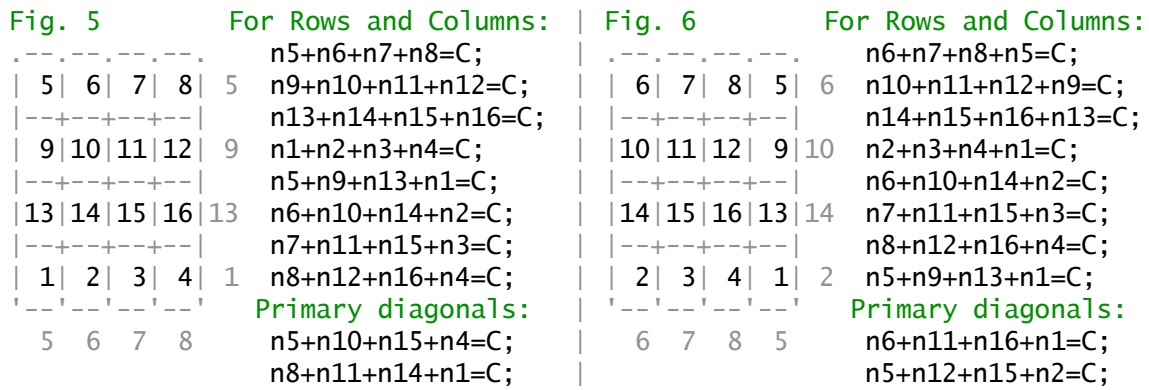
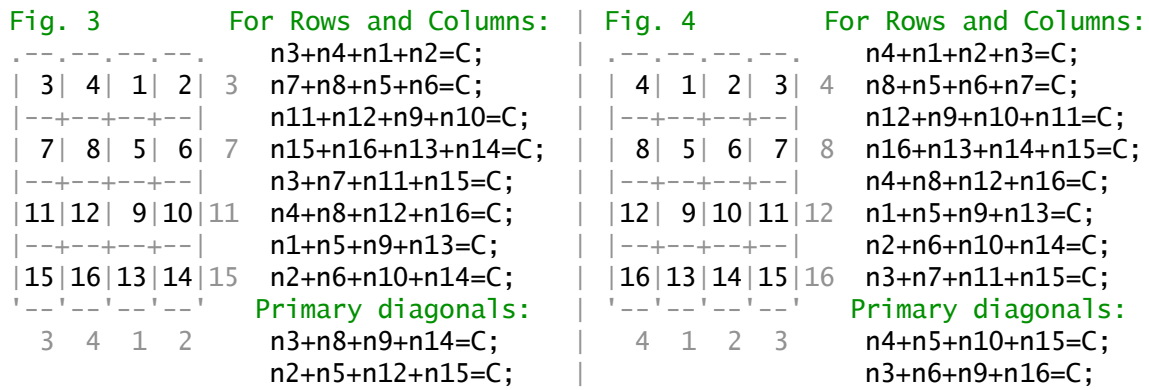
- (1) Select the row on the top floor.
- (2) Move it down with the 4 numbers to the bottom end under the ground.
- (3) Put it there for a new row.

How many different forms can we make by those two types of Pan-magic Shift, if we can repeat any type and combine them in any order?

How many possible primary diagonals should we define to add up to the constant?

**\*\* Diagrams of the Pan-magic Shifts for Pan-magic Squares of Order 4 \*\***  
**\*\* Made by Kanji Setsuda on July 7, 2010 with MacOSX & Xcode 3.1.2 \*\***

Fig. 1	For Rows and Columns:	Fig. 2	For Rows and Columns:
<pre> -----   1  2  3  4  1  ---+---+---+---    5  6  7  8  5  ---+---+---+---    9 10 11 12  9  ---+---+---+---   13 14 15 16 13  -----  1 2 3 4 </pre>	$n1+n2+n3+n4=C;$ $n5+n6+n7+n8=C;$ $n9+n10+n11+n12=C;$ $n13+n14+n15+n16=C;$ $n1+n5+n9+n13=C;$ $n2+n6+n10+n14=C;$ $n3+n7+n11+n15=C;$ $n4+n8+n12+n16=C;$ <b>Primary diagonals:</b> $n1+n6+n11+n16=C;$ $n4+n7+n10+n13=C;$	<pre> -----   2  3  4  1  2  ---+---+---+---    6  7  8  5  6  ---+---+---+---   10 11 12  9 10  ---+---+---+---   14 15 16 13 14  -----  2 3 4 1 </pre>	$n2+n3+n4+n1=C;$ $n6+n7+n8+n5=C;$ $n10+n11+n12+n9=C;$ $n14+n15+n16+n13=C;$ $n2+n6+n10+n14=C;$ $n3+n7+n11+n15=C;$ $n4+n8+n12+n16=C;$ $n1+n5+n9+n13=C;$ <b>Primary diagonals:</b> $n2+n7+n12+n13=C;$ $n1+n8+n11+n14=C;$



---+---+---+---	n9+n13+n1+n5=C;	---+---+---+---	n10+n14+n2+n6=C;
7  8  5  6  7	n10+n14+n2+n6=C;	8  5  6  7  8	n11+n15+n3+n7=C;
'---'---'---'---'	<b>Primary diagonals:</b>	'---'---'---'---'	<b>Primary diagonals:</b>
11 12 9 10	n11+n16+n1+n6=C;	12 9 10 11	n12+n13+n2+n7=C;
	n10+n13+n4+n7=C;		n11+n14+n1+n8=C;

<b>Fig.13</b>	<b>For Rows and Columns:</b>	<b>Fig.14</b>	<b>For Rows and Columns:</b>
---+---+---+---	n13+n14+n15+n16=C;	---+---+---+---	n14+n15+n16+n13=C;
13 14 15 16 13	n1+n2+n3+n4=C;	14 15 16 13 14	n2+n3+n4+n1=C;
---+---+---+---	n5+n6+n7+n8=C;	---+---+---+---	n6+n7+n8+n5=C;
1  2  3  4  1	n9+n10+n11+n12=C;	2  3  4  1  2	n10+n11+n12+n9=C;
---+---+---+---	n13+n1+n5+n9=C;	---+---+---+---	n14+n2+n6+n10=C;
5  6  7  8  5	n14+n2+n6+n10=C;	6  7  8  5  6	n15+n3+n7+n11=C;
---+---+---+---	n15+n3+n7+n11=C;	---+---+---+---	n16+n4+n8+n12=C;
9 10 11 12  9	n16+n4+n8+n12=C;	10 11 12  9 10	n13+n1+n5+n9=C;
'---'---'---'---'	<b>Primary diagonals:</b>	'---'---'---'---'	<b>Primary diagonals:</b>
13 14 15 16	n13+n2+n7+n12=C;	14 15 16 13	n14+n3+n8+n9=C;
	n16+n3+n6+n9=C;		n13+n4+n7+n10=C;

<b>Fig.15</b>	<b>For Rows and Columns:</b>	<b>Fig.16</b>	<b>For Rows and Columns:</b>
---+---+---+---	n15+n16+n13+n14=C;	---+---+---+---	n16+n13+n14+n15=C;
15 16 13 14 15	n3+n4+n1+n2=C;	16 13 14 15 16	n4+n1+n2+n3=C;
---+---+---+---	n7+n8+n5+n6=C;	---+---+---+---	n8+n5+n6+n7=C;
3  4  1  2  3	n11+n12+n9+n10=C;	4  1  2  3  4	n12+n9+n10+n11=C;
---+---+---+---	n15+n3+n7+n11=C;	---+---+---+---	n16+n4+n8+n12=C;
7  8  5  6  7	n16+n4+n8+n12=C;	8  5  6  7  8	n13+n1+n5+n9=C;
---+---+---+---	n13+n1+n5+n9=C;	---+---+---+---	n14+n2+n6+n10=C;
11 12  9 10 11	n14+n2+n6+n10=C;	12  9 10 11 12	n15+n3+n7+n11=C;
'---'---'---'---'	<b>Primary diagonals:</b>	'---'---'---'---'	<b>Primary diagonals:</b>
15 16 13 14	n15+n4+n5+n10=C;	16 13 14 15	n16+n1+n6+n11=C;
	n14+n1+n8+n11=C;		n15+n2+n5+n12=C;

We can make 16 different forms in all as shown above from the Fig.1 including itself. No more or no less. Can you see every set of the conditions for rows and columns always have the same content in common? Yes. The contents are kept unchanged, while the equations and variables themselves appear in different orders.

How about the conditions of our new primary diagonals I collected here as follows?

**\*\* Collective List of all the Possible Primary diagonals: C=34 \*\***

n1+n6+n11+n16=C ...pd11;	n4+n7+n10+n13=C ...pd12;
n2+n7+n12+n13=C ...pd21;	n1+n8+n11+n14=C ...pd22;
n3+n8+n9+n14=C ...pd31;	n2+n5+n12+n15=C ...pd32;
n4+n5+n10+n15=C ...pd41;	n3+n6+n9+n16=C ...pd42;
n4+n5+n10+n15=C ...pd51;	n1+n8+n11+n14=C ...pd52;
n1+n6+n11+n16=C ...pd61;	n2+n5+n12+n15=C ...pd62;
n2+n7+n12+n13=C ...pd71;	n3+n6+n9+n16=C ...pd72;
n3+n8+n9+n14=C ...pd81;	n4+n7+n10+n13=C ...pd82;
n3+n8+n9+n14=C ...pd91;	n2+n5+n12+n15=C ...pd92;
n4+n5+n10+n15=C ...pd101;	n3+n6+n9+n16=C ...pd102;
n1+n6+n11+n16=C ...pd111;	n4+n7+n10+n13=C ...pd112;
n2+n7+n12+n13=C ...pd121;	n1+n8+n11+n14=C ...pd122;
n2+n7+n12+n13=C ...pd131;	n3+n6+n9+n16=C ...pd132;
n3+n8+n9+n14=C ...pd141;	n4+n7+n10+n13=C ...pd142;
n4+n5+n10+n15=C ...pd151;	n1+n8+n11+n14=C ...pd152;
n1+n6+n11+n16=C ...pd161;	n2+n5+n12+n15=C ...pd162;

The new conditions have come all at the first four cases and the rest of conditions appeared with the same contents as the first eight. You can certainly find repetition.

We really need those eight conditions for diagonals [pd11~pd42](#). When we have all of them from the beginning, we can always compose any new magic squares of order 4 by our Pan-magic Shifts anytime. It is these eight that surely make 16 squares.

Let's call the "Pan-diagonals" for the eight above. Let's add all the eight conditions above to our definition list at the first stage.

### #1. New Concept of Pan-Diagonal Magic Square of Order 4

Let's start now simply with these diagrams and simultaneous equations as follows.

#### [Basic Form and Basic Definitions]

	* Rows and Columns *
n1 n2 n3 n4	n1+ n2+ n3+ n4 = C ... rw1
---+---+---+---	n5+ n6+ n7+ n8 = C ... rw2
n5 n6 n7 n8	n9+n10+n11+n12 = C ... rw3
---+---+---+---	n13+n14+n15+n16 = C ... rw4
n9 10 11 12	n1+ n5+ n9+n13 = C ... c1
---+---+---+---	n2+ n6+n10+n14 = C ... c2
13 14 15 16	n3+ n7+n11+n15 = C ... c3
---+---+---+---	n4+ n8+n12+n16 = C ... c4

#### [Extended Space, Moving Frame and Pan-diagonals]

11 12 n9 10 11 12 n9 10 11	
15 16 13 14 15 16 13 14 15	* Pan-diagonal Equations *
n3 n4 n1 n2 n3 n4 n1 n2 n3	n1+ n6+n11+n16 = C ... pd1
. -- ---+---+---+--- -. -	n1+ n8+n11+n14 = C ... pd2
n7 n8 n5 n6 n7 n8 n5 n6 n7	n2+ n7+n12+n13 = C ... pd3
- ---+---+---+---  -  -	n2+ n5+n12+n15 = C ... pd4
11 12 n9 10 11 12 n9 10 11	n3+ n8+ n9+n14 = C ... pd5
- ---+---+---+---  -' -	n3+ n6+ n9+n16 = C ... pd6
15 16 13 14 15 16 13 14 15	n4+ n5+n10+n15 = C ... pd7
---+---+---+---	n4+ n7+n10+n13 = C ... pd8
n3 n4 n1 n2 n3 n4 n1 n2 n3	(C means the magic constant 34.)
---+---+---+---	

The lower figure above shows the Extended Space and the moving frame of 4x4 square. This diagram is very useful for us to find any pan-diagonals, because we can always see them in 'straight lines' instead of those notorious 'broken, wrapping lines.'

Pan-diagonal Equations [pd1~pd8](#) can be easily taken out of the Extended Space.

You can move any last row or column of the supposed square far to the opposite side and put it beyond the first, and you can make a new Pan-magic square, as you know. In the Extended Space you can realize the same thing by moving the 'square frame' as illustrated above. New squares can be taken out only by picking up any 4x4 entries with the new frames moved to anywhere else.

#### \* 4x4 New Squares Taken out of the Extended Space \*

	00/	01/	02/	03/
n1 n2 n3 n4	13 14 15 16	9 10 11 12	5 6 7 8	
n5 n6 n7 n8	1 2 3 4	13 14 15 16	9 10 11 12	
n9 10 11 12	5 6 7 8	1 2 3 4	13 14 15 16	
13 14 15 16	9 10 11 12	5 6 7 8	1 2 3 4	

		10/		11/		12/		13/							
4	1	2	3	16	13	14	15	12	9	10	11	8	5	6	7
8	5	6	7	4	1	2	3	16	13	14	15	12	9	10	11
12	9	10	11	8	5	6	7	4	1	2	3	16	13	14	15
16	13	14	15	12	9	10	11	8	5	6	7	4	1	2	3
		20/		21/		22/		23/							
3	4	1	2	15	16	13	14	11	12	9	10	7	8	5	6
7	8	5	6	3	4	1	2	15	16	13	14	11	12	9	10
11	12	9	10	7	8	5	6	3	4	1	2	15	16	13	14
15	16	13	14	11	12	9	10	7	8	5	6	3	4	1	2
		30/		31/		32/		33/							
2	3	4	1	14	15	16	13	10	11	12	9	6	7	8	5
6	7	8	5	2	3	4	1	14	15	16	13	10	11	12	9
10	11	12	9	6	7	8	5	2	3	4	1	14	15	16	13
14	15	16	13	10	11	12	9	6	7	8	5	2	3	4	1

You can finally take out those new Pan-magic squares above as many as 4x4. In any form two diagonals must add up to the magic constant C(=34), since it should just satisfy the original concept of 'pan-magic square' of order 4. The constant sum of every pandiagonal makes it possible.

## #2. How many Solutions can we find?

What does it actually look like? Now let's make our computer draw and count all pan-magic squares 4x4 at first, shall we?

We know there are three levels of solutions as follows: (1) 'Primitive', (2) 'Standard', and (3) 'Fundamental'. Let's have them all here.

### \*\* 384 'Primitive Solutions' of Pan-Diagonal Magic Squares 4x4 \*\*

		1/		2/		3/		4/		5/		6/											
1	8	10	15	1	8	10	15	1	8	11	14	1	8	13	12	1	8	13	12				
12	13	3	6	14	11	5	4	12	13	2	7	15	10	5	4	14	11	2	7	15	10	3	6
7	2	16	9	7	2	16	9	6	3	16	9	6	3	16	9	4	5	16	9	4	5	16	9
14	11	5	4	12	13	3	6	15	10	5	4	12	13	2	7	15	10	3	6	14	11	2	7
		7/		8/		9/		10/		11/		12/											
1	12	6	15	1	12	6	15	1	12	7	14	1	12	7	14	1	12	13	8	1	12	13	8
8	13	3	10	14	7	9	4	8	13	2	11	15	6	9	4	14	7	2	11	15	6	3	10
11	2	16	5	11	2	16	5	10	3	16	5	10	3	16	5	4	9	16	5	4	9	16	5
14	7	9	4	8	13	3	10	15	6	9	4	8	13	2	11	15	6	3	10	14	7	2	11
		13/		14/		15/		16/		17/		18/											
1	14	4	15	1	14	4	15	1	14	7	12	1	14	7	12	1	14	11	8	1	14	11	8
8	11	5	10	12	7	9	6	8	11	2	13	15	4	9	6	12	7	2	13	15	4	5	10
13	2	16	3	13	2	16	3	10	5	16	3	10	5	16	3	6	9	16	3	6	9	16	3
12	7	9	6	8	11	5	10	15	4	9	6	8	11	2	13	15	4	5	10	12	7	2	13
		19/		20/		21/		22/		23/		24/											
1	15	4	14	1	15	4	14	1	15	6	12	1	15	6	12	1	15	10	8	1	15	10	8
8	10	5	11	12	6	9	7	8	10	3	13	14	4	9	7	12	6	3	13	14	4	5	11
13	3	16	2	13	3	16	2	11	5	16	2	11	5	16	2	7	9	16	2	7	9	16	2
12	6	9	7	8	10	5	11	14	4	9	7	8	10	3	13	14	4	5	11	12	6	3	13
		25/		26/		27/		28/		29/		30/											
2	7	9	16	2	7	9	16	2	7	12	13	2	7	12	13	2	7	14	11	2	7	14	11
11	14	4	5	13	12	6	3	11	14	1	8	16	9	6	3	13	12	1	8	16	9	4	5
8	1	15	10	8	1	15	10	5	4	15	10	5	4	15	10	3	6	15	10	3	6	15	10
13	12	6	3	11	14	4	5	16	9	6	3	11	14	1	8	16	9	4	5	13	12	1	8

	31/	32/	33/	34/	35/	36/
2 11 5 16	2 11 5 16	2 11 8 13	2 11 8 13	2 11 14 7	2 11 14 7	
7 14 4 9	13 8 10 3	7 14 1 12	16 5 10 3	13 8 1 12	16 5 4 9	
12 1 15 6	12 1 15 6	9 4 15 6	9 4 15 6	3 10 15 6	3 10 15 6	
13 8 10 3	7 14 4 9	16 5 10 3	7 14 1 12	16 5 4 9	13 8 1 12	
	37/	38/	39/	40/	41/	42/
2 13 3 16	2 13 3 16	2 13 8 11	2 13 8 11	2 13 12 7	2 13 12 7	
7 12 6 9	11 8 10 5	7 12 1 14	16 3 10 5	11 8 1 14	16 3 6 9	
14 1 15 4	14 1 15 4	9 6 15 4	9 6 15 4	5 10 15 4	5 10 15 4	
11 8 10 5	7 12 6 9	16 3 10 5	7 12 1 14	16 3 6 9	11 8 1 14	
	43/	44/	45/	46/	47/	48/
2 16 3 13	2 16 3 13	2 16 5 11	2 16 5 11	2 16 9 7	2 16 9 7	
7 9 6 12	11 5 10 8	7 9 4 14	13 3 10 8	11 5 4 14	13 3 6 12	
14 4 15 1	14 4 15 1	12 6 15 1	12 6 15 1	8 10 15 1	8 10 15 1	
11 5 10 8	7 9 6 12	13 3 10 8	7 9 4 14	13 3 6 12	11 5 4 14	
	49/	50/	51/	52/	53/	54/
3 6 9 16	3 6 9 16	3 6 12 13	3 6 12 13	3 6 15 10	3 6 15 10	
10 15 4 5	13 12 7 2	10 15 1 8	16 9 7 2	13 12 1 8	16 9 4 5	
8 1 14 11	8 1 14 11	5 4 14 11	5 4 14 11	2 7 14 11	2 7 14 11	
13 12 7 2	10 15 4 5	16 9 7 2	10 15 1 8	16 9 4 5	13 12 1 8	
	55/	56/	57/	58/	59/	60/
3 10 5 16	3 10 5 16	3 10 8 13	3 10 8 13	3 10 15 6	3 10 15 6	
6 15 4 9	13 8 11 2	6 15 1 12	16 5 11 2	13 8 1 12	16 5 4 9	
12 1 14 7	12 1 14 7	9 4 14 7	9 4 14 7	2 11 14 7	2 11 14 7	
13 8 11 2	6 15 4 9	16 5 11 2	6 15 1 12	16 5 4 9	13 8 1 12	
	61/	62/	63/	64/	65/	66/
3 13 2 16	3 13 2 16	3 13 8 10	3 13 8 10	3 13 12 6	3 13 12 6	
6 12 7 9	10 8 11 5	6 12 1 15	16 2 11 5	10 8 1 15	16 2 7 9	
15 1 14 4	15 1 14 4	9 7 14 4	9 7 14 4	5 11 14 4	5 11 14 4	
10 8 11 5	6 12 7 9	16 2 11 5	6 12 1 15	16 2 7 9	10 8 1 15	
	67/	68/	69/	70/	71/	72/
3 16 2 13	3 16 2 13	3 16 5 10	3 16 5 10	3 16 9 6	3 16 9 6	
6 9 7 12	10 5 11 8	6 9 4 15	13 2 11 8	10 5 4 15	13 2 7 12	
15 4 14 1	15 4 14 1	12 7 14 1	12 7 14 1	8 11 14 1	8 11 14 1	
10 5 11 8	6 9 7 12	13 2 11 8	6 9 4 15	13 2 7 12	10 5 4 15	
	73/	74/	75/	76/	77/	78/
4 5 10 15	4 5 10 15	4 5 11 14	4 5 11 14	4 5 16 9	4 5 16 9	
9 16 3 6	14 11 8 1	9 16 2 7	15 10 8 1	14 11 2 7	15 10 3 6	
7 2 13 12	7 2 13 12	6 3 13 12	6 3 13 12	1 8 13 12	1 8 13 12	
14 11 8 1	9 16 3 6	15 10 8 1	9 16 2 7	15 10 3 6	14 11 2 7	
	79/	80/	81/	82/	83/	84/
4 9 6 15	4 9 6 15	4 9 7 14	4 9 7 14	4 9 16 5	4 9 16 5	
5 16 3 10	14 7 12 1	5 16 2 11	15 6 12 1	14 7 2 11	15 6 3 10	
11 2 13 8	11 2 13 8	10 3 13 8	10 3 13 8	1 12 13 8	1 12 13 8	
14 7 12 1	5 16 3 10	15 6 12 1	5 16 2 11	15 6 3 10	14 7 2 11	
	85/	86/	87/	88/	89/	90/
4 14 1 15	4 14 1 15	4 14 7 9	4 14 7 9	4 14 11 5	4 14 11 5	
5 11 8 10	9 7 12 6	5 11 2 16	15 1 12 6	9 7 2 16	15 1 8 10	
16 2 13 3	16 2 13 3	10 8 13 3	10 8 13 3	6 12 13 3	6 12 13 3	
9 7 12 6	5 11 8 10	15 1 12 6	5 11 2 16	15 1 8 10	9 7 2 16	
	91/	92/	93/	94/	95/	96/
4 15 1 14	4 15 1 14	4 15 6 9	4 15 6 9	4 15 10 5	4 15 10 5	
5 10 8 11	9 6 12 7	5 10 3 16	14 1 12 7	9 6 3 16	14 1 8 11	
16 3 13 2	16 3 13 2	11 8 13 2	11 8 13 2	7 12 13 2	7 12 13 2	
9 6 12 7	5 10 8 11	14 1 12 7	5 10 3 16	14 1 8 11	9 6 3 16	

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          97/          98/          99/          100/          101/          102/
5  4  9 16  5  4  9 16  5  4 14 11  5  4 14 11  5  4 15 10  5  4 15 10
10 15  6  3 11 14  7  2 10 15  1  8 16  9  7  2 11 14  1  8 16  9  6  3
  8  1 12 13  8  1 12 13  3  6 12 13  3  6 12 13  2  7 12 13  2  7 12 13
11 14  7  2 10 15  6  3 16  9  7  2 10 15  1  8 16  9  6  3 11 14  1  8

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          355/          356/          357/          358/          359/          360/
15 10  3  6 15 10  3  6 15 10  5  4 15 10  5  4 15 10  8  1 15 10  8  1
  1  8 13 12  4  5 16  9  1  8 11 14  6  3 16  9  4  5 11 14  6  3 13 12
14 11  2  7 14 11  2  7 12 13  2  7 12 13  2  7  9 16  2  7  9 16  2  7
  4  5 16  9  1  8 13 12  6  3 16  9  1  8 11 14  6  3 13 12  4  5 11 14
          361/          362/          363/          364/          365/          366/
16  2  7  9 16  2  7  9 16  2 11  5 16  2 11  5 16  2 13  3 16  2 13  3
  3 13 12  6  5 11 14  4  3 13  8 10  9  7 14  4  5 11  8 10  9  7 12  6
10  8  1 15 10  8  1 15  6 12  1 15  6 12  1 15  4 14  1 15  4 14  1 15
  5 11 14  4  3 13 12  6  9  7 14  4  3 13  8 10  9  7 12  6  5 11  8 10
          367/          368/          369/          370/          371/          372/
16  3  6  9 16  3  6  9 16  3 10  5 16  3 10  5 16  3 13  2 16  3 13  2
  2 13 12  7  5 10 15  4  2 13  8 11  9  6 15  4  5 10  8 11  9  6 12  7
11  8  1 14 11  8  1 14  7 12  1 14  7 12  1 14  4 15  1 14  4 15  1 14
  5 10 15  4  2 13 12  7  9  6 15  4  2 13  8 11  9  6 12  7  5 10  8 11
          373/          374/          375/          376/          377/          378/
16  5  4  9 16  5  4  9 16  5 10  3 16  5 10  3 16  5 11  2 16  5 11  2
  2 11 14  7  3 10 15  6  2 11  8 13  9  4 15  6  3 10  8 13  9  4 14  7
13  8  1 12 13  8  1 12  7 14  1 12  7 14  1 12  6 15  1 12  6 15  1 12
  3 10 15  6  2 11 14  7  9  4 15  6  2 11  8 13  9  4 14  7  3 10  8 13
          379/          380/          381/          382/          383/          384/
16  9  4  5 16  9  4  5 16  9  6  3 16  9  6  3 16  9  7  2 16  9  7  2
  2  7 14 11  3  6 15 10  2  7 12 13  5  4 15 10  3  6 12 13  5  4 14 11
13 12  1  8 13 12  1  8 11 14  1  8 11 14  1  8 10 15  1  8 10 15  1  8
  3  6 15 10  2  7 14 11  5  4 15 10  2  7 12 13  5  4 14 11  3  6 12 13

```

\* Counts according to the value of n1 \*

```

[ 1: 24] [ 2: 24] [ 3: 24] [ 4: 24] [ 5: 24] [ 6: 24] [ 7: 24] [ 8: 24]
[ 9: 24] [10: 24] [11: 24] [12: 24] [13: 24] [14: 24] [15: 24] [16: 24]

```

\* Total Count = 384 \*

The next list is a sample program dictated for the purpose of making the 'Standard Solutions' of pan-diagonal magic squares of order 4. The list-forming inequality conditions I put for them are { $n_1 < n_4$ ;  $n_1 < n_{13}$ ;  $n_1 < n_{16}$  and  $n_2 < n_5$ }.

```

/** Pandiagonal Magic Squares of Order 4 */
/** 'MS44PD48.c' built by Kanji Setsuda */
/** on Aug. 8, 2000; Dec.16, 2005; */
/** Worked on MacOSX and Xcode 1.5 */
/**/
#include <stdio.h>
/**/
short cnt, cnt2;
short LSM;
short nm[17], uflg[17];
short anm[9][17];
/**/
void stp01(void), stp02(void), stp03(void), stp04(void);

```

```

void stp05(void), stp06(void), stp07(void), stp08(void);
void stp09(void), stp10(void), stp11(void), stp12(void);
void stp13(void), stp14(void), stp15(void), stp16(void);
void ansprint(void);
void printans(short x);
/**/
/* Main Program */
int main(){
  short n;
  printf("\n** 48 Standard Solutions of Pan-diagonal Magic Squares 4x4 **\n");
  for(n=0;n<17;n++){nm[n]=0; uflg[n]=0;}
  LSM=34; cnt=0; cnt2=0;
  stp01();/* Begin the Calculations */
  if(cnt2>0){printans(cnt2);}
  printf(" [Count = %d]\n",cnt);
  printf("  OK!\n");
  return 0;
}
/* Begin the Calculations */
/* Set n1 */
void stp01(){
  short a;
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[1]=a; uflg[a]=1;
      stp02();
      uflg[a]=0;}
  }
}
/* Set n2 */
void stp02(){
  short a;
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[2]=a; uflg[a]=1;
      stp03();
      uflg[a]=0;}
  }
}
/* Set n5 & n5>n2 */
void stp03(){
  short a;
  for(a=nm[2]+1;a<17;a++){
    if(uflg[a]==0){
      nm[5]=a; uflg[a]=1;
      stp04();
      uflg[a]=0;}
  }
}
/* Set n6 */
void stp04(){
  short a;
  for(a=1;a<17;a++){
    if(uflg[a]==0){
      nm[6]=a; uflg[a]=1;
      stp05();
      uflg[a]=0;}
  }
}
}

```



```

/* Set n4 & n1<n4 */
void stp05(){
  short a;
  for(a=nm[1]+1;a<17;a++){
    if(uflg[a]==0){
      nm[4]=a; uflg[a]=1;
      stp06();
      uflg[a]=0;}
  }
}
/* Set n3=LSM-n1-n2-n4 */
void stp06(){
  short a;
  a=LSM-nm[1]-nm[2]-nm[4];
  if((0<a)&&(a<17)&&(uflg[a]==0)){
    nm[3]=a; uflg[a]=1;
    stp07();
    uflg[a]=0;}
}
/* Set n13 & n1<n13 */
void stp07(){
  short a;
  for(a=nm[1]+1;a<17;a++){
    if(uflg[a]==0){
      nm[13]=a; uflg[a]=1;
      stp08();
      uflg[a]=0;}
  }
}
/* Set n9=LSM-n1-n5-n13 */
void stp08(){
  short a;
  a=LSM-nm[1]-nm[5]-nm[13];
  if((0<a)&&(a<17)&&(uflg[a]==0)){
    nm[9]=a; uflg[a]=1;
    stp09();
    uflg[a]=0;}
}
/* Set n16=LSM-n3-n6-n9 & n1<n16 */
void stp09(){
  short a;
  a=LSM-nm[3]-nm[6]-nm[9];
  if((nm[1]<a)&&(a<17)&&(uflg[a]==0)){
    nm[16]=a; uflg[a]=1;
    stp10();
    uflg[a]=0;}
}
/* Set n11=LSM-n1-n6-n16 */
void stp10(){
  short a;
  a=LSM-nm[1]-nm[6]-nm[16];
  if((0<a)&&(a<17)&&(uflg[a]==0)){
    nm[11]=a; uflg[a]=1;
    stp11();
    uflg[a]=0;}
}
/* Set n7 */
void stp11(){
  short a;

```

```

    for(a=1;a<17;a++){
        if(uflg[a]==0){
            nm[7]=a; uflg[a]=1;
            stp12();
            uflg[a]=0;}
    }
}
/* Set n8=LSM-n5-n6-n7 */
void stp12(){
    short a;
    a=LSM-nm[5]-nm[6]-nm[7];
    if((0<a)&&(a<17)&&(uflg[a]==0)){
        nm[8]=a; uflg[a]=1;
        stp13();
        uflg[a]=0;}
}
/* Set n10=LSM-n4-n7-n13 */
void stp13(){
    short a;
    a=LSM-nm[4]-nm[7]-nm[13];
    if((0<a)&&(a<17)&&(uflg[a]==0)){
        nm[10]=a; uflg[a]=1;
        stp14();
        uflg[a]=0;}
}
/* Set n12=LSM-n9-n10-n11 */
void stp14(){
    short a,b,c;
    a=LSM-nm[9]-nm[10]-nm[11];
    b=LSM-nm[4]-nm[8]-nm[16];
    c=LSM-nm[2]-nm[7]-nm[13];
    if((0<a)&&(a<17)){
        if((a==b)&&(a==c)&&(uflg[a]==0)){
            nm[12]=a; uflg[a]=1;
            stp15();
            uflg[a]=0;}}
}
/* Set n14=LSM-n2-n6-n10 */
void stp15(){
    short a,b,c;
    a=LSM-nm[2]-nm[6]-nm[10];
    b=LSM-nm[1]-nm[8]-nm[11];
    c=LSM-nm[3]-nm[8]-nm[9];
    if((0<a)&&(a<17)){
        if((a==b)&&(a==c)&&(uflg[a]==0)){
            nm[14]=a; uflg[a]=1;
            stp16();
            uflg[a]=0;}}
}
/* Set n15=LSM-n3-n7-n11 */
void stp16(){
    short a,b,c,d;
    a=LSM-nm[3]-nm[7]-nm[11];
    b=LSM-nm[2]-nm[5]-nm[12];
    c=LSM-nm[4]-nm[5]-nm[10];
    d=LSM-nm[13]-nm[14]-nm[16];
    if((0<a)&&(a<17)&&(uflg[a]==0)){
        if((a==b)&&(a==c)&&(a==d)){
            nm[15]=a; uflg[a]=1;

```

```

    ansprint();
    uflg[a]=0;}}
}
/**/
/* Print the Answers */
void ansprint(){
short n;
cnt++;
anm[cnt2][0]=cnt;
for(n=1;n<17;n++){anm[cnt2][n]=nm[n];}
cnt2++; if(cnt2==6){printans(6); cnt2=0;}
}
/* Print the Answers */
void printans(short x){
short m,n;
for(m=0;m<x;m++){printf("%13d/",anm[m][0]);}
printf("\n");
for(m=0;m<x;m++){printf(" .-.-.-.-.-.");}
printf("\n");
for(m=0;m<x;m++){
printf(" |%2d|%2d|%2d|%2d|",anm[m][1],anm[m][2],anm[m][3],anm[m][4]);}
printf("\n");
for(m=0;m<x;m++){printf(" |---+---+---+---|");}
printf("\n");
for(m=0;m<x;m++){
printf(" |%2d|%2d|%2d|%2d|",anm[m][5],anm[m][6],anm[m][7],anm[m][8]);}
printf("\n");
for(m=0;m<x;m++){printf(" |---+---+---+---|");}
printf("\n");
for(m=0;m<x;m++){
printf(" |%2d|%2d|%2d|%2d|",anm[m][9],anm[m][10],anm[m][11],anm[m][12]);}
printf("\n");
for(m=0;m<x;m++){printf(" |---+---+---+---|");}
printf("\n");
for(m=0;m<x;m++){
printf(" |%2d|%2d|%2d|%2d|",anm[m][13],anm[m][14],anm[m][15],anm[m][16]);}
printf("\n");
for(m=0;m<x;m++){printf(" '---'---'---'---'");}
printf("\n");
}
/**/

```

\*\*\* 48 'Standard Solutions' of Pan-diagonal Magic Squares 4x4 \*\*\*

1/	2/	3/	4/	5/	6/
1   8   11   14	1   8   10   15	1   8   13   12	1   8   10   15	1   8   13   12	1   8   11   14
12   13   2   7	12   13   3   6	14   11   2   7	14   11   5   4	15   10   3   6	15   10   5   4
6   3   16   9	7   2   16   9	4   5   16   9	7   2   16   9	4   5   16   9	6   3   16   9
15   10   5   4	14   11   5   4	15   10   3   6	12   13   3   6	14   11   2   7	12   13   2   7
7/	8/	9/	10/	11/	12/
1   12   13   8	1   12   6   15	1   12   13   8	1   12   7   14	1   14   11   8	1   14   7   12
14   7   2   11	14   7   9   4	15   6   3   10	15   6   9   4	15   4   5   10	15   4   9   6
4   9   16   5	11   2   16   5	4   9   16   5	10   3   16   5	6   9   16   3	10   5   16   3
15   6   3   10	8   13   3   10	14   7   2   11	8   13   2   11	12   7   2   13	8   11   2   13

13/ 2   7   12   13 11   14   1   8 5   4   15   10 16   9   6   3	14/ 2   7   9   16 11   14   4   5 8   1   15   10 13   12   6   3	15/ 2   7   14   11 13   12   1   8 3   6   15   10 16   9   4   5	16/ 2   7   9   16 13   12   6   3 8   1   15   10 11   14   4   5	17/ 2   7   14   11 16   9   4   5 3   6   15   10 13   12   1   8	18/ 2   7   12   13 16   9   6   3 5   4   15   10 11   14   1   8
19/ 2   11   14   7 13   8   1   12 3   10   15   6 16   5   4   9	20/ 2   11   5   16 13   8   10   3 12   1   15   6 7   14   4   9	21/ 2   11   14   7 16   5   4   9 3   10   15   6 13   8   1   12	22/ 2   11   8   13 16   5   10   3 9   4   15   6 7   14   1   12	23/ 2   13   12   7 16   3   6   9 5   10   15   4 11   8   1   14	24/ 2   13   8   11 16   3   10   5 9   6   15   4 7   12   1   14
25/ 3   6   15   10 13   12   1   8 2   7   14   11 16   9   4   5	26/ 3   6   9   16 13   12   7   2 8   1   14   11 10   15   4   5	27/ 3   6   15   10 16   9   4   5 2   7   14   11 13   12   1   8	28/ 3   6   12   13 16   9   7   2 5   4   14   11 10   15   1   8	29/ 3   10   15   6 13   8   1   12 2   11   14   7 16   5   4   9	30/ 3   10   5   16 13   8   11   2 12   1   14   7 6   15   4   9
31/ 3   10   15   6 16   5   4   9 2   11   14   7 13   8   1   12	32/ 3   10   8   13 16   5   11   2 9   4   14   7 6   15   1   12	33/ 3   13   12   6 16   2   7   9 5   11   14   4 10   8   1   15	34/ 3   13   8   10 16   2   11   5 9   7   14   4 6   12   1   15	35/ 4   5   16   9 14   11   2   7 1   8   13   12 15   10   3   6	36/ 4   5   10   15 14   11   8   1 7   2   13   12 9   16   3   6
37/ 4   5   16   9 15   10   3   6 1   8   13   12 14   11   2   7	38/ 4   5   11   14 15   10   8   1 6   3   13   12 9   16   2   7	39/ 4   9   16   5 14   7   2   11 1   12   13   8 15   6   3   10	40/ 4   9   6   15 14   7   12   1 11   2   13   8 5   16   3   10	41/ 4   9   16   5 15   6   3   10 1   12   13   8 14   7   2   11	42/ 4   9   7   14 15   6   12   1 10   3   13   8 5   16   2   11
43/ 4   14   11   5 15   1   8   10 6   12   13   3 9   7   2   16	44/ 4   14   7   9 15   1   12   6 10   8   13   3 5   11   2   16	45/ 5   4   15   10 16   9   6   3 2   7   12   13 11   14   1   8	46/ 5   4   14   11 16   9   7   2 3   6   12   13 10   15   1   8	47/ 6   3   16   9 15   10   5   4 1   8   11   14 12   13   2   7	48/ 6   3   13   12 15   10   8   1 4   5   11   14 9   16   2   7

[Count = 48] OK!

It may be of this type what you often see for the solution list of our object. But it really changes its 'faces' according to the list forming inequality conditions taken.

The next list is made up under the conditions:  $n_1=1$ ;  $n_2<n_4$ ;  $n_5<n_{13}$  and  $n_2<n_5$

\*\* 'Fundamental Three' of Pan-diagonal Magic Squares 4x4 \*\*

[1]

<pre> 5  4 15 10  5  4 15 10 11 14  1  8 11 14  1  8     ---+---+---+---    2  7 12 13  2  7 12 13     ---+---+---+---  16  9  6  3 16  9  6  3     ---+---+---+---    5  4 15 10  5  4 15 10     ---+---+---+---  11 14  1  8 11 14  1  8                 </pre>	<p>* Sums of Rows and Columns *</p> <p>1+ 8+11+14 = 34 ... rw1  12+13+ 2+ 7 = 34 ... rw2  6+ 3+16+ 9 = 34 ... rw3  15+10+ 5+ 4 = 34 ... rw4  1+12+ 6+15 = 34 ... c11  8+13+ 3+10 = 34 ... c12  11+ 2+16+ 5 = 34 ... c13  14+ 7+ 9+ 4 = 34 ... c14</p>
-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

\* Sums of Pan-diagonals \*

1+13+16+ 4 = 34 ... pd1; 1+ 7+16+10 = 34 ... pb1;  
8+ 2+ 9+15 = 34 ... pd2; 8+12+ 9+ 5 = 34 ... pb2;  
11+ 7+ 6+10 = 34 ... pd3; 11+13+ 6+ 4 = 34 ... pb3;  
14+12+ 3+ 5 = 34 ... pd4; 14+ 2+ 3+15 = 34 ... pb4;  
... This is certainly a Pan-diagonal Magic Type!

[2]

<pre> 5  4 14 11  5  4 14 11 10 15  1  8 10 15  1  8     ---+---+---+---    3  6 12 13  3  6 12 13     ---+---+---+---  16  9  7  2 16  9  7  2     ---+---+---+---    5  4 14 11  5  4 14 11     ---+---+---+---  10 15  1  8 10 15  1  8                 </pre>	<p>* Sums of Rows and Columns *</p> <p>1+ 8+10+15 = 34 ... rw1  12+13+ 3+ 6 = 34 ... rw2  7+ 2+16+ 9 = 34 ... rw3  14+11+ 5+ 4 = 34 ... rw4  1+12+ 7+14 = 34 ... c11  8+13+ 2+11 = 34 ... c12  10+ 3+16+ 5 = 34 ... c13  15+ 6+ 9+ 4 = 34 ... c14</p>
-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

\* Sums of Pan-diagonals \*

1+13+16+ 4 = 34 ... pd1; 1+ 6+16+11 = 34 ... pb1;  
8+ 3+ 9+14 = 34 ... pd2; 8+12+ 9+ 5 = 34 ... pb2;  
10+ 6+ 7+11 = 34 ... pd3; 10+13+ 7+ 4 = 34 ... pb3;  
15+12+ 2+ 5 = 34 ... pd4; 15+ 3+ 2+14 = 34 ... pb4;  
... This is certainly a Pan-diagonal Magic Type!

[3]

<pre> 3  6 15 10  3  6 15 10 13 12  1  8 13 12  1  8     ---+---+---+---    2  7 14 11  2  7 14 11     ---+---+---+---  16  9  4  5 16  9  4  5     ---+---+---+---    3  6 15 10  3  6 15 10     ---+---+---+---  13 12  1  8 13 12  1  8                 </pre>	<p>* Sums of Rows and Columns *</p> <p>1+ 8+13+12 = 34 ... rw1  14+11+ 2+ 7 = 34 ... rw2  4+ 5+16+ 9 = 34 ... rw3  15+10+ 3+ 6 = 34 ... rw4  1+14+ 4+15 = 34 ... c11  8+11+ 5+10 = 34 ... c12  13+ 2+16+ 3 = 34 ... c13  12+ 7+ 9+ 6 = 34 ... c14</p>
-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

\* Sums of Pan-diagonals \*

1+11+16+ 6 = 34 ... pd1; 1+ 7+16+10 = 34 ... pb1;  
8+ 2+ 9+15 = 34 ... pd2; 8+14+ 9+ 3 = 34 ... pb2;  
13+ 7+ 4+10 = 34 ... pd3; 13+11+ 4+ 6 = 34 ... pb3;  
12+14+ 5+ 3 = 34 ... pd4; 12+ 2+ 5+15 = 34 ... pb4;  
... This is certainly a Pan-diagonal Magic Type!

[Count = 3] OK!

### #3. Various Types of Pan-magic Squares have the Same Solution Set.

Can you imagine what solution sets you might make for pan-magic squares 4x4 under such four different definitions as listed below?

No.1 is just the same with the previous definition of this section, as you see.

In the case of order 8, we know we could have four independent sets of solutions, different from each other, under similar style of definitions. How about order 4?

#### 1. [Basic Form and Basic Definitions] for 'Pan-diagonal' Type:

<pre> 15 16 13 14 15 16 13 14              3  4 n1 n2 n3 n4  1  2    ---+---+---+---   7  8 n5 n6 n7 n8  5  6    ---+---+---+---  11 12 n9 10 11 12  9 10    ---+---+---+---  15 16 13 14 15 16 13 14    ---+---+---+---   3  4  1  2  3  4  1  2                 </pre>	<pre> * Rows and Columns * n1+ n2+ n3+ n4 = C ... rw1 n5+ n6+ n7+ n8 = C ... rw2 n9+n10+n11+n12 = C ... rw3 n13+n14+n15+n16 = C ... rw4 n1+ n5+ n9+n13 = C ... c11 n2+ n6+n10+n14 = C ... c12 n3+ n7+n11+n15 = C ... c13 n4+ n8+n12+n16 = C ... c14                 </pre>
--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

#### \* Pan-diagonal Conditions \*

$n1+ n6+n11+n16 = C \dots pd1;$      $n1+ n8+n11+n14 = C \dots pb1;$   
 $n2+ n7+n12+n13 = C \dots pd2;$      $n2+ n5+n12+n15 = C \dots pb2;$   
 $n3+ n8+ n9+n14 = C \dots pd3;$      $n3+ n6+ n9+n16 = C \dots pb3;$   
 $n4+ n5+n10+n15 = C \dots pd4;$      $n4+ n7+n10+n13 = C \dots pb4;$

#### 2. [Basic Form and Basic Definitions] for 'Complete' Type:

<pre>  n1 n2 n3 n4   ---+---+---+---   n5 n6 n7 n8   ---+---+---+---   n9 10 11 12   ---+---+---+---   13 14 15 16   ---+---+---+---                  </pre>	<pre> * Rows and Columns * n1+ n2+ n3+ n4 = C ... rw1 n5+ n6+ n7+ n8 = C ... rw2 n9+n10+n11+n12 = C ... rw3 n13+n14+n15+n16 = C ... rw4 n1+ n5+ n9+n13 = C ... c11 n2+ n6+n10+n14 = C ... c12 n3+ n7+n11+n15 = C ... c13 n4+ n8+n12+n16 = C ... c14                 </pre>
--------------------------------------------------------------------------------------------------------------------------------------------------------------	----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

#### \* Complete Conditions \*

$n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC=C/2 \dots cmlt$

#### 3. [Basic Form and Basic Definitions] for 'Composite' Type:

<pre>  n1 n2 n3 n4   ---+---+---+---   n5 n6 n7 n8   ---+---+---+---   n9 10 11 12   ---+---+---+---   13 14 15 16   ---+---+---+---                  </pre>	<pre> * Composite Conditions * n1+ n2+ n5+ n6 = C ... cst1 n2+ n3+ n6+ n7 = C ... cst2 n3+ n4+ n7+ n8 = C ... cst3 n5+ n6+ n9+n10 = C ... cst4 n6+ n7+n10+n11 = C ... cst5 n7+ n8+n11+n12 = C ... cst6 n9+n10+n13+n14 = C ... cst7 n10+n11+n14+n15 = C ... cst8 n11+n12+n15+n16 = C ... cst9                 </pre>
--------------------------------------------------------------------------------------------------------------------------------------------------------------	---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

#### \* Additional Conditions \*

$n1+n2+n3+n4=C \dots rw1;$      $n1+n5+n9+n13=C \dots c11;$      $n1+n6+n11+n16=C \dots pd1;$

4. [Basic Form and Basic Definitions] for 'Composite and Complete' Type:

<pre> . . . . .  n1 n2 n3 n4   --+--+--+--+  n5 n6 n7 n8   --+--+--+--+  n9 10 11 12   --+--+--+--+  13 14 15 16  '-'-'-'-'-'                 </pre>	<p><b>* Composite Conditions *</b></p> <p>n1+ n2+ n5+ n6 = C ...cst1  n2+ n3+ n6+ n7 = C ...cst2  n3+ n4+ n7+ n8 = C ...cst3  n5+ n6+ n9+n10 = C ...cst4  n6+ n7+n10+n11 = C ...cst5  n7+ n8+n11+n12 = C ...cst6  n9+n10+n13+n14 = C ...cst7  n10+n11+n14+n15 = C ...cst8  n11+n12+n15+n16 = C ...cst9</p>	<p><b>* Row and Column *</b></p> <p>n1+ n2+ n3+ n4 = C ...rw1  n1+ n5+ n9+n13 = C ...c11</p>
------------------------------------------------------------------------------------------------------------------------------------------------------	--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	--------------------------------------------------------------------------------------------------

**\* Complete Conditions \***  
n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC=C/2 ... cmlt

In the case of order 4, whenever we start making under any of those four different definitions above, we cannot but make the only one solution set, completely the same in three levels. We cannot make any other solution set at all.

It is truly amazing. Every time when I saw the same result of those four types, I used to be extremely surprised. I began to think about what it really means.

Let's have some algebraic study of our object here for a while.

**#4. Important Properties of Pan-Diagonal Magic Squares**

I want to start our argument with the Definition No.1 for 'Pan-diagonal' type.

<pre> 15 16 13 14 15 16 13 14  3  4  n1 n2 n3 n4   1  2      ---+---+---+---   7  8  n5 n6 n7 n8   5  6      ---+---+---+---  11 12  n9 10 11 12   9 10      ---+---+---+---  15 16  13 14 15 16  13 14     '---'-'---'-'---'-'---'  3  4  1  2  3  4  1  2                 </pre>	<p><b>* Rows and Columns *</b></p> <p>n1+ n2+ n3+ n4 = C ... rw1  n5+ n6+ n7+ n8 = C ... rw2  n9+n10+n11+n12 = C ... rw3  n13+n14+n15+n16 = C ... rw4  n1+ n5+ n9+n13 = C ... c11  n2+ n6+n10+n14 = C ... c12  n3+ n7+n11+n15 = C ... c13  n4+ n8+n12+n16 = C ... c14</p>
------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

n1+ n6+n11+n16 = C ... pd1;	n1+ n8+n11+n14 = C ... pb1;
n2+ n7+n12+n13 = C ... pd2;	n2+ n5+n12+n15 = C ... pb2;
n3+ n8+ n9+n14 = C ... pd3;	n3+ n6+ n9+n16 = C ... pb3;
n4+ n5+n10+n15 = C ... pd4;	n4+ n7+n10+n13 = C ... pb4;

rw1+rw2+rw3+rw4  
n1+n2+n3+n4+n5+n6+n7+n8+n9+ ... +n15+n16 = 4\*C  
1+2+3+4+5+6+7+8+9+ ... +15+16 = 4\*C  
(1+16)\*16/2 = 4\*C

Therefore C = 34 ... r0

Compare and Calculate the Pandiagonal equations as follows:

n1+ n6+n11+n16=C ... pd1 -->	(n1+n11)+(n6+n16)=C
n1+ n8+n11+n14=C ... pb1 -->	(n1+n11)+(n8+n14)=C
Therefore	n6+n16=n8+n14 (=CC1)
n2+ n7+n12+n13=C ... pd2 -->	(n2+n12)+(n7+n13)=C
n2+ n5+n12+n15=C ... pb2 -->	(n2+n12)+(n5+n15)=C
Therefore	n5+n15=n7+n13 (=CC2)
n3+ n8+ n9+n14=C ... pd3 -->	(n3+n9)+(n8+n14)=C





n1   n2   n3   n4	$bA+bB=(n1+n2+n3+n4)+(n5+n6+n7+n8)=2*C;$
-bA--   --bB-	$bA+bD=(n1+n\underline{n1})+(n2+n\underline{n2})+(n5+n\underline{n5})+(n6+n\underline{n6})=4x17=68=2*C;$
n5   n6   n7   n8	$bB+bD=(n3+n7+n11+n15)+(n4+n8+n12+n16)=2*C;$
-----+-----	$bB+bC=(n3+n\underline{n3})+(n4+n\underline{n4})+(n7+n\underline{n7})+(n8+n\underline{n8})=4x17=68=2*C;$
n3   n4   n1   n2	The first two shows $bB=bD=2*C/2=C (=34)$
-bC--   --bD-	The second two shows $bA=bB=2*C/2=C (=34)$
n7   n8   n5   n6	.....
	Consequently $bA=bB=bC=bD=C (=34)$

This means half of 'Composite Conditions' in Definition No.4 are always true.

What about {n2, n3, n6, n7}?

$$n1+n2+n3+n4=C;$$

$$+) n5+n6+n7+n8=C;$$


---


$$(n2+n3+n6+n7)+(n1+n4+n5+n8)=2*C$$

But  $(n2+n3+n6+n7)+(n\underline{n1}+n\underline{n4}+n\underline{n5}+n\underline{n8})=2*C$

Therefore  $n1+n4+n5+n8 = n\underline{n1}+n\underline{n4}+n\underline{n5}+n\underline{n8}$

But  $(n1+n5+n4+n8)+(n4+n8+n1+n5)$

$$=(n1+n\underline{n1})+(n4+n\underline{n4})+(n5+n\underline{n5})+(n8+n\underline{n8})=4x17=68=2*C$$

As a consequence  $n1+n4+n5+n8 = n\underline{n1}+n\underline{n4}+n\underline{n5}+n\underline{n8} = C$

That means  $n2+n3+n6+n7 = C$

In the same way all the other half of 'Composite Conditions' are always true.

After all we can say any Pan-diagonal magic square proves to be always the 'Composite' pan-magic type, and also the 'Composite and Complete' type at the same time, in the case of order 4. This makes it possible that every type has the same solution set. Only our algebraic study could reveal this inner relations among them.

### #6. What Transformation System can we Assume?

Let's go on our algebraic study and step to the transformation system.

Which lines can we exchange in our object? Won't you see the next diagrams using the new notation for 'Complementary Pairs of 17'?

Let me skip all those boring explanations and jump over to the conclusion.

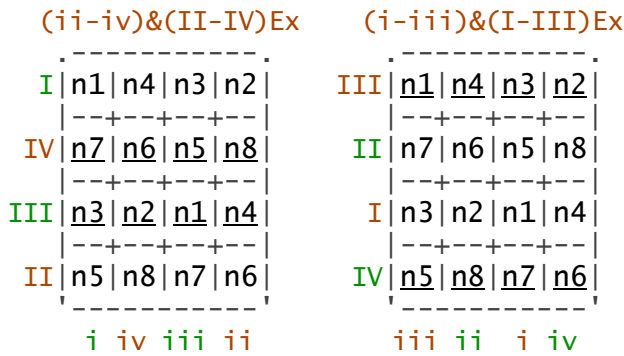
We know we can exchange the 2nd<->4th row/column with each other, and also exchange the 1st<->3rd row/column with each other.

We could even combine those two types of exchanges without destroying anything about the constant sums we always expect.

Therefore, we can take such a powerful transformation system with this possibility of line exchanges as follows.

#### [Possibility of Line Exchange]

Basic Form	(ii-iv)Exchange	(II-IV)Exchange																											
<table style="width: 100%; border-collapse: collapse;"> <tr><td style="border-right: 1px solid black; padding: 2px;">I   n1   n2   n3   n4  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">II   n5   n6   n7   n8  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">III   n3   n4   n1   n2  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">IV   n7   n8   n5   n6  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">i   ii   iii   iv  </td></tr> </table>	I   n1   n2   n3   n4	---+---+---+---	II   n5   n6   n7   n8	---+---+---+---	III   n3   n4   n1   n2	---+---+---+---	IV   n7   n8   n5   n6	---+---+---+---	i   ii   iii   iv	<table style="width: 100%; border-collapse: collapse;"> <tr><td style="border-right: 1px solid black; padding: 2px;">I   n1   n4   n3   n2  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">II   n5   n8   n7   n6  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">III   n3   n2   n1   n4  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">IV   n7   n6   n5   n8  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">i   iv   iii   ii  </td></tr> </table>	I   n1   n4   n3   n2	---+---+---+---	II   n5   n8   n7   n6	---+---+---+---	III   n3   n2   n1   n4	---+---+---+---	IV   n7   n6   n5   n8	---+---+---+---	i   iv   iii   ii	<table style="width: 100%; border-collapse: collapse;"> <tr><td style="border-right: 1px solid black; padding: 2px;">I   n1   n2   n3   n4  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">IV   n7   n8   n5   n6  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">III   n3   n4   n1   n2  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">II   n5   n6   n7   n8  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">  ---+---+---+---  </td></tr> <tr><td style="border-right: 1px solid black; padding: 2px;">i   ii   iii   iv  </td></tr> </table>	I   n1   n2   n3   n4	---+---+---+---	IV   n7   n8   n5   n6	---+---+---+---	III   n3   n4   n1   n2	---+---+---+---	II   n5   n6   n7   n8	---+---+---+---	i   ii   iii   iv
I   n1   n2   n3   n4																													
---+---+---+---																													
II   n5   n6   n7   n8																													
---+---+---+---																													
III   n3   n4   n1   n2																													
---+---+---+---																													
IV   n7   n8   n5   n6																													
---+---+---+---																													
i   ii   iii   iv																													
I   n1   n4   n3   n2																													
---+---+---+---																													
II   n5   n8   n7   n6																													
---+---+---+---																													
III   n3   n2   n1   n4																													
---+---+---+---																													
IV   n7   n6   n5   n8																													
---+---+---+---																													
i   iv   iii   ii																													
I   n1   n2   n3   n4																													
---+---+---+---																													
IV   n7   n8   n5   n6																													
---+---+---+---																													
III   n3   n4   n1   n2																													
---+---+---+---																													
II   n5   n6   n7   n8																													
---+---+---+---																													
i   ii   iii   iv																													



Please see the next diagram I presented at the beginning of this section here again.

\* 4x4 New Squares Taken out of the Extended Space \*

00/	01/	02/	03/
n1 n2 n3 n4	13 14 15 16	9 10 11 12	5 6 7 8
n5 n6 n7 n8	1 2 3 4	13 14 15 16	9 10 11 12
n9 10 11 12	5 6 7 8	1 2 3 4	13 14 15 16
13 14 15 16	9 10 11 12	5 6 7 8	1 2 3 4
10/	11/	12/	13/
4 1 2 3	16 13 14 15	12 9 10 11	8 5 6 7
8 5 6 7	4 1 2 3	16 13 14 15	12 9 10 11
12 9 10 11	8 5 6 7	4 1 2 3	16 13 14 15
16 13 14 15	12 9 10 11	8 5 6 7	4 1 2 3
20/	21/	22/	23/
3 4 1 2	15 16 13 14	11 12 9 10	7 8 5 6
7 8 5 6	3 4 1 2	15 16 13 14	11 12 9 10
11 12 9 10	7 8 5 6	3 4 1 2	15 16 13 14
15 16 13 14	11 12 9 10	7 8 5 6	3 4 1 2
30/	31/	32/	33/
2 3 4 1	14 15 16 13	10 11 12 9	6 7 8 5
6 7 8 5	2 3 4 1	14 15 16 13	10 11 12 9
10 11 12 9	6 7 8 5	2 3 4 1	14 15 16 13
14 15 16 13	10 11 12 9	6 7 8 5	2 3 4 1

We can explain what it really means by the line exchangeability, can't we?

Take 4 examples 00/, 03/, 30/ and 33/ and get them smart with n1=1 always on the left top as shown below. What parts of 03/, 30/ and 33/ are different from the ones of original 00/?

Yes. We can transform the original into others with those line exchangeabilities between the 2nd<->4th rows/columns and the combination of two types.

00/	03/	30/	33/
n1   n2   n3   n4	n1   n2   n3   n4	n1   n4   n3   n2	n1   n4   n3   n2
n5   n6   n7   n8	13   14   15   16	n5   n8   n7   n6	13   16   15   14
n9   10   11   12	n9   10   11   12	n9   12   11   10	n9   12   11   10
13   14   15   16	n5   n6   n7   n8	13   16   15   14	n5   n8   n7   n6

## #7. Reconstruction of All Pandiagonal MS44 from the Fundamental Three

Now let's have an experiment to reconstruct all 384 'primitive solutions' from the 'Fundamental Three' by such transformation system as listed below.

**\* Transformation System for Pan-diagonal MS44 \***

**1. [4x4 Transformations by the 'Row/Column Shift']**

	00/	01/	02/	03/											
1	2	3	4	13	14	15	16	9	10	11	12	5	6	7	8
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
9	10	11	12	5	6	7	8	1	2	3	4	13	14	15	16
13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4
	10/	11/	12/	13/											
4	1	2	3	16	13	14	15	12	9	10	11	8	5	6	7
8	5	6	7	4	1	2	3	16	13	14	15	12	9	10	11
12	9	10	11	8	5	6	7	4	1	2	3	16	13	14	15
16	13	14	15	12	9	10	11	8	5	6	7	4	1	2	3
	20/	21/	22/	23/											
3	4	1	2	15	16	13	14	11	12	9	10	7	8	5	6
7	8	5	6	3	4	1	2	15	16	13	14	11	12	9	10
11	12	9	10	7	8	5	6	3	4	1	2	15	16	13	14
15	16	13	14	11	12	9	10	7	8	5	6	3	4	1	2
	30/	31/	32/	33/											
2	3	4	1	14	15	16	13	10	11	12	9	6	7	8	5
6	7	8	5	2	3	4	1	14	15	16	13	10	11	12	9
10	11	12	9	6	7	8	5	2	3	4	1	14	15	16	13
14	15	16	13	10	11	12	9	6	7	8	5	2	3	4	1

**2. [Mirror Reflection and Rotation by 90 Degrees Clockwise]**

	00/	01/	02/	03/											
1	2	3	4	13	9	5	1	16	15	14	13	4	8	12	16
5	6	7	8	14	10	6	2	12	11	10	9	3	7	11	15
9	10	11	12	15	11	7	3	8	7	6	5	2	6	10	14
13	14	15	16	16	12	8	4	4	3	2	1	1	5	9	13
	10/	11/	12/	13/											
1	5	9	13	4	3	2	1	16	12	8	4	13	14	15	16
2	6	10	14	8	7	6	5	15	11	7	3	9	10	11	12
3	7	11	15	12	11	10	9	14	10	6	2	5	6	7	8
4	8	12	16	16	15	14	13	13	9	5	1	1	2	3	4

We plan to make each Fundamental into 16x8 forms by the combination of these two types of transformation, and finally make 3x16x8=384 'primitive solutions'.

First of all, let me show you our starting points: the Fundamental Three.  
And then you see the long list demonstrating the result of our reconstruction.

**\*\* Reconstruct 'Pan-diagonal' Magic Squares 4x4 \*\***  
**\*\* Start: The 'Fundamental' Three Solutions: \*\***

	1/	2/	3/								
1	8	10	15	1	8	11	14	1	8	13	12
12	13	3	6	12	13	2	7	14	11	2	7
7	2	16	9	6	3	16	9	4	5	16	9
14	11	5	4	15	10	5	4	15	10	3	6

**\*\* Transform them into 'Primitive' Solutions and Compare them with the Old List \*\***

[Fundamenta] #1]

	1/	330/	74/	353/	8/	360/	79/	335/
1	8 10 15	14 7 12 1	4 5 11 14	15 6 9 4	1 12 7 14	15 10 8 1	4 9 6 15	14 11 5 4
12	13 3 6	11 2 13 8	9 16 2 7	10 3 16 5	8 13 2 11	6 3 13 12	5 16 3 10	7 2 16 9
7	2 16 9	5 16 3 10	6 3 13 12	8 13 2 11	10 3 16 5	9 16 2 7	11 2 13 8	12 13 3 6
14	11 5 4	4 9 6 15	15 10 8 1	1 12 7 14	15 6 9 4	4 5 11 14	14 7 12 1	1 8 10 15
	332/	157/	213/	92/	317/	77/	204/	148/
14	11 5 4	7 12 1 14	9 16 2 7	4 15 6 9	14 1 12 7	4 5 11 14	9 6 15 4	7 2 16 9
1	8 10 15	2 13 8 11	6 3 13 12	5 10 3 16	11 8 13 2	15 10 8 1	16 3 10 5	12 13 3 6
12	13 3 6	16 3 10 5	15 10 8 1	11 8 13 2	5 10 3 16	6 3 13 12	2 13 8 11	1 8 10 15
7	2 16 9	9 6 15 4	4 5 11 14	14 1 12 7	4 15 6 9	9 16 2 7	7 12 1 14	14 11 5 4
	150/	269/	125/	198/	163/	211/	140/	284/
7	2 16 9	12 1 14 7	6 3 13 12	9 4 15 6	7 14 1 12	9 16 2 7	6 15 4 9	12 13 3 6
14	11 5 4	13 8 11 2	15 10 8 1	16 5 10 3	2 11 8 13	4 5 11 14	3 10 5 16	1 8 10 15
1	8 10 15	3 10 5 16	4 5 11 14	2 11 8 13	16 5 10 3	15 10 8 1	13 8 11 2	14 11 5 4
12	13 3 6	6 15 4 9	9 16 2 7	7 14 1 12	9 4 15 6	6 3 13 12	12 1 14 7	7 2 16 9
	287/	14/	358/	127/	282/	122/	347/	3/
12	13 3 6	1 14 7 12	15 10 8 1	6 9 4 15	12 7 14 1	6 3 13 12	15 4 9 6	1 8 10 15
7	2 16 9	8 11 2 13	4 5 11 14	3 16 5 10	13 2 11 8	9 16 2 7	10 5 16 3	14 11 5 4
14	11 5 4	10 5 16 3	9 16 2 7	13 2 11 8	3 16 5 10	4 5 11 14	8 11 2 13	7 2 16 9
1	8 10 15	15 4 9 6	6 3 13 12	12 7 14 1	6 9 4 15	15 10 8 1	1 14 7 12	12 13 3 6
	339/	81/	114/	220/	350/	229/	119/	88/
15	1 8 10	4 9 6 15	5 11 14 4	10 3 16 5	15 6 9 4	10 8 1 15	5 16 3 10	4 14 11 5
6	12 13 3	14 7 12 1	16 2 7 9	8 13 2 11	1 12 7 14	3 13 12 6	11 2 13 8	9 7 2 16
9	7 2 16	11 2 13 8	3 13 12 6	1 12 7 14	8 13 2 11	16 2 7 9	14 7 12 1	6 12 13 3
4	14 11 5	5 16 3 10	10 8 1 15	15 6 9 4	10 3 16 5	5 11 14 4	4 9 6 15	15 1 8 10
	90/	202/	361/	105/	95/	112/	368/	207/
4	14 11 5	9 6 15 4	16 2 7 9	5 10 3 16	4 15 6 9	5 11 14 4	16 3 10 5	9 7 2 16
15	1 8 10	7 12 1 14	3 13 12 6	11 8 13 2	14 1 12 7	10 8 1 15	2 13 8 11	6 12 13 3
6	12 13 3	2 13 8 11	10 8 1 15	14 1 12 7	11 8 13 2	3 13 12 6	7 12 1 14	15 1 8 10
9	7 2 16	16 3 10 5	5 11 14 4	4 15 6 9	5 10 3 16	16 2 7 9	9 6 15 4	4 14 11 5
	205/	143/	64/	374/	196/	363/	57/	138/
9	7 2 16	6 15 4 9	3 13 12 6	16 5 10 3	9 4 15 6	16 2 7 9	3 10 5 16	6 12 13 3
4	14 11 5	12 1 14 7	10 8 1 15	2 11 8 13	7 14 1 12	5 11 14 4	13 8 11 2	15 1 8 10
15	1 8 10	13 8 11 2	5 11 14 4	7 14 1 12	2 11 8 13	10 8 1 15	12 1 14 7	4 14 11 5
6	12 13 3	3 10 5 16	16 2 7 9	9 4 15 6	16 5 10 3	3 13 12 6	6 15 4 9	9 7 2 16
	136/	344/	231/	71/	129/	66/	226/	337/
6	12 13 3	15 4 9 6	10 8 1 15	3 16 5 10	6 9 4 15	3 13 12 6	10 5 16 3	15 1 8 10
9	7 2 16	1 14 7 12	5 11 14 4	13 2 11 8	12 7 14 1	16 2 7 9	8 11 2 13	4 14 11 5
4	14 11 5	8 11 2 13	16 2 7 9	12 7 14 1	13 2 11 8	5 11 14 4	1 14 7 12	9 7 2 16
15	1 8 10	10 5 16 3	3 13 12 6	6 9 4 15	3 16 5 10	10 8 1 15	15 4 9 6	6 12 13 3
	235/	116/	260/	187/	222/	174/	245/	101/
10	15 1 8	5 16 3 10	11 14 4 5	8 13 2 11	10 3 16 5	8 1 15 10	11 2 13 8	5 4 14 11
3	6 12 13	4 9 6 15	2 7 9 16	1 12 7 14	15 6 9 4	13 12 6 3	14 7 12 1	16 9 7 2
16	9 7 2	14 7 12 1	13 12 6 3	15 6 9 4	1 12 7 14	2 7 9 16	4 9 6 15	3 6 12 13
5	4 14 11	11 2 13 8	8 1 15 10	10 3 16 5	8 13 2 11	11 14 4 5	5 16 3 10	10 15 1 8
	98/	371/	27/	258/	103/	263/	38/	382/
5	4 14 11	16 3 10 5	2 7 9 16	11 8 13 2	5 10 3 16	11 14 4 5	2 13 8 11	16 9 7 2
10	15 1 8	9 6 15 4	13 12 6 3	14 1 12 7	4 15 6 9	8 1 15 10	7 12 1 14	3 6 12 13
3	6 12 13	7 12 1 14	8 1 15 10	4 15 6 9	14 1 12 7	13 12 6 3	9 6 15 4	10 15 1 8
16	9 7 2	2 13 8 11	11 14 4 5	5 10 3 16	11 8 13 2	2 7 9 16	16 3 10 5	5 4 14 11
	384/	55/	311/	32/	377/	25/	306/	50/
16	9 7 2	3 10 5 16	13 12 6 3	2 11 8 13	16 5 10 3	2 7 9 16	13 8 11 2	3 6 12 13
5	4 14 11	6 15 4 9	8 1 15 10	7 14 1 12	9 4 15 6	11 14 4 5	12 1 14 7	10 15 1 8
10	15 1 8	12 1 14 7	11 14 4 5	9 4 15 6	7 14 1 12	8 1 15 10	6 15 4 9	5 4 14 11
3	6 12 13	13 8 11 2	2 7 9 16	16 5 10 3	2 11 8 13	13 12 6 3	3 10 5 16	16 9 7 2
	53/	228/	172/	293/	68/	308/	181/	237/
3	6 12 13	10 5 16 3	8 1 15 10	13 2 11 8	3 16 5 10	13 12 6 3	8 11 2 13	10 15 1 8
16	9 7 2	15 4 9 6	11 14 4 5	12 7 14 1	6 9 4 15	2 7 9 16	1 14 7 12	5 4 14 11
5	4 14 11	1 14 7 12	2 7 9 16	6 9 4 15	12 7 14 1	11 14 4 5	15 4 9 6	16 9 7 2
10	15 1 8	8 11 2 13	13 12 6 3	3 16 5 10	13 2 11 8	8 1 15 10	10 5 16 3	3 6 12 13

180/	242/	321/	11/	189/	22/	328/	247/
8 10 15 1	11 2 13 8	14 4 5 11	1 12 7 14	8 13 2 11	1 15 10 8	14 7 12 1	11 5 4 14
13 3 6 12	5 16 3 10	7 9 16 2	15 6 9 4	10 3 16 5	12 6 3 13	4 9 6 15	2 16 9 7
2 16 9 7	4 9 6 15	12 6 3 13	10 3 16 5	15 6 9 4	7 9 16 2	5 16 3 10	13 3 6 12
11 5 4 14	14 7 12 1	1 15 10 8	8 13 2 11	1 12 7 14	14 4 5 11	11 2 13 8	8 10 15 1
249/	41/	154/	314/	256/	319/	159/	48/
11 5 4 14	2 13 8 11	7 9 16 2	14 1 12 7	11 8 13 2	14 4 5 11	7 12 1 14	2 16 9 7
8 10 15 1	16 3 10 5	12 6 3 13	4 15 6 9	5 10 3 16	1 15 10 8	9 6 15 4	13 3 6 12
13 3 6 12	9 6 15 4	1 15 10 8	5 10 3 16	4 15 6 9	12 6 3 13	16 3 10 5	8 10 15 1
2 16 9 7	7 12 1 14	14 4 5 11	11 8 13 2	14 1 12 7	7 9 16 2	2 13 8 11	11 5 4 14
46/	304/	271/	165/	35/	156/	266/	297/
2 16 9 7	13 8 11 2	12 6 3 13	7 14 1 12	2 11 8 13	7 9 16 2	12 1 14 7	13 3 6 12
11 5 4 14	3 10 5 16	1 15 10 8	9 4 15 6	16 5 10 3	14 4 5 11	6 15 4 9	8 10 15 1
8 10 15 1	6 15 4 9	14 4 5 11	16 5 10 3	9 4 15 6	1 15 10 8	3 10 5 16	11 5 4 14
13 3 6 12	12 1 14 7	7 9 16 2	2 11 8 13	7 14 1 12	12 6 3 13	13 8 11 2	2 16 9 7
295/	183/	24/	280/	290/	273/	17/	178/
13 3 6 12	8 11 2 13	1 15 10 8	12 7 14 1	13 2 11 8	12 6 3 13	1 14 7 12	8 10 15 1
2 16 9 7	10 5 16 3	14 4 5 11	6 9 4 15	3 16 5 10	7 9 16 2	15 4 9 6	11 5 4 14
11 5 4 14	15 4 9 6	7 9 16 2	3 16 5 10	6 9 4 15	14 4 5 11	10 5 16 3	2 16 9 7
8 10 15 1	1 14 7 12	12 6 3 13	13 2 11 8	12 7 14 1	1 15 10 8	8 11 2 13	13 3 6 12

[Fundamental #2]

2/	354/	73/	329/	7/	336/	80/	359/
1 8 11 14	15 6 12 1	4 5 10 15	14 7 9 4	1 12 6 15	14 11 8 1	4 9 7 14	15 10 5 4
12 13 2 7	10 3 13 8	9 16 3 6	11 2 16 5	8 13 3 10	7 2 13 12	5 16 2 11	6 3 16 9
6 3 16 9	5 16 2 11	7 2 13 12	8 13 3 10	11 2 16 5	9 16 3 6	10 3 13 8	12 13 2 7
15 10 5 4	4 9 7 14	14 11 8 1	1 12 6 15	14 7 9 4	4 5 10 15	15 6 12 1	1 8 11 14
356/	133/	215/	86/	341/	75/	210/	124/
15 10 5 4	6 12 1 15	9 16 3 6	4 14 7 9	15 1 12 6	4 5 10 15	9 7 14 4	6 3 16 9
1 8 11 14	3 13 8 10	7 2 13 12	5 11 2 16	10 8 13 3	14 11 8 1	16 2 11 5	12 13 2 7
12 13 2 7	16 2 11 5	14 11 8 1	10 8 13 3	5 11 2 16	7 2 13 12	3 13 8 10	1 8 11 14
6 3 16 9	9 7 14 4	4 5 10 15	15 1 12 6	4 14 7 9	9 16 3 6	6 12 1 15	15 10 5 4
126/	270/	149/	197/	139/	212/	164/	283/
6 3 16 9	12 1 15 6	7 2 13 12	9 4 14 7	6 15 1 12	9 16 3 6	7 14 4 9	12 13 2 7
15 10 5 4	13 8 10 3	14 11 8 1	16 5 11 2	3 10 8 13	4 5 10 15	2 11 5 16	1 8 11 14
1 8 11 14	2 11 5 16	4 5 10 15	3 10 8 13	16 5 11 2	14 11 8 1	13 8 10 3	15 10 5 4
12 13 2 7	7 14 4 9	9 16 3 6	6 15 1 12	9 4 14 7	7 2 13 12	12 1 15 6	6 3 16 9
285/	20/	334/	151/	276/	146/	323/	5/
12 13 2 7	1 15 6 12	14 11 8 1	7 9 4 14	12 6 15 1	7 2 13 12	14 4 9 7	1 8 11 14
6 3 16 9	8 10 3 13	4 5 10 15	2 16 5 11	13 3 10 8	9 16 3 6	11 5 16 2	15 10 5 4
15 10 5 4	11 5 16 2	9 16 3 6	13 3 10 8	2 16 5 11	4 5 10 15	8 10 3 13	6 3 16 9
1 8 11 14	14 4 9 7	7 2 13 12	12 6 15 1	7 9 4 14	14 11 8 1	1 15 6 12	12 13 2 7
315/	83/	108/	244/	326/	253/	117/	94/
14 1 8 11	4 9 7 14	5 10 15 4	11 2 16 5	14 7 9 4	11 8 1 14	5 16 2 11	4 15 10 5
7 12 13 2	15 6 12 1	16 3 6 9	8 13 3 10	1 12 6 15	2 13 12 7	10 3 13 8	9 6 3 16
9 6 3 16	10 3 13 8	2 13 12 7	1 12 6 15	8 13 3 10	16 3 6 9	15 6 12 1	7 12 13 2
4 15 10 5	5 16 2 11	11 8 1 14	14 7 9 4	11 2 16 5	5 10 15 4	4 9 7 14	14 1 8 11
96/	208/	367/	111/	89/	106/	362/	201/
4 15 10 5	9 7 14 4	16 3 6 9	5 11 2 16	4 14 7 9	5 10 15 4	16 2 11 5	9 6 3 16
14 1 8 11	6 12 1 15	2 13 12 7	10 8 13 3	15 1 12 6	11 8 1 14	3 13 8 10	7 12 13 2
7 12 13 2	3 13 8 10	11 8 1 14	15 1 12 6	10 8 13 3	2 13 12 7	6 12 1 15	14 1 8 11
9 6 3 16	16 2 11 5	5 10 15 4	4 14 7 9	5 11 2 16	16 3 6 9	9 7 14 4	4 15 10 5
199/	167/	40/	376/	194/	369/	33/	162/
9 6 3 16	7 14 4 9	2 13 12 7	16 5 11 2	9 4 14 7	16 3 6 9	2 11 5 16	7 12 13 2
4 15 10 5	12 1 15 6	11 8 1 14	3 10 8 13	6 15 1 12	5 10 15 4	13 8 10 3	14 1 8 11
14 1 8 11	13 8 10 3	5 10 15 4	6 15 1 12	3 10 8 13	11 8 1 14	12 1 15 6	4 15 10 5
7 12 13 2	2 11 5 16	16 3 6 9	9 4 14 7	16 5 11 2	2 13 12 7	7 14 4 9	9 6 3 16
160/	320/	255/	47/	153/	42/	250/	313/
7 12 13 2	14 4 9 7	11 8 1 14	2 16 5 11	7 9 4 14	2 13 12 7	11 5 16 2	14 1 8 11
9 6 3 16	1 15 6 12	5 10 15 4	13 3 10 8	12 6 15 1	16 3 6 9	8 10 3 13	4 15 10 5
4 15 10 5	8 10 3 13	16 3 6 9	12 6 15 1	13 3 10 8	5 10 15 4	1 15 6 12	9 6 3 16
14 1 8 11	11 5 16 2	2 13 12 7	7 9 4 14	2 16 5 11	11 8 1 14	14 4 9 7	7 12 13 2

259/	115/	236/	188/	246/	173/	221/	102/
11 14 1 8 5 16 2 11 10 15 4 5 8 13 3 10 11 2 16 5 8 1 14 11 10 3 13 8 5 4 15 10							
2 7 12 13 4 9 7 14 3 6 9 16 1 12 6 15 14 7 9 4 13 12 7 2 15 6 12 1 16 9 6 3							
16 9 6 3 15 6 12 1 13 12 7 2 14 7 9 4 1 12 6 15 3 6 9 16 4 9 7 14 2 7 12 13							
5 4 15 10 10 3 13 8 8 1 14 11 11 2 16 5 8 13 3 10 10 15 4 5 5 16 2 11 11 14 1 8							
100/	365/	51/	234/	109/	239/	62/	380/
5 4 15 10 16 2 11 5 3 6 9 16 10 8 13 3 5 11 2 16 10 15 4 5 3 13 8 10 16 9 6 3							
11 14 1 8 9 7 14 4 13 12 7 2 15 1 12 6 4 14 7 9 8 1 14 11 6 12 1 15 2 7 12 13							
2 7 12 13 6 12 1 15 8 1 14 11 4 14 7 9 15 1 12 6 13 12 7 2 9 7 14 4 11 14 1 8							
16 9 6 3 3 13 8 10 10 15 4 5 5 11 2 16 10 8 13 3 3 6 9 16 16 2 11 5 5 4 15 10							
383/	31/	312/	56/	378/	49/	305/	26/
16 9 6 3 2 11 5 16 13 12 7 2 3 10 8 13 16 5 11 2 3 6 9 16 13 8 10 3 2 7 12 13							
5 4 15 10 7 14 4 9 8 1 14 11 6 15 1 12 9 4 14 7 10 15 4 5 12 1 15 6 11 14 1 8							
11 14 1 8 12 1 15 6 10 15 4 5 9 4 14 7 6 15 1 12 8 1 14 11 7 14 4 9 5 4 15 10							
2 7 12 13 13 8 10 3 3 6 9 16 16 5 11 2 3 10 8 13 13 12 7 2 2 11 5 16 16 9 6 3							
29/	252/	170/	299/	44/	310/	175/	261/
2 7 12 13 11 5 16 2 8 1 14 11 13 3 10 8 2 16 5 11 13 12 7 2 8 10 3 13 11 14 1 8							
16 9 6 3 14 4 9 7 10 15 4 5 12 6 15 1 7 9 4 14 3 6 9 16 1 15 6 12 5 4 15 10							
5 4 15 10 1 15 6 12 3 6 9 16 7 9 4 14 12 6 15 1 10 15 4 5 14 4 9 7 16 9 6 3							
11 14 1 8 8 10 3 13 13 12 7 2 2 16 5 11 13 3 10 8 8 1 14 11 11 5 16 2 2 7 12 13							
186/	218/	345/	9/	191/	16/	352/	223/
8 11 14 1 10 3 13 8 15 4 5 10 1 12 6 15 8 13 3 10 1 14 11 8 15 6 12 1 10 5 4 15							
13 2 7 12 5 16 2 11 6 9 16 3 14 7 9 4 11 2 16 5 12 7 2 13 4 9 7 14 3 16 9 6							
3 16 9 6 4 9 7 14 12 7 2 13 11 2 16 5 14 7 9 4 6 9 16 3 5 16 2 11 13 2 7 12							
10 5 4 15 15 6 12 1 1 14 11 8 8 13 3 10 1 12 6 15 15 4 5 10 10 3 13 8 8 11 14 1							
225/	65/	130/	338/	232/	343/	135/	72/
10 5 4 15 3 13 8 10 6 9 16 3 15 1 12 6 10 8 13 3 15 4 5 10 6 12 1 15 3 16 9 6							
8 11 14 1 16 2 11 5 12 7 2 13 4 14 7 9 5 11 2 16 1 14 11 8 9 7 14 4 13 2 7 12							
13 2 7 12 9 7 14 4 1 14 11 8 5 11 2 16 4 14 7 9 12 7 2 13 16 2 11 5 8 11 14 1							
3 16 9 6 6 12 1 15 15 4 5 10 10 8 13 3 15 1 12 6 6 9 16 3 3 13 8 10 10 5 4 15							
70/	302/	277/	141/	59/	132/	268/	291/
3 16 9 6 13 8 10 3 12 7 2 13 6 15 1 12 3 10 8 13 6 9 16 3 12 1 15 6 13 2 7 12							
10 5 4 15 2 11 5 16 1 14 11 8 9 4 14 7 16 5 11 2 15 4 5 10 7 14 4 9 8 11 14 1							
8 11 14 1 7 14 4 9 15 4 5 10 16 5 11 2 9 4 14 7 1 14 11 8 2 11 5 16 10 5 4 15							
13 2 7 12 12 1 15 6 6 9 16 3 3 10 8 13 6 15 1 12 12 7 2 13 13 8 10 3 3 16 9 6							
289/	177/	18/	274/	296/	279/	23/	184/
13 2 7 12 8 10 3 13 1 14 11 8 12 6 15 1 13 3 10 8 12 7 2 13 1 15 6 12 8 11 14 1							
3 16 9 6 11 5 16 2 15 4 5 10 7 9 4 14 2 16 5 11 6 9 16 3 14 4 9 7 10 5 4 15							
10 5 4 15 14 4 9 7 6 9 16 3 2 16 5 11 7 9 4 14 15 4 5 10 11 5 16 2 3 16 9 6							
8 11 14 1 1 15 6 12 12 7 2 13 13 3 10 8 12 6 15 1 1 14 11 8 8 10 3 13 13 2 7 12							

[Fundamenta] #3]

4/	348/	121/	281/	13/	288/	128/	357/
1 8 13 12 15 4 14 1 6 3 10 15 12 7 9 6 1 14 4 15 12 13 8 1 6 9 7 12 15 10 3 6							
14 11 2 7 10 5 11 8 9 16 5 4 13 2 16 3 8 11 5 10 7 2 11 14 3 16 2 13 4 5 16 9							
4 5 16 9 3 16 2 13 7 2 11 14 8 11 5 10 13 2 16 3 9 16 5 4 10 5 11 8 14 11 2 7							
15 10 3 6 6 9 7 12 12 13 8 1 1 14 4 15 12 7 9 6 6 3 10 15 15 4 14 1 1 8 13 12							
355/	85/	216/	134/	342/	123/	209/	76/
15 10 3 6 4 14 1 15 9 16 5 4 6 12 7 9 15 1 14 4 6 3 10 15 9 7 12 6 4 5 16 9							
1 8 13 12 5 11 8 10 7 2 11 14 3 13 2 16 10 8 11 5 12 13 8 1 16 2 13 3 14 11 2 7							
14 11 2 7 16 2 13 3 12 13 8 1 10 8 11 5 3 13 2 16 7 2 11 14 5 11 8 10 1 8 13 12							
4 5 16 9 9 7 12 6 6 3 10 15 15 1 14 4 6 12 7 9 9 16 5 4 4 14 1 15 15 10 3 6							
78/	318/	147/	203/	91/	214/	158/	331/
4 5 16 9 14 1 15 4 7 2 11 14 9 6 12 7 4 15 1 14 9 16 5 4 7 12 6 9 14 11 2 7							
15 10 3 6 11 8 10 5 12 13 8 1 16 3 13 2 5 10 8 11 6 3 10 15 2 13 3 16 1 8 13 12							
1 8 13 12 2 13 3 16 6 3 10 15 5 10 8 11 16 3 13 2 12 13 8 1 11 8 10 5 15 10 3 6							
14 11 2 7 7 12 6 9 9 16 5 4 4 15 1 14 9 6 12 7 7 2 11 14 14 1 15 4 4 5 16 9							
333/	19/	286/	152/	324/	145/	275/	6/
14 11 2 7 1 15 4 14 12 13 8 1 7 9 6 12 14 4 15 1 7 2 11 14 12 6 9 7 1 8 13 12							
4 5 16 9 8 10 5 11 6 3 10 15 2 16 3 13 11 5 10 8 9 16 5 4 13 3 16 2 15 10 3 6							
15 10 3 6 13 3 16 2 9 16 5 4 11 5 10 8 2 16 3 13 6 3 10 15 8 10 5 11 4 5 16 9							
1 8 13 12 12 6 9 7 7 2 11 14 14 4 15 1 7 9 6 12 12 13 8 1 1 15 4 14 14 11 2 7							

267/	131/	60/	292/	278/	301/	69/	142/
12 1 8 13	6 9 7 12	3 10 15 6	13 2 16 3	12 7 9 6	13 8 1 12	3 16 2 13	6 15 10 3
7 14 11 2	15 4 14 1	16 5 4 9	8 11 5 10	1 14 4 15	2 11 14 7	10 5 11 8	9 4 5 16
9 4 5 16	10 5 11 8	2 11 14 7	1 14 4 15	8 11 5 10	16 5 4 9	15 4 14 1	7 14 11 2
6 15 10 3	3 16 2 13	13 8 1 12	12 7 9 6	13 2 16 3	3 10 15 6	6 9 7 12	12 1 8 13
144/	206/	373/	63/	137/	58/	364/	195/
6 15 10 3	9 7 12 6	16 5 4 9	3 13 2 16	6 12 7 9	3 10 15 6	16 2 13 3	9 4 5 16
12 1 8 13	4 14 1 15	2 11 14 7	10 8 11 5	15 1 14 4	13 8 1 12	5 11 8 10	7 14 11 2
7 14 11 2	5 11 8 10	13 8 1 12	15 1 14 4	10 8 11 5	2 11 14 7	4 14 1 15	12 1 8 13
9 4 5 16	16 2 13 3	3 10 15 6	6 12 7 9	3 13 2 16	16 5 4 9	9 7 12 6	6 15 10 3
193/	161/	34/	370/	200/	375/	39/	168/
9 4 5 16	7 12 6 9	2 11 14 7	16 3 13 2	9 6 12 7	16 5 4 9	2 13 3 16	7 14 11 2
6 15 10 3	14 1 15 4	13 8 1 12	5 10 8 11	4 15 1 14	3 10 15 6	11 8 10 5	12 1 8 13
12 1 8 13	11 8 10 5	3 10 15 6	4 15 1 14	5 10 8 11	13 8 1 12	14 1 15 4	6 15 10 3
7 14 11 2	2 13 3 16	16 5 4 9	9 6 12 7	16 3 13 2	2 11 14 7	7 12 6 9	9 4 5 16
166/	272/	303/	45/	155/	36/	298/	265/
7 14 11 2	12 6 9 7	13 8 1 12	2 16 3 13	7 9 6 12	2 11 14 7	13 3 16 2	12 1 8 13
9 4 5 16	1 15 4 14	3 10 15 6	11 5 10 8	14 4 15 1	16 5 4 9	8 10 5 11	6 15 10 3
6 15 10 3	8 10 5 11	16 5 4 9	14 4 15 1	11 5 10 8	3 10 15 6	1 15 4 14	9 4 5 16
12 1 8 13	13 3 16 2	2 11 14 7	7 9 6 12	2 16 3 13	13 8 1 12	12 6 9 7	7 14 11 2
307/	67/	238/	182/	294/	171/	227/	54/
13 12 1 8	3 16 2 13	10 15 6 3	8 11 5 10	13 2 16 3	8 1 12 13	10 5 11 8	3 6 15 10
2 7 14 11	6 9 7 12	5 4 9 16	1 14 4 15	12 7 9 6	11 14 7 2	15 4 14 1	16 9 4 5
16 9 4 5	15 4 14 1	11 14 7 2	12 7 9 6	1 14 4 15	5 4 9 16	6 9 7 12	2 7 14 11
3 6 15 10	10 5 11 8	8 1 12 13	13 2 16 3	8 11 5 10	10 15 6 3	3 16 2 13	13 12 1 8
52/	366/	99/	233/	61/	240/	110/	379/
3 6 15 10	16 2 13 3	5 4 9 16	10 8 11 5	3 13 2 16	10 15 6 3	5 11 8 10	16 9 4 5
13 12 1 8	9 7 12 6	11 14 7 2	15 1 14 4	6 12 7 9	8 1 12 13	4 14 1 15	2 7 14 11
2 7 14 11	4 14 1 15	8 1 12 13	6 12 7 9	15 1 14 4	11 14 7 2	9 7 12 6	13 12 1 8
16 9 4 5	5 11 8 10	10 15 6 3	3 13 2 16	10 8 11 5	5 4 9 16	16 2 13 3	3 6 15 10
381/	37/	264/	104/	372/	97/	257/	28/
16 9 4 5	2 13 3 16	11 14 7 2	5 10 8 11	16 3 13 2	5 4 9 16	11 8 10 5	2 7 14 11
3 6 15 10	7 12 6 9	8 1 12 13	4 15 1 14	9 6 12 7	10 15 6 3	14 1 15 4	13 12 1 8
13 12 1 8	14 1 15 4	10 15 6 3	9 6 12 7	4 15 1 14	8 1 12 13	7 12 6 9	3 6 15 10
2 7 14 11	11 8 10 5	5 4 9 16	16 3 13 2	5 10 8 11	11 14 7 2	2 13 3 16	16 9 4 5
30/	300/	169/	251/	43/	262/	176/	309/
2 7 14 11	13 3 16 2	8 1 12 13	11 5 10 8	2 16 3 13	11 14 7 2	8 10 5 11	13 12 1 8
16 9 4 5	12 6 9 7	10 15 6 3	14 4 15 1	7 9 6 12	5 4 9 16	1 15 4 14	3 6 15 10
3 6 15 10	1 15 4 14	5 4 9 16	7 9 6 12	14 4 15 1	10 15 6 3	12 6 9 7	16 9 4 5
13 12 1 8	8 10 5 11	11 14 7 2	2 16 3 13	11 5 10 8	8 1 12 13	13 3 16 2	2 7 14 11
192/	224/	351/	15/	185/	10/	346/	217/
8 13 12 1	10 5 11 8	15 6 3 10	1 14 4 15	8 11 5 10	1 12 13 8	15 4 14 1	10 3 6 15
11 2 7 14	3 16 2 13	4 9 16 5	12 7 9 6	13 2 16 3	14 7 2 11	6 9 7 12	5 16 9 4
5 16 9 4	6 9 7 12	14 7 2 11	13 2 16 3	12 7 9 6	4 9 16 5	3 16 2 13	11 2 7 14
10 3 6 15	15 4 14 1	1 12 13 8	8 11 5 10	1 14 4 15	15 6 3 10	10 5 11 8	8 13 12 1
219/	113/	82/	340/	230/	349/	87/	120/
10 3 6 15	5 11 8 10	4 9 16 5	15 1 14 4	10 8 11 5	15 6 3 10	4 14 1 15	5 16 9 4
8 13 12 1	16 2 13 3	14 7 2 11	6 12 7 9	3 13 2 16	1 12 13 8	9 7 12 6	11 2 7 14
11 2 7 14	9 7 12 6	1 12 13 8	3 13 2 16	6 12 7 9	14 7 2 11	16 2 13 3	8 13 12 1
5 16 9 4	4 14 1 15	15 6 3 10	10 8 11 5	15 1 14 4	4 9 16 5	5 11 8 10	10 3 6 15
118/	254/	325/	93/	107/	84/	316/	243/
5 16 9 4	11 8 10 5	14 7 2 11	4 15 1 14	5 10 8 11	4 9 16 5	14 1 15 4	11 2 7 14
10 3 6 15	2 13 3 16	1 12 13 8	9 6 12 7	16 3 13 2	15 6 3 10	7 12 6 9	8 13 12 1
8 13 12 1	7 12 6 9	15 6 3 10	16 3 13 2	9 6 12 7	1 12 13 8	2 13 3 16	10 3 6 15
11 2 7 14	14 1 15 4	4 9 16 5	5 10 8 11	4 15 1 14	14 7 2 11	11 8 10 5	5 16 9 4
241/	179/	12/	322/	248/	327/	21/	190/
11 2 7 14	8 10 5 11	1 12 13 8	14 4 15 1	11 5 10 8	14 7 2 11	1 15 4 14	8 13 12 1
5 16 9 4	13 3 16 2	15 6 3 10	7 9 6 12	2 16 3 13	4 9 16 5	12 6 9 7	10 3 6 15
10 3 6 15	12 6 9 7	4 9 16 5	2 16 3 13	7 9 6 12	15 6 3 10	13 3 16 2	5 16 9 4
8 13 12 1	1 15 4 14	14 7 2 11	11 5 10 8	14 4 15 1	1 12 13 8	8 10 5 11	11 2 7 14

\* Monitor List of Correspondence between Old and New \*

```

?: 0
1: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
25: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
49: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
73: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
97: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
121: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
145: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
169: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
193: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
217: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
241: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
265: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
289: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
313: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
337: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
361: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

```

[OK!]

The last beautiful result of Monitor list tells us of our success in reconstructing all 384 primitive solutions from the Fundamental Three. We could make no other type of solutions, nor lose any one we expected at all.

The Fundamental Three prove to be really the representative solutions of all.

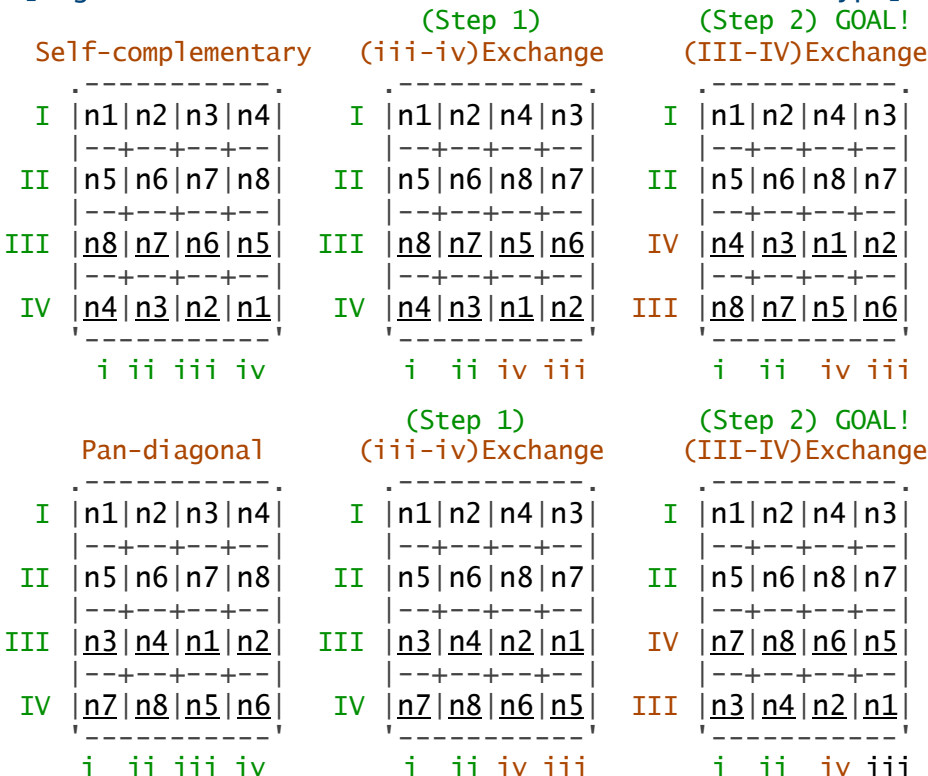
We know what the solution counts in three levels really mean:  $3 \times 16 \times 8 = 48 \times 8 = 384$ .

You may be surprised at the similarity between a self-complementary type and a pan-diagonal magic one. Yes. There are so many similar properties between them.

### #8. Type Conversion between S-C and P-D

I must mention now about the possibility of type conversion here. You can indeed transform one type into another by some simple line exchanges. Take your careful look at the next figures.

[Figure 5: How to Transform One into the Other Type]





The step-figures above show how to transform a self-complementary type into a pan-magic one. Exchange the 3rd<->4th rows and columns at the same time. Watch the result and find all complementary pairs are located only on pan-diagonals.

The lower step-figures show how to transform a pan-diagonal magic type back into a self-complementary one. Exchange the 3rd<->4th rows and columns at the same time. Watch the result and find all complementary pairs are located symmetrically.

Though each result is not completely the same with the other start form, it is still a meaningful transformation. The solution counts of those two types are also the same. We may well accept the one-to-one correspondence should exist between them.

\*\*\* Comparison between 'Self-Complementary' and 'Pan-diagonal' Types: \*\*\*  
 \*\* List of the Fundamental Three Solutions of Magic Squares of Order 4 \*\*

S/1/P		S/2/P		S/3/P	
1   8   15   10	1   8   10   15	1   8   14   11	1   8   11   14	1   8   12   13	1   8   13   12
12   13   6   3	12   13   3   6	12   13   7   2	12   13   2   7	14   11   7   2	14   11   2   7
14   11   4   5	7   2   16   9	15   10   4   5	6   3   16   9	15   10   6   3	4   5   16   9
7   2   9   16	14   11   5   4	6   3   9   16	15   10   5   4	4   5   9   16	15   10   3   6

[Count = S/3:3/P]  
 OK!

### #9. Is One-to-one Correspondence broken between the Standard Solution Sets?

But, I noticed the one-to-one correspondence is broken between the two sets of 48 Standard Solutions afterward. Look carefully at the next list shown below.

\*\* Transformation of MS44: Between 'Self-Complementary' and 'Complete' \*\*  
 \*\* Comparison between /Self-Complementary and /Complete Types \*\*

S/ 1/C		S/ 2/C		S/ 3/C	
1 8 15 10	1 8 10 15	1 8 14 11	1 8 11 14	1 8 15 10	1 8 10 15
12 13 6 3	12 13 3 6	12 13 7 2	12 13 2 7	14 11 4 5	14 11 5 4
14 11 4 5	7 2 16 9	15 10 4 5	6 3 16 9	12 13 6 3	7 2 16 9
7 2 9 16	14 11 5 4	6 3 9 16	15 10 5 4	7 2 9 16	12 13 3 6
S/ 4/C		S/ 5/C		S/ 6/C	
1 8 12 13	1 8 13 12	1 8 14 11	1 8 11 14	1 8 12 13	1 8 13 12
14 11 7 2	14 11 2 7	15 10 4 5	15 10 5 4	15 10 6 3	15 10 3 6
15 10 6 3	4 5 16 9	12 13 7 2	6 3 16 9	14 11 7 2	4 5 16 9
4 5 9 16	15 10 3 6	6 3 9 16	12 13 2 7	4 5 9 16	14 11 2 7
S/ 7/C		S/ 8/C		S/ 9/C	
1 12 15 6	1 12 6 15	1 12 8 13	1 12 13 8	1 12 14 7	1 12 7 14
14 7 4 9	14 7 9 4	14 7 11 2	14 7 2 11	15 6 4 9	15 6 9 4
8 13 10 3	11 2 16 5	15 6 10 3	4 9 16 5	8 13 11 2	10 3 16 5
11 2 5 16	8 13 3 10	4 9 5 16	15 6 3 10	10 3 5 16	8 13 2 11
S/10/C		S/11/C		S/12/C	
1 12 8 13	1 12 13 8	1 14 12 7	1 14 7 12	1 14 8 11	1 14 11 8
15 6 10 3	15 6 3 10	15 4 6 9	15 4 9 6	15 4 10 5	15 4 5 10
14 7 11 2	4 9 16 5	8 11 13 2	10 5 16 3	12 7 13 2	6 9 16 3
4 9 5 16	14 7 2 11	10 5 3 16	8 11 2 13	6 9 3 16	12 7 2 13
S/13/C		S/14/C		S/15/C	
2 7 16 9	2 7 9 16	2 7 13 12	2 7 12 13	2 7 16 9	2 7 9 16
11 14 5 4	11 14 4 5	11 14 8 1	11 14 1 8	13 12 3 6	13 12 6 3
13 12 3 6	8 1 15 10	16 9 3 6	5 4 15 10	11 14 5 4	8 1 15 10
8 1 10 15	13 12 6 3	5 4 10 15	16 9 6 3	8 1 10 15	11 14 4 5

S/16/C			S/17/C			S/18/C			
2 7 11 14	2 7 14 11	2 7 13 12	2 7 12 13	2 7 11 14	2 7 14 11	2 7 11 14	2 7 14 11	2 7 14 11	
13 12 8 1	13 12 1 8	16 9 3 6	16 9 6 3	16 9 5 4	16 9 4 5	16 9 5 4	16 9 4 5	16 9 4 5	
16 9 5 4	3 6 15 10	11 14 8 1	5 4 15 10	13 12 8 1	3 6 15 10	13 12 8 1	3 6 15 10	13 12 8 1	
3 6 10 15	16 9 4 5	5 4 10 15	11 14 1 8	3 6 10 15	13 12 1 8	3 6 10 15	13 12 1 8	3 6 10 15	
S/19/C			S/20/C			S/21/C			
2 11 16 5	2 11 5 16	2 11 7 14	2 11 14 7	2 11 13 8	2 11 8 13	2 11 13 8	2 11 8 13	2 11 8 13	
13 8 3 10	13 8 10 3	13 8 12 1	13 8 1 12	16 5 3 10	16 5 10 3	16 5 3 10	16 5 10 3	16 5 10 3	
7 14 9 4	12 1 15 6	16 5 9 4	3 10 15 6	7 14 12 1	9 4 15 6	7 14 12 1	9 4 15 6	7 14 12 1	
12 1 6 15	7 14 4 9	3 10 6 15	16 5 4 9	9 4 6 15	7 14 1 12	9 4 6 15	7 14 1 12	9 4 6 15	
S/22/C			S/23/C			S/24/C			
2 11 7 14	2 11 14 7	2 13 11 8	2 13 8 11	2 13 7 12	2 13 12 7	2 13 7 12	2 13 12 7	2 13 12 7	
16 5 9 4	16 5 4 9	16 3 5 10	16 3 10 5	16 3 9 6	16 3 6 9	16 3 9 6	16 3 6 9	16 3 6 9	
13 8 12 1	3 10 15 6	7 12 14 1	9 6 15 4	11 8 14 1	5 10 15 4	11 8 14 1	5 10 15 4	11 8 14 1	
3 10 6 15	13 8 1 12	9 6 4 15	7 12 1 14	5 10 4 15	11 8 1 14	5 10 4 15	11 8 1 14	5 10 4 15	
25/S		0/T	25/C		26/S		0/T	26/C	
3 6 16 9	3 6 9 16	3 6 9 16	3 6 9 16	3 6 13 12	3 6 12 13	3 6 13 12	3 6 12 13	3 6 15 10	3 6 15 10
10 15 5 4	10 15 4 5	13 12 7 2	13 12 7 2	10 15 8 1	10 15 1 8	10 15 8 1	10 15 1 8	13 12 1 8	13 12 1 8
13 12 2 7	8 1 14 11	8 1 14 11	8 1 14 11	16 9 2 7	5 4 14 11	16 9 2 7	5 4 14 11	2 7 14 11	2 7 14 11
8 1 11 14	13 12 7 2	10 15 4 5	10 15 4 5	5 4 11 14	16 9 7 2	5 4 11 14	16 9 7 2	16 9 4 5	16 9 4 5
27/S		25/T	27/C		28/S		27/T	28/C	
3 6 16 9	3 6 9 16	3 6 12 13	3 6 12 13	3 6 13 12	3 6 12 13	3 6 13 12	3 6 12 13	3 6 15 10	3 6 15 10
13 12 2 7	13 12 7 2	16 9 7 2	16 9 7 2	16 9 2 7	16 9 7 2	16 9 2 7	16 9 7 2	16 9 4 5	16 9 4 5
10 15 5 4	8 1 14 11	5 4 14 11	5 4 14 11	10 15 8 1	5 4 14 11	10 15 8 1	5 4 14 11	2 7 14 11	2 7 14 11
8 1 11 14	10 15 4 5	10 15 1 8	10 15 1 8	5 4 11 14	10 15 1 8	5 4 11 14	10 15 1 8	13 12 1 8	13 12 1 8
29/S		29/T	29/C		30/S		31/T	30/C	
3 10 16 5	3 10 5 16	3 10 5 16	3 10 5 16	3 10 13 8	3 10 8 13	3 10 13 8	3 10 8 13	3 10 15 6	3 10 15 6
13 8 2 11	13 8 11 2	13 8 11 2	13 8 11 2	16 5 2 11	16 5 11 2	16 5 2 11	16 5 11 2	13 8 1 12	13 8 1 12
6 15 9 4	12 1 14 7	12 1 14 7	12 1 14 7	6 15 12 1	9 4 14 7	6 15 12 1	9 4 14 7	2 11 14 7	2 11 14 7
12 1 7 14	6 15 4 9	6 15 4 9	6 15 4 9	9 4 7 14	6 15 1 12	9 4 7 14	6 15 1 12	16 5 4 9	16 5 4 9
31/S		33/T	31/C		32/S		34/T	32/C	
3 13 10 8	3 13 8 10	3 10 8 13	3 10 8 13	3 13 6 12	3 13 12 6	3 13 6 12	3 13 12 6	3 10 15 6	3 10 15 6
16 2 5 11	16 2 11 5	16 5 11 2	16 5 11 2	16 2 9 7	16 2 7 9	16 2 9 7	16 2 7 9	16 5 4 9	16 5 4 9
6 12 15 1	9 7 14 4	9 4 14 7	9 4 14 7	10 8 15 1	5 11 14 4	10 8 15 1	5 11 14 4	2 11 14 7	2 11 14 7
9 7 4 14	6 12 1 15	6 15 1 12	6 15 1 12	5 11 4 14	10 8 1 15	5 11 4 14	10 8 1 15	13 8 1 12	13 8 1 12
33/S		0/T	33/C		34/S		0/T	34/C	
4 5 15 10	4 5 10 15	3 13 8 10	3 13 8 10	4 5 14 11	4 5 11 14	4 5 14 11	4 5 11 14	3 13 12 6	3 13 12 6
9 16 6 3	9 16 3 6	16 2 11 5	16 2 11 5	9 16 7 2	9 16 2 7	9 16 7 2	9 16 2 7	16 2 7 9	16 2 7 9
14 11 1 8	7 2 13 12	9 7 14 4	9 7 14 4	15 10 1 8	6 3 13 12	15 10 1 8	6 3 13 12	5 11 14 4	5 11 14 4
7 2 12 13	14 11 8 1	6 12 1 15	6 12 1 15	6 3 12 13	15 10 8 1	6 3 12 13	15 10 8 1	10 8 1 15	10 8 1 15
35/S		35/T	35/C		36/S		37/T	36/C	
4 5 15 10	4 5 10 15	4 5 10 15	4 5 10 15	4 5 14 11	4 5 11 14	4 5 14 11	4 5 11 14	4 5 16 9	4 5 16 9
14 11 1 8	14 11 8 1	14 11 8 1	14 11 8 1	15 10 1 8	15 10 8 1	15 10 1 8	15 10 8 1	14 11 2 7	14 11 2 7
9 16 6 3	7 2 13 12	7 2 13 12	7 2 13 12	9 16 7 2	6 3 13 12	9 16 7 2	6 3 13 12	1 8 13 12	1 8 13 12
7 2 12 13	9 16 3 6	9 16 3 6	9 16 3 6	6 3 12 13	9 16 2 7	6 3 12 13	9 16 2 7	15 10 3 6	15 10 3 6
37/S		39/T	37/C		38/S		41/T	38/C	
4 9 15 6	4 9 6 15	4 5 11 14	4 5 11 14	4 9 14 7	4 9 7 14	4 9 14 7	4 9 7 14	4 5 16 9	4 5 16 9
14 7 1 12	14 7 12 1	15 10 8 1	15 10 8 1	15 6 1 12	15 6 12 1	15 6 1 12	15 6 12 1	15 10 3 6	15 10 3 6
5 16 10 3	11 2 13 8	6 3 13 12	6 3 13 12	5 16 11 2	10 3 13 8	5 16 11 2	10 3 13 8	1 8 13 12	1 8 13 12
11 2 8 13	5 16 3 10	9 16 2 7	9 16 2 7	10 3 8 13	5 16 2 11	10 3 8 13	5 16 2 11	14 11 2 7	14 11 2 7
39/S		43/T	39/C		40/S		44/T	40/C	
4 14 9 7	4 14 7 9	4 9 6 15	4 9 6 15	4 14 5 11	4 14 11 5	4 14 5 11	4 14 11 5	4 9 16 5	4 9 16 5
15 1 6 12	15 1 12 6	14 7 12 1	14 7 12 1	15 1 10 8	15 1 8 10	15 1 10 8	15 1 8 10	14 7 2 11	14 7 2 11
5 11 16 2	10 8 13 3	11 2 13 8	11 2 13 8	9 7 16 2	6 12 13 3	9 7 16 2	6 12 13 3	1 12 13 8	1 12 13 8
10 8 3 13	5 11 2 16	5 16 3 10	5 16 3 10	6 12 3 13	9 7 2 16	6 12 3 13	9 7 2 16	15 6 3 10	15 6 3 10

41/S	0/T	41/C	42/S	0/T	42/C
5 4 16 9	5 4 9 16	4 9 7 14	5 4 16 9	5 4 9 16	4 9 16 5
10 15 3 6	10 15 6 3	15 6 12 1	11 14 2 7	11 14 7 2	15 6 3 10
11 14 2 7	8 1 12 13	10 3 13 8	10 15 3 6	8 1 12 13	1 12 13 8
8 1 13 12	11 14 7 2	5 16 2 11	8 1 13 12	10 15 6 3	14 7 2 11
43/S	0/T	43/C	44/S	0/T	44/C
5 10 11 8	5 10 8 11	4 14 7 9	5 11 10 8	5 11 8 10	4 14 11 5
16 3 2 13	16 3 13 2	15 1 12 6	16 2 3 13	16 2 13 3	15 1 8 10
4 15 14 1	9 6 12 7	10 8 13 3	4 14 15 1	9 7 12 6	6 12 13 3
9 6 7 12	4 15 1 14	5 11 2 16	9 7 6 12	4 14 1 15	9 7 2 16
45/S	0/T	45/C	46/S	0/T	46/C
6 3 15 10	6 3 10 15	5 4 14 11	6 3 15 10	6 3 10 15	5 4 15 10
9 16 4 5	9 16 5 4	16 9 7 2	12 13 1 8	12 13 8 1	16 9 6 3
12 13 1 8	7 2 11 14	3 6 12 13	9 16 4 5	7 2 11 14	2 7 12 13
7 2 14 11	12 13 8 1	10 15 1 8	7 2 14 11	9 16 5 4	11 14 1 8
47/S	0/T	47/C	48/S	0/T	48/C
6 9 12 7	6 9 7 12	6 3 13 12	6 12 9 7	6 12 7 9	6 3 16 9
15 4 1 14	15 4 14 1	15 10 8 1	15 1 4 14	15 1 14 4	15 10 5 4
3 16 13 2	10 5 11 8	4 5 11 14	3 13 16 2	10 8 11 5	1 8 11 14
10 5 8 11	3 16 2 13	9 16 2 7	10 8 5 11	3 13 2 16	12 13 2 7

\* Monitor List of Solution Correspondence between /T and /C \*

?: 12

1: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 25: 1 0 1 0 1 0 1 0 1 1 1 0 1 0 1 0 1 1 0 0 0 0

[Count = S/48:48/C] OK!

I could find no problem in the first half of the two lists. But I found the one-to-one correspondence was partially broken in the last half. I could not find any 'perfect partners' for twelve solutions there. I was shocked. What was wrong with them?

The problem is the list-forming inequality conditions. When they are transformed, some results in the last half should get against the conditions:  $n1 < n4$ ;  $n1 < n13$ ;  $n1 < n16$  and  $n2 < n5$ , and cannot finally find any 'perfect partners' in our list.

When I examined between the two sets of 'Primitive solutions' (that are listed under no inequality conditions), I could find no problem there.

See the next list shown below. You can certainly see there exists the complete one-to-one correspondence between the 'self-complementary' magic squares 4x4 and the 'complete' ones (same with 'pan-diagonal' type).

\*\* Transformation of MS44: Between 'Self-Complementary' and 'Complete' \*\*

\*\* Comparison between /S-C and /C in the List of Primitive Solutions \*\*

S/ 1/C	S/ 2/C	S/ 3/C
1 8 15 10	1 8 14 11	1 8 15 10
12 13 6 3	12 13 7 2	14 11 4 5
14 11 4 5	15 10 4 5	12 13 6 3
7 2 9 16	6 3 9 16	7 2 9 16
S/ 4/C	S/ 5/C	S/ 6/C
1 8 12 13	1 8 14 11	1 8 12 13
14 11 7 2	15 10 4 5	15 10 6 3
15 10 6 3	12 13 7 2	14 11 7 2
4 5 9 16	6 3 9 16	4 5 9 16
S/ 7/C	S/ 8/C	S/ 9/C
1 12 15 6	1 12 14 7	1 12 15 6
8 13 10 3	8 13 11 2	14 7 4 9
14 7 4 9	15 6 4 9	8 13 10 3
11 2 5 16	10 3 5 16	11 2 5 16

S/10/C	S/11/C	S/12/C
1 12 8 13 1 12 13 8	1 12 14 7 1 12 7 14	1 12 8 13 1 12 13 8
14 7 11 2 14 7 2 11	15 6 4 9 15 6 9 4	15 6 10 3 15 6 3 10
15 6 10 3 4 9 16 5	8 13 11 2 10 3 16 5	14 7 11 2 4 9 16 5
4 9 5 16 15 6 3 10	10 3 5 16 8 13 2 11	4 9 5 16 14 7 2 11
S/13/C	S/14/C	S/15/C
1 14 15 4 1 14 4 15	1 14 12 7 1 14 7 12	1 14 15 4 1 14 4 15
8 11 10 5 8 11 5 10	8 11 13 2 8 11 2 13	12 7 6 9 12 7 9 6
12 7 6 9 13 2 16 3	15 4 6 9 10 5 16 3	8 11 10 5 13 2 16 3
13 2 3 16 12 7 9 6	10 5 3 16 15 4 9 6	13 2 3 16 8 11 5 10
S/16/C	S/17/C	S/18/C
1 14 8 11 1 14 11 8	1 14 12 7 1 14 7 12	1 14 8 11 1 14 11 8
12 7 13 2 12 7 2 13	15 4 6 9 15 4 9 6	15 4 10 5 15 4 5 10
15 4 10 5 6 9 16 3	8 11 13 2 10 5 16 3	12 7 13 2 6 9 16 3
6 9 3 16 15 4 5 10	10 5 3 16 8 11 2 13	6 9 3 16 12 7 2 13
S/19/C	S/20/C	S/21/C
1 15 14 4 1 15 4 14	1 15 12 6 1 15 6 12	1 15 14 4 1 15 4 14
8 10 11 5 8 10 5 11	8 10 13 3 8 10 3 13	12 6 7 9 12 6 9 7
12 6 7 9 13 3 16 2	14 4 7 9 11 5 16 2	8 10 11 5 13 3 16 2
13 3 2 16 12 6 9 7	11 5 2 16 14 4 9 7	13 3 2 16 8 10 5 11
S/22/C	S/23/C	S/24/C
1 15 8 10 1 15 10 8	1 15 12 6 1 15 6 12	1 15 8 10 1 15 10 8
12 6 13 3 12 6 3 13	14 4 7 9 14 4 9 7	14 4 11 5 14 4 5 11
14 4 11 5 7 9 16 2	8 10 13 3 11 5 16 2	12 6 13 3 7 9 16 2
7 9 2 16 14 4 5 11	11 5 2 16 8 10 3 13	7 9 2 16 12 6 3 13
S/25/C	S/26/C	S/27/C
2 7 16 9 2 7 9 16	2 7 13 12 2 7 12 13	2 7 16 9 2 7 9 16
11 14 5 4 11 14 4 5	11 14 8 1 11 14 1 8	13 12 3 6 13 12 6 3
13 12 3 6 8 1 15 10	16 9 3 6 5 4 15 10	11 14 5 4 8 1 15 10
8 1 10 15 13 12 6 3	5 4 10 15 16 9 6 3	8 1 10 15 11 14 4 5
. . . . .		
S/193/C	S/194/C	S/195/C
9 4 16 5 9 4 5 16	9 4 7 14 9 4 14 7	9 4 16 5 9 4 5 16
6 15 3 10 6 15 10 3	6 15 12 1 6 15 1 12	7 14 2 11 7 14 11 2
7 14 2 11 12 1 8 13	16 5 2 11 3 10 8 13	6 15 3 10 12 1 8 13
12 1 13 8 7 14 11 2	3 10 13 8 16 5 11 2	12 1 13 8 6 15 10 3
S/196/C	S/197/C	S/198/C
9 4 6 15 9 4 15 6	9 4 7 14 9 4 14 7	9 4 6 15 9 4 15 6
7 14 12 1 7 14 1 12	16 5 2 11 16 5 11 2	16 5 3 10 16 5 10 3
16 5 3 10 2 11 8 13	6 15 12 1 3 10 8 13	7 14 12 1 2 11 8 13
2 11 13 8 16 5 10 3	3 10 13 8 6 15 1 12	2 11 13 8 7 14 1 12
S/199/C	S/200/C	S/201/C
9 6 16 3 9 6 3 16	9 6 7 12 9 6 12 7	9 6 16 3 9 6 3 16
4 15 5 10 4 15 10 5	4 15 14 1 4 15 1 14	7 12 2 13 7 12 13 2
7 12 2 13 14 1 8 11	16 3 2 13 5 10 8 11	4 15 5 10 14 1 8 11
14 1 11 8 7 12 13 2	5 10 11 8 16 3 13 2	14 1 11 8 4 15 10 5
S/202/C	S/203/C	S/204/C
9 6 4 15 9 6 15 4	9 6 7 12 9 6 12 7	9 6 4 15 9 6 15 4
7 12 14 1 7 12 1 14	16 3 2 13 16 3 13 2	16 3 5 10 16 3 10 5
16 3 5 10 2 13 8 11	4 15 14 1 5 10 8 11	7 12 14 1 2 13 8 11
2 13 11 8 16 3 10 5	5 10 11 8 4 15 1 14	2 13 11 8 7 12 1 14
S/205/C	S/206/C	S/207/C
9 7 16 2 9 7 2 16	9 7 6 12 9 7 12 6	9 7 16 2 9 7 2 16
4 14 5 11 4 14 11 5	4 14 15 1 4 14 1 15	6 12 3 13 6 12 13 3
6 12 3 13 15 1 8 10	16 2 3 13 5 11 8 10	4 14 5 11 15 1 8 10
15 1 10 8 6 12 13 3	5 11 10 8 16 2 13 3	15 1 10 8 4 14 11 5

S/208/C	S/209/C	S/210/C
9 7 4 14 9 7 14 4	9 7 6 12 9 7 12 6	9 7 4 14 9 7 14 4
6 12 15 1 6 12 1 15	16 2 3 13 16 2 13 3	16 2 5 11 16 2 11 5
16 2 5 11 3 13 8 10	4 14 15 1 5 11 8 10	6 12 15 1 3 13 8 10
3 13 10 8 16 2 11 5	5 11 10 8 4 14 1 15	3 13 10 8 6 12 1 15
S/211/C	S/212/C	S/213/C
9 16 7 2 9 16 2 7	9 16 6 3 9 16 3 6	9 16 7 2 9 16 2 7
4 5 14 11 4 5 11 14	4 5 15 10 4 5 10 15	6 3 12 13 6 3 13 12
6 3 12 13 15 10 8 1	7 2 12 13 14 11 8 1	4 5 14 11 15 10 8 1
15 10 1 8 6 3 13 12	14 11 1 8 7 2 13 12	15 10 1 8 4 5 11 14
S/214/C	S/215/C	S/216/C
9 16 4 5 9 16 5 4	9 16 6 3 9 16 3 6	9 16 4 5 9 16 5 4
6 3 15 10 6 3 10 15	7 2 12 13 7 2 13 12	7 2 14 11 7 2 11 14
7 2 14 11 12 13 8 1	4 5 15 10 14 11 8 1	6 3 15 10 12 13 8 1
12 13 1 8 7 2 11 14	14 11 1 8 4 5 10 15	12 13 1 8 6 3 10 15
S/217/C	S/218/C	S/219/C
10 3 15 6 10 3 6 15	10 3 8 13 10 3 13 8	10 3 15 6 10 3 6 15
5 16 4 9 5 16 9 4	5 16 11 2 5 16 2 11	8 13 1 12 8 13 12 1
8 13 1 12 11 2 7 14	15 6 1 12 4 9 7 14	5 16 4 9 11 2 7 14
11 2 14 7 8 13 12 1	4 9 14 7 15 6 12 1	11 2 14 7 5 16 9 4

.....

S/370/C	S/371/C	S/372/C
16 3 2 13 16 3 13 2	16 3 5 10 16 3 10 5	16 3 2 13 16 3 13 2
5 10 11 8 5 10 8 11	9 6 4 15 9 6 15 4	9 6 7 12 9 6 12 7
9 6 7 12 4 15 1 14	2 13 11 8 7 12 1 14	5 10 11 8 4 15 1 14
4 15 14 1 9 6 12 7	7 12 14 1 2 13 8 11	4 15 14 1 5 10 8 11
S/373/C	S/374/C	S/375/C
16 5 9 4 16 5 4 9	16 5 3 10 16 5 10 3	16 5 9 4 16 5 4 9
2 11 7 14 2 11 14 7	2 11 13 8 2 11 8 13	3 10 6 15 3 10 15 6
3 10 6 15 13 8 1 12	9 4 6 15 7 14 1 12	2 11 7 14 13 8 1 12
13 8 12 1 3 10 15 6	7 14 12 1 9 4 15 6	13 8 12 1 2 11 14 7
S/376/C	S/377/C	S/378/C
16 5 2 11 16 5 11 2	16 5 3 10 16 5 10 3	16 5 2 11 16 5 11 2
3 10 13 8 3 10 8 13	9 4 6 15 9 4 15 6	9 4 7 14 9 4 14 7
9 4 7 14 6 15 1 12	2 11 13 8 7 14 1 12	3 10 13 8 6 15 1 12
6 15 12 1 9 4 14 7	7 14 12 1 2 11 8 13	6 15 12 1 3 10 8 13
S/379/C	S/380/C	S/381/C
16 9 5 4 16 9 4 5	16 9 3 6 16 9 6 3	16 9 5 4 16 9 4 5
2 7 11 14 2 7 14 11	2 7 13 12 2 7 12 13	3 6 10 15 3 6 15 10
3 6 10 15 13 12 1 8	5 4 10 15 11 14 1 8	2 7 11 14 13 12 1 8
13 12 8 1 3 6 15 10	11 14 8 1 5 4 15 10	13 12 8 1 2 7 14 11
S/382/C	S/383/C	S/384/C
16 9 2 7 16 9 7 2	16 9 3 6 16 9 6 3	16 9 2 7 16 9 7 2
3 6 13 12 3 6 12 13	5 4 10 15 5 4 15 10	5 4 11 14 5 4 14 11
5 4 11 14 10 15 1 8	2 7 13 12 11 14 1 8	3 6 13 12 10 15 1 8
10 15 8 1 5 4 14 11	11 14 8 1 2 7 12 13	10 15 8 1 3 6 12 13

[Count = S/384:384/C]

\* Monitor List of Solution Correspondence between /S and /C \*

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??: 0
1: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
25: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
49: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
73: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
97: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
121: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

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145: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 169: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 193: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 217: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 241: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 265: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 289: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 313: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 337: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  
 361: 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

OK!

How impressive our Monitor list is!

It moved me very much, but puzzled me more. I could not help thinking about what the true reason is for such a beautiful concordance. I had to study for a long time.

At last I came up to the idea of true origin, 4 dimensional extra-cubic object of order 2. I could find one of the answers for that, I believe.

(Original written in English in 2002; Revised on Sep.3, 2003, Dec.18, 2005;  
 Newest revision on July 11, 2010 by Kanji Setsuda on MacOSX Xcode 3.1.2)

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E-Mail Address: [jag12001@nifty.com](mailto:jag12001@nifty.com)