

Chapter 2 : New Study of Magic Squares 4x4: Kanji Setsuda

Section 4-2: 'Self-Complementary' Magic Squares of Order 4 and the meaning of 'Complementary Pairs'

#1. Let's study the 'Self-Complementary' Magic Squares of Order 4 right here in this section and know what the meaning of 'Complementary Pairs' is for them.

Since this article was written at the same time with the last section "Composite and Complete Pan-magic Squares 4x4 and the meaning of 'Complementary Pairs'" from the same view point, will you read the Section 4 before all, if you have not yet read it?

And you will understand that I discuss almost the common points in both articles.

At first as usual, let's define the present object 'Self-Complementary' type of MS44 with the Basic Form and the Simultaneous Equations, and get the sample solutions of it, but comparing a little with the last 'Complete' type of PMS44.

**** 'Self-Complementary' and 'Complete' Types; ****

*** Basic Forms of Definitions ***

S-c/0/Cmplt

n1 n2 n3 n4	n1 n2 n4 n3
n5 n6 n7 n8	n5 n6 n8 n7
n9 10 11 12	13 14 16 15
13 14 15 16	n9 10 12 11

**** Basic Conditions: C=34;**

n1+n2+n3+n4=C	...rw1;
n5+n6+n7+n8=C	...rw2;
n9+n10+n11+n12=C	...rw3;
n13+n14+n15+n16=C	...rw4;
n1+n5+n9+n13=C	...c11;
n2+n6+n10+n14=C	...c12;
n3+n7+n11+n15=C	...c13;
n4+n8+n12+n16=C	...c14;

Self-Complementary Conditions: CC=17; **

n1+n16=CC	...Cmp1;
n2+n15=CC	...Cmp2;
n3+n14=CC	...Cmp3;
n4+n13=CC	...Cmp4;
n5+n12=CC	...Cmp5;
n6+n11=CC	...Cmp6;
n7+n10=CC	...Cmp7;
n8+n9=CC	...Cmp8;

(Every Complementary Pair must be placed symmetrically with respect to the center of square.)

*** List of the 384 Primitive Solutions: 'Self-Complementary' and 'Complete' Types; ***

S/1/C

1 15 12 6	1 15 6 12
14 4 7 9	14 4 9 7
8 10 13 3	11 5 16 2
11 5 2 16	8 10 3 13

S/2/C

1 15 8 10	1 15 10 8
14 4 11 5	14 4 5 11
12 6 13 3	7 9 16 2
7 9 2 16	12 6 3 13

S/3/C

1 15 14 4	1 15 4 14
12 6 7 9	12 6 9 7
8 10 11 5	13 3 16 2
13 3 2 16	8 10 5 11

S/4/C

1 15 8 10	1 15 10 8
12 6 13 3	12 6 3 13
14 4 11 5	7 9 16 2
7 9 2 16	14 4 5 11

S/5/C

1 15 14 4	1 15 4 14
8 10 11 5	8 10 5 11
12 6 7 9	13 3 16 2
13 3 2 16	12 6 9 7

S/6/C

1 15 12 6	1 15 6 12
8 10 13 3	8 10 3 13
14 4 7 9	11 5 16 2
11 5 2 16	14 4 9 7

S/7/C		S/8/C		S/9/C	
1 14 12 7 15 4 6 9 8 11 13 2 10 5 3 16	1 14 7 12 15 4 9 6 10 5 16 3 8 11 2 13	1 14 8 11 15 4 10 5 12 7 13 2 6 9 3 16	1 14 11 8 15 4 5 10 6 9 16 3 12 7 2 13	1 14 15 4 12 7 6 9 8 11 10 5 13 2 3 16	1 14 4 15 12 7 9 6 13 2 16 3 8 11 5 10

S/10/C		S/11/C		S/12/C	
1 14 8 11 12 7 13 2 15 4 10 5 6 9 3 16	1 14 11 8 12 7 2 13 6 9 16 3 15 4 5 10	1 14 15 4 8 11 10 5 12 7 6 9 13 2 3 16	1 14 4 15 8 11 5 10 13 2 16 3 12 7 9 6	1 14 12 7 8 11 13 2 15 4 6 9 10 5 3 16	1 14 7 12 8 11 2 13 10 5 16 3 15 4 9 6

S/13/C		S/14/C		S/15/C	
1 12 14 7 15 6 4 9 8 13 11 2 10 3 5 16	1 12 7 14 15 6 9 4 10 3 16 5 8 13 2 11	1 12 8 13 15 6 10 3 14 7 11 2 4 9 5 16	1 12 13 8 15 6 3 10 4 9 16 5 14 7 2 11	1 12 15 6 14 7 4 9 8 13 10 3 11 2 5 16	1 12 6 15 14 7 9 4 11 2 16 5 8 13 3 10

S/16/C		S/17/C		S/18/C	
1 12 8 13 14 7 11 2 15 6 10 3 4 9 5 16	1 12 13 8 14 7 2 11 4 9 16 5 15 6 3 10	1 12 15 6 8 13 10 3 14 7 4 9 11 2 5 16	1 12 6 15 8 13 3 10 11 2 16 5 14 7 9 4	1 12 14 7 8 13 11 2 15 6 4 9 10 3 16 5	1 12 7 14 8 13 2 11 10 3 16 5 15 6 9 4

S/19/C		S/20/C		S/21/C	
1 8 14 11 15 10 4 5 12 13 7 2 6 3 9 16	1 8 11 14 15 10 5 4 6 3 16 9 12 13 2 7	1 8 12 13 15 10 6 3 14 11 7 2 4 5 9 16	1 8 13 12 15 10 3 6 4 5 16 9 14 11 2 7	1 8 15 10 14 11 4 5 12 13 6 3 7 2 9 16	1 8 10 15 14 11 5 4 7 2 16 9 12 13 3 6

S/22/C		S/23/C		S/24/C	
1 8 12 13 14 11 7 2 15 10 6 3 4 5 9 16	1 8 13 12 14 11 2 7 4 5 16 9 15 10 3 6	1 8 15 10 12 13 6 3 14 11 4 5 7 2 9 16	1 8 10 15 12 13 3 6 7 2 16 9 14 11 5 4	1 8 14 11 12 13 7 2 15 10 4 5 6 3 9 16	1 8 11 14 12 13 2 7 6 3 16 9 15 10 5 4

S/25/C		S/31/C		S/37/C	
2 16 11 5 13 3 8 10 7 9 14 4 12 6 1 15	2 16 5 11 13 3 10 8 12 6 15 1 7 9 4 14	2 13 11 8 16 3 5 10 7 12 14 1 9 6 4 15	2 13 8 11 16 3 10 5 9 6 15 4 7 12 1 14	2 11 13 8 16 5 3 10 7 14 12 1 9 4 6 15	2 11 8 13 16 5 10 3 9 4 15 6 7 14 1 12

(S means 'Self-Complementary' Type, while C means 'Complete' type.)

<p>S/43/C</p> <p>2 7 13 12 2 7 12 13 16 9 3 6 16 9 6 3 11 14 8 1 5 4 15 10 5 4 10 15 11 14 1 8</p>	<p>S/49/C</p> <p>3 16 10 5 3 16 5 10 13 2 8 11 13 2 11 8 6 9 15 4 12 7 14 1 12 7 1 14 6 9 4 15</p>	<p>S/55/C</p> <p>3 13 10 8 3 13 8 10 16 2 5 11 16 2 11 5 6 12 15 1 9 7 14 4 9 7 4 14 6 12 1 15</p>
<p>S/61/C</p> <p>3 10 13 8 3 10 8 13 16 5 2 11 16 5 11 2 6 15 12 1 9 4 14 7 9 4 7 14 6 15 1 12</p>	<p>S/67/C</p> <p>3 6 13 12 3 6 12 13 16 9 2 7 16 9 7 2 10 15 8 1 5 4 14 11 5 4 11 14 10 15 1 8</p>	<p>S/73/C</p> <p>4 15 9 6 4 15 6 9 14 1 7 12 14 1 12 7 5 10 16 3 11 8 13 2 11 8 2 13 5 10 3 16</p>
<p>S/79/C</p> <p>4 14 9 7 4 14 7 9 15 1 6 12 15 1 12 6 5 11 16 2 10 8 13 3 10 8 3 13 5 11 2 16</p>	<p>S/85/C</p> <p>4 9 14 7 4 9 7 14 15 6 1 12 15 6 12 1 5 16 11 2 10 3 13 8 10 3 8 13 5 16 2 11</p>	<p>S/91/C</p> <p>4 5 14 11 4 5 11 14 15 10 1 8 15 10 8 1 9 16 7 2 6 3 13 12 6 3 12 13 9 16 2 7</p>
<p>S/97/C</p> <p>5 16 10 3 5 16 3 10 11 2 8 13 11 2 13 8 4 9 15 6 14 7 12 1 14 7 1 12 4 9 6 15</p>	<p>S/103/C</p> <p>5 11 10 8 5 11 8 10 16 2 3 13 16 2 13 3 4 14 15 1 9 7 12 6 9 7 6 12 4 14 1 15</p>	<p>S/109/C</p> <p>5 10 11 8 5 10 8 11 16 3 2 13 16 3 13 2 4 15 14 1 9 6 12 7 9 6 7 12 4 15 1 14</p>
<p>S/115/C</p> <p>5 4 11 14 5 4 14 11 16 9 2 7 16 9 7 2 10 15 8 1 3 6 12 13 3 6 13 12 10 15 1 8</p>	<p>S/121/C</p> <p>6 15 9 4 6 15 4 9 12 1 7 14 12 1 14 7 3 10 16 5 13 8 11 2 13 8 2 11 3 10 5 16</p>	<p>S/127/C</p> <p>6 12 9 7 6 12 7 9 15 1 4 14 15 1 14 4 3 13 16 2 10 8 11 5 10 8 5 11 3 13 2 16</p>
<p>S/133/C</p> <p>6 9 12 7 6 9 7 12 15 4 1 14 15 4 14 1 3 16 13 2 10 5 11 8 10 5 8 11 3 16 2 13</p>	<p>S/139/C</p> <p>6 3 12 13 6 3 13 12 15 10 1 8 15 10 8 1 9 16 7 2 4 5 11 14 4 5 14 11 9 16 2 7</p>	<p>S/145/C</p> <p>7 14 9 4 7 14 4 9 12 1 6 15 12 1 15 6 2 11 16 5 13 8 10 3 13 8 3 10 2 11 5 16</p>
<p>S/151/C</p> <p>7 12 9 6 7 12 6 9 14 1 4 15 14 1 15 4 2 13 16 3 11 8 10 5 11 8 5 10 2 13 3 16</p>	<p>S/157/C</p> <p>7 9 12 6 7 9 6 12 14 4 1 15 14 4 15 1 2 16 13 3 11 5 10 8 11 5 8 10 2 16 3 13</p>	<p>S/163/C</p> <p>7 2 12 13 7 2 13 12 14 11 1 8 14 11 8 1 9 16 6 3 4 5 10 15 4 5 15 10 9 16 3 6</p>
<p>S/169/C</p> <p>8 13 10 3 8 13 3 10 11 2 5 16 11 2 16 5 1 12 15 6 14 7 9 4 14 7 4 9 1 12 6 15</p>	<p>S/175/C</p> <p>8 11 10 5 8 11 5 10 13 2 3 16 13 2 16 3 1 14 15 4 12 7 9 6 12 7 6 9 1 14 4 15</p>	<p>S/181/C</p> <p>8 10 11 5 8 10 5 11 13 3 2 16 13 3 16 2 1 15 14 4 12 6 9 7 12 6 7 9 1 15 4 14</p>
<p>S/187/C</p> <p>8 1 11 14 8 1 14 11 13 12 2 7 13 12 7 2 10 15 5 4 3 6 9 16 3 6 16 9 10 15 4 5</p>	<p>S/193/C</p> <p>9 16 6 3 9 16 3 6 7 2 12 13 7 2 13 12 4 5 15 10 14 11 8 1 14 11 1 8 4 5 10 15</p>	<p>S/199/C</p> <p>9 7 6 12 9 7 12 6 16 2 3 13 16 2 13 3 4 14 15 1 5 11 8 10 5 11 10 8 4 14 1 15</p>
<p>S/205/C</p> <p>9 6 7 12 9 6 12 7 16 3 2 13 16 3 13 2 4 15 14 1 5 10 8 11 5 10 11 8 4 15 1 14</p>	<p>S/211/C</p> <p>9 4 7 14 9 4 14 7 16 5 2 11 16 5 11 2 6 15 12 1 3 10 8 13 3 10 13 8 6 15 1 12</p>	<p>S/217/C</p> <p>10 15 5 4 10 15 4 5 8 1 11 14 8 1 14 11 3 6 16 9 13 12 7 2 13 12 2 7 3 6 9 16</p>
<p>S/223/C</p> <p>10 8 5 11 10 8 11 5 15 1 4 14 15 1 14 4 3 13 16 2 6 12 7 9 6 12 9 7 3 13 2 16</p>	<p>S/229/C</p> <p>10 5 8 11 10 5 11 8 15 4 1 14 15 4 14 1 3 16 13 2 6 9 7 12 6 9 12 7 3 16 2 13</p>	<p>S/235/C</p> <p>10 3 8 13 10 3 13 8 15 6 1 12 15 6 12 1 5 16 11 2 4 9 7 14 4 9 14 7 5 16 2 11</p>

(S means 'Self-Complementary' MS44, while C means 'Complete' MS44.)

<p>S/241/C</p> <p>11 14 5 4 11 14 4 5 8 1 10 15 8 1 15 10 2 7 16 9 13 12 6 3 13 12 3 6 2 7 9 16</p>	<p>S/247/C</p> <p>11 8 5 10 11 8 10 5 14 1 4 15 14 1 15 4 2 13 16 3 7 12 6 9 7 12 9 6 2 13 3 16</p>	<p>S/253/C</p> <p>11 5 8 10 11 5 10 8 14 4 1 15 14 4 15 1 2 16 13 3 7 9 6 12 7 9 12 6 2 16 3 13</p>
<p>S/259/C</p> <p>11 2 8 13 11 2 13 8 14 7 1 12 14 7 12 1 5 16 10 3 4 9 6 15 4 9 15 6 5 16 3 10</p>	<p>S/265/C</p> <p>12 13 6 3 12 13 3 6 7 2 9 16 7 2 16 9 1 8 15 10 14 11 5 4 14 11 4 5 1 8 10 15</p>	<p>S/271/C</p> <p>12 7 6 9 12 7 9 6 13 2 3 16 13 2 16 3 1 14 15 4 8 11 5 10 8 11 10 5 1 14 4 15</p>
<p>S/277/C</p> <p>12 6 7 9 12 6 9 7 13 3 2 16 13 3 16 2 1 15 14 4 8 10 5 11 8 10 11 5 1 15 4 14</p>	<p>S/283/C</p> <p>12 1 7 14 12 1 14 7 13 8 2 11 13 8 11 2 6 15 9 4 3 10 5 16 3 10 16 5 6 15 4 9</p>	<p>S/289/C</p> <p>13 12 3 6 13 12 6 3 8 1 10 15 8 1 15 10 2 7 16 9 11 14 4 5 11 14 5 4 2 7 9 16</p>
<p>S/295/C</p> <p>13 8 3 10 13 8 10 3 12 1 6 15 12 1 15 6 2 11 16 5 7 14 4 9 7 14 9 4 2 11 5 16</p>	<p>S/301/C</p> <p>13 3 8 10 13 3 10 8 12 6 1 15 12 6 15 1 2 16 11 5 7 9 4 14 7 9 14 4 2 16 5 11</p>	<p>S/307/C</p> <p>13 2 8 11 13 2 11 8 12 7 1 14 12 7 14 1 3 16 10 5 6 9 4 15 6 9 15 4 3 16 5 10</p>
<p>S/313/C</p> <p>14 11 4 5 14 11 5 4 7 2 9 16 7 2 16 9 1 8 15 10 12 13 3 6 12 13 6 3 1 8 10 15</p>	<p>S/319/C</p> <p>14 7 4 9 14 7 9 4 11 2 5 16 11 2 16 5 1 12 15 6 8 13 3 10 8 13 10 3 1 12 6 15</p>	<p>S/325/C</p> <p>14 4 7 9 14 4 9 7 11 5 2 16 11 5 16 2 1 15 12 6 8 10 3 13 8 10 13 3 1 15 6 12</p>
<p>S/331/C</p> <p>14 1 7 12 14 1 12 7 11 8 2 13 11 8 13 2 4 15 9 6 5 10 3 16 5 10 16 3 4 15 6 9</p>	<p>S/337/C</p> <p>15 10 4 5 15 10 5 4 6 3 9 16 6 3 16 9 1 8 14 11 12 13 2 7 12 13 7 2 1 8 11 14</p>	<p>S/343/C</p> <p>15 6 4 9 15 6 9 4 10 3 5 16 10 3 16 5 1 12 14 7 8 13 2 11 8 13 11 2 1 12 7 14</p>
<p>S/349/C</p> <p>15 4 6 9 15 4 9 6 10 5 3 16 10 5 16 3 1 14 12 7 8 11 2 13 8 11 13 2 1 14 7 12</p>	<p>S/355/C</p> <p>15 1 6 12 15 1 12 6 10 8 3 13 10 8 13 3 4 14 9 7 5 11 2 16 5 11 16 2 4 14 7 9</p>	<p>S/361/C</p> <p>16 9 3 6 16 9 6 3 5 4 10 15 5 4 15 10 2 7 13 12 11 14 1 8 11 14 8 1 2 7 12 13</p>
<p>S/367/C</p> <p>16 5 3 10 16 5 10 3 9 4 6 15 9 4 15 6 2 11 13 8 7 14 1 12 7 14 12 1 2 11 8 13</p>	<p>S/373/C</p> <p>16 3 5 10 16 3 10 5 9 6 4 15 9 6 15 4 2 13 11 8 7 12 1 14 7 12 14 1 2 13 8 11</p>	<p>S/379/C</p> <p>16 2 5 11 16 2 11 5 9 7 4 14 9 7 14 4 3 13 10 8 6 12 1 15 6 12 15 1 3 13 8 10</p>

[Count = 384] [OK!]

** Calculated and Listed by Kanji Setsuda on
Nov.29, 2014 with MacOSX 10.10.1 & Xcode 6.1 **

As you watch the list above, you might probably think both of the two types are quite similar to each other.

- (1) In the case of Self-Complementary type of MS44 each numerical value **1** has also its own fixed followers as many as 4 near around it. The members {**8**, **12**, **14** and **15**} are quite the same with the 'Complete' Type. The number **16**, Complementary Pair of **1**, is also the same, but its position is symmetrical to **1**.
- (2) The Old Good Fellows {**8**, **12**, **14** and **15**} sit on the same axes, either same row or same column where the **1** is situated on, nearest enough from it.
- (3) How many ways of sitting order do they have around **1**, when $n1=1$? You would know 24 ways, if you count them carefully in the following list.

* Positions of Five Fellow Numbers for the Self-Complementary Type:
 Master **1**, **Four Fixed Followers** and **[Symmetric Pair]** *

1#				2#				3#				4#				5#				6#			
1	15	12	0	1	15	8	0	1	15	14	0	1	15	8	0	1	15	14	0	1	15	12	0
14	0	0	0	14	0	0	0	12	0	0	0	12	0	0	0	8	0	0	0	8	0	0	0
8	0	0	0	12	0	0	0	8	0	0	0	14	0	0	0	12	0	0	0	14	0	0	0
0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16

7#				8#				9#				10#				11#				12#			
1	14	12	0	1	14	8	0	1	14	15	0	1	14	8	0	1	14	15	0	1	14	12	0
15	0	0	0	15	0	0	0	12	0	0	0	12	0	0	0	8	0	0	0	8	0	0	0
8	0	0	0	12	0	0	0	8	0	0	0	15	0	0	0	12	0	0	0	15	0	0	0
0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16

13#				14#				15#				16#				17#				18#			
1	12	14	0	1	12	8	0	1	12	15	0	1	12	8	0	1	12	15	0	1	12	14	0
15	0	0	0	15	0	0	0	14	0	0	0	14	0	0	0	8	0	0	0	8	0	0	0
8	0	0	0	14	0	0	0	8	0	0	0	15	0	0	0	14	0	0	0	15	0	0	0
0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16

19#				20#				21#				22#				23#				24#			
1	8	14	0	1	8	12	0	1	8	15	0	1	8	12	0	1	8	15	0	1	8	14	0
15	0	0	0	15	0	0	0	14	0	0	0	14	0	0	0	12	0	0	0	12	0	0	0
12	0	0	0	14	0	0	0	12	0	0	0	15	0	0	0	14	0	0	0	15	0	0	0
0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16	0	0	0	16

25#				49#				73#				97#				121#				145#			
0	16	0	0	0	16	0	0	0	15	0	0	0	16	0	0	0	15	0	0	0	14	0	0
0	0	8	0	0	0	8	0	14	1	0	12	0	0	8	0	12	1	0	14	12	1	0	15
0	0	14	0	0	0	15	0	0	0	16	0	0	0	15	0	0	0	16	0	0	0	16	0
12	0	1	15	12	0	1	14	0	8	0	0	14	0	1	12	0	8	0	0	0	8	0	0

169#				193#				217#				241#				265#				289#			
8	0	0	0	0	16	0	0	0	15	0	0	0	14	0	0	12	0	0	0	0	12	0	0
0	0	0	16	0	0	12	0	8	1	0	14	8	1	0	15	0	0	0	16	8	1	0	15
1	12	15	0	0	0	15	0	0	0	16	0	0	0	16	0	1	8	15	0	0	0	16	0
14	0	0	0	14	0	1	8	0	12	0	0	0	12	0	0	14	0	0	0	0	14	0	0

313#				337#				361#			
14	0	0	0	15	0	0	0	16	0	0	0
0	0	0	16	0	0	0	16	0	0	0	15
1	8	15	0	1	8	14	0	0	0	0	12
12	0	0	0	12	0	0	0	0	14	8	1

[Count = 384]

* Solution Counts according to the Positions of Value 1 [Position: Counts] *

1: 24, 2: 24, 3: 24, 4: 24, 5: 24, 6: 24, 7: 24, 8: 24,
 9: 24, 10: 24, 11: 24, 12: 24, 13: 24, 14: 24, 15: 24, 16: 24;

[OK!]

** Calculated and Listed by Kanji Setsuda on

Dec. 2, 2014 with MacOSX 10.10.1 & Xcode 6.1 **

It is the problem of permutation: $4P_4 = 4 \times 3 \times 2 \times 1 = 24$;

- (4) The Monitor List just above tells us on the whole that number **1** can go anywhere equally with its 5 fixed fellow numbers, starting from Origin n_1 to every position $n_2, n_3, n_4, n_5, \dots, n_{15}$ and n_{16} .

Therefore, you can calculate the total count of primitive solutions of our object as $24 \times 16 = 384$;

How interesting it is! Don't you think so?

Is it only an accidental coincidence? No. It is necessary to be true, I believe.

But what makes it possible?

We have to find any reason why they are made so.

#2. In the case of 'Self-Complementary' type of MS44, can we also find anything like that finest structure characterized by the multiple sets of 'Complimentary Pairs'?

And do the Natural Numbers 1~16 give them the same kind of real Complementary Pairs again with those for the 'Complete' type?

We must find them by making our elaborate algebraic calculations here again.

**** Self-Complementary Magic Squares of Order 4 ****

[Basic Form and a Model Solution]

0#	1#
n1 n2 n3 n4	1 15 12 6
n5 n6 n7 n8	14 4 7 9
n9 10 11 12	8 10 13 3
13 14 15 16	11 5 2 16

**** Basic Conditions: C=34; | Self-Complementary Conditions: CC=17; ****

n1+n2+n3+n4=C ...rw1;	n1+n16=CC ...Cmp1;
n5+n6+n7+n8=C ...rw2;	n2+n15=CC ...Cmp2;
n9+n10+n11+n12=C ...rw3;	n3+n14=CC ...Cmp3;
n13+n14+n15+n16=C ...rw4;	n4+n13=CC ...Cmp4;
n1+n5+n9+n13=C ...c11;	n5+n12=CC ...Cmp5;
n2+n6+n10+n14=C ...c12;	n6+n11=CC ...Cmp6;
n3+n7+n11+n15=C ...c13;	n7+n10=CC ...Cmp7;
n4+n8+n12+n16=C ...c14;	n8+n9=CC ...Cmp8;

**** Algebraic Calculations: ****

(1)

Cmp1+Cmp2

(n1+n16)+(n2+n15)=(n1+n2)+(n15+n16)=2CC=C;

But, n1+n2+n3+n4=C ...rw1;

Therefore, n3+n4=n15+n16=c1;

Cmp3+Cmp4

(n3+n14)+(n4+n13)=(n3+n4)+(n13+n14)=2CC=C;

But, n1+n2+n3+n4=C ...rw1;

Therefore, n1+n2=n13+n14=c2;

Cmp5+Cmp6

(n5+n12)+(n6+n11)=(n5+n6)+(n11+n12)=2CC=C;

But, n5+n6+n7+n8=C ...rw1;

Therefore, n7+n8=n11+n12=c3;

Cmp7+Cmp8

(n7+n10)+(n8+n9)=(n7+n8)+(n9+n10)=2CC=C;

But, n5+n6+n7+n8=C ...rw2;

Therefore, n5+n6=n9+n10=c4;

But, n9+n10+n11+n12=C ...rw3;

It means n7+n8=n11+n12=c5=c3;

c13+c14

n3+n7+n11+n15=C

+)n4+n8+n12+n16=C

n3+n4+n7+n8+n11+n12+n15+n16=2C

But, n3+n4=n15+n16=c1; n7+n8=n11+n12=c3;

Therefore, 2(n3+n4)+2(n7+n8)=2C

It means n3+n4+n7+n8=C;

But, n1+n2+n3+n4=C ...rw1;

Therefore, n1+n2=n7+n8=c6;

Summary 1:

n1+n2=n7+n8=n11+n12=n13+n14=c6=c2=c3=cp1;

And then

n3+n4=C-(n1+n2)=C-cp1; n5+n6=C-(n7+n8)=C-cp1;

n9+n10=C-(n11+n12)=C-cp1; n15+n16=C-(n13+n14)=C-cp1;

Therefore, $n_3+n_4=n_5+n_6=n_9+n_{10}=n_{15}+n_{16}=cp_2$;
 And $cp_1+cp_2=C$;

(2)

Cmp1+Cmp3

$$(n_1+n_{16})+(n_3+n_{14})=(n_1+n_3)+(n_{14}+n_{16})=2CC=C;$$

But, $n_1+n_2+n_3+n_4=C \dots rw1$;

Therefore, $n_2+n_4=n_{14}+n_{16}=c_7$;

But, $n_1+n_2+n_3+n_4=C \dots rw1$; $n_{13}+n_{14}+n_{15}+n_{16}=C \dots rw4$;

It means $C-(n_2+n_4)=C-(n_{14}+n_{16})=C-c_7$;

Therefore, $n_1+n_3=n_{13}+n_{15}=c_8$;

And $c_7+c_8=C$;

Cmp6+Cmp8

$$(n_6+n_{11})+(n_8+n_9)=(n_6+n_8)+(n_9+n_{11})=2CC=C;$$

But, $n_5+n_6+n_7+n_8=C \dots rw2$;

Therefore, $n_5+n_7=n_9+n_{11}=c_9$;

But, $n_5+n_6+n_7+n_8=C \dots rw2$; $n_9+n_{10}+n_{11}+n_{12}=C \dots rw3$;

It means $C-(n_5+n_7)=C-(n_9+n_{11})=C-c_9$;

Therefore, $n_6+n_8=n_{10}+n_{12}=c_{10}$;

And $c_9+c_{10}=C$;

c12+c13

$$n_2+n_6+n_{10}+n_{14}=C \dots c12$$

$$+n_4+n_8+n_{12}+n_{16}=C \dots c13$$

$$n_2+n_4+n_6+n_8+n_{10}+n_{12}+n_{14}+n_{16}=2C$$

But, $n_2+n_4=n_{14}+n_{16}=c_7$; $n_6+n_8=n_{10}+n_{12}=c_{10}$;

Therefore, $2(n_2+n_4)+2(n_6+n_8)=2C$

It means $n_2+n_4+n_6+n_8=C$;

But, $n_1+n_2+n_3+n_4=C \dots rw1$;

Therefore, $n_1+n_3=n_6+n_8=c_{11}$;

Summary 2:

$$n_1+n_3=n_6+n_8=n_{10}+n_{12}=n_{13}+n_{15}=c_{11}=c_{10}=c_8=cp_3$$
;

And then

$$n_2+n_4=C-(n_1+n_3)=C-cp_3$$
; $n_5+n_7=C-(n_6+n_8)=C-cp_3$;

$$n_9+n_{11}=C-(n_{10}+n_{12})=C-cp_3$$
; $n_{14}+n_{16}=C-(n_{13}+n_{15})=C-cp_3$;

Therefore, $n_2+n_4=n_5+n_7=n_9+n_{11}=n_{14}+n_{16}=cp_4$;

And $cp_3+cp_4=C$;

The rest 2 sets of Complementary Pairs are available by making the Basic Forms of the former 2 sets reflect with respect to the Primary Diagonal $\{n_1, n_6, n_{11}, n_{16}\}$.

1P/	2P/	3P/	4P/	5P/
1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
5 6 7 8	5 6 7 8	5 6 7 8	5 6 7 8	5 6 7 8
9 10 11 12	9 10 11 12	9 10 11 12	9 10 11 12	9 10 11 12
13 14 15 16	13 14 15 16	13 14 15 16	13 14 15 16	13 14 15 16

Thus you can easily manage to list out the rest sets.

(3) $n_1+n_5=n_4+n_8=n_{10}+n_{14}=n_{11}+n_{15}=cp_5$;
 $n_2+n_6=n_3+n_7=n_9+n_{13}=n_{12}+n_{16}=cp_6$; and $cp_5+cp_6=C$;

(4) $n_1+n_9=n_4+n_{12}=n_6+n_{14}=n_7+n_{15}=cp_7$;
 $n_2+n_{10}=n_3+n_{11}=n_5+n_{13}=n_8+n_{16}=cp_8$; and $cp_7+cp_8=C$;

** List of All the 5 Sets of 2x4 Complementary Pairs (C=34; CC=17;) **

(1) $n_1+n_2=n_7+n_8=n_{11}+n_{12}=n_{13}+n_{14}=cp_1$;
 $n_3+n_4=n_5+n_6=n_9+n_{10}=n_{15}+n_{16}=cp_2$; and $cp_1+cp_2=C$;

(2) $n_1+n_3=n_6+n_8=n_{10}+n_{12}=n_{13}+n_{15}=cp_3$;
 $n_2+n_4=n_5+n_7=n_9+n_{11}=n_{14}+n_{16}=cp_4$; and $cp_3+cp_4=C$;

(3) $n_1+n_5=n_4+n_8=n_{10}+n_{14}=n_{11}+n_{15}=cp_5$;
 $n_2+n_6=n_3+n_7=n_9+n_{13}=n_{12}+n_{16}=cp_6$; and $cp_5+cp_6=C$;

- (4) $n1+n9=n4+n12=n6+n14=n7+n15=cp7$;
 $n2+n10=n3+n11=n5+n13=n8+n16=cp8$; and $cp7+cp8=C$;
- (5) $n1+n16=n2+n15=n3+n14=n4+n13=$
 $n5+n12=n6+n11=n7+n10=n8+n9=CC=cp9$;

1P/	2P/	3P/	4P/	5P/
1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3 4
5 6 7 8	5 6 7 8	5 6 7 8	5 6 7 8	5 6 7 8
9 10 11 12	9 10 11 12	9 10 11 12	9 10 11 12	9 10 11 12
13 14 15 16	13 14 15 16	13 14 15 16	13 14 15 16	13 14 15 16

[OK!]

* Calculated and Listed by Kanji Setsuda on
 Dec. 2, 2014 with MacOSX 10.10.1 & Xcode 6.1 **

We could also find the 4 groups of Complementary Pairs: 2 sets for each group: 4 pairs for each set. Each set has its own unique constant sum, while two constant sums in each group must add up to the magic constant $C(=34)$.

The last 5-th group listed above contains the 8 pairs whose sum is equal to 17. Yes. They come from the 'Self-Complementary Conditions' listed at the definition stage.

And we know the positions where all the pairs should be placed as shown above.

We need next to have all these 5 groups of Complementary Pairs with real values and have to make them appropriately located in any piece of Self-Complementary MS44 according to the five diagrams above. We must neither use any number twice or more often in any group of complementary pairs, nor un-use any number there.

We don't yet know the real sums of $cp1$, $cp2$, $cp3$, $cp4$, ..., $cp7$ and $cp8$ at the definition stage, but only know about the constant sum(=17) of the fifth group.

#3. Let me omit my explanations here about how the Natural Numbers 1~16 give all the 5 groups of real Complementary Pairs. It is because the process itself is just the same with the case of 'Complete' type in the last section. Read the Section 4, please.

Natural Numbers 1~16 themselves do never mind the difference between the MS types. They always mind only the sums of pairs, nothing else.

Right here let me copy only the conclusion of the last study as follows.

**** 5 Groups of 16 Essential Complementary Pairs ****

[1]: [$S1=09$, $S2=25$]

(1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (7, 2), (8, 1),
 (9, 16), (10, 15), (11, 14), (12, 13), (13, 12), (14, 11), (15, 10), (16, 9);

[2]: [$S1=13$, $S2=21$]

(1, 12), (2, 11), (3, 10), (4, 9), (5, 16), (6, 15), (7, 14), (8, 13),
 (9, 4), (10, 3), (11, 2), (12, 1), (13, 8), (14, 7), (15, 6), (16, 5);

[3]: [$S1=15$, $S2=19$]

(1, 14), (2, 13), (3, 16), (4, 15), (5, 10), (6, 9), (7, 12), (8, 11),
 (9, 6), (10, 5), (11, 8), (12, 7), (13, 2), (14, 1), (15, 4), (16, 3);

[4]: [$S1=16$, $S2=18$]

(1, 15), (2, 16), (3, 13), (4, 14), (5, 11), (6, 12), (7, 9), (8, 10),
 (9, 7), (10, 8), (11, 5), (12, 6), (13, 3), (14, 4), (15, 1), (16, 2);

[5]: [$S1=17$, $S2=17$]

(1, 16), (2, 15), (3, 14), (4, 13), (5, 12), (6, 11), (7, 10), (8, 9),
 (9, 8), (10, 7), (11, 6), (12, 5), (13, 4), (14, 3), (15, 2), (16, 1);

* Re-arranged List of Essential Complementary Pairs led by their Masters *

5 Groups: (S09, S25) (S13, S21) (S15, S19) (S16, S18) (S17=S17)
 (1, X) = {(1, 8), (1, 12), (1, 14), (1, 15), (1, 16)};
 (2, X) = {(2, 7), (2, 11), (2, 13), (2, 16), (2, 15)};
 (3, X) = {(3, 6), (3, 10), (3, 16), (3, 13), (3, 14)};
 (4, X) = {(4, 5), (4, 9), (4, 15), (4, 14), (4, 13)};
 (5, X) = {(5, 4), (5, 16), (5, 10), (5, 11), (5, 12)};
 (6, X) = {(6, 3), (6, 15), (6, 9), (6, 12), (6, 11)};
 (7, X) = {(7, 2), (7, 14), (7, 12), (7, 9), (7, 10)};
 (8, X) = {(8, 1), (8, 13), (8, 11), (8, 10), (8, 9)};
 (9, X) = {(9, 16), (9, 4), (9, 6), (9, 7), (9, 8)};
 (10, X) = {(10, 15), (10, 3), (10, 5), (10, 8), (10, 7)};
 (11, X) = {(11, 14), (11, 2), (11, 8), (11, 5), (11, 6)};
 (12, X) = {(12, 13), (12, 1), (12, 7), (12, 6), (12, 5)};
 (13, X) = {(13, 12), (13, 8), (13, 2), (13, 3), (13, 4)};
 (14, X) = {(14, 11), (14, 7), (14, 1), (14, 4), (14, 3)};
 (15, X) = {(15, 10), (15, 6), (15, 4), (15, 1), (15, 2)};
 (16, X) = {(16, 9), (16, 5), (16, 3), (16, 2), (16, 1)};

** Calculated and Listed by Kanji Setsuda on
 Dec. 9, 2014 with MacOSX 10.10.1 & Xcode 6.1.1 **

As you see above, Natural Numbers 1~16 really specify what are the fixed followers around the number 1. Self-complementary MS44 also obey this actual instruction.

#4. What about the other numbers? Do all 2, 3, 4, 5, ... , 15 and 16 also accept this instruction given by the natural numbers equally? Watch the next abstract list.

* Contents of Five Fellow Numbers:

Master 2, Four Fixed Followers and [Symmetric Pair: 15] *

1#		25#		26#		27#		28#		29#	
0	15	0	0	2	16	11	0	2	16	7	0
0	0	7	0	13	0	0	0	13	0	0	0
0	0	13	0	7	0	0	0	11	0	0	0
11	0	2	16	0	0	0	15	0	0	0	15
30#		31#		32#		33#		34#		35#	
2	16	11	0	2	13	11	0	2	13	7	0
7	0	0	0	16	0	0	0	11	0	0	0
13	0	0	0	7	0	0	0	16	0	0	0
0	0	0	15	0	0	0	15	0	0	0	15
36#		37#		38#		39#		40#		41#	
2	13	11	0	2	11	13	0	2	11	7	0
7	0	0	0	16	0	0	0	13	0	0	0
16	0	0	0	7	0	0	0	16	0	0	0
0	0	0	15	0	0	0	15	0	0	0	15
42#		43#		44#		45#		46#		47#	
2	11	13	0	2	7	13	0	2	7	11	0
7	0	0	0	16	0	0	0	13	0	0	0
16	0	0	0	11	0	0	0	16	0	0	0
0	0	0	15	0	0	0	15	0	0	0	15
48#		49#		73#		97#		121#		145#	
2	7	13	0	0	15	0	0	0	15	0	0
11	0	0	0	13	2	0	11	0	0	7	0
16	0	0	0	0	0	16	0	0	0	16	0
0	0	0	15	0	0	11	0	2	13	0	0

169#	193#	217#	241#	265#	289#
0 13 0 0	0 16 0 0	0 15 0 0	11 0 0 0	0 13 0 0	13 0 0 0
11 2 0 16	7 2 0 13	0 0 11 0	0 0 0 15	7 2 0 16	0 0 0 15
0 0 15 0	0 0 15 0	0 0 16 0	2 7 16 0	0 0 15 0	2 7 16 0
0 7 0 0	0 11 0 0	13 0 2 7	13 0 0 0	0 11 0 0	11 0 0 0
313#	337#	361#			
0 11 0 0	15 0 0 0	16 0 0 0			
7 2 0 16	0 0 0 16	0 0 0 15			
0 0 15 0	0 0 0 11	2 7 13 0			
0 13 0 0	0 13 7 2	11 0 0 0			

[Count = 384]

* Contents of Five Fellow Numbers:

Master 3, Four Fixed Followers and [Symmetric Pair:14] *

1#	25#	49#	73#	97#	121#
0 0 0 6	0 16 0 0	3 16 10 0	0 0 0 6	0 16 10 3	6 0 0 0
14 0 0 0	13 3 0 10	13 0 0 0	14 0 0 0	0 0 0 13	0 0 0 14
0 10 13 3	0 0 14 0	6 0 0 0	0 10 16 3	0 0 0 6	3 10 16 0
0 0 0 16	0 6 0 0	0 0 0 14	0 0 0 13	14 0 0 0	13 0 0 0
145#	169#	193#	217#	241#	265#
0 14 0 0	0 13 10 3	0 16 6 3	10 0 0 0	0 14 0 0	0 13 6 3
0 0 6 0	0 0 0 16	0 0 0 13	0 0 0 14	0 0 10 0	0 0 0 16
0 0 16 0	0 0 0 6	0 0 0 10	3 6 16 0	0 0 16 0	0 0 0 10
13 0 3 10	14 0 0 0	14 0 0 0	13 0 0 0	13 0 3 6	14 0 0 0
289#	313#	337#	361#		
13 0 3 6	14 0 0 0	0 10 0 0	16 0 3 6		
0 0 10 0	0 0 0 16	6 3 0 16	0 0 10 0		
0 0 16 0	0 0 0 10	0 0 14 0	0 0 13 0		
0 14 0 0	0 13 6 3	0 13 0 0	0 14 0 0		

[Count = 384]

* Contents of Five Fellow Numbers:

Master 4, Four Fixed Followers and [Symmetric Pair:13] *

1#	25#	49#	73#	97#	121#
0 15 0 0	0 0 0 5	0 0 0 5	4 15 9 0	5 0 0 0	0 15 9 4
14 4 0 9	13 0 0 0	13 0 0 0	14 0 0 0	0 0 0 13	0 0 0 14
0 0 13 0	0 9 14 4	0 9 15 4	5 0 0 0	4 9 15 0	0 0 0 5
0 5 0 0	0 0 0 15	0 0 0 14	0 0 0 13	14 0 0 0	13 0 0 0
145#	169#	193#	217#	241#	265#
0 14 9 4	0 13 0 0	9 0 0 0	0 15 5 4	0 14 5 4	0 13 0 0
0 0 0 15	0 0 5 0	0 0 0 13	0 0 0 14	0 0 0 15	0 0 9 0
0 0 0 5	0 0 15 0	4 5 15 0	0 0 0 9	0 0 0 9	0 0 15 0
13 0 0 0	14 0 4 9	14 0 0 0	13 0 0 0	13 0 0 0	14 0 4 5
289#	313#	337#	361#		
13 0 0 0	14 0 4 5	15 0 4 5	0 9 0 0		
0 0 0 15	0 0 9 0	0 0 9 0	5 4 0 15		
0 0 0 9	0 0 15 0	0 0 14 0	0 0 13 0		
0 14 5 4	0 13 0 0	0 13 0 0	0 14 0 0		

[Count = 384]

* Contents of Five Fellow Numbers:

Master 5, Four Fixed Followers and [Symmetric Pair:12] *

1#	25#	49#	73#	97#	121#
0 0 12 0	0 16 11 5	0 16 10 5	4 0 0 0	5 16 10 0	0 0 0 4
0 4 0 0	0 0 0 10	0 0 0 11	0 0 0 12	11 0 0 0	12 0 0 0
0 10 0 0	0 0 0 4	0 0 0 4	5 10 16 0	4 0 0 0	0 10 16 5
11 5 0 16	12 0 0 0	12 0 0 0	11 0 0 0	0 0 0 12	0 0 0 11
145#	169#	193#	217#	241#	265#
0 0 0 4	0 0 10 0	0 16 0 0	10 0 5 4	11 0 5 4	12 0 0 0
12 0 0 0	11 0 5 16	0 0 12 0	0 0 11 0	0 0 10 0	0 0 0 16
0 11 16 5	0 12 0 0	4 5 0 10	0 0 16 0	0 0 16 0	0 0 0 10
0 0 0 10	0 0 4 0	0 11 0 0	0 12 0 0	0 12 0 0	0 11 4 5

289#	313#	337#	361#	
0 12 0 0	0 11 4 5	0 10 4 5	16 0 0 0	
0 0 10 0	0 0 0 16	0 0 0 16	5 4 10 0	
0 0 16 0	0 0 0 10	0 0 0 11	0 0 0 12	
11 0 5 4	12 0 0 0	12 0 0 0	11 0 0 0	[Count = 384]

* Contents of Five Fellow Numbers:

Master 9, Four Fixed Followers and [Symmetric Pair: 8] *

193#	194#	195#	196#	197#	198#
9 16 6 0	9 16 4 0	9 16 7 0	9 16 4 0	9 16 7 0	9 16 6 0
7 0 0 0	7 0 0 0	6 0 0 0	6 0 0 0	4 0 0 0	4 0 0 0
4 0 0 0	6 0 0 0	4 0 0 0	7 0 0 0	6 0 0 0	7 0 0 0
0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8
199#	200#	201#	202#	203#	204#
9 7 6 0	9 7 4 0	9 7 16 0	9 7 4 0	9 7 16 0	9 7 6 0
16 0 0 0	16 0 0 0	6 0 0 0	6 0 0 0	4 0 0 0	4 0 0 0
4 0 0 0	6 0 0 0	4 0 0 0	16 0 0 0	6 0 0 0	16 0 0 0
0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8
205#	206#	207#	208#	209#	210#
9 6 7 0	9 6 4 0	9 6 16 0	9 6 4 0	9 6 16 0	9 6 7 0
16 0 0 0	16 0 0 0	7 0 0 0	7 0 0 0	4 0 0 0	4 0 0 0
4 0 0 0	7 0 0 0	4 0 0 0	16 0 0 0	7 0 0 0	16 0 0 0
0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8
211#	212#	213#	214#	215#	216#
9 4 7 0	9 4 6 0	9 4 16 0	9 4 6 0	9 4 16 0	9 4 7 0
16 0 0 0	16 0 0 0	7 0 0 0	7 0 0 0	6 0 0 0	6 0 0 0
6 0 0 0	7 0 0 0	6 0 0 0	16 0 0 0	7 0 0 0	16 0 0 0
0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8	0 0 0 8

You can imagine what the total list should be like, can't you?

Every numbers really respect and accept the Natural Number's Specification equally and simultaneously. Each of them tells us the total count of primitive solutions is:

$$4P_4 \times 16 = 24 \times 16 = 384;$$

#5. Let me prove $24 \times 16 = 384$ in another way here, making some experiment.

Let's reconstruct all the 384 solutions by some special Transformation Method making from the basic data 24 solutions when $n_1=1$.

What does that transformation method look like, then?

Let me show you what is in my mind now by the conceptual diagrams as follows.

** Basic Form of SCMS44(0#)
and Concepts of 15 Transformations **

0#	1#	2#	3#
1 2 3 4	2 1 4 3	3 4 1 2	4 3 2 1
5 6 7 8	6 5 8 7	7 8 5 6	8 7 6 5
9 10 11 12	10 9 12 11	11 12 9 10	12 11 10 9
13 14 15 16	14 13 16 15	15 16 13 14	16 15 14 13
4#	5#	6#	7#
5 6 7 8	6 5 8 7	7 8 5 6	8 7 6 5
1 2 3 4	2 1 4 3	3 4 1 2	4 3 2 1
13 14 15 16	14 13 16 15	15 16 13 14	16 15 14 13
9 10 11 12	10 9 12 11	11 12 9 10	12 11 10 9

8#	9#	10#	11#
9 10 11 12 13 14 15 16 1 2 3 4 5 6 7 8	10 9 12 11 14 13 16 15 2 1 4 3 6 5 8 7	11 12 9 10 15 16 13 14 3 4 1 2 7 8 5 6	12 11 10 9 16 15 14 13 4 3 2 1 8 7 6 5
12#	13#	14#	15#
13 14 15 16 9 10 11 12 5 6 7 8 1 2 3 4	14 13 16 15 10 9 12 11 6 5 8 7 2 1 4 3	15 16 13 14 11 12 9 10 7 8 5 6 3 4 1 2	16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

I have wanted to move the number 1 from the original position n1 to n2, n3, n4, n5, ..., n15, n16 in a consecutive order regularly along with its fixed 5 'followers'.

This method is one of my original inventions I developed.

Let's adopt it for our transformation method now and reconstruct all the 384 solutions from the 24 basic data when n1=1, shall we?

All the results of my reconstruction are demonstrated as follows.

*** Reconstruction of SCMS44 by our Original Transformation Method ***
 ** List of 24 Solutions when n1=1 and the 15 Transformations of each **

Group 1:

1#	360#	280#	129#	330#	79#
1 15 12 6 14 4 7 9 8 10 13 3 11 5 2 16	15 1 6 12 4 14 9 7 10 8 3 13 5 11 16 2	12 6 1 15 7 9 14 4 13 3 8 10 2 16 11 5	6 12 15 1 9 7 4 14 3 13 10 8 16 2 5 11	14 4 7 9 1 15 12 6 11 5 2 16 8 10 13 3	4 14 9 7 15 1 6 12 5 11 16 2 10 8 3 13
159#	202#	183#	226#	306#	55#
7 9 14 4 12 6 1 15 2 16 11 5 13 3 8 10	9 7 4 14 6 12 15 1 16 2 5 11 3 13 10 8	8 10 13 3 11 5 2 16 1 15 12 6 14 4 7 9	10 8 3 13 5 11 16 2 15 1 6 12 4 14 9 7	13 3 8 10 2 16 11 5 12 6 1 15 7 9 14 4	3 13 10 8 16 2 5 11 6 12 15 1 9 7 4 14
256#	105#	25#	384#		
11 5 2 16 8 10 13 3 14 4 7 9 1 15 12 6	5 11 16 2 10 8 3 13 4 14 9 7 15 1 6 12	2 16 11 5 13 3 8 10 7 9 14 4 12 6 1 15	16 2 5 11 3 13 10 8 9 7 4 14 6 12 15 1		

Group 2:

2#	359#	184#	225#	329#	80#
1 15 8 10 14 4 11 5 12 6 13 3 7 9 2 16	15 1 10 8 4 14 5 11 6 12 3 13 9 7 16 2	8 10 1 15 11 5 14 4 13 3 12 6 2 16 7 9	10 8 15 1 5 11 4 14 3 13 6 12 16 2 9 7	14 4 11 5 1 15 8 10 7 9 2 16 12 6 13 3	4 14 5 11 15 1 10 8 9 7 16 2 6 12 3 13

255#	106#	279#	130#	305#	56#
11 5 14 4 8 10 1 15 2 16 7 9 13 3 12 6	5 11 4 14 10 8 15 1 16 2 9 7 3 13 6 12	12 6 13 3 7 9 2 16 1 15 8 10 14 4 11 5	6 12 3 13 9 7 16 2 15 1 10 8 4 14 5 11	13 3 12 6 2 16 7 9 8 10 1 15 11 5 14 4	3 13 6 12 16 2 9 7 10 8 15 1 5 11 4 14
160#	201#	26#	383#		
7 9 2 16 12 6 13 3 14 4 11 5 1 15 8 10	9 7 16 2 6 12 3 13 4 14 5 11 15 1 10 8	2 16 7 9 13 3 12 6 11 5 14 4 8 10 1 15	16 2 9 7 3 13 6 12 5 11 4 14 10 8 15 1		

Group 3:

3#	358#	328#	81#	282#	127#
1 15 14 4 12 6 7 9 8 10 11 5 13 3 2 16	15 1 4 14 6 12 9 7 10 8 5 11 3 13 16 2	14 4 1 15 7 9 12 6 11 5 8 10 2 16 13 3	4 14 15 1 9 7 6 12 5 11 10 8 16 2 3 13	12 6 7 9 1 15 14 4 13 3 2 16 8 10 11 5	6 12 9 7 15 1 4 14 3 13 16 2 10 8 5 11
157#	204#	181#	228#	258#	103#
7 9 12 6 14 4 1 15 2 16 13 3 11 5 8 10	9 7 6 12 4 14 15 1 16 2 3 13 5 11 10 8	8 10 11 5 13 3 2 16 1 15 14 4 12 6 7 9	10 8 5 11 3 13 16 2 15 1 4 14 6 12 9 7	11 5 8 10 2 16 13 3 14 4 1 15 7 9 12 6	5 11 10 8 16 2 3 13 4 14 15 1 9 7 6 12
304#	57#	27#	382#		
13 3 2 16 8 10 11 5 12 6 7 9 1 15 14 4	3 13 16 2 10 8 5 11 6 12 9 7 15 1 4 14	2 16 13 3 11 5 8 10 7 9 12 6 14 4 1 15	16 2 3 13 5 11 10 8 9 7 6 12 4 14 15 1		

Group 4:

4#	357#	182#	227#	281#	128#
1 15 8 10 12 6 13 3 14 4 11 5 7 9 2 16	15 1 10 8 6 12 3 13 4 14 5 11 9 7 16 2	8 10 1 15 13 3 12 6 11 5 14 4 2 16 7 9	10 8 15 1 3 13 6 12 5 11 4 14 16 2 9 7	12 6 13 3 1 15 8 10 7 9 2 16 14 4 11 5	6 12 3 13 15 1 10 8 9 7 16 2 4 14 5 11
303#	58#	327#	82#	257#	104#
13 3 12 6 8 10 1 15 2 16 7 9 11 5 14 4	3 13 6 12 10 8 15 1 16 2 9 7 5 11 4 14	14 4 11 5 7 9 2 16 1 15 8 10 12 6 13 3	4 14 5 11 9 7 16 2 15 1 10 8 6 12 3 13	11 5 14 4 2 16 7 9 8 10 1 15 13 3 12 6	5 11 4 14 16 2 9 7 10 8 15 1 3 13 6 12
158#	203#	28#	381#		
7 9 2 16 14 4 11 5 12 6 13 3 1 15 8 10	9 7 16 2 4 14 5 11 6 12 3 13 15 1 10 8	2 16 7 9 11 5 14 4 13 3 12 6 8 10 1 15	16 2 9 7 5 11 4 14 3 13 6 12 10 8 15 1		

Group 5:

5#	356#	326#	83#	186#	223#	253#	108#
1 15 14 4	15 1 4 14	14 4 1 15	4 14 15 1	8 10 11 5	10 8 5 11	11 5 8 10	5 11 10 8
8 10 11 5	10 8 5 11	11 5 8 10	5 11 10 8	1 15 14 4	15 1 4 14	14 4 1 15	4 14 15 1
12 6 7 9	6 12 9 7	7 9 12 6	9 7 6 12	13 3 2 16	3 13 16 2	2 16 13 3	16 2 3 13
13 3 2 16	3 13 16 2	2 16 13 3	16 2 3 13	12 6 7 9	6 12 9 7	7 9 12 6	9 7 6 12
277#	132#	162#	199#	302#	59#	29#	380#
12 6 7 9	6 12 9 7	7 9 12 6	9 7 6 12	13 3 2 16	3 13 16 2	2 16 13 3	16 2 3 13
13 3 2 16	3 13 16 2	2 16 13 3	16 2 3 13	12 6 7 9	6 12 9 7	7 9 12 6	9 7 6 12
1 15 14 4	15 1 4 14	14 4 1 15	4 14 15 1	8 10 11 5	10 8 5 11	11 5 8 10	5 11 10 8
8 10 11 5	10 8 5 11	11 5 8 10	5 11 10 8	1 15 14 4	15 1 4 14	14 4 1 15	4 14 15 1

Group 6:

6#	355#	278#	131#	185#	224#	301#	60#
1 15 12 6	15 1 6 12	12 6 1 15	6 12 15 1	8 10 13 3	10 8 3 13	13 3 8 10	3 13 10 8
8 10 13 3	10 8 3 13	13 3 8 10	3 13 10 8	1 15 12 6	15 1 6 12	12 6 1 15	6 12 15 1
14 4 7 9	4 14 9 7	7 9 14 4	9 7 4 14	11 5 2 16	5 11 16 2	2 16 11 5	16 2 5 11
11 5 2 16	5 11 16 2	2 16 11 5	16 2 5 11	14 4 7 9	4 14 9 7	7 9 14 4	9 7 4 14
325#	84#	161#	200#	254#	107#	30#	379#
14 4 7 9	4 14 9 7	7 9 14 4	9 7 4 14	11 5 2 16	5 11 16 2	2 16 11 5	16 2 5 11
11 5 2 16	5 11 16 2	2 16 11 5	16 2 5 11	14 4 7 9	4 14 9 7	7 9 14 4	9 7 4 14
1 15 12 6	15 1 6 12	12 6 1 15	6 12 15 1	8 10 13 3	10 8 3 13	13 3 8 10	3 13 10 8
8 10 13 3	10 8 3 13	13 3 8 10	3 13 10 8	1 15 12 6	15 1 6 12	12 6 1 15	6 12 15 1

Group 7:

7#	336#	274#	153#	354#	73#	135#	208#
1 14 12 7	14 1 7 12	12 7 1 14	7 12 14 1	15 4 6 9	4 15 9 6	6 9 15 4	9 6 4 15
15 4 6 9	4 15 9 6	6 9 15 4	9 6 4 15	1 14 12 7	14 1 7 12	12 7 1 14	7 12 14 1
8 11 13 2	11 8 2 13	13 2 8 11	2 13 11 8	10 5 3 16	5 10 16 3	3 16 10 5	16 3 5 10
10 5 3 16	5 10 16 3	3 16 10 5	16 3 5 10	8 11 13 2	11 8 2 13	13 2 8 11	2 13 11 8
177#	250#	312#	31#	232#	111#	49#	378#
8 11 13 2	11 8 2 13	13 2 8 11	2 13 11 8	10 5 3 16	5 10 16 3	3 16 10 5	16 3 5 10
10 5 3 16	5 10 16 3	3 16 10 5	16 3 5 10	8 11 13 2	11 8 2 13	13 2 8 11	2 13 11 8
1 14 12 7	14 1 7 12	12 7 1 14	7 12 14 1	15 4 6 9	4 15 9 6	6 9 15 4	9 6 4 15
15 4 6 9	4 15 9 6	6 9 15 4	9 6 4 15	1 14 12 7	14 1 7 12	12 7 1 14	7 12 14 1

Group 8:

8#	335#	178#	249#	353#	74#	231#	112#
1 14 8 11	14 1 11 8	8 11 1 14	11 8 14 1	15 4 10 5	4 15 5 10	10 5 15 4	5 10 4 15
15 4 10 5	4 15 5 10	10 5 15 4	5 10 4 15	1 14 8 11	14 1 11 8	8 11 1 14	11 8 14 1
12 7 13 2	7 12 2 13	13 2 12 7	2 13 7 12	6 9 3 16	9 6 16 3	3 16 6 9	16 3 9 6
6 9 3 16	9 6 16 3	3 16 6 9	16 3 9 6	12 7 13 2	7 12 2 13	13 2 12 7	2 13 7 12
273#	154#	311#	32#	136#	207#	50#	377#
12 7 13 2	7 12 2 13	13 2 12 7	2 13 7 12	6 9 3 16	9 6 16 3	3 16 6 9	16 3 9 6
6 9 3 16	9 6 16 3	3 16 6 9	16 3 9 6	12 7 13 2	7 12 2 13	13 2 12 7	2 13 7 12
1 14 8 11	14 1 11 8	8 11 1 14	11 8 14 1	15 4 10 5	4 15 5 10	10 5 15 4	5 10 4 15
15 4 10 5	4 15 5 10	10 5 15 4	5 10 4 15	1 14 8 11	14 1 11 8	8 11 1 14	11 8 14 1

Group 9:

9#	334#	352#	75#	276#	151#	133#	210#
1 14 15 4	14 1 4 15	15 4 1 14	4 15 14 1	12 7 6 9	7 12 9 6	6 9 12 7	9 6 7 12
12 7 6 9	7 12 9 6	6 9 12 7	9 6 7 12	1 14 15 4	14 1 4 15	15 4 1 14	4 15 14 1
8 11 10 5	11 8 5 10	10 5 8 11	5 10 11 8	13 2 3 16	2 13 16 3	3 16 13 2	16 3 2 13
13 2 3 16	2 13 16 3	3 16 13 2	16 3 2 13	8 11 10 5	11 8 5 10	10 5 8 11	5 10 11 8
175#	252#	234#	109#	310#	33#	51#	376#
8 11 10 5	11 8 5 10	10 5 8 11	5 10 11 8	13 2 3 16	2 13 16 3	3 16 13 2	16 3 2 13
13 2 3 16	2 13 16 3	3 16 13 2	16 3 2 13	8 11 10 5	11 8 5 10	10 5 8 11	5 10 11 8
1 14 15 4	14 1 4 15	15 4 1 14	4 15 14 1	12 7 6 9	7 12 9 6	6 9 12 7	9 6 7 12
12 7 6 9	7 12 9 6	6 9 12 7	9 6 7 12	1 14 15 4	14 1 4 15	15 4 1 14	4 15 14 1

Group 10:

10#	333#	176#	251#	275#	152#	309#	34#
1 14 8 11	14 1 11 8	8 11 1 14	11 8 14 1	12 7 13 2	7 12 2 13	13 2 12 7	2 13 7 12
12 7 13 2	7 12 2 13	13 2 12 7	2 13 7 12	1 14 8 11	14 1 11 8	8 11 1 14	11 8 14 1
15 4 10 5	4 15 5 10	10 5 15 4	5 10 4 15	6 9 3 16	9 6 16 3	3 16 6 9	16 3 9 6
6 9 3 16	9 6 16 3	3 16 6 9	16 3 9 6	15 4 10 5	4 15 5 10	10 5 15 4	5 10 4 15
351#	76#	233#	110#	134#	209#	52#	375#
15 4 10 5	4 15 5 10	10 5 15 4	5 10 4 15	6 9 3 16	9 6 16 3	3 16 6 9	16 3 9 6
6 9 3 16	9 6 16 3	3 16 6 9	16 3 9 6	15 4 10 5	4 15 5 10	10 5 15 4	5 10 4 15
1 14 8 11	14 1 11 8	8 11 1 14	11 8 14 1	12 7 13 2	7 12 2 13	13 2 12 7	2 13 7 12
12 7 13 2	7 12 2 13	13 2 12 7	2 13 7 12	1 14 8 11	14 1 11 8	8 11 1 14	11 8 14 1

Group 11:

11#	332#	350#	77#	180#	247#	229#	114#
1 14 15 4	14 1 4 15	15 4 1 14	4 15 14 1	8 11 10 5	11 8 5 10	10 5 8 11	5 10 11 8
8 11 10 5	11 8 5 10	10 5 8 11	5 10 11 8	1 14 15 4	14 1 4 15	15 4 1 14	4 15 14 1
12 7 6 9	7 12 9 6	6 9 12 7	9 6 7 12	13 2 3 16	2 13 16 3	3 16 13 2	16 3 2 13
13 2 3 16	2 13 16 3	3 16 13 2	16 3 2 13	12 7 6 9	7 12 9 6	6 9 12 7	9 6 7 12
271#	156#	138#	205#	308#	35#	53#	374#
12 7 6 9	7 12 9 6	6 9 12 7	9 6 7 12	13 2 3 16	2 13 16 3	3 16 13 2	16 3 2 13
13 2 3 16	2 13 16 3	3 16 13 2	16 3 2 13	12 7 6 9	7 12 9 6	6 9 12 7	9 6 7 12
1 14 15 4	14 1 4 15	15 4 1 14	4 15 14 1	8 11 10 5	11 8 5 10	10 5 8 11	5 10 11 8
8 11 10 5	11 8 5 10	10 5 8 11	5 10 11 8	1 14 15 4	14 1 4 15	15 4 1 14	4 15 14 1

Group 12:

12#	331#	272#	155#	179#	248#	307#	36#
1 14 12 7	14 1 7 12	12 7 1 14	7 12 14 1	8 11 13 2	11 8 2 13	13 2 8 11	2 13 11 8
8 11 13 2	11 8 2 13	13 2 8 11	2 13 11 8	1 14 12 7	14 1 7 12	12 7 1 14	7 12 14 1
15 4 6 9	4 15 9 6	6 9 15 4	9 6 4 15	10 5 3 16	5 10 16 3	3 16 10 5	16 3 5 10
10 5 3 16	5 10 16 3	3 16 10 5	16 3 5 10	15 4 6 9	4 15 9 6	6 9 15 4	9 6 4 15
349#	78#	137#	206#	230#	113#	54#	373#
15 4 6 9	4 15 9 6	6 9 15 4	9 6 4 15	10 5 3 16	5 10 16 3	3 16 10 5	16 3 5 10
10 5 3 16	5 10 16 3	3 16 10 5	16 3 5 10	15 4 6 9	4 15 9 6	6 9 15 4	9 6 4 15
1 14 12 7	14 1 7 12	12 7 1 14	7 12 14 1	8 11 13 2	11 8 2 13	13 2 8 11	2 13 11 8
8 11 13 2	11 8 2 13	13 2 8 11	2 13 11 8	1 14 12 7	14 1 7 12	12 7 1 14	7 12 14 1

Group 13:

13#	288#	322#	147#	348#	121#	87#	214#
1 12 14 7	12 1 7 14	14 7 1 12	7 14 12 1	15 6 4 9	6 15 9 4	4 9 15 6	9 4 6 15
15 6 4 9	6 15 9 4	4 9 15 6	9 4 6 15	1 12 14 7	12 1 7 14	14 7 1 12	7 14 12 1
8 13 11 2	13 8 2 11	11 2 8 13	2 11 13 8	10 3 5 16	3 10 16 5	5 16 10 3	16 5 3 10
10 3 5 16	3 10 16 5	5 16 10 3	16 5 3 10	8 13 11 2	13 8 2 11	11 2 8 13	2 11 13 8
171#	298#	264#	37#	238#	63#	97#	372#
8 13 11 2	13 8 2 11	11 2 8 13	2 11 13 8	10 3 5 16	3 10 16 5	5 16 10 3	16 5 3 10
10 3 5 16	3 10 16 5	5 16 10 3	16 5 3 10	8 13 11 2	13 8 2 11	11 2 8 13	2 11 13 8
1 12 14 7	12 1 7 14	14 7 1 12	7 14 12 1	15 6 4 9	6 15 9 4	4 9 15 6	9 4 6 15
15 6 4 9	6 15 9 4	4 9 15 6	9 4 6 15	1 12 14 7	12 1 7 14	14 7 1 12	7 14 12 1

Group 14:

14#	287#	172#	297#	347#	122#	237#	64#
1 12 8 13	12 1 13 8	8 13 1 12	13 8 12 1	15 6 10 3	6 15 3 10	10 3 15 6	3 10 6 15
15 6 10 3	6 15 3 10	10 3 15 6	3 10 6 15	1 12 8 13	12 1 13 8	8 13 1 12	13 8 12 1
14 7 11 2	7 14 2 11	11 2 14 7	2 11 7 14	4 9 5 16	9 4 16 5	5 16 4 9	16 5 9 4
4 9 5 16	9 4 16 5	5 16 4 9	16 5 9 4	14 7 11 2	7 14 2 11	11 2 14 7	2 11 7 14
321#	148#	263#	38#	88#	213#	98#	371#
14 7 11 2	7 14 2 11	11 2 14 7	2 11 7 14	4 9 5 16	9 4 16 5	5 16 4 9	16 5 9 4
4 9 5 16	9 4 16 5	5 16 4 9	16 5 9 4	14 7 11 2	7 14 2 11	11 2 14 7	2 11 7 14
1 12 8 13	12 1 13 8	8 13 1 12	13 8 12 1	15 6 10 3	6 15 3 10	10 3 15 6	3 10 6 15
15 6 10 3	6 15 3 10	10 3 15 6	3 10 6 15	1 12 8 13	12 1 13 8	8 13 1 12	13 8 12 1

Group 15:

15#	286#	346#	123#	324#	145#	85#	216#
1 12 15 6	12 1 6 15	15 6 1 12	6 15 12 1	14 7 4 9	7 14 9 4	4 9 14 7	9 4 7 14
14 7 4 9	7 14 9 4	4 9 14 7	9 4 7 14	1 12 15 6	12 1 6 15	15 6 1 12	6 15 12 1
8 13 10 3	13 8 3 10	10 3 8 13	3 10 13 8	11 2 5 16	2 11 16 5	5 16 11 2	16 5 2 11
11 2 5 16	2 11 16 5	5 16 11 2	16 5 2 11	8 13 10 3	13 8 3 10	10 3 8 13	3 10 13 8
169#	300#	240#	61#	262#	39#	99#	370#
8 13 10 3	13 8 3 10	10 3 8 13	3 10 13 8	11 2 5 16	2 11 16 5	5 16 11 2	16 5 2 11
11 2 5 16	2 11 16 5	5 16 11 2	16 5 2 11	8 13 10 3	13 8 3 10	10 3 8 13	3 10 13 8
1 12 15 6	12 1 6 15	15 6 1 12	6 15 12 1	14 7 4 9	7 14 9 4	4 9 14 7	9 4 7 14
14 7 4 9	7 14 9 4	4 9 14 7	9 4 7 14	1 12 15 6	12 1 6 15	15 6 1 12	6 15 12 1

Group 16:

16#	285#	170#	299#	323#	146#	261#	40#
1 12 8 13	12 1 13 8	8 13 1 12	13 8 12 1	14 7 11 2	7 14 2 11	11 2 14 7	2 11 7 14
14 7 11 2	7 14 2 11	11 2 14 7	2 11 7 14	1 12 8 13	12 1 13 8	8 13 1 12	13 8 12 1
15 6 10 3	6 15 3 10	10 3 15 6	3 10 6 15	4 9 5 16	9 4 16 5	5 16 4 9	16 5 9 4
4 9 5 16	9 4 16 5	5 16 4 9	16 5 9 4	15 6 10 3	6 15 3 10	10 3 15 6	3 10 6 15
345#	124#	239#	62#	86#	215#	100#	369#
15 6 10 3	6 15 3 10	10 3 15 6	3 10 6 15	4 9 5 16	9 4 16 5	5 16 4 9	16 5 9 4
4 9 5 16	9 4 16 5	5 16 4 9	16 5 9 4	15 6 10 3	6 15 3 10	10 3 15 6	3 10 6 15
1 12 8 13	12 1 13 8	8 13 1 12	13 8 12 1	14 7 11 2	7 14 2 11	11 2 14 7	2 11 7 14
14 7 11 2	7 14 2 11	11 2 14 7	2 11 7 14	1 12 8 13	12 1 13 8	8 13 1 12	13 8 12 1

Group 17:

17#	284#	344#	125#	174#	295#	235#	66#
1 12 15 6	12 1 6 15	15 6 1 12	6 15 12 1	8 13 10 3	13 8 3 10	10 3 8 13	3 10 13 8
8 13 10 3	13 8 3 10	10 3 8 13	3 10 13 8	1 12 15 6	12 1 6 15	15 6 1 12	6 15 12 1
14 7 4 9	7 14 9 4	4 9 14 7	9 4 7 14	11 2 5 16	2 11 16 5	5 16 11 2	16 5 2 11
11 2 5 16	2 11 16 5	5 16 11 2	16 5 2 11	14 7 4 9	7 14 9 4	4 9 14 7	9 4 7 14
319#	150#	90#	211#	260#	41#	101#	368#
14 7 4 9	7 14 9 4	4 9 14 7	9 4 7 14	11 2 5 16	2 11 16 5	5 16 11 2	16 5 2 11
11 2 5 16	2 11 16 5	5 16 11 2	16 5 2 11	14 7 4 9	7 14 9 4	4 9 14 7	9 4 7 14
1 12 15 6	12 1 6 15	15 6 1 12	6 15 12 1	8 13 10 3	13 8 3 10	10 3 8 13	3 10 13 8
8 13 10 3	13 8 3 10	10 3 8 13	3 10 13 8	1 12 15 6	12 1 6 15	15 6 1 12	6 15 12 1

Group 18:

18#	283#	320#	149#	173#	296#	259#	42#
1 12 14 7	12 1 7 14	14 7 1 12	7 14 12 1	8 13 11 2	13 8 2 11	11 2 8 13	2 11 13 8
8 13 11 2	13 8 2 11	11 2 8 13	2 11 13 8	1 12 14 7	12 1 7 14	14 7 1 12	7 14 12 1
15 6 4 9	6 15 9 4	4 9 15 6	9 4 6 15	10 3 5 16	3 10 16 5	5 16 10 3	16 5 3 10
10 3 5 16	3 10 16 5	5 16 10 3	16 5 3 10	15 6 4 9	6 15 9 4	4 9 15 6	9 4 6 15
343#	126#	89#	212#	236#	65#	102#	367#
15 6 4 9	6 15 9 4	4 9 15 6	9 4 6 15	10 3 5 16	3 10 16 5	5 16 10 3	16 5 3 10
10 3 5 16	3 10 16 5	5 16 10 3	16 5 3 10	15 6 4 9	6 15 9 4	4 9 15 6	9 4 6 15
1 12 14 7	12 1 7 14	14 7 1 12	7 14 12 1	8 13 11 2	13 8 2 11	11 2 8 13	2 11 13 8
8 13 11 2	13 8 2 11	11 2 8 13	2 11 13 8	1 12 14 7	12 1 7 14	14 7 1 12	7 14 12 1

Group 19:

19#	192#	316#	243#	342#	217#	93#	118#
1 8 14 11	8 1 11 14	14 11 1 8	11 14 8 1	15 10 4 5	10 15 5 4	4 5 15 10	5 4 10 15
15 10 4 5	10 15 5 4	4 5 15 10	5 4 10 15	1 8 14 11	8 1 11 14	14 11 1 8	11 14 8 1
12 13 7 2	13 12 2 7	7 2 12 13	2 7 13 12	6 3 9 16	3 6 16 9	9 16 6 3	16 9 3 6
6 3 9 16	3 6 16 9	9 16 6 3	16 9 3 6	12 13 7 2	13 12 2 7	7 2 12 13	2 7 13 12
12 13 7 2	13 12 2 7	7 2 12 13	2 7 13 12	6 3 9 16	3 6 16 9	9 16 6 3	16 9 3 6
6 3 9 16	3 6 16 9	9 16 6 3	16 9 3 6	12 13 7 2	13 12 2 7	7 2 12 13	2 7 13 12
1 8 14 11	8 1 11 14	14 11 1 8	11 14 8 1	15 10 4 5	10 15 5 4	4 5 15 10	5 4 10 15
15 10 4 5	10 15 5 4	4 5 15 10	5 4 10 15	1 8 14 11	8 1 11 14	14 11 1 8	11 14 8 1

Group 20:

20#	191#	268#	291#	341#	218#	141#	70#
1 8 12 13	8 1 13 12	12 13 1 8	13 12 8 1	15 10 6 3	10 15 3 6	6 3 15 10	3 6 10 15
15 10 6 3	10 15 3 6	6 3 15 10	3 6 10 15	1 8 12 13	8 1 13 12	12 13 1 8	13 12 8 1
14 11 7 2	11 14 2 7	7 2 14 11	2 7 11 14	4 5 9 16	5 4 16 9	9 16 4 5	16 9 5 4
4 5 9 16	5 4 16 9	9 16 4 5	16 9 5 4	14 11 7 2	11 14 2 7	7 2 14 11	2 7 11 14
14 11 7 2	11 14 2 7	7 2 14 11	2 7 11 14	4 5 9 16	5 4 16 9	9 16 4 5	16 9 5 4
4 5 9 16	5 4 16 9	9 16 4 5	16 9 5 4	14 11 7 2	11 14 2 7	7 2 14 11	2 7 11 14
1 8 12 13	8 1 13 12	12 13 1 8	13 12 8 1	15 10 6 3	10 15 3 6	6 3 15 10	3 6 10 15
15 10 6 3	10 15 3 6	6 3 15 10	3 6 10 15	1 8 12 13	8 1 13 12	12 13 1 8	13 12 8 1

Group 21:

21#	190#	340#	219#	318#	241#	91#	120#
1 8 15 10	8 1 10 15	15 10 1 8	10 15 8 1	14 11 4 5	11 14 5 4	4 5 14 11	5 4 11 14
14 11 4 5	11 14 5 4	4 5 14 11	5 4 11 14	1 8 15 10	8 1 10 15	15 10 1 8	10 15 8 1
12 13 6 3	13 12 3 6	6 3 12 13	3 6 13 12	7 2 9 16	2 7 16 9	9 16 7 2	16 9 2 7
7 2 9 16	2 7 16 9	9 16 7 2	16 9 2 7	12 13 6 3	13 12 3 6	6 3 12 13	3 6 13 12
12 13 6 3	13 12 3 6	6 3 12 13	3 6 13 12	7 2 9 16	2 7 16 9	9 16 7 2	16 9 2 7
7 2 9 16	2 7 16 9	9 16 7 2	16 9 2 7	12 13 6 3	13 12 3 6	6 3 12 13	3 6 13 12
1 8 15 10	8 1 10 15	15 10 1 8	10 15 8 1	14 11 4 5	11 14 5 4	4 5 14 11	5 4 11 14
14 11 4 5	11 14 5 4	4 5 14 11	5 4 11 14	1 8 15 10	8 1 10 15	15 10 1 8	10 15 8 1

Group 22:

22#	189#	266#	293#	317#	242#	165#	46#
1 8 12 13	8 1 13 12	12 13 1 8	13 12 8 1	14 11 7 2	11 14 2 7	7 2 14 11	2 7 11 14
14 11 7 2	11 14 2 7	7 2 14 11	2 7 11 14	1 8 12 13	8 1 13 12	12 13 1 8	13 12 8 1
15 10 6 3	10 15 3 6	6 3 15 10	3 6 15 10	4 5 9 16	5 4 16 9	9 16 4 5	16 9 5 4
4 5 9 16	5 4 16 9	9 16 4 5	16 9 5 4	15 10 6 3	10 15 3 6	6 3 15 10	3 6 10 15
15 10 6 3	10 15 3 6	6 3 15 10	3 6 10 15	4 5 9 16	5 4 16 9	9 16 4 5	16 9 5 4
4 5 9 16	5 4 16 9	9 16 4 5	16 9 5 4	15 10 6 3	10 15 3 6	6 3 15 10	3 6 10 15
1 8 12 13	8 1 13 12	12 13 1 8	13 12 8 1	14 11 7 2	11 14 2 7	7 2 14 11	2 7 11 14
14 11 7 2	11 14 2 7	7 2 14 11	2 7 11 14	1 8 12 13	8 1 13 12	12 13 1 8	13 12 8 1

Group 23:

23#	188#	338#	221#	270#	289#	139#	72#
1 8 15 10	8 1 10 15	15 10 1 8	10 15 8 1	12 13 6 3	13 12 3 6	6 3 12 13	3 6 13 12
12 13 6 3	13 12 3 6	6 3 12 13	3 6 13 12	1 8 15 10	8 1 10 15	15 10 1 8	10 15 8 1
14 11 4 5	11 14 5 4	4 5 14 11	5 4 11 14	7 2 9 16	2 7 16 9	9 16 7 2	16 9 2 7
7 2 9 16	2 7 16 9	9 16 7 2	16 9 2 7	14 11 4 5	11 14 5 4	4 5 14 11	5 4 11 14
14 11 4 5	11 14 5 4	4 5 14 11	5 4 11 14	7 2 9 16	2 7 16 9	9 16 7 2	16 9 2 7
7 2 9 16	2 7 16 9	9 16 7 2	16 9 2 7	14 11 4 5	11 14 5 4	4 5 14 11	5 4 11 14
1 8 15 10	8 1 10 15	15 10 1 8	10 15 8 1	12 13 6 3	13 12 3 6	6 3 12 13	3 6 13 12
12 13 6 3	13 12 3 6	6 3 12 13	3 6 13 12	1 8 15 10	8 1 10 15	15 10 1 8	10 15 8 1

Group 24:

24#	187#	314#	245#	269#	290#	163#	48#
1 8 14 11	8 1 11 14	14 11 1 8	11 14 8 1	12 13 7 2	13 12 2 7	7 2 12 13	2 7 13 12
12 13 7 2	13 12 2 7	7 2 12 13	2 7 13 12	1 8 14 11	8 1 11 14	14 11 1 8	11 14 8 1
15 10 4 5	10 15 5 4	4 5 15 10	5 4 10 15	6 3 9 16	3 6 16 9	9 16 6 3	16 9 3 6
6 3 9 16	3 6 16 9	9 16 6 3	16 9 3 6	15 10 4 5	10 15 5 4	4 5 15 10	5 4 10 15
15 10 4 5	10 15 5 4	4 5 15 10	5 4 10 15	6 3 9 16	3 6 16 9	9 16 6 3	16 9 3 6
6 3 9 16	3 6 16 9	9 16 6 3	16 9 3 6	15 10 4 5	10 15 5 4	4 5 15 10	5 4 10 15
1 8 14 11	8 1 11 14	14 11 1 8	11 14 8 1	12 13 7 2	13 12 2 7	7 2 12 13	2 7 13 12
12 13 7 2	13 12 2 7	7 2 12 13	2 7 13 12	1 8 14 11	8 1 11 14	14 11 1 8	11 14 8 1

**** Monitor List-1: Check the Old Numbers of Solutions Reconstructed ****

?: 0

1: 1
 33: 1
 65: 1
 97: 1
 129: 1
 161: 1

193: 1
 225: 1
 257: 1
 289: 1
 321: 1
 353: 1

[OK!]

** Calculated and Listed by Kanji Setsuda on

Dec. 9, 2014 with MacOSX 10.10.1 & Xcode 6.1.1 **

I wanted to know the solution numbers of our reconstructions, so that I referred to the old list of Primitive Solutions, found and counted them one by one as above.

The Monitor List-1 tells us that we could have really succeeded in reconstructing all the solutions we have wanted. It directly proves that the equation $24 \times 16 = 384$ is true.

#6. How about the case of 24 basic data when $n_1=2$? Can we also reconstruct all?

*** Reconstruction of SCMS44 by our Original Transformation Method ***

* Abstract List of 24 Solutions when $n_1=2$ and the 15 Transformations of each *

Group 1:

25# 2 16 11 5 13 3 8 10 7 9 14 4 12 6 1 15	384# 16 2 5 11 3 13 10 8 9 7 4 14 6 12 15 1	256# 11 5 2 16 8 10 13 3 14 4 7 9 1 15 12 6	105# 5 11 16 2 10 8 3 13 4 14 9 7 15 1 6 12	306# 13 3 8 10 2 16 11 5 12 6 1 15 7 9 14 4	55# 3 13 10 8 16 2 5 11 6 12 15 1 9 7 4 14
183# 8 10 13 3 11 5 2 16 1 15 12 6 14 4 7 9	226# 10 8 3 13 5 11 16 2 15 1 6 12 4 14 9 7	159# 7 9 14 4 12 6 1 15 2 16 11 5 13 3 8 10	202# 9 7 4 14 6 12 15 1 16 2 5 11 3 13 10 8	330# 14 4 7 9 1 15 12 6 11 5 2 16 8 10 13 3	79# 4 14 9 7 15 1 6 12 5 11 16 2 10 8 3 13
280# 12 6 1 15 7 9 14 4 13 3 8 10 2 16 11 5	129# 6 12 15 1 9 7 4 14 3 13 10 8 16 2 5 11	1# 1 15 12 6 14 4 7 9 8 10 13 3 11 5 2 16	360# 15 1 6 12 4 14 9 7 10 8 3 13 5 11 16 2		

Group 2:

26# 2 16 7 9 13 3 12 6 11 5 14 4 8 10 1 15	383# 16 2 9 7 3 13 6 12 5 11 4 14 10 8 15 1	160# 7 9 2 16 12 6 13 3 14 4 11 5 1 15 8 10	201# 9 7 16 2 6 12 3 13 4 14 5 11 15 1 10 8	305# 13 3 12 6 2 16 7 9 8 10 1 15 11 5 14 4	56# 3 13 6 12 16 2 9 7 10 8 15 1 5 11 4 14	279# 12 6 13 3 7 9 2 16 1 15 8 10 14 4 11 5	130# 6 12 3 13 9 7 16 2 15 1 10 8 4 14 5 11
255# 11 5 14 4 8 10 1 15 2 16 7 9 13 3 12 6	106# 5 11 4 14 10 8 15 1 16 2 9 7 3 13 6 12	329# 14 4 11 5 1 15 8 10 7 9 2 16 12 6 13 3	80# 4 14 5 11 15 1 10 8 9 7 16 2 6 12 3 13	184# 8 10 1 15 11 5 14 4 13 3 12 6 2 16 7 9	225# 10 8 15 1 5 11 4 14 3 13 6 12 16 2 9 7	2# 1 15 8 10 14 4 11 5 12 6 13 3 7 9 2 16	359# 15 1 10 8 4 14 5 11 6 12 3 13 9 7 16 2

Group 3:

27# 2 16 13 3 11 5 8 10 7 9 12 6 14 4 1 15	382# 16 2 3 13 5 11 10 8 9 7 6 12 4 14 15 1	304# 13 3 2 16 8 10 11 5 12 6 7 9 1 15 14 4	57# 3 13 16 2 10 8 5 11 6 12 9 7 15 1 4 14	258# 11 5 8 10 2 16 13 3 14 4 1 15 7 9 12 6	103# 5 11 10 8 16 2 3 13 4 14 15 1 9 7 6 12	181# 8 10 11 5 13 3 2 16 1 15 14 4 12 6 7 9	228# 10 8 5 11 3 13 16 2 15 1 4 14 6 12 9 7
157# 7 9 12 6 14 4 1 15 2 16 13 3 11 5 8 10	204# 9 7 6 12 4 14 15 1 16 2 3 13 5 11 10 8	282# 12 6 7 9 1 15 14 4 13 3 2 16 8 10 11 5	127# 6 12 9 7 15 1 4 14 3 13 16 2 10 8 5 11	328# 14 4 1 15 7 9 12 6 11 5 8 10 2 16 13 3	81# 4 14 15 1 9 7 6 12 5 11 10 8 16 2 3 13	3# 1 15 14 4 12 6 7 9 8 10 11 5 13 3 2 16	358# 15 1 4 14 6 12 9 7 10 8 5 11 3 13 16 2

Group 4:

28# 2 16 7 9 11 5 14 4 13 3 12 6 8 10 1 15	381# 16 2 9 7 5 11 4 14 3 13 6 12 10 8 15 1	158# 7 9 2 16 14 4 11 5 12 6 13 3 1 15 8 10	203# 9 7 16 2 4 14 5 11 6 12 3 13 15 1 10 8	257# 11 5 14 4 2 16 7 9 8 10 1 15 13 3 12 6	104# 5 11 4 14 16 2 9 7 10 8 15 1 3 13 6 12	327# 14 4 11 5 7 9 2 16 1 15 8 10 12 6 13 3	82# 4 14 5 11 9 7 16 2 15 1 10 8 6 12 3 13
303# 13 3 12 6 8 10 1 15 2 16 7 9 11 5 14 4	58# 3 13 6 12 10 8 15 1 16 2 9 7 5 11 4 14	281# 12 6 13 3 1 15 8 10 7 9 2 16 14 4 11 5	128# 6 12 3 13 15 1 10 8 9 7 16 2 4 14 5 11	182# 8 10 1 15 13 3 12 6 11 5 14 4 2 16 7 9	227# 10 8 15 1 3 13 6 12 5 11 4 14 16 2 9 7	4# 1 15 8 10 12 6 13 3 14 4 11 5 7 9 2 16	357# 15 1 10 8 6 12 3 13 4 14 5 11 9 7 16 2

Group 5:

29#	380#	302#	59#	162#	199#	277#	132#
2 16 13 3	16 2 3 13	13 3 2 16	3 13 16 2	7 9 12 6	9 7 6 12	12 6 7 9	6 12 9 7
7 9 12 6	9 7 6 12	12 6 7 9	6 12 9 7	2 16 13 3	16 2 3 13	13 3 2 16	3 13 16 2
11 5 8 10	5 11 10 8	8 10 11 5	10 8 5 11	14 4 1 15	4 14 15 1	1 15 14 4	15 1 4 14
14 4 1 15	4 14 15 1	1 15 14 4	15 1 4 14	11 5 8 10	5 11 10 8	8 10 11 5	10 8 5 11
253#	108#	186#	223#	326#	83#	5#	356#
11 5 8 10	5 11 10 8	8 10 11 5	10 8 5 11	14 4 1 15	4 14 15 1	1 15 14 4	15 1 4 14
14 4 1 15	4 14 15 1	1 15 14 4	15 1 4 14	11 5 8 10	5 11 10 8	8 10 11 5	10 8 5 11
2 16 13 3	16 2 3 13	13 3 2 16	3 13 16 2	7 9 12 6	9 7 6 12	12 6 7 9	6 12 9 7
7 9 12 6	9 7 6 12	12 6 7 9	6 12 9 7	2 16 13 3	16 2 3 13	13 3 2 16	3 13 16 2

Group 6:

30#	379#	254#	107#	161#	200#	325#	84#
2 16 11 5	16 2 5 11	11 5 2 16	5 11 16 2	7 9 14 4	9 7 4 14	14 4 7 9	4 14 9 7
7 9 14 4	9 7 4 14	14 4 7 9	4 14 9 7	2 16 11 5	16 2 5 11	11 5 2 16	5 11 16 2
13 3 8 10	3 13 10 8	8 10 13 3	10 8 3 13	12 6 1 15	6 12 15 1	1 15 12 6	15 1 6 12
12 6 1 15	6 12 15 1	1 15 12 6	15 1 6 12	13 3 8 10	3 13 10 8	8 10 13 3	10 8 3 13
301#	60#	185#	224#	278#	131#	6#	355#
13 3 8 10	3 13 10 8	8 10 13 3	10 8 3 13	12 6 1 15	6 12 15 1	1 15 12 6	15 1 6 12
12 6 1 15	6 12 15 1	1 15 12 6	15 1 6 12	13 3 8 10	3 13 10 8	8 10 13 3	10 8 3 13
2 16 11 5	16 2 5 11	11 5 2 16	5 11 16 2	7 9 14 4	9 7 4 14	14 4 7 9	4 14 9 7
7 9 14 4	9 7 4 14	14 4 7 9	4 14 9 7	2 16 11 5	16 2 5 11	11 5 2 16	5 11 16 2

Group 7:

31#	312#	250#	177#	378#	49#	111#	232#
2 13 11 8	13 2 8 11	11 8 2 13	8 11 13 2	16 3 5 10	3 16 10 5	5 10 16 3	10 5 3 16
16 3 5 10	3 16 10 5	5 10 16 3	10 5 3 16	2 13 11 8	13 2 8 11	11 8 2 13	8 11 13 2
7 12 14 1	12 7 1 14	14 1 7 12	1 14 12 7	9 6 4 15	6 9 15 4	4 15 9 6	15 4 6 9
9 6 4 15	6 9 15 4	4 15 9 6	15 4 6 9	7 12 14 1	12 7 1 14	14 1 7 12	1 14 12 7
153#	274#	336#	7#	208#	135#	73#	354#
7 12 14 1	12 7 1 14	14 1 7 12	1 14 12 7	9 6 4 15	6 9 15 4	4 15 9 6	15 4 6 9
9 6 4 15	6 9 15 4	4 15 9 6	15 4 6 9	7 12 14 1	12 7 1 14	14 1 7 12	1 14 12 7
2 13 11 8	13 2 8 11	11 8 2 13	8 11 13 2	16 3 5 10	3 16 10 5	5 10 16 3	10 5 3 16
16 3 5 10	3 16 10 5	5 10 16 3	10 5 3 16	2 13 11 8	13 2 8 11	11 8 2 13	8 11 13 2

Group 8:

32#	311#	154#	273#	377#	50#	207#	136#
2 13 7 12	13 2 12 7	7 12 2 13	12 7 13 2	16 3 9 6	3 16 6 9	9 6 16 3	6 9 3 16
16 3 9 6	3 16 6 9	9 6 16 3	6 9 3 16	2 13 7 12	13 2 12 7	7 12 2 13	12 7 13 2
11 8 14 1	8 11 1 14	14 1 11 8	1 14 8 11	5 10 4 15	10 5 15 4	4 15 5 10	15 4 10 5
5 10 4 15	10 5 15 4	4 15 5 10	15 4 10 5	11 8 14 1	8 11 1 14	14 1 11 8	1 14 8 11
249#	178#	335#	8#	112#	231#	74#	353#
11 8 14 1	8 11 1 14	14 1 11 8	1 14 8 11	5 10 4 15	10 5 15 4	4 15 5 10	15 4 10 5
5 10 4 15	10 5 15 4	4 15 5 10	15 4 10 5	11 8 14 1	8 11 1 14	14 1 11 8	1 14 8 11
2 13 7 12	13 2 12 7	7 12 2 13	12 7 13 2	16 3 9 6	3 16 6 9	9 6 16 3	6 9 3 16
16 3 9 6	3 16 6 9	9 6 16 3	6 9 3 16	2 13 7 12	13 2 12 7	7 12 2 13	12 7 13 2

Group 9:

33#	310#	376#	51#	252#	175#	109#	234#
2 13 16 3	13 2 3 16	16 3 2 13	3 16 13 2	11 8 5 10	8 11 10 5	5 10 11 8	10 5 8 11
11 8 5 10	8 11 10 5	5 10 11 8	10 5 8 11	2 13 16 3	13 2 3 16	16 3 2 13	3 16 13 2
7 12 9 6	12 7 6 9	9 6 7 12	6 9 12 7	14 1 4 15	1 14 15 4	4 15 14 1	15 4 1 14
14 1 4 15	1 14 15 4	4 15 14 1	15 4 1 14	7 12 9 6	12 7 6 9	9 6 7 12	6 9 12 7
151#	276#	210#	133#	334#	9#	75#	352#
7 12 9 6	12 7 6 9	9 6 7 12	6 9 12 7	14 1 4 15	1 14 15 4	4 15 14 1	15 4 1 14
14 1 4 15	1 14 15 4	4 15 14 1	15 4 1 14	7 12 9 6	12 7 6 9	9 6 7 12	6 9 12 7
2 13 16 3	13 2 3 16	16 3 2 13	3 16 13 2	11 8 5 10	8 11 10 5	5 10 11 8	10 5 8 11
11 8 5 10	8 11 10 5	5 10 11 8	10 5 8 11	2 13 16 3	13 2 3 16	16 3 2 13	3 16 13 2

Group 10:

34#	309#	152#	275#	251#	176#	333#	10#
2 13 7 12	13 2 12 7	7 12 2 13	12 7 13 2	11 8 14 1	8 11 1 14	14 1 11 8	1 14 8 11
11 8 14 1	8 11 1 14	14 1 11 8	1 14 8 11	2 13 7 12	13 2 12 7	7 12 2 13	12 7 13 2
16 3 9 6	3 16 6 9	9 6 16 3	6 9 3 16	5 10 4 15	10 5 15 4	4 15 5 10	15 4 10 5
5 10 4 15	10 5 15 4	4 15 5 10	15 4 10 5	16 3 9 6	3 16 6 9	9 6 16 3	6 9 3 16
375#	52#	209#	134#	110#	233#	76#	351#
16 3 9 6	3 16 6 9	9 6 16 3	6 9 3 16	5 10 4 15	10 5 15 4	4 15 5 10	15 4 10 5
5 10 4 15	10 5 15 4	4 15 5 10	15 4 10 5	16 3 9 6	3 16 6 9	9 6 16 3	6 9 3 16
2 13 7 12	13 2 12 7	7 12 2 13	12 7 13 2	11 8 14 1	8 11 1 14	14 1 11 8	1 14 8 11
11 8 14 1	8 11 1 14	14 1 11 8	1 14 8 11	2 13 7 12	13 2 12 7	7 12 2 13	12 7 13 2

... (Skip) ...

Group 22:

46#	165#	242#	317#	293#	266#	189#	22#
2 7 11 14	7 2 14 11	11 14 2 7	14 11 7 2	13 12 8 1	12 13 1 8	8 1 13 12	1 8 12 13
13 12 8 1	12 13 1 8	8 1 13 12	1 8 12 13	2 7 11 14	7 2 14 11	11 14 2 7	14 11 7 2
16 9 5 4	9 16 4 5	5 4 16 9	4 5 9 16	3 6 10 15	6 3 15 10	10 15 3 6	15 10 6 3
3 6 10 15	6 3 15 10	10 15 3 6	15 10 6 3	16 9 5 4	9 16 4 5	5 4 16 9	4 5 9 16

The factor 3 proves to mean the 'inheritance' from the $4P_4$ with a big loss. And this 3 implies the count of 'Most Fundamental Solutions', I believe.

#8. Let me introduce you something extra here at the end of this article.

The Method: How to make the 'Self-Complementary' PMS44 by your own hand; It is completely different from the well-known method of 'Classical Composition'. You can make it only with your pen, paper and your brain, without using any PC.

(1) You must start everything with your good memory:

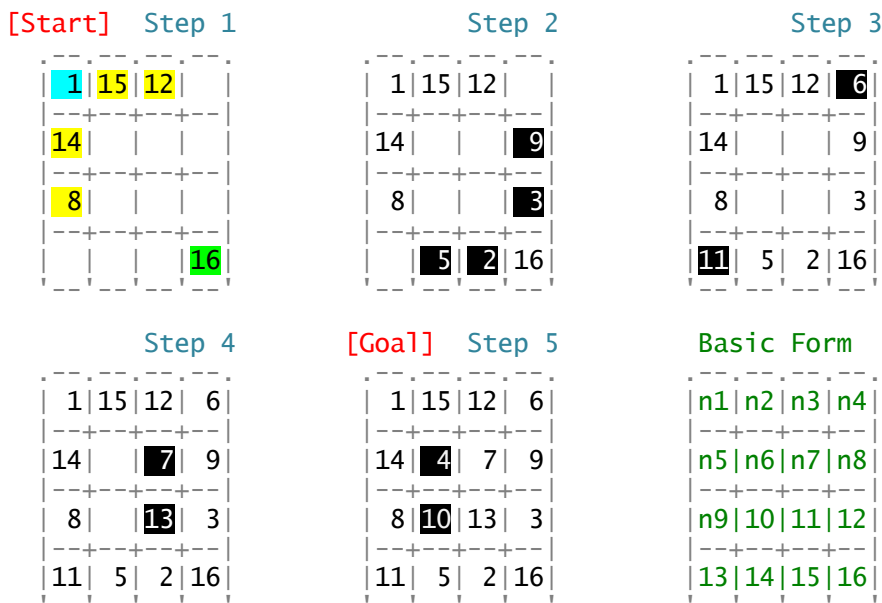
Put the number **1** on the Origin n_1 as the Basic Diagram shows as follows.

Place the 4 numbers { **8**, **12**, **14** and **15** } around neighborhood of the number **1**.

You don't have to mind where they should sit among those 4 positions. They can sit freely as you like.

Put the number **16** (Complementary Pair of 1) On n_{16} (at the Symmetric Point).

*** How to make the Solution of 'Self-complementary' MS44 by your own hand ***



(2) From now on you have to do all only by your calculations.

You can fill up the 4 numbers to the first 4 blank positions n_8, n_{12}, n_{14} and n_{15} .

Use the definition of constant sums of Symmetric Pairs: $C_{Sum}=17$;

$8+X=17$; $X=17-8=9$; $14+Y=17$; $Y=17-14=3$; Put the number **9** on n_8 , **3** on n_{12} .

$12+X=17$; $X=17-12=5$; $15+Y=17$; $Y=17-15=2$; Put the number **5** on n_{14} , **2** on n_{15} .

(3) You can fill the 2 numbers up to the second 2 blank positions n_4 and n_{13} .

Use the definitions of 'Basic Conditions' of row and column: $C_{Sum}=34$;

$1+15+12+X=34$; $X=34-1-15-12=6$; Put the Number **6** on n_4 .

$1+14+8+Y=34$; $Y=34-1-14-8=11$; Put the Number **11** on n_{13} .

(4) You can fill up the next 4 blank positions in several ways, but here let's use the newest conclusion: $n_1+n_2=n_7+n_8=n_{11}+n_{12}=n_{13}+n_{14}=cp_1$;

$n_3+n_4=n_5+n_6=n_9+n_{10}=n_{15}+n_{16}=cp_2$; and $cp_1+cp_2=C$;

$1+15=X+9$; $X=1+15-9=7$; Put the Number **7** on n_7 .

$Y+3=11+5$; $Y=11+5+3=13$; Put the Number **13** on n_{10} .

(5) You need no more explanation.

$12+6=14+X$; $X=12+6-14=4$; Put the Number **4** on n_6 .

$8+Y=2+16$; $Y=2+16-8=10$; Put the Number **10** on n_{10} .

At last you can get the wonderful piece of 'S-C' Magic Square of Order 4. You would be proud of your smartest demonstration by your own hand.

If you change the sitting order of those 4 good fellows around 1, you can get your 24 different solutions in all. This is one of the best points of this method.

Written on Dec.23, 2014 with MacOSX 10.10.1 and Xcode 6.1.1

by Kanji Setsuda: jag12001@nifty.com: E-Mail Address

<< Appendix: List of New Program >>

```
/** 'Self-Complementary' Magic Squares of Order 4: **
/** Reconstruction of 384 by the Transformation of 24; **
/** File 'SCMS44PT.c': Dictated by Kanji Setsuda **
/** on Dec.18, 2004; Dec. 9, 2014; **
/** with MacOSX 10.10.1 and Xcode 6.1.1 **
//
/** Definition: 'Self-Complementary' Magic Squares of Order 4 **
// ** Basic Form and a Model Solution **
//          0#          1#
// .---.---.---. .---.---.---.
// | 1| 2| 3| 4| | 1|15|12| 6|
// |---+---+---+---| |---+---+---+---|
// | 5| 6| 7| 8| |14| 4| 7| 9|
// |---+---+---+---| |---+---+---+---|
// | 9|10|11|12| | 8|10|13| 3|
// |---+---+---+---| |---+---+---+---|
// |13|14|15|16| |11| 5| 2|16|
// |'-'-'-'-'-'-'-'-'| |'-'-'-'-'-'-'-'-'|
//[Basic Conditions: C=34] [Self-Complementary Conditions: SC=17]
// n1+n2+n3+n4=C ...rw1; n1+n16=SC ...cmp1;
// n5+n6+n7+n8=C ...rw2; n2+n15=SC ...cmp2;
// n9+n10+n11+n12=C ...rw3; n3+n14=SC ...cmp3;
// n13+n14+n15+n16=C ...rw4; n4+n13=SC ...cmp4;
// n1+n5+n9+n13=C ...cl1; n5+n12=SC ...cmp5;
// n2+n6+n10+n14=C ...cl2; n6+n11=SC ...cmp6;
// n3+n7+n11+n15=C ...cl3; n7+n10=SC ...cmp7;
// n4+n8+n12+n16=C ...cl4; n8+n9=SC ...cmp8;
//
#include <stdio.h>
//
short cnt, cnt3;
short LSM, SC, typ;
short nm[17], uflg[17], mn[17];
short anm[9][17];
short snm[385][17], tnm[17][17];
short cnt01[17];
short refnmb[385];
//
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void svsol(void), ansprint(void);
void reprint(void), reprint2(void);
void printsol(short x), prsolc(short x);
void fnd01(void);
void prcnt01(void);
void prtrns(void), prtrlst(void), trnsprc(void);
void trnsf01(void), trnsf02(void), trnsf03(void), trnsf04(void);
void trnsf05(void), trnsf06(void), trnsf07(void), trnsf08(void);
void trnsf09(void), trnsf10(void), trnsf11(void), trnsf12(void);
void trnsf13(void), trnsf14(void), trnsf15(void);
void prmon(void);
//
short fndon(short x);
```

```

//
/** Main Program *
int main(){
short n;
printf("\n");
printf("*** 'Self-Complementary' Magic Squares of Order 4 **\n\n");
printf(" * Making the Primitive Solutions ... *\n");
for(n=0;n<17;n++){nm[n]=0; uflg[n]=0;}
LSM=34; SC=17; cnt=0; cnt3=0;
stp01(); /* Begin the Calculations *
printf(" [Count = %d]\n",cnt);
printf("\n");
printf("*** Compact List of the 384 Primitive Solutions of 'Self-Complementary' MS44 **\n");
ansprint();
if(cnt3>0){prsolc(cnt3);}
printf(" [Solution Counts = %d]\n",cnt);
printf("\n");
printf("* Solution Counts according to the Positions of Value 1 [Position: Counts] *\n");
prcnt01();
printf("\n");
for(n=0;n<=cnt;n++){refnmb[n]=0;}
printf("*** Basic Form of SCMS44 and Concepts of 15 Transformations **\n");
cnt3=0; prtrns();
printf("\n");
printf("*** Reconstruction of SCMS44 by our Original Transformation Method ***\n");
printf("*** List of 24 Solutions when n1=1 and the 15 Transformations of each **\n");
cnt3=0;
reprint(); /* Transform 24 when n1=1 into 384
printf("\n");
printf(" ** Monitor List: Check the Old Numbers of Solutions Reconstructed **\n");
prmon();
printf("\n");
printf(" OK!\n");
printf("*** Calculated and Listed by Kanji Setsuda on\n");
printf(" Dec. 9, 2014 with MacOSX 10.10.1 & Xcode 6.1.1 **\n");
printf("\n");
return 0;
}
/** Begin the Calculations *
/** Set N1 & n16=SC-n1 *
void stp01(){
short a,b;
for(a=1;a<17;a++){b=SC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[1]=a; nm[16]=b;
uflg[a]=1; uflg[b]=1;
stp02();
uflg[b]=0; uflg[a]=0;}
}
}
/** Set N2 & n15=SC-n2 *
void stp02(){
short a,b;
for(a=16;a>0;a--){b=SC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[2]=a; nm[15]=b;
uflg[a]=1; uflg[b]=1;
stp03();
uflg[b]=0; uflg[a]=0;}
}
}
/** Set N5 & n12=SC-n5 *
void stp03(){
short a,b;
for(a=16;a>0;a--){b=SC-a;

```

```

        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[5]=a; nm[12]=b;
            uflg[a]=1; uflg[b]=1;
            stp04();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set N3 & n14 *
void stp04(){
    short a,b;
    for(a=16;a>0;a--){b=SC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            uflg[a]=1; uflg[b]=1;
            nm[3]=a; nm[14]=b;
            stp05();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set n4=LSM-n1-n2-n3 & n13 *
void stp05(){
    short a,b;
    a=LSM-nm[1]-nm[2]-nm[3];
    if((0<a)&&(a<17)){
        b=LSM-nm[14]-nm[15]-nm[16];
        if(a+b==SC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                uflg[a]=1; uflg[b]=1;
                nm[4]=a; nm[13]=b;
                stp06();
                uflg[b]=0; uflg[a]=0;}}
    }
}
/** Set n9=LSM-n1-n5-n13 & n8 *
void stp06(){
    short a,b;
    a=LSM-nm[1]-nm[5]-nm[13];
    if((0<a)&&(a<17)){
        b=LSM-nm[4]-nm[12]-nm[16];
        if(a+b==SC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                uflg[a]=1; uflg[b]=1;
                nm[9]=a; nm[8]=b;
                stp07();
                uflg[b]=0; uflg[a]=0;}}
    }
}
/** Set N6 & n11 *
void stp07(){
    short a,b;
    for(a=1;a<17;a++){b=SC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            uflg[a]=1; uflg[b]=1;
            nm[6]=a; nm[11]=b;
            stp08();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set n7=LSM-n5-n6-n8 & n10 *
void stp08(){
    short a,b,c,d;
    a=LSM-nm[5]-nm[6]-nm[8];
    if((0<a)&&(a<17)){
        b=LSM-nm[9]-nm[11]-nm[12];
        if(a+b==SC){
            c=LSM-nm[3]-nm[11]-nm[15];
            d=LSM-nm[2]-nm[6]-nm[14];
            if((a==c)&&(b==d)){

```

```

        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[7]=a; nm[10]=b;
            uflg[a]=1; uflg[b]=1;
            svsol();
            uflg[b]=0; uflg[a]=0;}}}}
    }
    //
    /* Save the Solutions *
void svsol(){
    short n;
    cnt++; snm[cnt][0]=cnt;
    for(n=1;n<17;n++){snm[cnt][n]=nm[n];}
    n=0; /* Find 1 and Count solutions with that position *
    for(n=1;n<17;n++){
        if(nm[n]==1){cnt01[n]++;}
    }
}
//
/* Print the Answers *
void ansprint(){
    short m,n;
    for(m=1;m<=24;m++){
        anm[cnt3][0]=snm[m][0];
        for(n=1;n<17;n++){anm[cnt3][n]=snm[m][n];}
        cnt3++; if(cnt3==6){printsol(cnt3); cnt3=0;}
    }
    for(m=25;m<=cnt;m=m+12){
        anm[cnt3][0]=snm[m][0];
        for(n=1;n<17;n++){anm[cnt3][n]=snm[m][n];}
        cnt3++; if(cnt3==6){prsolc(cnt3); cnt3=0;}
    }
}
//
/* Print the Answers *
void printsol(short x){
    short m;
    for(m=0;m<x;m++){printf("%13d#",anm[m][0]);}; printf("\n");
    for(m=0;m<x;m++){printf(" .--.--.--.--.");}; printf("\n");
    for(m=0;m<x;m++){
        printf(" |%2d|%2d|%2d|%2d|",
            anm[m][1],anm[m][2],anm[m][3],anm[m][4]);}
    printf("\n");
    for(m=0;m<x;m++){printf(" |---+---+---+|");}; printf("\n");
    for(m=0;m<x;m++){
        printf(" |%2d|%2d|%2d|%2d|",
            anm[m][5],anm[m][6],anm[m][7],anm[m][8]);}
    printf("\n");
    for(m=0;m<x;m++){printf(" |---+---+---+|");}; printf("\n");
    for(m=0;m<x;m++){
        printf(" |%2d|%2d|%2d|%2d|",
            anm[m][9],anm[m][10],anm[m][11],anm[m][12]);}
    printf("\n");
    for(m=0;m<x;m++){printf(" |---+---+---+|");}; printf("\n");
    for(m=0;m<x;m++){
        printf(" |%2d|%2d|%2d|%2d|",
            anm[m][13],anm[m][14],anm[m][15],anm[m][16]);}
    printf("\n");
    for(m=0;m<x;m++){printf(" '---'---'---'");}
    printf("\n");
}
//
/* Print the Answers *
void prsolc(short x){
    short m;
    for(m=0;m<x;m++){

```



```

    printf("%12d#", anm[m][0]);
    if(m+1<x){printf(" ");}};
printf("\n");
for(m=0;m<x;m++){
    printf(" %2d %2d %2d %2d",
        anm[m][1], anm[m][2], anm[m][3], anm[m][4]);
    if(m+1<x){printf(" ");}};
printf("\n");
for(m=0;m<x;m++){
    printf(" %2d %2d %2d %2d",
        anm[m][5], anm[m][6], anm[m][7], anm[m][8]);
    if(m+1<x){printf(" ");}};
printf("\n");
for(m=0;m<x;m++){
    printf(" %2d %2d %2d %2d",
        anm[m][9], anm[m][10], anm[m][11], anm[m][12]);
    if(m+1<x){printf(" ");}};
printf("\n");
for(m=0;m<x;m++){
    printf(" %2d %2d %2d %2d",
        anm[m][13], anm[m][14], anm[m][15], anm[m][16]);
    if(m+1<x){printf(" ");}};
printf("\n");
}
//
/* Find 1 and Count solutions with that position */
void fnd01(){
    short m;
    m=0;
    for(m=1;m<17;m++){
        if(nm[m]==1){cnt01[m]++;}
    }
}
//
/* Solution Counts according to the Positions of Value 1 */
void prcnt01(){
    short m,n;
    for(m=1;m<17;m++){n=cnt01[m];
        printf("%4d:%3d", m,n);
        if((m%8)==0){printf("\n");}
    }
    printf("\n");
}
//
/* Print the Reconstruction transformed from the 24 solutions with n1=1 */
void reprint(){
    short m,n;
    for(m=1;m<=24;m++){
        printf(" Group %d:\n", snm[m][0]);
        for(n=1;n<17;n++){nm[n]=snm[m][n]; tnm[0][n]=snm[m][n];};
        trnsprc();
        prtrlst();
    }
}
//
/* Conceptual Diagrams of our original Transformations */
void prtrns(){
    short m,n;
    for(n=0;n<17;n++){nm[n]=n; tnm[0][n]=n;};
    trnsprc();
    cnt3=0; for(n=0;n<17;n++){tnm[n][0]=n;}
    for(m=0;m<16;m++){
        for(n=0;n<17;n++){anm[cnt3][n]=tnm[m][n];}
        cnt3++; if(cnt3==4){printsol(cnt3); cnt3=0;}
    }
}

```

```

}
//
/* Transform them by our original Transformations *
void trnsprc(){
short n;
for(n=1;n<=16;n++){nm[n]=tnm[0][n];}
trnsf01(); for(n=1;n<=16;n++){tnm[ 1][n]=mn[n];}
trnsf02(); for(n=1;n<=16;n++){tnm[ 2][n]=mn[n];}
trnsf03(); for(n=1;n<=16;n++){tnm[ 3][n]=mn[n];}
trnsf04(); for(n=1;n<=16;n++){tnm[ 4][n]=mn[n];}
trnsf05(); for(n=1;n<=16;n++){tnm[ 5][n]=mn[n];}
trnsf06(); for(n=1;n<=16;n++){tnm[ 6][n]=mn[n];}
trnsf07(); for(n=1;n<=16;n++){tnm[ 7][n]=mn[n];}
trnsf08(); for(n=1;n<=16;n++){tnm[ 8][n]=mn[n];}
trnsf09(); for(n=1;n<=16;n++){tnm[ 9][n]=mn[n];}
trnsf10(); for(n=1;n<=16;n++){tnm[10][n]=mn[n];}
trnsf11(); for(n=1;n<=16;n++){tnm[11][n]=mn[n];}
trnsf12(); for(n=1;n<=16;n++){tnm[12][n]=mn[n];}
trnsf13(); for(n=1;n<=16;n++){tnm[13][n]=mn[n];}
trnsf14(); for(n=1;n<=16;n++){tnm[14][n]=mn[n];}
trnsf15(); for(n=1;n<=16;n++){tnm[15][n]=mn[n];}
}
//
/* 15 types of Transformation *
void trnsf01(){
mn[ 1]=nm[ 2]; mn[ 2]=nm[ 1]; mn[ 3]=nm[ 4]; mn[ 4]=nm[ 3];
mn[ 5]=nm[ 6]; mn[ 6]=nm[ 5]; mn[ 7]=nm[ 8]; mn[ 8]=nm[ 7];
mn[ 9]=nm[10]; mn[10]=nm[ 9]; mn[11]=nm[12]; mn[12]=nm[11];
mn[13]=nm[14]; mn[14]=nm[13]; mn[15]=nm[16]; mn[16]=nm[15];
};
void trnsf02(){
mn[ 1]=nm[ 3]; mn[ 2]=nm[ 4]; mn[ 3]=nm[ 1]; mn[ 4]=nm[ 2];
mn[ 5]=nm[ 7]; mn[ 6]=nm[ 8]; mn[ 7]=nm[ 5]; mn[ 8]=nm[ 6];
mn[ 9]=nm[11]; mn[10]=nm[12]; mn[11]=nm[ 9]; mn[12]=nm[10];
mn[13]=nm[15]; mn[14]=nm[16]; mn[15]=nm[13]; mn[16]=nm[14];
};
void trnsf03(){
mn[ 1]=nm[ 4]; mn[ 2]=nm[ 3]; mn[ 3]=nm[ 2]; mn[ 4]=nm[ 1];
mn[ 5]=nm[ 8]; mn[ 6]=nm[ 7]; mn[ 7]=nm[ 6]; mn[ 8]=nm[ 5];
mn[ 9]=nm[12]; mn[10]=nm[11]; mn[11]=nm[10]; mn[12]=nm[ 9];
mn[13]=nm[16]; mn[14]=nm[15]; mn[15]=nm[14]; mn[16]=nm[13];
};
void trnsf04(){
mn[ 1]=nm[ 5]; mn[ 2]=nm[ 6]; mn[ 3]=nm[ 7]; mn[ 4]=nm[ 8];
mn[ 5]=nm[ 1]; mn[ 6]=nm[ 2]; mn[ 7]=nm[ 3]; mn[ 8]=nm[ 4];
mn[ 9]=nm[13]; mn[10]=nm[14]; mn[11]=nm[15]; mn[12]=nm[16];
mn[13]=nm[ 9]; mn[14]=nm[10]; mn[15]=nm[11]; mn[16]=nm[12];
};
void trnsf05(){
mn[ 1]=nm[ 6]; mn[ 2]=nm[ 5]; mn[ 3]=nm[ 8]; mn[ 4]=nm[ 7];
mn[ 5]=nm[ 2]; mn[ 6]=nm[ 1]; mn[ 7]=nm[ 4]; mn[ 8]=nm[ 3];
mn[ 9]=nm[14]; mn[10]=nm[13]; mn[11]=nm[16]; mn[12]=nm[15];
mn[13]=nm[10]; mn[14]=nm[ 9]; mn[15]=nm[12]; mn[16]=nm[11];
};
void trnsf06(){
mn[ 1]=nm[ 7]; mn[ 2]=nm[ 8]; mn[ 3]=nm[ 5]; mn[ 4]=nm[ 6];
mn[ 5]=nm[ 3]; mn[ 6]=nm[ 4]; mn[ 7]=nm[ 1]; mn[ 8]=nm[ 2];
mn[ 9]=nm[15]; mn[10]=nm[16]; mn[11]=nm[13]; mn[12]=nm[14];
mn[13]=nm[11]; mn[14]=nm[12]; mn[15]=nm[ 9]; mn[16]=nm[10];
};
void trnsf07(){
mn[ 1]=nm[ 8]; mn[ 2]=nm[ 7]; mn[ 3]=nm[ 6]; mn[ 4]=nm[ 5];
mn[ 5]=nm[ 4]; mn[ 6]=nm[ 3]; mn[ 7]=nm[ 2]; mn[ 8]=nm[ 1];
mn[ 9]=nm[16]; mn[10]=nm[15]; mn[11]=nm[14]; mn[12]=nm[13];
mn[13]=nm[12]; mn[14]=nm[11]; mn[15]=nm[10]; mn[16]=nm[ 9];
};

```

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};
void trnsf08(){
    mn[ 1]=nm[ 9]; mn[ 2]=nm[10]; mn[ 3]=nm[11]; mn[ 4]=nm[12];
    mn[ 5]=nm[13]; mn[ 6]=nm[14]; mn[ 7]=nm[15]; mn[ 8]=nm[16];
    mn[ 9]=nm[ 1]; mn[10]=nm[ 2]; mn[11]=nm[ 3]; mn[12]=nm[ 4];
    mn[13]=nm[ 5]; mn[14]=nm[ 6]; mn[15]=nm[ 7]; mn[16]=nm[ 8];
};
void trnsf09(){
    mn[ 1]=nm[10]; mn[ 2]=nm[ 9]; mn[ 3]=nm[12]; mn[ 4]=nm[11];
    mn[ 5]=nm[14]; mn[ 6]=nm[13]; mn[ 7]=nm[16]; mn[ 8]=nm[15];
    mn[ 9]=nm[ 2]; mn[10]=nm[ 1]; mn[11]=nm[ 4]; mn[12]=nm[ 3];
    mn[13]=nm[ 6]; mn[14]=nm[ 5]; mn[15]=nm[ 8]; mn[16]=nm[ 7];
};
void trnsf10(){
    mn[ 1]=nm[11]; mn[ 2]=nm[12]; mn[ 3]=nm[ 9]; mn[ 4]=nm[10];
    mn[ 5]=nm[15]; mn[ 6]=nm[16]; mn[ 7]=nm[13]; mn[ 8]=nm[14];
    mn[ 9]=nm[ 3]; mn[10]=nm[ 4]; mn[11]=nm[ 1]; mn[12]=nm[ 2];
    mn[13]=nm[ 7]; mn[14]=nm[ 8]; mn[15]=nm[ 5]; mn[16]=nm[ 6];
};
void trnsf11(){
    mn[ 1]=nm[12]; mn[ 2]=nm[11]; mn[ 3]=nm[10]; mn[ 4]=nm[ 9];
    mn[ 5]=nm[16]; mn[ 6]=nm[15]; mn[ 7]=nm[14]; mn[ 8]=nm[13];
    mn[ 9]=nm[ 4]; mn[10]=nm[ 3]; mn[11]=nm[ 2]; mn[12]=nm[ 1];
    mn[13]=nm[ 8]; mn[14]=nm[ 7]; mn[15]=nm[ 6]; mn[16]=nm[ 5];
};
void trnsf12(){
    mn[ 1]=nm[13]; mn[ 2]=nm[14]; mn[ 3]=nm[15]; mn[ 4]=nm[16];
    mn[ 5]=nm[ 9]; mn[ 6]=nm[10]; mn[ 7]=nm[11]; mn[ 8]=nm[12];
    mn[ 9]=nm[ 5]; mn[10]=nm[ 6]; mn[11]=nm[ 7]; mn[12]=nm[ 8];
    mn[13]=nm[ 1]; mn[14]=nm[ 2]; mn[15]=nm[ 3]; mn[16]=nm[ 4];
};
void trnsf13(){
    mn[ 1]=nm[14]; mn[ 2]=nm[13]; mn[ 3]=nm[16]; mn[ 4]=nm[15];
    mn[ 5]=nm[10]; mn[ 6]=nm[ 9]; mn[ 7]=nm[12]; mn[ 8]=nm[11];
    mn[ 9]=nm[ 6]; mn[10]=nm[ 5]; mn[11]=nm[ 8]; mn[12]=nm[ 7];
    mn[13]=nm[ 2]; mn[14]=nm[ 1]; mn[15]=nm[ 4]; mn[16]=nm[ 3];
};
void trnsf14(){
    mn[ 1]=nm[15]; mn[ 2]=nm[16]; mn[ 3]=nm[13]; mn[ 4]=nm[14];
    mn[ 5]=nm[11]; mn[ 6]=nm[12]; mn[ 7]=nm[ 9]; mn[ 8]=nm[10];
    mn[ 9]=nm[ 7]; mn[10]=nm[ 8]; mn[11]=nm[ 5]; mn[12]=nm[ 6];
    mn[13]=nm[ 3]; mn[14]=nm[ 4]; mn[15]=nm[ 1]; mn[16]=nm[ 2];
};
void trnsf15(){
    mn[ 1]=nm[16]; mn[ 2]=nm[15]; mn[ 3]=nm[14]; mn[ 4]=nm[13];
    mn[ 5]=nm[12]; mn[ 6]=nm[11]; mn[ 7]=nm[10]; mn[ 8]=nm[ 9];
    mn[ 9]=nm[ 8]; mn[10]=nm[ 7]; mn[11]=nm[ 6]; mn[12]=nm[ 5];
    mn[13]=nm[ 4]; mn[14]=nm[ 3]; mn[15]=nm[ 2]; mn[16]=nm[ 1];
};
//
/** Print the Result List of Transformations */
void prtrlst(){
    short m,n,oldn;
    for(m=0;m<16;m++){
        for(n=0;n<17;n++){anm[cnt3][n]=tnm[m][n];}
        oldn=fndon(cnt3); anm[cnt3][0]=oldn; refnmb[oldn]++;
        cnt3++; if(cnt3==6){prsolc(cnt3); cnt3=0;}
    }
    if(cnt3>0){prsolc(cnt3); cnt3=0;}
}
//
/** Find the Relative Solution Number in the Old List */
short fndon(short x){
    short d,m;
    short n,f;

```

```

d=0;
for(m=1;m<=cnt;m++){f=0;
  for(n=1;n<17;n++){
    if(anm[x][n]==snm[m][n]){f++;}
    else{break;}
  }
  if(f==16){d=snm[m][0]; break;}
}
return d;
}
//
/** Monitor of the Old Numbers of the Reconstructed **
void prmon(){
short n;
printf(" ??: %d",refnmb[0]);
for(n=1;n<385;n++){
  if(n%32==1){printf("\n%4d:",n);}
  printf("%2d",refnmb[n]);
}
printf("\n");
}
//

```