

Part 4: "New Advanced Study of Magic Squares and Cubes"

Chapter 7: New Method of Composing High-Dimensional Extra-Cubic Objects and their Developed Forms: Kanji Setsuda

Section 2: Composition of Four-Dimensional Extra-Cubic Objects of Order 2 and their Developed Forms

#1. Our Purpose

I have found we can draw many pictures of the same Extra-Cubic Object. We can use these pictures for some 'basic diagrams' or 'prototype forms' or anything else.

This time let's use these pictures for the Basic Diagrams of our object. Let's design and compose $ECO2^4$ again more strictly than before, and count and know how many primitive solutions it has in its 'Mother Set' in reality.

First of all we have to know how many simultaneous equations are really necessary for our definition stage.

#2. How do you define $ECO2^4$?

We now know we can draw so many different pictures as 24 for our object by the 4-time loops of $for(\dots)\{ \dots \}$ sentences and with a little change in the order of variables $\{d_0, d_1, d_2, d_3\}$. Permutation taking 4 out of 4 is ${}_4P_4=4 \times 3 \times 2 \times 1=24$.

Let me list out all 24 pictures here before all.

** All Possible View-Forms with 'Composite Conditions' **

1/d0 1---- 2 3---+ 4 5-- - 6 7----- 8	/d1 9-----10 11---+12 13-- -14 15-----16	n1+n2+n3+n4=C n1+n2+n5+n6=C n1+n3+n5+n7=C n2+n4+n6+n8=C n3+n4+n7+n8=C n5+n6+n7+n8=C	n9+n10+n11+n12=C n9+n10+n13+n14=C n9+n11+n13+n15=C n10+n12+n14+n16=C n11+n12+n15+n16=C n13+n14+n15+n16=C
2/d0 1----- 3 2---+ 4 5-- - 7 6----- 8	/d1 9-----11 10---+12 13-- -15 14-----16	n1+n2+n3+n4=C n1+n3+n5+n7=C n1+n2+n5+n6=C n3+n4+n7+n8=C n2+n4+n6+n8=C n5+n6+n7+n8=C	n9+n10+n11+n12=C n9+n11+n13+n15=C n9+n10+n13+n14=C n11+n12+n15+n16=C n10+n12+n14+n16=C n13+n14+n15+n16=C
3/d0 1----- 2 5---+ 6 3-- - 4 7----- 8	/d1 9-----10 13---+14 11-- -12 15-----16	n1+n2+n5+n6=C n1+n2+n3+n4=C n1+n3+n5+n7=C n2+n4+n6+n8=C n5+n6+n7+n8=C n3+n4+n7+n8=C	n9+n10+n13+n14=C n9+n10+n11+n12=C n9+n11+n13+n15=C n10+n12+n14+n16=C n13+n14+n15+n16=C n11+n12+n15+n16=C
4/d0 1----- 3 5---+ 7 2-- - 4 6----- 8	/d1 9-----11 13---+15 10-- -12 14-----16	n1+n3+n5+n7=C n1+n2+n3+n4=C n1+n2+n5+n6=C n3+n4+n7+n8=C n5+n6+n7+n8=C n2+n4+n6+n8=C	n9+n11+n13+n15=C n9+n10+n11+n12=C n9+n10+n13+n14=C n11+n12+n15+n16=C n13+n14+n15+n16=C n10+n12+n14+n16=C
5/d0 1----- 5 2---+ 6 3-- - 7 4----- 8	/d1 9-----13 10---+14 11-- -15 12-----16	n1+n2+n5+n6=C n1+n3+n5+n7=C n1+n2+n3+n4=C n5+n6+n7+n8=C n2+n4+n6+n8=C n3+n4+n7+n8=C	n9+n10+n13+n14=C n9+n11+n13+n15=C n9+n10+n11+n12=C n13+n14+n15+n16=C n10+n12+n14+n16=C n11+n12+n15+n16=C

6/d0	/d1	n1+n3+n5+n7=C	n9+n11+n13+n15=C
1---- 5	9----13	n1+n2+n5+n6=C	n9+n10+n13+n14=C
3---+ 7	11---+15	n1+n2+n3+n4=C	n9+n10+n11+n12=C
2-- - 6	10-- -14	n5+n6+n7+n8=C	n13+n14+n15+n16=C
4---- 8	12----16	n3+n4+n7+n8=C	n11+n12+n15+n16=C
		n2+n4+n6+n8=C	n10+n12+n14+n16=C
7/d0	/d1	n1+n2+n3+n4=C	n5+n6+n7+n8=C
1---- 2	5---- 6	n1+n2+n9+n10=C	n5+n6+n13+n14=C
3---+ 4	7---+ 8	n1+n3+n9+n11=C	n5+n7+n13+n15=C
9-- -10	13-- -14	n2+n4+n10+n12=C	n6+n8+n14+n16=C
11----12	15----16	n3+n4+n11+n12=C	n7+n8+n15+n16=C
		n9+n10+n11+n12=C	n13+n14+n15+n16=C
8/d0	/d1	n1+n2+n3+n4=C	n5+n6+n7+n8=C
1---- 3	5---- 7	n1+n3+n9+n11=C	n5+n7+n13+n15=C
2---+ 4	6---+ 8	n1+n2+n9+n10=C	n5+n6+n13+n14=C
9-- -11	13-- -15	n3+n4+n11+n12=C	n7+n8+n15+n16=C
10----12	14----16	n2+n4+n10+n12=C	n6+n8+n14+n16=C
		n9+n10+n11+n12=C	n13+n14+n15+n16=C
9/d0	/d1	n1+n2+n5+n6=C	n3+n4+n7+n8=C
1---- 2	3---- 4	n1+n2+n9+n10=C	n3+n4+n11+n12=C
5---+ 6	7---+ 8	n1+n5+n9+n13=C	n3+n7+n11+n15=C
9-- -10	11-- -12	n2+n6+n10+n14=C	n4+n8+n12+n16=C
13----14	15----16	n5+n6+n13+n14=C	n7+n8+n15+n16=C
		n9+n10+n13+n14=C	n11+n12+n15+n16=C
10/d0	/d1	n1+n3+n5+n7=C	n2+n4+n6+n8=C
1---- 3	2---- 4	n1+n3+n9+n11=C	n2+n4+n10+n12=C
5---+ 7	6---+ 8	n1+n5+n9+n13=C	n2+n6+n10+n14=C
9-- -11	10-- -12	n3+n7+n11+n15=C	n4+n8+n12+n16=C
13----15	14----16	n5+n7+n13+n15=C	n6+n8+n14+n16=C
		n9+n11+n13+n15=C	n10+n12+n14+n16=C
11/d0	/d1	n1+n2+n5+n6=C	n3+n4+n7+n8=C
1---- 5	3---- 7	n1+n5+n9+n13=C	n3+n7+n11+n15=C
2---+ 6	4---+ 8	n1+n2+n9+n10=C	n3+n4+n11+n12=C
9-- -13	11-- -15	n5+n6+n13+n14=C	n7+n8+n15+n16=C
10----14	12----16	n2+n6+n10+n14=C	n4+n8+n12+n16=C
		n9+n10+n13+n14=C	n11+n12+n15+n16=C
12/d0	/d1	n1+n3+n5+n7=C	n2+n4+n6+n8=C
1---- 5	2---- 6	n1+n5+n9+n13=C	n2+n6+n10+n14=C
3---+ 7	4---+ 8	n1+n3+n9+n11=C	n2+n4+n10+n12=C
9-- -13	10-- -14	n5+n7+n13+n15=C	n6+n8+n14+n16=C
11----15	12----16	n3+n7+n11+n15=C	n4+n8+n12+n16=C
		n9+n11+n13+n15=C	n10+n12+n14+n16=C
13/d0	/d1	n1+n2+n9+n10=C	n5+n6+n13+n14=C
1---- 2	5---- 6	n1+n2+n3+n4=C	n5+n6+n7+n8=C
9---+10	13---+14	n1+n3+n9+n11=C	n5+n7+n13+n15=C
3-- - 4	7-- - 8	n2+n4+n10+n12=C	n6+n8+n14+n16=C
11----12	15----16	n9+n10+n11+n12=C	n13+n14+n15+n16=C
		n3+n4+n11+n12=C	n7+n8+n15+n16=C
14/d0	/d1	n1+n3+n9+n11=C	n5+n7+n13+n15=C
1---- 3	5---- 7	n1+n2+n3+n4=C	n5+n6+n7+n8=C
9---+11	13---+15	n1+n2+n9+n10=C	n5+n6+n13+n14=C
2-- - 4	6-- - 8	n3+n4+n11+n12=C	n7+n8+n15+n16=C
10----12	14----16	n9+n10+n11+n12=C	n13+n14+n15+n16=C
		n2+n4+n10+n12=C	n6+n8+n14+n16=C

15/d0 1---- 2 9---+10 5-- - 6 13----14	/d1 3---- 4 11---+12 7-- - 8 15----16	n1+n2+n9+n10=C n1+n2+n5+n6=C n1+n5+n9+n13=C n2+n6+n10+n14=C n9+n10+n13+n14=C n5+n6+n13+n14=C	n3+n4+n11+n12=C n3+n4+n7+n8=C n3+n7+n11+n15=C n4+n8+n12+n16=C n11+n12+n15+n16=C n7+n8+n15+n16=C
16/d0 1---- 3 9---+11 5-- - 7 13----15	/d1 2---- 4 10---+12 6-- - 8 14----16	n1+n3+n9+n11=C n1+n3+n5+n7=C n1+n5+n9+n13=C n3+n7+n11+n15=C n9+n11+n13+n15=C n5+n7+n13+n15=C	n2+n4+n10+n12=C n2+n4+n6+n8=C n2+n6+n10+n14=C n4+n8+n12+n16=C n10+n12+n14+n16=C n6+n8+n14+n16=C
17/d0 1---- 5 9---+13 2-- - 6 10----14	/d1 3---- 7 11---+15 4-- - 8 12----16	n1+n5+n9+n13=C n1+n2+n5+n6=C n1+n2+n9+n10=C n5+n6+n13+n14=C n9+n10+n13+n14=C n2+n6+n10+n14=C	n3+n7+n11+n15=C n3+n4+n7+n8=C n3+n4+n11+n12=C n7+n8+n15+n16=C n11+n12+n15+n16=C n4+n8+n12+n16=C
18/d0 1---- 5 9---+13 3-- - 7 11----15	/d1 2---- 6 10---+14 4-- - 8 12----16	n1+n5+n9+n13=C n1+n3+n5+n7=C n1+n3+n9+n11=C n5+n7+n13+n15=C n9+n11+n13+n15=C n3+n7+n11+n15=C	n2+n6+n10+n14=C n2+n4+n6+n8=C n2+n4+n10+n12=C n6+n8+n14+n16=C n10+n12+n14+n16=C n4+n8+n12+n16=C
19/d0 1---- 9 2---+10 3-- -11 4----12	/d1 5----13 6---+14 7-- -15 8----16	n1+n2+n9+n10=C n1+n3+n9+n11=C n1+n2+n3+n4=C n9+n10+n11+n12=C n2+n4+n10+n12=C n3+n4+n11+n12=C	n5+n6+n13+n14=C n5+n7+n13+n15=C n5+n6+n7+n8=C n13+n14+n15+n16=C n6+n8+n14+n16=C n7+n8+n15+n16=C
20/d0 1---- 9 3---+11 2-- -10 4----12	/d1 5----13 7---+15 6-- -14 8----16	n1+n3+n9+n11=C n1+n2+n9+n10=C n1+n2+n3+n4=C n9+n10+n11+n12=C n3+n4+n11+n12=C n2+n4+n10+n12=C	n5+n7+n13+n15=C n5+n6+n13+n14=C n5+n6+n7+n8=C n13+n14+n15+n16=C n7+n8+n15+n16=C n6+n8+n14+n16=C
21/d0 1---- 9 2---+10 5-- -13 6----14	/d1 3----11 4---+12 7-- -15 8----16	n1+n2+n9+n10=C n1+n5+n9+n13=C n1+n2+n5+n6=C n9+n10+n13+n14=C n2+n6+n10+n14=C n5+n6+n13+n14=C	n3+n4+n11+n12=C n3+n7+n11+n15=C n3+n4+n7+n8=C n11+n12+n15+n16=C n4+n8+n12+n16=C n7+n8+n15+n16=C
22/d0 1---- 9 3---+11 5-- -13 7----15	/d1 2----10 4---+12 6-- -14 8----16	n1+n3+n9+n11=C n1+n5+n9+n13=C n1+n3+n5+n7=C n9+n11+n13+n15=C n3+n7+n11+n15=C n5+n7+n13+n15=C	n2+n4+n10+n12=C n2+n6+n10+n14=C n2+n4+n6+n8=C n10+n12+n14+n16=C n4+n8+n12+n16=C n6+n8+n14+n16=C
23/d0 1---- 9 5---+13 2-- -10 6----14	/d1 3----11 7---+15 4-- -12 8----16	n1+n5+n9+n13=C n1+n2+n9+n10=C n1+n2+n5+n6=C n9+n10+n13+n14=C n5+n6+n13+n14=C n2+n6+n10+n14=C	n3+n7+n11+n15=C n3+n4+n11+n12=C n3+n4+n7+n8=C n11+n12+n15+n16=C n7+n8+n15+n16=C n4+n8+n12+n16=C

24/d0	/d1		n1+n5+n9+n13=C		n2+n6+n10+n14=C
1---- 9	2----10		n1+n3+n9+n11=C		n2+n4+n10+n12=C
5---+13	6---+14		n1+n3+n5+n7=C		n2+n4+n6+n8=C
3-- -11	4-- -12		n9+n11+n13+n15=C		n10+n12+n14+n16=C
7----15	8----16		n5+n7+n13+n15=C		n6+n8+n14+n16=C
			n3+n7+n11+n15=C		n4+n8+n12+n16=C

We want to put those equations all to every little cube of order 2 just as above. Every 4 numbers on each surface of those little ones must add up to C(=34: the Magic Constant). But $24 \times 12 = 288$ equations are too many for us to deal with.

What do we have to do, then? Can we cut any equations?

If you watch the list above carefully, you will find many duplications of any kind. As you see, every simple reflection or rotation of a representative pattern has the same set of equations. You can remove those duplications for our purpose.

The next 4 view-forms below are supposed to be really necessary for us to define our ECO with.

**** The Selected 4 View-Forms with 'Composite Conditions' ****

1/d0	/d1		n1+n2+n3+n4=C		n9+n10+n11+n12=C
1---- 2	9----10		n1+n2+n5+n6=C		n9+n10+n13+n14=C
3---+ 4	11---+12		n1+n3+n5+n7=C		n9+n11+n13+n15=C
5-- - 6	13-- -14		n2+n4+n6+n8=C		n10+n12+n14+n16=C
7---- 8	15----16		n3+n4+n7+n8=C		n11+n12+n15+n16=C
			n5+n6+n7+n8=C		n13+n14+n15+n16=C

2/d0	/d1		n1+n2+n3+n4=C		n5+n6+n7+n8=C
1---- 2	5---- 6		n1+n2+n9+n10=C		n5+n6+n13+n14=C
3---+ 4	7---+ 8		n1+n3+n9+n11=C		n5+n7+n13+n15=C
9-- -10	13-- -14		n2+n4+n10+n12=C		n6+n8+n14+n16=C
11----12	15----16		n3+n4+n11+n12=C		n7+n8+n15+n16=C
			n9+n10+n11+n12=C		n13+n14+n15+n16=C

3/d0	/d1		n1+n2+n5+n6=C		n3+n4+n7+n8=C
1---- 2	3---- 4		n1+n2+n9+n10=C		n3+n4+n11+n12=C
5---+ 6	7---+ 8		n1+n5+n9+n13=C		n3+n7+n11+n15=C
9-- -10	11-- -12		n2+n6+n10+n14=C		n4+n8+n12+n16=C
13----14	15----16		n5+n6+n13+n14=C		n7+n8+n15+n16=C
			n9+n10+n13+n14=C		n11+n12+n15+n16=C

4/d0	/d1		n1+n3+n5+n7=C		n2+n4+n6+n8=C
1---- 3	2---- 4		n1+n3+n9+n11=C		n2+n4+n10+n12=C
5---+ 7	6---+ 8		n1+n5+n9+n13=C		n2+n6+n10+n14=C
9-- -11	10-- -12		n3+n7+n11+n15=C		n4+n8+n12+n16=C
13----15	14----16		n5+n7+n13+n15=C		n6+n8+n14+n16=C
			n9+n11+n13+n15=C		n10+n12+n14+n16=C

But you may still find some duplications of the same equation. Of course, you can remove them all right and get the smart list of simultaneous equations.

On the other hand, we also want to add the next equations to our definition.

$$n1+n16=n2+n15=n3+n14=n4+n13=n5+n12=n6+n11=n7+n10=n8+n9=CC=C/2=17$$

These equations are so called 'Self-Complementary Conditions'. They mean:

$$n16=CC-n1; n15=CC-n2; n14=CC-n3; \dots n10=CC-n7; n9=CC-n8;$$

Therefore $n13+n14+n15+n16$

$$=(CC-n4)+(CC-n3)+(CC-n2)+(CC-n1)=4*CC-(n1+n2+n3+n4)=C$$

$$68 \quad 34 \quad 34$$

In the same way $n9+n10+n11+n12=4*CC-(n5+n6+n7+n8)=C$

$$n7+n8+n15+n16=4*C-(n1+n2+n9+n10)=C$$

.....

As a consequence, when you assume $n1+n2+n3+n4=C$ at first, you don't have to define $n13+n14+n15+n16=C$ any longer. If you assume $n1+n2+n9+n10=C$ in advance, you don't have to define $n7+n8+n15+n16=C$ any longer. ...

You can remove half of the equations above by these reasons.

**** Collection of 'Composite Conditions' ****

$$\begin{array}{lll} n1+n2+n3+n4=C & \dots(1); & n1+n2+n5+n6=C & \dots(2); & n1+n2+n9+n10=C & \dots(3); \\ n1+n3+n5+n7=C & \dots(4); & n1+n3+n9+n11=C & \dots(5); & n1+n5+n9+n13=C & \dots(6); \\ n2+n4+n6+n8=C & \dots(7); & n2+n4+n10+n12=C & \dots(8); & n2+n6+n10+n14=C & \dots(9); \\ n3+n4+n7+n8=C & \dots(10); & n3+n4+n11+n12=C & \dots(11); & n5+n6+n7+n8=C & \dots(12); \end{array}$$

By the way, do we really have to define 6 equations for each little cube? See the next algebraic calculations. They show if you assume any 4 equations on each little cube beforehand, you don't have to define the rest 2 any longer.

$$\begin{array}{lll} 1-----5 & n1+n2+n3+n4=C & \dots(1) \\ | \quad \backslash \quad | \quad \backslash & n1+n2+n5+n6=C & \dots(2) \\ | \quad 2-----+---6 & n1+n3+n5+n7=C & \dots(3) \\ 3-|-----7 \quad | & n2+n4+n6+n8=C & \dots(4) \\ \quad \backslash \quad \quad \backslash & & \\ \quad \quad 4-----8 & n1+n3+n5+n7=C & \dots(3) \\ & +)n2+n4+n6+n8=C & \dots(4) \\ \hline & n1+n2+n3+n4+n5+n6+n7+n8=2*C & \\ & (n1+n2+n3+n4)+(n5+n6+n7+n8)=2*C & \\ \text{But} & n1+n2+n3+n4=C & \dots(1) \\ \text{Therefore} & n5+n6+n7+n8=C & \dots(5) \\ & & \\ & n1+n3+n5+n7=C & \dots(3) \\ & +)n2+n4+n6+n8=C & \dots(4) \\ \hline & n1+n2+n3+n4+n5+n6+n7+n8=2*C & \\ & (n1+n2+n5+n6)+(n3+n4+n7+n8)=2*C & \\ \text{But} & n1+n2+n5+n6=C & \dots(2) \\ \text{Therefore} & n3+n4+n7+n8=C & \dots(6) \end{array}$$

You can remove more by this reason and get such the smartest list as follows.

**** Essential 9 Equations for 'Composite Conditions' ****

$$\begin{array}{lll} n1+n2+n3+n4=C & \dots(1); & n1+n2+n5+n6=C & \dots(2); & n1+n2+n9+n10=C & \dots(3); \\ n1+n3+n5+n7=C & \dots(4); & n1+n3+n9+n11=C & \dots(5); & n1+n5+n9+n13=C & \dots(6); \\ n2+n4+n6+n8=C & \dots(7); & n2+n4+n10+n12=C & \dots(8); & n2+n6+n10+n14=C & \dots(9); \end{array}$$

These 9 equations are logically equivalent even to the 288 equations of the first diagrams. It means if you assume 9 beforehand, then you don't have to define anything more. But it doesn't mean that you must not define anything else.

You may well add any equations you have already removed from the lists above to our first definition. These additions will make no difference at all. You will have the same result constantly, even if you revive many other equations.

#3. Composition of ECO2⁴ Again

Let's design and compose our ECO2⁴ again strictly under these simultaneous equations with the Basic Diagrams below. It is almost all the same as Chapter 6.

The two diagrams on the right hand side show how we can convert it down into

2-dimensions and get the two types of magic squares of order 4. We really want to have these objects: 'Self-Complementary' and 'Composite & Complete' MS44.

**** Definition ****

BD/d0	/d1	SC/	CC/
1---- 2	3---- 4	1 2 3 4	1 2 4 3
9---+10	11---+12	5 6 7 8	5 6 8 7
5-- - 6	7-- - 8	9 10 11 12	13 14 16 15
13----14	15----16	13 14 15 16	9 10 12 11

$n1+n2+n3+n4=C \dots(1); n1+n2+n5+n6=C \dots(2); n1+n2+n9+n10=C \dots(3);$
 $n1+n3+n5+n7=C \dots(4); n1+n3+n9+n11=C \dots(5); n1+n5+n9+n13=C \dots(6);$
 $n2+n4+n6+n8=C \dots(7); n2+n4+n10+n12=C \dots(8); n2+n6+n10+n14=C \dots(9);$
 $n1+n16=n2+n15=n3+n14=n4+n13=n5+n12=n6+n11=n7+n10=n8+n9=PSM=C/2 \dots(10);$

Let me show you a sample list of computer program of mine.
It is written in the smallest size so that you might understand it easily.

```

/** Compose the 4-Dimensional Magic Objects of Order 2 and **
/** Transform them into Two Types of MS44:'S-C' and 'C&C' **
/** File:'EC024A384.c' Dictated by Kanji Setsuda **
/** on Jan.14, 2006; Revised on March 3, 2013; Recompiled **
/** on Mar. 2, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 **
//
#include <stdio.h>
//
/** Variables *
short int cnt;
short CC, LSM, cnt2;
short nm[17], uflg[17];
short t[384][17];
//
/** Sub-Routines *
void stp01(void), stp02(void), stp03(void);
void stp04(void), stp05(void), stp06(void);
void stp07(void), stp08(void), stp09(void);
void stp10(void), stp11(void), stp12(void);
void ansreocrd(void);
void pr4f24(void);
//
/** Main Program *
int main(){
short n;
printf("\n");
printf("** Compose the 4-Dimensional Magic Objects of Order 2 and **\n");
printf("** Transform them into Two Types of MS44:'S-C' and 'C&C' **\n");
for(n=0;n<17;n++){nm[n]=0; uflg[n]=0;}
CC=17; LSM=34; cnt=0;
stp01();          /** Begin the Calculation *
pr4f24();        /** Print the Answer in 4 Forms *
printf(" [Count = %d] OK!\n",cnt);
printf("\n");
printf("** Recompiled and Listed by Kanji Setsuda on\n");
printf("   March 2, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 **\n");
printf("\n");
return 0;
}
//
/** Calculations *
/** Set n1 & n16 *
void stp01(){
short a,b;

```

```

    for(a=1;a<17;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[1]=a; nm[16]=b;
            uflg[a]=1; uflg[b]=1;
            stp02();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set n2 & n15 *
void stp02(){
    short a,b;
    for(a=16;a>0;a--){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[2]=a; nm[15]=b;
            uflg[a]=1; uflg[b]=1;
            stp03();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set n5 & n12 *
void stp03(){
    short a,b;
    for(a=16;a>0;a--){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[5]=a; nm[12]=b;
            uflg[a]=1; uflg[b]=1;
            stp04();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set n6=LSM-n1-n2-n5 & n11 *
void stp04(){
    short a,b;
    a=LSM-nm[1]-nm[2]-nm[5];
    if((0<a)&&(a<17)){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[6]=a; nm[11]=b;
            uflg[a]=1; uflg[b]=1;
            stp05();
            uflg[b]=0; uflg[a]=0;}}}
}
/** Set n3 & n14 *
void stp05(){
    short a,b;
    for(a=16;a>0;a--){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[3]=a; nm[14]=b;
            uflg[a]=1; uflg[b]=1;
            stp06();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set n4=LSM-n1-n2-n3 & n13 *
void stp06(){
    short a,b;
    a=LSM-nm[1]-nm[2]-nm[3];
    if((0<a)&&(a<17)){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[4]=a; nm[13]=b;
            uflg[a]=1; uflg[b]=1;
            stp07();
            uflg[b]=0; uflg[a]=0;}}}
}
/** Set n7=LSM-n1-n3-n5 & n10 *
void stp07(){

```

```

short a,b;
a=LSM-nm[1]-nm[3]-nm[5];
if((0<a)&&(a<17)){b=CC-a;
    if((0<b)&&(uflg[a]==0)&&(uflg[b]==0)){
        nm[7]=a; nm[10]=b;
        uflg[a]=1; uflg[b]=1;
        stp08();
        uflg[b]=0; uflg[a]=0;}}
}
/* Set n8=LSM-n2-n4-n6 & & n9 *
void stp08(){
short a,b,c;
a=LSM-nm[2]-nm[4]-nm[6];
b=LSM-nm[3]-nm[4]-nm[7];
c=LSM-nm[5]-nm[6]-nm[7];
if((0<a)&&(a<17)&&(a==b)&&(a==c)){b=CC-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
        nm[8]=a; nm[9]=b;
        uflg[a]=1; uflg[b]=1; cnt2=0;
        stp09();
        uflg[b]=0; uflg[a]=0;}}
}
/* Check some Line-sums *
void stp09(){
short sm1,sm2,sm3;
sm1=nm[1]+nm[2]+nm[9]+nm[10];
sm2=nm[1]+nm[3]+nm[9]+nm[11];
sm3=nm[1]+nm[5]+nm[9]+nm[13];
if((sm1==LSM)&&(sm2==LSM)&&(sm3==LSM)){stp10();}
}
/* Check some Line-sums *
void stp10(){
short sm1,sm2,sm3;
sm1=nm[2]+nm[4]+nm[10]+nm[12];
sm2=nm[2]+nm[6]+nm[10]+nm[14];
sm3=nm[3]+nm[4]+nm[11]+nm[12];
if((sm1==LSM)&&(sm2==LSM)&&(sm3==LSM)){ansreocrd();}
}
//
void ansreocrd(void){
short n;
t[cnt][0]=cnt+1;
for(n=1;n<17;n++){t[cnt][n]=nm[n];}
cnt++;
}
//
/* Print the Answer of Order 2^4 in 4 Forms *
void pr4f24(void){
short m,n;
short tn[17];
for(m=0;m<cnt;m++){
    for(n=0;n<17;n++){tn[n]=t[m][n];}
    printf("%3d/d0          /d1          SC/          CC^n",tn[0]);
    printf("%4d----%2d%7d----%2d  ",tn[1],tn[2],tn[3],tn[4]);
    printf(" %3d%3d%3d%3d %3d%3d%3d%3d\n",
        tn[1],tn[2],tn[3],tn[4],tn[1],tn[2],tn[4],tn[3]);
    printf(" | %2d---%2d | %2d---%2d",tn[9],tn[10],tn[11],tn[12]);
    printf(" %3d%3d%3d%3d %3d%3d%3d%3d\n",
        tn[5],tn[6],tn[7],tn[8],tn[5],tn[6],tn[8],tn[7]);
    printf("%4d--l-%2d l%4d--l-%2d l",tn[5],tn[6],tn[7],tn[8]);
    printf(" %3d%3d%3d%3d %3d%3d%3d%3d\n",
        tn[9],tn[10],tn[11],tn[12],tn[13],tn[14],tn[16],tn[15]);
    printf("%7d----%2d%7d----%2d",tn[13],tn[14],tn[15],tn[16]);
    printf(" %3d%3d%3d%3d %3d%3d%3d%3d\n",
        tn[13],tn[14],tn[15],tn[16],tn[9],tn[10],tn[12],tn[11]);
}
}

```



```
}
}
```

Let me show you part of the execution result of this program as follows.

```
** Compose the 4-Dimensional Magic Objects of Order 2 and **
** Transform them into Two Types of MS44: 'S-C' and 'C&C' **
```

```
1/d0      /d1      SC/      CC/
 1----15   12---- 6     1 15 12 6    1 15 6 12
 | 8---+-10 | 13---+- 3    14 4 7 9    14 4 9 7
14--|- 4 | 7--|- 9 |    8 10 13 3   11 5 16 2
 11---- 5     2----16   11 5 2 16    8 10 3 13

2/d0      /d1      SC/      CC/
 1----15   8----10     1 15 8 10    1 15 10 8
 | 12---+- 6 | 13---+- 3    14 4 11 5    14 4 5 11
14--|- 4 | 11--|- 5 |    12 6 13 3    7 9 16 2
 7---- 9     2----16   7 9 2 16    12 6 3 13

3/d0      /d1      SC/      CC/
 1----15   14---- 4     1 15 14 4    1 15 4 14
 | 8---+-10 | 11---+- 5    12 6 7 9    12 6 9 7
12--|- 6 | 7--|- 9 |    8 10 11 5    13 3 16 2
 13---- 3     2----16   13 3 2 16    8 10 5 11

4/d0      /d1      SC/      CC/
 1----15   8----10     1 15 8 10    1 15 10 8
 | 14---+- 4 | 11---+- 5    12 6 13 3    12 6 3 13
12--|- 6 | 13--|- 3 |    14 4 11 5    7 9 16 2
 7---- 9     2----16   7 9 2 16    14 4 5 11

5/d0      /d1      SC/      CC/
 1----15   14---- 4     1 15 14 4    1 15 4 14
 | 12---+- 6 | 7---+- 9     8 10 11 5    8 10 5 11
 8--|-10 | 11--|- 5 |    12 6 7 9    13 3 16 2
 13---- 3     2----16   13 3 2 16    12 6 9 7

6/d0      /d1      SC/      CC/
 1----15   12---- 6     1 15 12 6    1 15 6 12
 | 14---+- 4 | 7---+- 9     8 10 13 3    8 10 3 13
 8--|-10 | 13--|- 3 |    14 4 7 9    11 5 16 2
 11---- 5     2----16   11 5 2 16    14 4 9 7

7/d0      /d1      SC/      CC/
 1----14   12---- 7     1 14 12 7    1 14 7 12
 | 8---+-11 | 13---+- 2    15 4 6 9    15 4 9 6
15--|- 4 | 6--|- 9 |    8 11 13 2    10 5 16 3
 10---- 5     3----16   10 5 3 16    8 11 2 13

8/d0      /d1      SC/      CC/
 1----14   8----11     1 14 8 11    1 14 11 8
 | 12---+- 7 | 13---+- 2    15 4 10 5    15 4 5 10
15--|- 4 | 10--|- 5 |    12 7 13 2    6 9 16 3
 6---- 9     3----16   6 9 3 16    12 7 2 13

9/d0      /d1      SC/      CC/
 1----14   15---- 4     1 14 15 4    1 14 4 15
 | 8---+-11 | 10---+- 5    12 7 6 9    12 7 9 6
12--|- 7 | 6--|- 9 |    8 11 10 5    13 2 16 3
 13---- 2     3----16   13 2 3 16    8 11 5 10

10/d0     /d1      SC/      CC/
 1----14   8----11     1 14 8 11    1 14 11 8
 | 15---+- 4 | 10---+- 5    12 7 13 2    12 7 2 13
12--|- 7 | 13--|- 2 |    15 4 10 5    6 9 16 3
 6---- 9     3----16   6 9 3 16    15 4 5 10
```

11/d0	/d1	SC/	CC/
1----14	15---- 4	1 14 15 4	1 14 4 15
12---+ 7	6---+ 9	8 11 10 5	8 11 5 10
8-- -11	10-- - 5	12 7 6 9	13 2 16 3
13---- 2	3----16	13 2 3 16	12 7 9 6
12/d0	/d1	SC/	CC/
1----14	12---- 7	1 14 12 7	1 14 7 12
15---+ 4	6---+ 9	8 11 13 2	8 11 2 13
8-- -11	13-- - 2	15 4 6 9	10 5 16 3
10---- 5	3----16	10 5 3 16	15 4 9 6
13/d0	/d1	SC/	CC/
1----12	14---- 7	1 12 14 7	1 12 7 14
8---+13	11---+ 2	15 6 4 9	15 6 9 4
15-- - 6	4-- - 9	8 13 11 2	10 3 16 5
10---- 3	5----16	10 3 5 16	8 13 2 11
14/d0	/d1	SC/	CC/
1----12	8----13	1 12 8 13	1 12 13 8
14---+ 7	11---+ 2	15 6 10 3	15 6 3 10
15-- - 6	10-- - 3	14 7 11 2	4 9 16 5
4---- 9	5----16	4 9 5 16	14 7 2 11
15/d0	/d1	SC/	CC/
1----12	15---- 6	1 12 15 6	1 12 6 15
8---+13	10---+ 3	14 7 4 9	14 7 9 4
14-- - 7	4-- - 9	8 13 10 3	11 2 16 5
11---- 2	5----16	11 2 5 16	8 13 3 10
16/d0	/d1	SC/	CC/
1----12	8----13	1 12 8 13	1 12 13 8
15---+ 6	10---+ 3	14 7 11 2	14 7 2 11
14-- - 7	11-- - 2	15 6 10 3	4 9 16 5
4---- 9	5----16	4 9 5 16	15 6 3 10
17/d0	/d1	SC/	CC/
1----12	15---- 6	1 12 15 6	1 12 6 15
14---+ 7	4---+ 9	8 13 10 3	8 13 3 10
8-- -13	10-- - 3	14 7 4 9	11 2 16 5
11---- 2	5----16	11 2 5 16	14 7 9 4
18/d0	/d1	SC/	CC/
1----12	14---- 7	1 12 14 7	1 12 7 14
15---+ 6	4---+ 9	8 13 11 2	8 13 2 11
8-- -13	11-- - 2	15 6 4 9	10 3 16 5
10---- 3	5----16	10 3 5 16	15 6 9 4
19/d0	/d1	SC/	CC/
1---- 8	14----11	1 8 14 11	1 8 11 14
12---+13	7---+ 2	15 10 4 5	15 10 5 4
15-- -10	4-- - 5	12 13 7 2	6 3 16 9
6---- 3	9----16	6 3 9 16	12 13 2 7
20/d0	/d1	SC/	CC/
1---- 8	12----13	1 8 12 13	1 8 13 12
14---+11	7---+ 2	15 10 6 3	15 10 3 6
15-- -10	6-- - 3	14 11 7 2	4 5 16 9
4---- 5	9----16	4 5 9 16	14 11 2 7
21/d0	/d1	SC/	CC/
1---- 8	15----10	1 8 15 10	1 8 10 15
12---+13	6---+ 3	14 11 4 5	14 11 5 4
14-- -11	4-- - 5	12 13 6 3	7 2 16 9
7---- 2	9----16	7 2 9 16	12 13 3 6

22/d0	/d1	SC/	CC/
1---- 8	12----13	1 8 12 13	1 8 13 12
15---+10	6---+ 3	14 11 7 2	14 11 2 7
14-- -11	7-- - 2	15 10 6 3	4 5 16 9
4---- 5	9----16	4 5 9 16	15 10 3 6

23/d0	/d1	SC/	CC/
1---- 8	15----10	1 8 15 10	1 8 10 15
14---+11	4---+ 5	12 13 6 3	12 13 3 6
12-- -13	6-- - 3	14 11 4 5	7 2 16 9
7---- 2	9----16	7 2 9 16	14 11 5 4

24/d0	/d1	SC/	CC/
1---- 8	14----11	1 8 14 11	1 8 11 14
15---+10	4---+ 5	12 13 7 2	12 13 2 7
12-- -13	7-- - 2	15 10 4 5	6 3 16 9
6---- 3	9----16	6 3 9 16	15 10 5 4

SC/ 25/CC	SC/ 26/CC	SC/ 27/CC
2 16 11 5	2 16 7 9	2 16 13 3
13 3 8 10	2 16 9 7	2 16 3 13
7 9 14 4	13 3 12 6	11 5 8 10
12 6 1 15	13 3 6 12	11 5 10 8
7 9 4 14	11 5 14 4	7 9 12 6
	8 10 15 1	14 4 15 1
	8 10 1 15	14 4 1 15
	11 5 4 14	7 9 6 12

SC/ 28/CC	SC/ 29/CC	SC/ 30/CC
2 16 7 9	2 16 13 3	2 16 11 5
11 5 14 4	2 16 3 13	2 16 5 11
13 3 12 6	7 9 12 6	7 9 14 4
8 10 1 15	7 9 6 12	7 9 4 14
13 3 6 12	11 5 8 10	13 3 8 10
	14 4 15 1	12 6 15 1
	14 4 1 15	12 6 1 15
	11 5 10 8	13 3 10 8

SC/ 31/CC	SC/ 32/CC	SC/ 33/CC
2 13 11 8	2 13 7 12	2 13 16 3
16 3 5 10	2 13 12 7	2 13 3 16
7 12 14 1	16 3 9 6	11 8 5 10
9 6 4 15	16 3 6 9	11 8 10 5
7 12 1 14	11 8 14 1	7 12 9 6
	5 10 15 4	14 1 15 4
	5 10 4 15	14 1 4 15
	11 8 1 14	7 12 6 9

SC/ 34/CC	SC/ 35/CC	SC/ 36/CC
2 13 7 12	2 13 16 3	2 13 11 8
11 8 14 1	2 13 3 16	2 13 8 11
16 3 9 6	7 12 9 6	7 12 14 1
5 10 4 15	7 12 6 9	7 12 1 14
16 3 6 9	11 8 5 10	16 3 5 10
	14 1 15 4	9 6 15 4
	14 1 4 15	9 6 4 15
	11 8 10 5	16 3 10 5

SC/ 37/CC	SC/ 38/CC	SC/ 39/CC
2 11 13 8	2 11 7 14	2 11 16 5
16 5 3 10	2 11 14 7	2 11 5 16
7 14 12 1	16 5 9 4	13 8 3 10
9 4 6 15	16 5 4 9	13 8 10 3
7 14 1 12	13 8 12 1	7 14 9 4
	3 10 15 6	12 1 15 6
	3 10 6 15	12 1 6 15
	13 8 1 12	7 14 4 9

SC/ 40/CC	SC/ 41/CC	SC/ 42/CC
2 11 7 14	2 11 16 5	2 11 13 8
13 8 12 1	2 11 5 16	2 11 8 13
16 5 9 4	7 14 9 4	7 14 12 1
3 10 6 15	7 14 4 9	7 14 1 12
16 5 4 9	13 8 3 10	16 5 3 10
	12 1 15 6	9 4 15 6
	12 1 6 15	9 4 6 15
	13 8 10 3	16 5 10 3

SC/ 43/CC	SC/ 44/CC	SC/ 45/CC
2 7 13 12	2 7 11 14	2 7 16 9
16 9 3 6	2 7 14 11	2 7 9 16
11 14 8 1	16 9 5 4	13 12 3 6
5 4 10 15	16 9 4 5	13 12 6 3
11 14 1 8	13 12 8 1	11 14 5 4
	3 6 15 10	8 1 15 10
	3 6 10 15	8 1 10 15
	13 12 1 8	11 14 4 5

SC/ 46/CC	SC/ 47/CC	SC/ 48/CC
2 7 11 14	2 7 16 9	2 7 13 12
13 12 8 1	2 7 9 16	2 7 12 13
16 9 5 4	11 14 5 4	11 14 8 1
3 6 10 15	11 14 4 5	11 14 1 8
16 9 4 5	13 12 3 6	16 9 3 6
	8 1 15 10	5 4 15 10
	8 1 10 15	5 4 10 15
	13 12 6 3	16 9 6 3

SC/ 49/CC	SC/ 50/CC	SC/ 51/CC
3 16 10 5	3 16 6 9	3 16 13 2
13 2 8 11	3 16 9 6	3 16 2 13
6 9 15 4	13 2 12 7	10 5 8 11
12 7 1 14	13 2 7 12	10 5 11 8
6 9 4 15	10 5 15 4	6 9 12 7
	8 11 14 1	15 4 14 1
	8 11 1 14	15 4 1 14
	10 5 4 15	6 9 7 12

SC/ 52/CC	SC/ 53/CC	SC/ 54/CC
3 16 6 9 3 16 9 6	3 16 13 2 3 16 2 13	3 16 10 5 3 16 5 10
10 5 15 4 10 5 4 15	6 9 12 7 6 9 7 12	6 9 15 4 6 9 4 15
13 2 12 7 8 11 14 1	10 5 8 11 15 4 14 1	13 2 8 11 12 7 14 1
8 11 1 14 13 2 7 12	15 4 1 14 10 5 11 8	12 7 1 14 13 2 11 8
SC/ 55/CC	SC/ 56/CC	SC/ 57/CC
3 13 10 8 3 13 8 10	3 13 6 12 3 13 12 6	3 13 16 2 3 13 2 16
16 2 5 11 16 2 11 5	16 2 9 7 16 2 7 9	10 8 5 11 10 8 11 5
6 12 15 1 9 7 14 4	10 8 15 1 5 11 14 4	6 12 9 7 15 1 14 4
9 7 4 14 6 12 1 15	5 11 4 14 10 8 1 15	15 1 4 14 6 12 7 9
SC/ 58/CC	SC/ 59/CC	SC/ 60/CC
3 13 6 12 3 13 12 6	3 13 16 2 3 13 2 16	3 13 10 8 3 13 8 10
10 8 15 1 10 8 1 15	6 12 9 7 6 12 7 9	6 12 15 1 6 12 1 15
16 2 9 7 5 11 14 4	10 8 5 11 15 1 14 4	16 2 5 11 9 7 14 4
5 11 4 14 16 2 7 9	15 1 4 14 10 8 11 5	9 7 4 14 16 2 11 5
SC/ 61/CC	SC/ 62/CC	SC/ 63/CC
3 10 13 8 3 10 8 13	3 10 6 15 3 10 15 6	3 10 16 5 3 10 5 16
16 5 2 11 16 5 11 2	16 5 9 4 16 5 4 9	13 8 2 11 13 8 11 2
6 15 12 1 9 4 14 7	13 8 12 1 2 11 14 7	6 15 9 4 12 1 14 7
9 4 7 14 6 15 1 12	2 11 7 14 13 8 1 12	12 1 7 14 6 15 4 9
SC/ 64/CC	SC/ 65/CC	SC/ 66/CC
3 10 6 15 3 10 15 6	3 10 16 5 3 10 5 16	3 10 13 8 3 10 8 13
13 8 12 1 13 8 1 12	6 15 9 4 6 15 4 9	6 15 12 1 6 15 1 12
16 5 9 4 2 11 14 7	13 8 2 11 12 1 14 7	16 5 2 11 9 4 14 7
2 11 7 14 16 5 4 9	12 1 7 14 13 8 11 2	9 4 7 14 16 5 11 2
SC/ 67/CC	SC/ 68/CC	SC/ 69/CC
3 6 13 12 3 6 12 13	3 6 10 15 3 6 15 10	3 6 16 9 3 6 9 16
16 9 2 7 16 9 7 2	16 9 5 4 16 9 4 5	13 12 2 7 13 12 7 2
10 15 8 1 5 4 14 11	13 12 8 1 2 7 14 11	10 15 5 4 8 1 14 11
5 4 11 14 10 15 1 8	2 7 11 14 13 12 1 8	8 1 11 14 10 15 4 5
SC/ 70/CC	SC/ 71/CC	SC/ 72/CC
3 6 10 15 3 6 15 10	3 6 16 9 3 6 9 16	3 6 13 12 3 6 12 13
13 12 8 1 13 12 1 8	10 15 5 4 10 15 4 5	10 15 8 1 10 15 1 8
16 9 5 4 2 7 14 11	13 12 2 7 8 1 14 11	16 9 2 7 5 4 14 11
2 7 11 14 16 9 4 5	8 1 11 14 13 12 7 2	5 4 11 14 16 9 7 2
SC/ 73/CC	SC/ 74/CC	SC/ 75/CC
4 15 9 6 4 15 6 9	4 15 5 10 4 15 10 5	4 15 14 1 4 15 1 14
14 1 7 12 14 1 12 7	14 1 11 8 14 1 8 11	9 6 7 12 9 6 12 7
5 10 16 3 11 8 13 2	9 6 16 3 7 12 13 2	5 10 11 8 16 3 13 2
11 8 2 13 5 10 3 16	7 12 2 13 9 6 3 16	16 3 2 13 5 10 8 11
.		
SC/358/CC	SC/359/CC	SC/360/CC
15 1 4 14 15 1 14 4	15 1 10 8 15 1 8 10	15 1 6 12 15 1 12 6
6 12 9 7 6 12 7 9	4 14 5 11 4 14 11 5	4 14 9 7 4 14 7 9
10 8 5 11 3 13 2 16	6 12 3 13 9 7 2 16	10 8 3 13 5 11 2 16
3 13 16 2 10 8 11 5	9 7 16 2 6 12 13 3	5 11 16 2 10 8 13 3
SC/361/CC	SC/362/CC	SC/363/CC
16 9 3 6 16 9 6 3	16 9 2 7 16 9 7 2	16 9 5 4 16 9 4 5
5 4 10 15 5 4 15 10	5 4 11 14 5 4 14 11	3 6 10 15 3 6 15 10
2 7 13 12 11 14 1 8	3 6 13 12 10 15 1 8	2 7 11 14 13 12 1 8
11 14 8 1 2 7 12 13	10 15 8 1 3 6 12 13	13 12 8 1 2 7 14 11
SC/364/CC	SC/365/CC	SC/366/CC
16 9 2 7 16 9 7 2	16 9 5 4 16 9 4 5	16 9 3 6 16 9 6 3
3 6 13 12 3 6 12 13	2 7 11 14 2 7 14 11	2 7 13 12 2 7 12 13
5 4 11 14 10 15 1 8	3 6 10 15 13 12 1 8	5 4 10 15 11 14 1 8
10 15 8 1 5 4 14 11	13 12 8 1 3 6 15 10	11 14 8 1 5 4 15 10
SC/367/CC	SC/368/CC	SC/369/CC
16 5 3 10 16 5 10 3	16 5 2 11 16 5 11 2	16 5 9 4 16 5 4 9
9 4 6 15 9 4 15 6	9 4 7 14 9 4 14 7	3 10 6 15 3 10 15 6
2 11 13 8 7 14 1 12	3 10 13 8 6 15 1 12	2 11 7 14 13 8 1 12
7 14 12 1 2 11 8 13	6 15 12 1 3 10 8 13	13 8 12 1 2 11 14 7

If you want to have the 48 'standard' solutions of any type, you may select them from this Mother Set by taking any 'normalizing inequality conditions' such as:
 $n_1 < n_4$; $n_1 < n_{13}$; $n_1 < n_{16}$; and $n_2 > n_5$ for two types of MS of order 4.

#4. Discussion

I am so impressed with the fact everything is realized here at the same time.

What does the result mean?

I think it directly implies that this ECO_2^4 is the origin of that 'miraculous unification' notified among various types of magic squares of order 4.

In other words of literature I say, while we, human being, enjoy every 384 solutions of various types in this world, Divine Being might enjoy the same thing in the only one original form in heaven.

(Written in English by Kanji Setsuda on May 10, 2005 with MacOSX & Xcode 1.5;
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