

# Chapter 2 : New Study of Magic Squares 4x4: Kanji Setsuda

## Section 4 : 'Composite and Complete' Pan-magic Squares 4x4 and the meaning of 'Complementary Pairs' : Revised Edition

#1. Let's study and know what is the true meaning of 'Complementary Pairs' for all types of Pan-diagonal Magic Squares of Order 4 right here in this section.

For that purpose, let's make 'Composite and Complete' Pan-magic Squares of Order 4 at first and study them precisely by our algebraic calculations.

\* 'Composite & Complete' Pan-magic Squares of Order 4 \*

\*\* Definition of Basic Form in the Extended Space: \*\*

```

14 15 16 13 14 15 16 13 14 15
  2  3 n4 |n1|n2|n3|n4|n1  2  3
  6  7 n8 |n5|n6|n7|n8|n5  6  7
 10 11 12 |n9|10|11|12|n9 10 11
 14 15 16 |13|14|15|16|13 14 15
  2  3 n4 |n1|n2|n3|n4|n1  2  3
  
```

\*\* Definition of Basic Conditions: C=34; \*\*

$n1+n2+n3+n4=C$       ...rw; |  $n1+n5+n9+n13=C$       ...c1;

\*\* Composite Conditions: C=34; \*\*

$n1+n2+n5+n6=C$       ...c1; |  $n2+n3+n6+n7=C$       ...c2;

$n3+n4+n7+n8=C$       ...c3; |  $n4+n1+n8+n5=C$       ...c4;

$n5+n6+n9+n10=C$       ...c5; |  $n6+n7+n10+n11=C$       ...c6;

$n7+n8+n11+n12=C$       ...c7; |  $n8+n5+n12+n9=C$       ...c8;

$n9+n10+n13+n14=C$       ...c9; |  $n10+n11+n14+n15=C$       ...c10;

$n11+n12+n15+n16=C$       ...c11; |  $n12+n9+n16+n13=C$       ...c12;

$n13+n14+n1+n2=C$       ...c13; |  $n14+n15+n2+n3=C$       ...c14;

$n15+n16+n3+n4=C$       ...c15; |  $n16+n13+n4+n1=C$       ...c16;

\*\* Complete Conditions: CC=17; \*\*

$n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC$       ...cc

\*\* Normalizing Inequality Conditions for Standard Solutions: \*\*

$n1 < n16, n1 < n4 < n13;$

The next list of solutions shows some specimens of our 'Composite and Complete' Pan-magic Squares 4x4 composed with the Basic Form and Definitions above.

\*\* Abstract List of the 48 Standard Solutions in the Extended Space \*\*

1#	4#	7#	10#
13 12 6 3 13 12	13 12 7 2 13 12	11 14 7 2 11 14	7 14 11 2 7 14
8   1   15   10   8   1	8   1   14   11   8   1	8   1   12   13   8   1	12   1   8   13   12   1
11   14   4   5   11   14	10   15   4   5   10   15	10   15   6   3   10   15	6   15   10   3   6   15
2   7   9   16   2   7	3   6   9   16   3   6	5   4   9   16   5   4	9   4   5   16   9   4
13   12   6   3   13   12	13   12   7   2   13   12	11   14   7   2   11   14	7   14   11   2   7   14
8 1 15 10 8 1	8 1 14 11 8 1	8 1 12 13 8 1	12 1 8 13 12 1

<p>13#</p> <pre> 14 11 5 4 14 11 7   2 16  9  7  2 ---+---+---+ 12 13  3  6 12 13 ---+---+---+ 1   8 10 15  1  8 ---+---+---+ 14 11  5  4 14 11 7 2 16 9 7 2 </pre>	<p>16#</p> <pre> 14 11 8 1 14 11 7   2 13 12  7  2 ---+---+---+ 9 16  3  6  9 16 ---+---+---+ 4   5 10 15  4  5 ---+---+---+ 14 11  8  1 14 11 7 2 13 12 7 2 </pre>	<p>19#</p> <pre> 12 13 8 1 12 13 7   2 11 14  7  2 ---+---+---+ 9 16  5  4  9 16 ---+---+---+ 6   3 10 15  6  3 ---+---+---+ 12 13  8  1 12 13 7 2 11 14 7 2 </pre>	<p>22#</p> <pre> 8 13 12 1 8 13 11  2  7 14 11  2 ---+---+---+ 5 16  9  4  5 16 ---+---+---+ 10  3  6 15 10  3 ---+---+---+ 8 13 12  1  8 13 11 2 7 14 11 2 </pre>
<p>25#</p> <pre> 15 10 5 4 15 10 6   3 16  9  6  3 ---+---+---+ 12 13  2  7 12 13 ---+---+---+ 1   8 11 14  1  8 ---+---+---+ 15 10  5  4 15 10 6 3 16 9 6 3 </pre>	<p>28#</p> <pre> 15 10 8 1 15 10 6   3 13 12  6  3 ---+---+---+ 9 16  2  7  9 16 ---+---+---+ 4   5 11 14  4  5 ---+---+---+ 15 10  8  1 15 10 6 3 13 12 6 3 </pre>	<p>31#</p> <pre> 12 13 8 1 12 13 6   3 10 15  6  3 ---+---+---+ 9 16  5  4  9 16 ---+---+---+ 7   2 11 14  7  2 ---+---+---+ 12 13  8  1 12 13 6 3 10 15 6 3 </pre>	<p>33#</p> <pre> 8 13 12 1 8 13 10  3  6 15 10  3 ---+---+---+ 5 16  9  4  5 16 ---+---+---+ 11  2  7 14 11  2 ---+---+---+ 8 13 12  1  8 13 10 3 6 15 10 3 </pre>
<p>35#</p> <pre> 16 9 6 3 16 9 5   4 15 10  5  4 ---+---+---+ 11 14  1  8 11 14 ---+---+---+ 2   7 12 13  2  7 ---+---+---+ 16  9  6  3 16  9 5 4 15 10 5 4 </pre>	<p>38#</p> <pre> 16 9 7 2 16 9 5   4 14 11  5  4 ---+---+---+ 10 15  1  8 10 15 ---+---+---+ 3   6 12 13  3  6 ---+---+---+ 16  9  7  2 16  9 5 4 14 11 5 4 </pre>	<p>41#</p> <pre> 11 14 7 2 11 14 5   4  9 16  5  4 ---+---+---+ 10 15  6  3 10 15 ---+---+---+ 8   1 12 13  8  1 ---+---+---+ 11 14  7  2 11 14 5 4 9 16 5 4 </pre>	<p>43#</p> <pre> 7 14 11 2 7 14 9   4  5 16  9  4 ---+---+---+ 6 15 10  3  6 15 ---+---+---+ 12  1  8 13 12  1 ---+---+---+ 7 14 11  2  7 14 9 4 5 16 9 4 </pre>
<p>45#</p> <pre> 8 11 2 13 8 11 10  5 16  3 10  5 ---+---+---+ 15  4  9  6 15  4 ---+---+---+ 1  14  7 12  1 14 ---+---+---+ 8 11  2 13  8 11 10 5 16 3 10 5 </pre>	<p>46#</p> <pre> 8 11 14 1 8 11 10  5  4 15 10  5 ---+---+---+ 3 16  9  6  3 16 ---+---+---+ 13  2  7 12 13  2 ---+---+---+ 8 11 14  1  8 11 10 5 4 15 10 5 </pre>	<p>47#</p> <pre> 7 12 1 14 7 12 9   6 15  4  9  6 ---+---+---+ 16  3 10  5 16  3 ---+---+---+ 2  13  8 11  2 13 ---+---+---+ 7 12  1 14  7 12 9 6 15 4 9 6 </pre>	<p>48#</p> <pre> 7 12 13 2 7 12 9   6  3 16  9  6 ---+---+---+ 4 15 10  5  4 15 ---+---+---+ 14  1  8 11 14  1 ---+---+---+ 7 12 13  2  7 12 9 6 3 16 9 6 </pre>

[Count = 48]

OK!

\*\* Calculated and Listed by Kanji Setsuda  
on Dec. 4, 2014 with MacOSX 10.10 & Xcode 6.1 \*\*

They should certainly make the same solution set with the ones of 'Pan-diagonal' type, 'Complete' or 'Composite and Pan-diagonal'. It has the 384 'Primitive' solutions and also the 48 'Standard' ones chosen by the Normalizing Inequality Conditions.

It is called the "Miraculous Unity" of all types of Pan-magic Squares of Order 4.

Take your careful look at the list of sample solutions above, and you can easily find the number 1 is always accompanied with the same members {8, 12, 14 and 15} on the left, right, upper and lower next. No one else would sit next to the number 1.

How interesting it is! Don't you think so?

What makes it possible? We have to find any reason why they are made so.

#2. The Miraculous Unity means that all of (1) Pan-diagonal type, (2) Complete one, (3) Composite Pan-magic one, and (4) Composite and Complete one are really equivalent. This implies the only same concept of all Pan-magic Squares of Order 4.

All those 4 types have the same set of 384 'Primitive Solutions' pre-normalized and consequently the same set of 48 'Standard Solutions' in common.

And you can easily transform those simultaneous equations at the early definition stage algebraically from each set to the others. Let me demonstrate one of my actual transformations here, for instance, from the definitions of 'Composite and Complete' type to the ones of ordinary Pan-diagonal type.

$n1+n2+n3+n4=C \dots rw$ ;  $n1+n2+n5+n6=C \dots c1$ ;  $\rightarrow n3+n4=n5+n6$   
 $n1+n2+n3+n4=C \dots rw$ ;  $n3+n4+n7+n8=C \dots c3$ ;  $\rightarrow n1+n2=n7+n8$   
 Therefore,  $n5+n6+n7+n8=n1+n2+n3+n4=C$   
 In the same way,  $n9+n10+n11+n12=n13+n14+n15+n16=n1+n2+n3+n4=C$   
 $n1+n5+n9+n13=C \dots c1$ ;  $n1+n2+n5+n6=C \dots c1$ ;  $\rightarrow n9+n13=n2+n6$   
 $n1+n5+n9+n13=C \dots c1$ ;  $n9+n10+n13+n14=C \dots c9$ ;  $\rightarrow n1+n5=n10+n14$   
 Therefore,  $n2+n6+n10+n14=n1+n5+n9+n13=C$   
 In the same way,  $n3+n7+n11+n15=n4+n8+n12+n16=n1+n5+n9+n13=C$

```

14 15 16 13 14 15 16 13 14 15
  2  3 n4 |n1|n2|n3|n4|n1  2  3
      |---+---+---+---|
  6  7 n8 |n5|n6|n7|n8|n5  6  7
      |---+---+---+---|
10 11 12 |n9|10|11|12|n9 10 11
      |---+---+---+---|
14 15 16 |13|14|15|16|13 14 15
      |---+---+---+---|
  2  3 n4 n1 n2 n3 n4 n1  2  3
    
```

Thus each row and each column should add up to the same constant sum  $C=34$ .

About pan-diagonals we can calculate the 'Complete Conditions' as follows:

$n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC \dots cc$ ;  
 $n1+n6+n11+n16=(n1+n11)+(n6+n16)=CC+CC=17+17=34=C \dots pd1$ ;  
 $n2+n7+n12+n13=(n2+n12)+(n7+n13)=CC+CC=17+17=34=C \dots pd2$ ;  
 $n3+n8+n9+n14=(n3+n9)+(n8+n14)=CC+CC=17+17=34=C \dots pd3$ ;  
 $n4+n5+n10+n15=(n4+n10)+(n5+n15)=CC+CC=17+17=34=C \dots pd4$ ;  
 $n1+n8+n11+n14=(n1+n11)+(n8+n14)=CC+CC=17+17=34=C \dots pb1$ ;  
 $n2+n5+n12+n15=(n2+n12)+(n5+n15)=CC+CC=17+17=34=C \dots pb2$ ;  
 $n3+n6+n9+n16=(n3+n9)+(n6+n16)=CC+CC=17+17=34=C \dots pb3$ ;  
 $n4+n7+n10+n13=(n4+n10)+(n7+n13)=CC+CC=17+17=34=C \dots pb4$ ;

Thus every pan-diagonals should add up to the same constant sum  $C(=34)$ .

Our 'Composite and Complete' Magic Squares of Order 4 prove to be one of the special types of ordinary Pan-diagonal Magic Squares.

But, as you know in the previous sections, all Pan-diagonal MS44 should naturally have the same properties with 'Composite' and 'Complete' Pan-magic Squares 4x4.

Therefore, we can say both of the two types are logically equivalent and naturally have the same properties in common as well as the same common solution-sets.

It means any type of Pan-magic Squares 4x4: (1) 'Pan-diagonal', (2) 'Complete', (3) 'Composite', and (4) 'Composite and Complete' is logically equivalent to one another.

If you would study one, then you could study all those four at the same time.

#3. Let's study further about our 'Composite and Complete' MS44, especially about their Complementary Pairs. How many pairs do we have to have to make this type?

When  $M+N=CC$ (Constant Complementary Sum), we use the word "Complementary Pairs" for (M, N), just like "(M, N) is a 'Complementary Pair' of CC".

```
* Algebraic Calculation [1]: C=34; *
n1+n2+n5+n6=C ...c1; n5+n6+n9+n10=C ...c5; -> n1+n2=n9+n10=cp1
n1+n2+n3+n4=C ...rw; n3+n4+n7+n8=C ...c3; -> n1+n2=n7+n8=cp1
n7+n8+n11+n12=C ...c7; n11+n12+n15+n16=C ..c11; -> n7+n8=n15+n16=cp1
n3+n4+n7+n8=C ...c3; n7+n8+n11+n12=C ...c7; -> n3+n4=n11+n12=cp2
n1+n2+n3+n4=C ...rw; n1+n2+n5+n6=C ...c1; -> n3+n4=n5+n6=cp2
n5+n6+n9+n10=C ...c5; n9+n10+n13+n14=C ...c9; -> n5+n6=n13+n14=cp2
Therefore n1+n2=n7+n8=n9+n10=n15+n16=cp1;
n3+n4=n5+n6=n11+n12=n13+n14=cp2; cp1+cp2=C;

* Algebraic Calculation [2]: C=34; *
n1+n2+n5+n6=C ...c1; n2+n3+n6+n7=C ...c2; -> n1+n5=n3+n7=cp3
n1+n5+n9+n13=C ...c1; n9+n10+n13+n14=C ...c9; -> n1+n5=n10+n14=cp3
n10+n11+n14+n15=C..c10; n11+n12+n15+n16=C..c11; -> n10+n14=n12+n16=cp3
n2+n3+n6+n7=C ...c2; n3+n4+n7+n8=C ...c3; -> n2+n6=n4+n8=cp4
n1+n2+n5+n6=C ...c1; n1+n5+n9+n13=C ...c1; -> n2+n6=n9+n13=cp4
n9+n10+n13+n14=C ...c9; n10+n11+n14+n15=C..c10; -> n9+n13=n11+n15=cp4
Therefore n1+n5=n3+n7=n10+n14=n12+n16=cp3;
n2+n6=n4+n8=n9+n13=n11+n15=cp4; cp3+cp4=C;

* Algebraic Calculation [3]: C=34; *
n1+n2+n3+n4=C ...rw; n2+n3+n6+n7=C ...c2; -> n1+n4=n6+n7=cp5
n6+n7+n10+n11=C ...c6; n10+n11+n14+n15=C..c10; -> n6+n7=n14+n15=cp5
n4+n1+n8+n5=C ...c4; n8+n5+n12+n9=C ...c8; -> n1+n4=n9+n12=cp5
n1+n2+n3+n4=C ...rw; n4+n1+n8+n5=C ...c4; -> n2+n3=n5+n8=cp6
n2+n3+n6+n7=C ...c2; n6+n7+n10+n11=C ...c6; -> n2+n3=n10+n11=cp6
n8+n5+n12+n9=C ...c8; n12+n9+n16+n13=C ...c12; -> n5+n8=n13+n16=cp6
Therefore n1+n4=n6+n7=n9+n12=n14+n15=cp5;
n2+n3=n5+n8=n10+n11=n13+n16=cp6; cp5+cp6=C;

* Algebraic Calculation [4]: C=34; *
n1+n5+n9+n13=C ...c1; n5+n6+n9+n10=C ...c5; -> n1+n13=n6+n10=cp7
n6+n7+n10+n11=C ...c6; n7+n8+n11+n12=C ...c7; -> n6+n10=n8+n12=cp7
n13+n14+n1+n2=C ...c13; n14+n15+n2+n3=C ...c14; -> n1+n13=n3+n15=cp7
n1+n5+n9+n13=C ...c1; n13+n14+n1+n2=C ...c13; -> n5+n9=n2+n14=cp8
n5+n6+n9+n10=C ...c5; n6+n7+n10+n11=C ...c6; -> n5+n9=n7+n11=cp8
n14+n15+n2+n3=C ...c14; n15+n16+n3+n4=C ...c15; -> n2+n14=n4+n16=cp8
Therefore n1+n13=n6+n10=n3+n15=n8+n12=cp7;
n5+n9=n2+n14=n7+n11=n4+n16=cp8; cp7+cp8=C;
```

\* Summary: Where are those Groups of Complementary Pairs? \*

```
[1] n1+n2=n7+n8=n9+n10=n15+n16=cp1;
n3+n4=n5+n6=n11+n12=n13+n14=cp2; cp1+cp2=C=34;
[2] n1+n5=n3+n7=n10+n14=n12+n16=cp3;
n2+n6=n4+n8=n9+n13=n11+n15=cp4; cp3+cp4=C=34;
[3] n1+n4=n6+n7=n9+n12=n14+n15=cp5;
n2+n3=n5+n8=n10+n11=n13+n16=cp6; cp5+cp6=C=34;
[4] n1+n13=n6+n10=n3+n15=n8+n12=cp7;
n5+n9=n2+n14=n7+n11=n4+n16=cp8; cp7+cp8=C=34;
```

[1]	[2]	[3]	[4]	[5]
16   13   14   15   16   13	16   13   14   15   16   13	16   13   14   15   16   13	16   13   14   15   16   13	16   13   14   15   16   13
n4   n1   n2   n3   n4   n1	n4   n1   n2   n3   n4   n1	n4   n1   n2   n3   n4   n1	n4   n1   n2   n3   n4   n1	n4   n1   n2   n3   n4   n1
n8   n5   n6   n7   n8   n5	n8   n5   n6   n7   n8   n5	n8   n5   n6   n7   n8   n5	n8   n5   n6   n7   n8   n5	n8   n5   n6   n7   n8   n5
12   n9   10   11   12   n9	12   n9   10   11   12   n9	12   n9   10   11   12   n9	12   n9   10   11   12   n9	12   n9   10   11   12   n9
16   13   14   15   16   13	16   13   14   15   16   13	16   13   14   15   16   13	16   13   14   15   16   13	16   13   14   15   16   13

[5]  $n_1+n_{11}=n_2+n_{12}=n_3+n_9=n_4+n_{10}=n_5+n_{15}=n_6+n_{16}=n_7+n_{13}=n_8+n_{14}=CC=17 \dots cc;$   
(cc: Complementary Conditions for our first Definition;)

We could find 4 groups of complementary pairs: 2 sets for each group: 4 pairs for each set. Each set has its own unique constant sum, while two constant sums in each group must add up to the magic constant  $C(=34)$ .

The last fifth group listed above contains the 8 pairs whose sum is equal to 17.

Yes. They come from the 'Complete Conditions' listed at the first definition stage.

We need to have all these 5 groups of real Complementary Pairs and we have to make them appropriately located in any 'Composite & Complete' MS44 according to the five diagrams above. We must neither use any number twice or more often in any group of complementary pairs, nor un-use any number there.

We don't yet know the real sums of cp1, cp2, cp3, cp4, ... , cp7 and cp8 at the definition stage, but only know about the constant sum(=17) of the fifth group.

#### #4. How can we have all the real complementary pairs correctly as many as $5 \times 8$ ?

We have to obey such the conditions that we should use any Natural Number from 1 to 16 strictly once and two constant sums should add up the magic constant 34 in each group: [1]  $cp_1+cp_2=34$ ; [2]  $cp_3+cp_4=34$ ; [3]  $cp_5+cp_6=34$ ; [4]  $cp_7+cp_8=34$ ;

Suppose  $cp_1=5$  and  $cp_2=29$ , for instance.

How can you make any set of real complementary pairs whose sums are 5 and 29, combining any two Natural Numbers from 1 to 16?

You can make  $\{(1, 4), (16, 13)\}$ ,  $\{(2, 3), (15, 14)\}$ ,  $\{(3, 2), (14, 15)\}$  and  $\{(4, 1), (13, 16)\}$  for your answers. But, the last half of those 8 pairs have the same combinations with the first half. You can only make the meaningful, compact pairs as many as 4 in all. They are not enough.

Suppose  $cp_1=6$  and  $cp_2=28$  next.

How can you make any real complementary pairs whose sums are 6 and 28, combining any two Natural Numbers from 1 to 16?

You can make  $\{(1, 5), (16, 12)\}$ ,  $\{(2, 4), (15, 13)\}$ ,  $\{\cancel{(4, 2)}, \cancel{(13, 15)}\}$  and  $\{\cancel{(5, 1)}, \cancel{(12, 16)}\}$  for your answers. You can really make only 4 compact pairs.

They are not enough.

.....

The next list collects the result of my calculations.

#### \*\* Possible Complementary Pairs for Pan-magic Squares of Order 4: \*\*

/CC= 5: C1( 1, 4), C2( 2, 3),  
\DD=29: D1(16,13), D2(15,14),

/CC= 6: C1( 1, 5), C2( 2, 4),  
\DD=28: D1(16,12), D2(15,13),

/CC= 7: C1( 1, 6), C2( 2, 5), C3( 3, 4),  
\DD=27: D1(16,11), D2(15,12), D3(14,13),

/CC= 8: C1( 1, 7), C2( 2, 6), C3( 3, 5),  
\DD=26: D1(16,10), D2(15,11), D3(14,12),

/CC= 9: C1( 1, 8), C2( 2, 7), C3( 3, 6), C4( 4, 5),  
\DD=25: D1(16, 9), D2(15,10), D3(14,11), D4(13,12); ... OK!

/CC=10: C1( 1, 9), C2( 3, 7), C3( 4, 6),  
\DD=24: D1(16, 8), D2(14,10), D3(13,11),

/CC=11: C1( 1,10), C2( 2, 9), C3( 5, 6),  
\DD=23: D1(16, 7), D2(15, 8), D3(12,11),

```

/CC=12: C1( 1,11), C2( 2,10), C3( 3, 9),
\DD=22: D1(16, 6), D2(15, 7), D3(14, 8),

/CC=13: C1( 1,12), C2( 2,11), C3( 3,10), C4( 4, 9),
\DD=21: D1(16, 5), D2(15, 6), D3(14, 7), D4(13, 8); ... OK!

/CC=14: C1( 1,13), C2( 2,12), C3( 3,11),
\DD=20: D1(16, 4), D2(15, 5), D3(14, 6),

/CC=15: C1( 1,14), C2( 2,13), C3( 5,10), C4( 6, 9),
\DD=19: D1(16, 3), D2(15, 4), D3(12, 7), D4(11, 8); ... OK!

/CC=16: C1( 1,15), C2( 3,13), C3( 5,11), C4( 7, 9),
\DD=18: D1(16, 2), D2(14, 4), D3(12, 6), D4(10, 8); ... OK!

{CC=DD=17}: CD1( 1,16), CD2( 2,15), CD3( 3,14), CD4( 4,13),
           CD5( 5,12), CD6( 6,11), CD7( 7,10), CD8( 8, 9); ... OK!

```

\*\* Calculated and Listed by Kanji Setsuda  
on May 16, 2007 with MacOSX & Xcode 2.2 \*\*

The following computer program would make such a similar list of Complementary Pairs as above for Pan-magic Squares of Order 4.

```

/** Possible Complementary Pairs for **
/** Pan-magic Squares of Order 4 **
/** 'CmpPairs4.c': Dictated by Kanji Setsuda *
/** on May 16, 2007 and on Nov.16, 2014 **
/** working with MacOSX & Xcode 2.2 **
//
#include <stdio.h>
//
short LSM, CSM;
short ctbl[17], dtbl[17];
short uflg[17];
//
int maktbl(short x);
void prans(short x, short y);
//
/** Main Program *
int main(void){
short cc, n;
printf("\n");
printf("** Possible Complementary Pairs for\n");
printf(" Pan-Magic Squares of Order 4 **\n");
LSM=34; CSM=17;
for(cc=5; cc<=CSM; cc++){
for(n=0; n<CSM; n++){ctbl[n]=0; dtbl[n]=0;}
for(n=0; n<CSM; n++){uflg[n]=0;}
n=maktbl(cc);
prans(cc,n);
}
printf("\n");
printf("** Calculated and Listed by Kanji Sesuda\n");
printf(" on May.16, 2007 with MacOSX & Xcode 2.2 **\n");
printf("\n");
return 0;
}
//
int maktbl(short x){
short c, d, cn, dn;
short m,n,p,q;
cn=0; dn=0; m=0; p=CSM;
c=x; d=LSM-x;
do{m++; n=c-m;

```

```

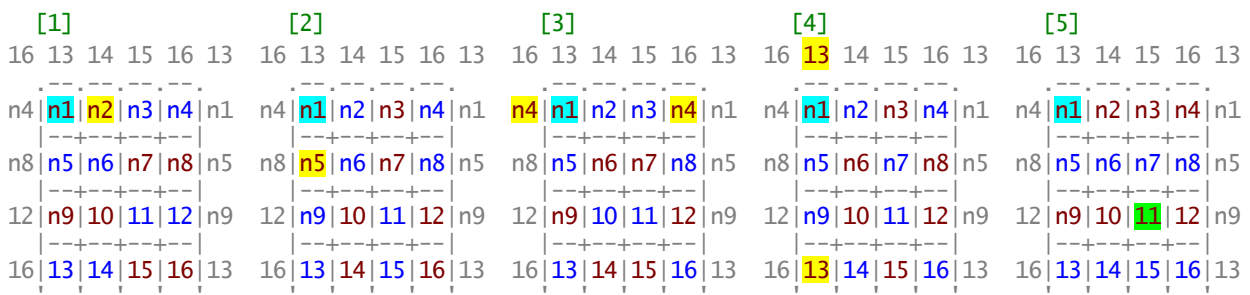
    if((m<n)&&(uflg[m]==0)&&(uflg[n]==0)){
        ctbl[cn]=m; cn++; uflg[m]=1;
        ctbl[cn]=n; cn++; uflg[n]=1;}
    p--; q=d-p;
    if((p>q)&&(uflg[p]==0)&&(uflg[q]==0)){
        dtbl[dn]=p; dn++; uflg[p]=1;
        dtbl[dn]=q; dn++; uflg[q]=1;}
}while((m<n)&&(p>q));
return cn;
}
//
void prans(short x, short y){
short cc, dd;
short m,n,c,d;
cc=x; dd=LSM-x;
if(cc==dd){printf("\n");
printf(" {CC=DD=%2d}:",cc);
for(m=0;m<y;m=m+2){
c=ctbl[m]; d=ctbl[m+1];
if((c>0)&&(d>0)){printf(" CD%d(%2d,%2d)",m/2+1,c,d);}
if(m%7==6){printf("\n ");}
}
printf(" ... OK!\n");}
else{printf("\n");
printf(" /CC=%2d:",cc);
for(m=0;m<y;m=m+2){
c=ctbl[m]; d=ctbl[m+1];
if((c>0)&&(d>0)){printf(" C%d(%2d,%2d)",m/2+1,c,d);}
}
printf("\n");
printf(" \DD=%2d:",dd);
for(n=0;n<y;n=n+2){
c=dtbl[n]; d=dtbl[n+1];
if((c>0)&&(d>0)){printf(" D%d(%2d,%2d)",n/2+1,c,d);}
}
if((m==8)&&(n==8)){printf(" ... OK!");}
printf("\n");}
}
//

```

Let me list out the next 5 groups of Essential Complementary Pairs for summary. They are all of our correct answers that would satisfy all the conditions we need. We could only find these 5 groups of 8x2 pairs in all, just as many as we need.

**\*\* 5 Groups of 16 Essential Complementary Pairs for 'C&C' MS44 \*\***

- [1]: [S1=09, S2=25]
  - (1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (7, 2), (8, 1),
  - (9, 16), (10, 15), (11, 14), (12, 13), (13, 12), (14, 11), (15, 10), (16, 9);
- [2]: [S1=13, S2=21]
  - (1, 12), (2, 11), (3, 10), (4, 9), (5, 16), (6, 15), (7, 14), (8, 13),
  - (9, 4), (10, 3), (11, 2), (12, 1), (13, 8), (14, 7), (15, 6), (16, 5);
- [3]: [S1=15, S2=19]
  - (1, 14), (2, 13), (3, 16), (4, 15), (5, 10), (6, 9), (7, 12), (8, 11),
  - (9, 6), (10, 5), (11, 8), (12, 7), (13, 2), (14, 1), (15, 4), (16, 3);
- [4]: [S1=16, S2=18]
  - (1, 15), (2, 16), (3, 13), (4, 14), (5, 11), (6, 12), (7, 9), (8, 10),
  - (9, 7), (10, 8), (11, 5), (12, 6), (13, 3), (14, 4), (15, 1), (16, 2);
- [5]: [S1=17, S2=17]
  - (1, 16), (2, 15), (3, 14), (4, 13), (5, 12), (6, 11), (7, 10), (8, 9),
  - (9, 8), (10, 7), (11, 6), (12, 5), (13, 4), (14, 3), (15, 2), (16, 1);



#5. How do we have to place all those complementary pairs in the right positions?

It is only the set of 8 pairs of the fifth group above that we know how to place.

Yes. We must place them on all pan-diagonals, according to the diagram [5] above.

How about the other 4 groups of real complementary pairs? We have to place [1], [2], [3] and [4] according to the position diagrams as [1], [2], [3] or [4] above.

When n1 takes the numerical value 1, the four diagrams above indicate where the complementary pairs of 1 should be placed: on either n2, n5, n4 or n13, the four adjacent positions next to n1.

The 'Complementary Pairs of 1' mean 8, 12, 14 or 15 as you know, when you see the list of possible complementary pairs, since the top pair of each group contains (1, Complementary Pair of 1).

$$\{(n1, n2), (n1, n4), (n1, n5), (n1, n13)\} = \{(1, 8), (1, 12), (1, 14), (1, 15)\}$$

Let me re-arrange the #4 list above into below exchanging X-Y axes of the form.

It would probably be more useful for designing how to place those pairs actually.

\* Re-arranged List of Essential Complementary Pairs led by their Masters \*

5 Groups: (S09, S25) (S13, S21) (S15, S19) (S16, S18) (S17=S17)

- ( 1, X) = {( 1, 8), ( 1, 12), ( 1, 14), ( 1, 15), ( 1, 16)};
- ( 2, X) = {( 2, 7), ( 2, 11), ( 2, 13), ( 2, 16), ( 2, 15)};
- ( 3, X) = {( 3, 6), ( 3, 10), ( 3, 16), ( 3, 13), ( 3, 14)};
- ( 4, X) = {( 4, 5), ( 4, 9), ( 4, 15), ( 4, 14), ( 4, 13)};
- ( 5, X) = {( 5, 4), ( 5, 16), ( 5, 10), ( 5, 11), ( 5, 12)};
- ( 6, X) = {( 6, 3), ( 6, 15), ( 6, 9), ( 6, 12), ( 6, 11)};
- ( 7, X) = {( 7, 2), ( 7, 14), ( 7, 12), ( 7, 9), ( 7, 10)};
- ( 8, X) = {( 8, 1), ( 8, 13), ( 8, 11), ( 8, 10), ( 8, 9)};
- ( 9, X) = {( 9, 16), ( 9, 4), ( 9, 6), ( 9, 7), ( 9, 8)};
- (10, X) = {(10, 15), (10, 3), (10, 5), (10, 8), (10, 7)};
- (11, X) = {(11, 14), (11, 2), (11, 8), (11, 5), (11, 6)};
- (12, X) = {(12, 13), (12, 1), (12, 7), (12, 6), (12, 5)};
- (13, X) = {(13, 12), (13, 8), (13, 2), (13, 3), (13, 4)};
- (14, X) = {(14, 11), (14, 7), (14, 1), (14, 4), (14, 3)};
- (15, X) = {(15, 10), (15, 6), (15, 4), (15, 1), (15, 2)};
- (16, X) = {(16, 9), (16, 5), (16, 3), (16, 2), (16, 1)};

\*\* Calculated and Listed by Kanji Setsuda on

Dec. 9, 2014 with MacOSX 10.10.1 & Xcode 6.1.1 \*\*

Watch the actual sample solutions below. Do you understand why 1 should be always accompanied with 8, 12, 14 and 15 on the left, right, upper or lower next?

It is because of the inner structure of Pan-magic Squares 4x4 that has those strict sets of Complementary Pairs. It is also because of the inherent properties of the serial Natural Numbers from 1 to 16 themselves that provides those 5 Essential Sets of real Complementary Pairs.



1/F	2/F	3/F	
13 8 10 3 13 8	13 12 6 3 13 12	11 8 10 5 11 8	
12   1   15   6   12   1	8   1   15   10   8   1	14   1   15   4   14   1	
7   14   4   9   7   14	11   14   4   5   11   14	7   12   6   9   7   12	
2   11   5   16   2   11	2   7   9   16   2   7	2   13   3   16   2   13	
13   8   10   3   13   8	13   12   6   3   13   12	11   8   10   5   11   8	
12 1 15 6 12 1	8 1 15 10 8 1	14 1 15 4 14 1	

#6. The next problem we must know about is how these 4 groups of Complementary Pairs should determine the total count of solutions of our object 'C&C' MS44.

When  $n_1=1$ , then  $\{n_2, n_4, n_5, n_{13}\}=\{8, 12, 14, 15\}$ .

This is the basic fact we can start with.

How many ways can we make these four fixed numbers  $\{8, 12, 14, 15\}$  placed on  $\{n_2, n_4, n_5, n_{13}\}$  differently from one another among those four?

It is the problem of permutation:  $4P_4 = 4 \times 3 \times 2 \times 1 = 24$

On the other hand the real numerical value 1 is expected to move freely from the Origin  $n_1$  to anywhere else such as  $n_2, n_3, n_4, \dots, n_{15}$  and  $n_{16}$ .

Then, we could count all the possible ways of arrangement as  $16 \times 4P_4 = 16 \times 24 = 384$ .

Yes. It is actually equal to the total count of 'Primitive Solutions' of this type.

We must study the Primitive Solution Set pre-normalized of C&C MS44 now.

**\*\* 'Composite & Complete' Magic Squares of Order 4 \*\***

**\*\* Basic Form and a Model Solution \*\***

0#	1#
16 13 14 15 16 13	13 8 10 3 13 8
n4   n1   n2   n3   n4   n1	12   1   15   6   12   1
n8   n5   n6   n7   n8   n5	7   14   4   9   7   14
12   n9   10   11   12   n9	2   11   5   16   2   11
16   13   14   15   16   13	13   8   10   3   13   8
4 1 2 3 4 1	12 1 15 6 12 1

**\*\* Abstract List of the 384 Primitive Solutions in the Extended Space \*\***

1#	2#	3#	4#
13 8 10 3 13 8	13 12 6 3 13 12	11 8 10 5 11 8	11 14 4 5 11 14
12   1   15   6   12   1	8   1   15   10   8   1	14   1   15   4   14   1	8   1   15   10   8   1
7   14   4   9   7   14	11   14   4   5   11   14	7   12   6   9   7   12	13   12   6   3   13   12
2   11   5   16   2   11	2   7   9   16   2   7	2   13   3   16   2   13	2   7   9   16   2   7
13   8   10   3   13   8	13   12   6   3   13   12	11   8   10   5   11   8	11   14   4   5   11   14
12 1 15 6 12 1	8 1 15 10 8 1	14 1 15 4 14 1	8 1 15 10 8 1
5#	6#	7#	8#
7 12 6 9 7 12	7 14 4 9 7 14	13 8 11 2 13 8	13 12 7 2 13 12
14   1   15   4   14   1	12   1   15   6   12   1	12   1   14   7   12   1	8   1   14   11   8   1
11   8   10   5   11   8	13   8   10   3   13   8	6   15   4   9   6   15	10   15   4   5   10   15
2   13   3   16   2   13	2   11   5   16   2   11	3   10   5   16   3   10	3   6   9   16   3   6
7   12   6   9   7   12	7   14   4   9   7   14	13   8   11   2   13   8	13   12   7   2   13   12
14 1 15 4 14 1	12 1 15 6 12 1	12 1 14 7 12 1	8 1 14 11 8 1

9#					10#					11#					12#								
10	8	11	5	10	8	10	15	4	5	10	15	6	12	7	9	6	12	6	15	4	9	6	15
15	1	14	4	15	1	8	1	14	11	8	1	15	1	14	4	15	1	12	1	14	7	12	1
6	12	7	9	6	12	13	12	7	2	13	12	10	8	11	5	10	8	13	8	11	2	13	8
3	13	2	16	3	13	3	6	9	16	3	6	3	13	2	16	3	13	3	10	5	16	3	10
10	8	11	5	10	8	10	15	4	5	10	15	6	12	7	9	6	12	6	15	4	9	6	15
15	1	14	4	15	1	8	1	14	11	8	1	15	1	14	4	15	1	12	1	14	7	12	1

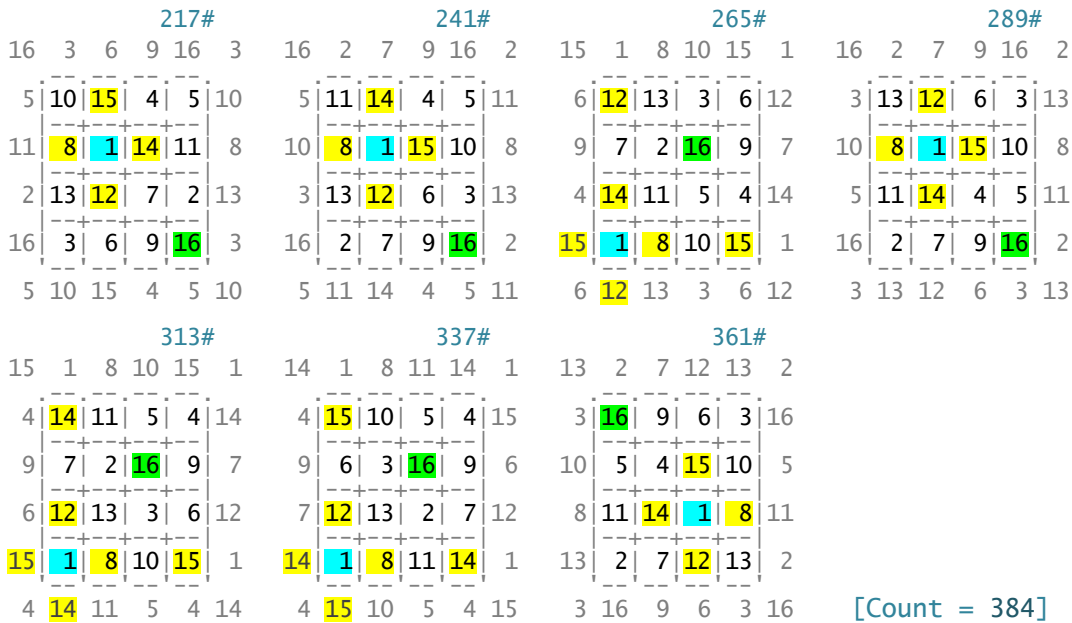
13#					14#					15#					16#								
11	8	13	2	11	8	11	14	7	2	11	14	10	8	13	3	10	8	10	15	6	3	10	15
14	1	12	7	14	1	8	1	12	13	8	1	15	1	12	6	15	1	8	1	12	13	8	1
4	15	6	9	4	15	10	15	6	3	10	15	4	14	7	9	4	14	11	14	7	2	11	14
5	10	3	16	5	10	5	4	9	16	5	4	5	11	2	16	5	11	5	4	9	16	5	4
11	8	13	2	11	8	11	14	7	2	11	14	10	8	13	3	10	8	10	15	6	3	10	15
14	1	12	7	14	1	8	1	12	13	8	1	15	1	12	6	15	1	8	1	12	13	8	1

17#					18#					19#					20#								
4	14	7	9	4	14	4	15	6	9	4	15	7	12	13	2	7	12	7	14	11	2	7	14
15	1	12	6	15	1	14	1	12	7	14	1	14	1	8	11	14	1	12	1	8	13	12	1
10	8	13	3	10	8	11	8	13	2	11	8	4	15	10	5	4	15	6	15	10	3	6	15
5	11	2	16	5	11	5	10	3	16	5	10	9	6	3	16	9	6	9	4	5	16	9	4
4	14	7	9	4	14	4	15	6	9	4	15	7	12	13	2	7	12	7	14	11	2	7	14
15	1	12	6	15	1	14	1	12	7	14	1	14	1	8	11	14	1	12	1	8	13	12	1

21#					22#					23#					24#								
6	12	13	3	6	12	6	15	10	3	6	15	4	14	11	5	4	14	4	15	10	5	4	15
15	1	8	10	15	1	12	1	8	13	12	1	15	1	8	10	15	1	14	1	8	11	14	1
4	14	11	5	4	14	7	14	11	2	7	14	6	12	13	3	6	12	7	12	13	2	7	12
9	7	2	16	9	7	9	4	5	16	9	4	9	7	2	16	9	7	9	6	3	16	9	6
6	12	13	3	6	12	6	15	10	3	6	15	4	14	11	5	4	14	4	15	10	5	4	15
15	1	8	10	15	1	12	1	8	13	12	1	15	1	8	10	15	1	14	1	8	11	14	1

25#					49#					73#					97#								
14	7	9	4	14	7	15	6	9	4	15	6	16	5	10	3	16	5	15	4	9	6	15	4
11	2	16	5	11	2	10	3	16	5	10	3	9	4	15	6	9	4	10	5	16	3	10	5
8	13	3	10	8	13	8	13	2	11	8	13	7	14	1	12	7	14	8	11	2	13	8	11
1	12	6	15	1	12	1	12	7	14	1	12	2	11	8	13	2	11	1	14	7	12	1	14
14	7	9	4	14	7	15	6	9	4	15	6	16	5	10	3	16	5	15	4	9	6	15	4
11	2	16	5	11	2	10	3	16	5	10	3	9	4	15	6	9	4	10	5	16	3	10	5

121#					145#					169#					193#								
16	3	10	5	16	3	16	2	11	5	16	2	15	1	12	6	15	1	15	4	5	10	15	4
9	6	15	4	9	6	9	7	14	4	9	7	10	8	13	3	10	8	6	9	16	3	6	9
7	12	1	14	7	12	6	12	1	15	6	12	5	11	2	16	5	11	12	7	2	13	12	7
2	13	8	11	2	13	3	13	8	10	3	13	4	14	7	9	4	14	1	14	11	8	1	14
16	3	10	5	16	3	16	2	11	5	16	2	15	1	12	6	15	1	15	4	5	10	15	4
9	6	15	4	9	6	9	7	14	4	9	7	10	8	13	3	10	8	6	9	16	3	6	9



\* Solution Counts according to the Positions of Value 1 [Position: Counts] \*  
 1: 24, 2: 24, 3: 24, 4: 24, 5: 24, 6: 24, 7: 24, 8: 24,  
 9: 24, 10: 24, 11: 24, 12: 24, 13: 24, 14: 24, 15: 24, 16: 24;

As you see in the Extended Space, every 1 is always surrounded with the fixed followers: {8, 12, 14 and 15} sitting left, right, upper and lower next to 1.

And the monitor list just above tells us the value 1 can go everywhere {n1, n2, n3, n4, ..., n15, n16} freely and equally.

Do you still think these two counts 384 mean only an accidental coincidence?

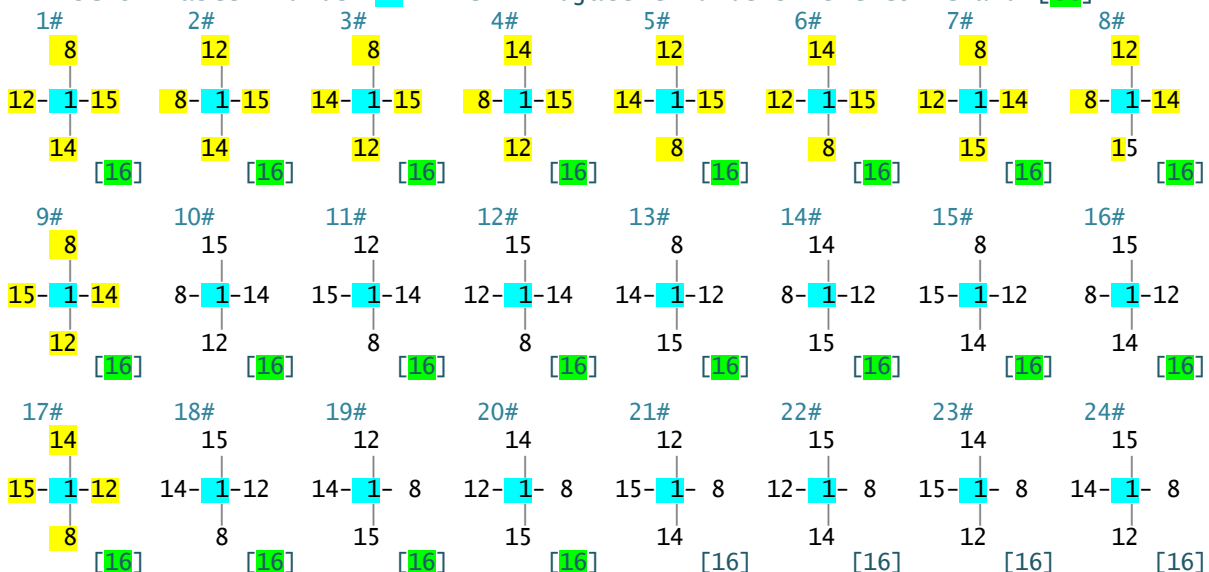
No. It is necessary that the number 1 and those 5 groups of Complementary Pairs should make the Primitive Solutions pre-normalized as many as 384.

The equation  $16 \times 4P_4 = 384$  must determine them directly.

#7. But why could only the team of number 1 and its old good fellows {8, 12, 14, 15} determine everything? Don't you think it seems too dictatorial and tyrannical?

We must know next about the cases of the other numbers 2, 3, 4, 5, ..., 15, 16 by checking the list of our 384 pre-normalized solutions precisely.

\*\* List of Master Number 1: with 4 Adjacent Numbers next to it and [CC] \*\*



25# 8 15-1-12 14 [16]	49# 8 14-1-12 15 [16]	73# 15 14-1-12 8 [16]	97# 8 12-1-14 15 [16]	121# 15 12-1-14 8 [16]	145# 14 12-1-15 8 [16]	169# 14 15-1-12 8 [16]	193# 12 8-1-14 15 [16]
217# 15 8-1-14 12 [16]	241# 14 8-1-15 12 [16]	265# 14 15-1-8 12 [16]	289# 12 8-1-15 14 [16]	313# 12 15-1-8 14 [16]	337# 12 14-1-8 15 [16]	361# 15 14-1-8 12 [16]	

[Count = 384]

\*\* List of Master Number 2: with 4 Adjacent Numbers next to it and [CC] \*\*

1# 7 16-2-11 13 [15]	25# 7 11-2-16 13 [15]	26# 11 7-2-16 13 [15]	27# 7 13-2-16 11 [15]	28# 13 7-2-16 11 [15]	29# 11 13-2-16 7 [15]	30# 13 11-2-16 7 [15]	31# 7 11-2-13 16 [15]
32# 11 7-2-13 16 [15]	33# 7 16-2-13 11 [15]	34# 16 7-2-13 11 [15]	35# 11 16-2-13 7 [15]	36# 16 11-2-13 7 [15]	37# 7 13-2-11 16 [15]	38# 13 7-2-11 16 [15]	39# 7 16-2-11 13 [15]
40# 16 7-2-11 13 [15]	41# 13 16-2-11 7 [15]	42# 16 13-2-11 7 [15]	43# 11 13-2-7 16 [15]	44# 13 11-2-7 16 [15]	45# 11 16-2-7 13 [15]	46# 16 11-2-7 13 [15]	47# 13 16-2-7 11 [15]

48# 2: 7, 13, 11, 16, [15];    49# 2: 11, 13, 7, 16, [15];    73# 2: 11, 13, 16, 7, [15];  
97# 2: 13, 11, 7, 16, [15];    121# 2: 13, 11, 16, 7, [15];    145# 2: 11, 16, 7, 13, [15];  
169# 2: 16, 11, 7, 13, [15];    193# 2: 13, 7, 11, 16, [15];    217# 2: 13, 7, 16, 11, [15];  
241# 2: 7, 16, 11, 13, [15];    265# 2: 16, 7, 11, 13, [15];    289# 2: 7, 16, 13, 11, [15];  
313# 2: 16, 7, 13, 11, [15];    337# 2: 7, 13, 11, 16, [15];    361# 2: 7, 13, 16, 11, [15];

[Count = 384]

\*\* List of Master Number 3: with 4 Adjacent Numbers next to it and [CC] \*\*

1# 16 10-3-13 6 [14]	25# 16 13-3-10 6 [14]	49# 6 10-3-16 13 [14]	50# 10 6-3-16 13 [14]	51# 6 13-3-16 10 [14]	52# 13 6-3-16 10 [14]	53# 10 13-3-16 6 [14]	54# 13 10-3-16 6 [14]
55# 6 10-3-13 16 [14]	56# 10 6-3-13 16 [14]	57# 6 16-3-13 10 [14]	58# 16 6-3-13 10 [14]	59# 10 16-3-13 6 [14]	60# 16 10-3-13 6 [14]	61# 6 13-3-10 16 [14]	62# 13 6-3-10 16 [14]
63# 6 16-3-10 13 [14]	64# 16 6-3-10 13 [14]	65# 13 16-3-10 6 [14]	66# 16 13-3-10 6 [14]	67# 10 13-3-6 16 [14]	68# 13 10-3-6 16 [14]	69# 10 16-3-6 13 [14]	70# 16 10-3-6 13 [14]

```

71# 3: 6,16,10,13,[14]; 72# 3: 6,13,10,16,[14]; 73# 3:16,10,6,13,[14];
97# 3:10,16,13,6,[14]; 121# 3:10,16,6,13,[14]; 145# 3:13,10,16,6,[14];
169# 3:10,13,16,6,[14]; 193# 3:6,16,13,10,[14]; 217# 3:6,16,10,13,[14];
241# 3:13,6,16,10,[14]; 265# 3:6,13,16,10,[14]; 289# 3:13,6,10,16,[14];
313# 3:6,13,10,16,[14]; 337# 3:16,6,13,10,[14]; 361# 3:16,6,10,13,[14];
[Count = 384]

```

```

** List of Master Number 4: with 4 Adjacent Numbers next to it and [CC] **
1# 4: 9,14,5,15,[13]; 25# 4:14,9,5,15,[13]; 49# 4:15,9,5,14,[13];
73# 4:15,9,14,5,[13]; 74# 4:15,5,14,9,[13]; 75# 4:15,14,9,5,[13];
76# 4:15,5,9,14,[13]; 77# 4:15,14,5,9,[13]; 78# 4:15,9,5,14,[13];
79# 4:14,9,15,5,[13]; 80# 4:14,5,15,9,[13]; 81# 4:14,15,9,5,[13];
82# 4:14,5,9,15,[13]; 83# 4:14,15,5,9,[13]; 84# 4:14,9,5,15,[13];
85# 4:9,14,15,5,[13]; 86# 4:9,5,15,14,[13]; 87# 4:9,15,14,5,[13];
88# 4:9,5,14,15,[13]; 89# 4:9,15,5,14,[13]; 90# 4:9,14,5,15,[13];
91# 4:5,14,15,9,[13]; 92# 4:5,9,15,14,[13]; 93# 4:5,15,14,9,[13];
94# 4:5,9,14,15,[13]; 95# 4:5,15,9,14,[13]; 96# 4:5,14,9,15,[13];
97# 4:9,15,5,14,[13]; 121# 4:9,15,14,5,[13]; 145# 4:9,14,15,5,[13];
169# 4:14,9,15,5,[13]; 193# 4:5,15,9,14,[13]; 217# 4:5,15,14,9,[13];
241# 4:5,14,15,9,[13]; 265# 4:14,5,15,9,[13]; 289# 4:5,14,9,15,[13];
313# 4:14,5,9,15,[13]; 337# 4:15,5,9,14,[13]; 361# 4:15,5,14,9,[13];
[Count = 384]

```

```

** List of Master Number 5: with 4 Adjacent Numbers next to it and [CC] **
1# 5:16,11,10,4,[12]; 25# 5:11,16,10,4,[12]; 49# 5:10,16,11,4,[12];
73# 5:10,16,4,11,[12]; 97# 5:16,10,11,4,[12]; 98# 5:16,4,11,10,[12];
99# 5:16,11,10,4,[12]; 100# 5:16,4,10,11,[12]; 101# 5:16,11,4,10,[12];
102# 5:16,10,4,11,[12]; 103# 5:11,10,16,4,[12]; 104# 5:11,4,16,10,[12];
105# 5:11,16,10,4,[12]; 106# 5:11,4,10,16,[12]; 107# 5:11,16,4,10,[12];
108# 5:11,10,4,16,[12]; 109# 5:10,11,16,4,[12]; 110# 5:10,4,16,11,[12];
111# 5:10,16,11,4,[12]; 112# 5:10,4,11,16,[12]; 113# 5:10,16,4,11,[12];
114# 5:10,11,4,16,[12]; 115# 5:4,11,16,10,[12]; 116# 5:4,10,16,11,[12];
117# 5:4,16,11,10,[12]; 118# 5:4,10,11,16,[12]; 119# 5:4,16,10,11,[12];
120# 5:4,11,10,16,[12]; 121# 5:16,10,4,11,[12]; 145# 5:16,11,4,10,[12];
169# 5:11,16,4,10,[12]; 193# 5:10,4,16,11,[12]; 217# 5:10,4,11,16,[12];
241# 5:11,4,10,16,[12]; 265# 5:4,11,10,16,[12]; 289# 5:11,4,16,10,[12];
313# 5:4,11,16,10,[12]; 337# 5:4,10,16,11,[12]; 361# 5:4,10,11,16,[12];
[Count = 384]

```

```

** List of Master Number 9: with 4 Adjacent Numbers next to it and [CC] **
1# 9: 7,4,16,6,[8]; 25# 9: 4,7,16,6,[8]; 49# 9: 4,6,16,7,[8];
73# 9: 4,6,7,16,[8]; 97# 9: 6,4,16,7,[8]; 121# 9: 6,4,7,16,[8];
145# 9: 7,4,6,16,[8]; 169# 9: 4,7,6,16,[8]; 193# 9: 16,6,7,4,[8];
194# 9:16,4,7,6,[8]; 195# 9:16,7,6,4,[8]; 196# 9:16,4,6,7,[8];
197# 9:16,7,4,6,[8]; 198# 9:16,6,4,7,[8]; 199# 9:7,6,16,4,[8];
200# 9:7,4,16,6,[8]; 201# 9:7,16,6,4,[8]; 202# 9:7,4,6,16,[8];
203# 9:7,16,4,6,[8]; 204# 9:7,6,4,16,[8]; 205# 9:6,7,16,4,[8];
206# 9:6,4,16,7,[8]; 207# 9:6,16,7,4,[8]; 208# 9:6,4,7,16,[8];
209# 9:6,16,4,7,[8]; 210# 9:6,7,4,16,[8]; 211# 9:4,7,16,6,[8];
212# 9:4,6,16,7,[8]; 213# 9:4,16,7,6,[8]; 214# 9:4,6,7,16,[8];
215# 9:4,16,6,7,[8]; 216# 9:4,7,6,16,[8]; 217# 9:16,6,4,7,[8];
241# 9:16,7,4,6,[8]; 265# 9:7,16,4,6,[8]; 289# 9:16,7,6,4,[8];
313# 9:7,16,6,4,[8]; 337# 9:6,16,7,4,[8]; 361# 9:6,16,4,7,[8];
[Count = 384]

```

```

** List of Master Number 13: with 4 Adjacent Numbers next to it and [CC] **
1# 13: 8,3,12,2,[4]; 25# 13: 3,8,12,2,[4]; 49# 13: 2,8,12,3,[4];
73# 13: 2,8,3,12,[4]; 97# 13: 8,2,12,3,[4]; 121# 13: 8,2,3,12,[4];
145# 13: 8,3,2,12,[4]; 169# 13: 3,8,2,12,[4]; 193# 13:12,2,8,3,[4];
217# 13:12,2,3,8,[4]; 241# 13:12,3,2,8,[4]; 265# 13:3,12,2,8,[4];
289# 13:12,3,8,2,[4]; 290# 13:12,2,8,3,[4]; 291# 13:12,8,3,2,[4];
292# 13:12,2,3,8,[4]; 293# 13:12,8,2,3,[4]; 294# 13:12,3,2,8,[4];
295# 13:8,3,12,2,[4]; 296# 13:8,2,12,3,[4]; 297# 13:8,12,3,2,[4];
298# 13:8,2,3,12,[4]; 299# 13:8,12,2,3,[4]; 300# 13:8,3,2,12,[4];
301# 13:3,8,12,2,[4]; 302# 13:3,2,12,8,[4]; 303# 13:3,12,8,2,[4];

```

```

304# 13: 3, 2, 8,12,[ 4]; 305# 13: 3,12, 2, 8,[ 4]; 306# 13: 3, 8, 2,12,[ 4];
307# 13: 2, 8,12, 3,[ 4]; 308# 13: 2, 3,12, 8,[ 4]; 309# 13: 2,12, 8, 3,[ 4];
310# 13: 2, 3, 8,12,[ 4]; 311# 13: 2,12, 3, 8,[ 4]; 312# 13: 2, 8, 3,12,[ 4];
313# 13: 3,12, 8, 2,[ 4]; 337# 13: 2,12, 8, 3,[ 4]; 361# 13: 2,12, 3, 8,[ 4];
[Count = 384]

```

\*\* List of Master Number 16: with 4 Adjacent Numbers next to it and [9] \*\*

```

1#      25#      49#      73#      97#      121#      145#      169#
  9      9      9      2      9      2      3      3
5-16-2  2-16-5  3-16-5  3-16-5  5-16-3  5-16-3  5-16-2  2-16-5
  3      3      2      9      2      9      9      9
  [ ]    [ ]    [ ]    [ 1]  [ 1]  [ 1]  [ 1]  [ 1]

193#    217#    241#    265#    289#    313#    337#
  5      2      3      3      5      5      5
9-16-3  9-16-3  9-16-2  2-16-9  9-16-2  2-16-9  3-16-9
  2      5      5      5      3      3      2
  [ ]    [ 1]  [ 1]  [ 1]  [ 1]  [ 1]  [ 1]

361# 16: 9, 3, 5, 2, [ 1]; 362# 16: 9, 2, 5, 3, [ 1]; 363# 16: 9, 5, 3, 2, [ 1];
364# 16: 9, 2, 3, 5, [ 1]; 365# 16: 9, 5, 2, 3, [ 1]; 366# 16: 9, 3, 2, 5, [ 1];
367# 16: 5, 3, 9, 2, [ 1]; 368# 16: 5, 2, 9, 3, [ 1]; 369# 16: 5, 9, 3, 2, [ 1];
370# 16: 5, 2, 3, 9, [ 1]; 371# 16: 5, 9, 2, 3, [ 1]; 372# 16: 5, 3, 2, 9, [ 1];
373# 16: 3, 5, 9, 2, [ 1]; 374# 16: 3, 2, 9, 5, [ 1]; 375# 16: 3, 9, 5, 2, [ 1];
376# 16: 3, 2, 5, 9, [ 1]; 377# 16: 3, 9, 2, 5, [ 1]; 378# 16: 3, 5, 2, 9, [ 1];
379# 16: 2, 5, 9, 3, [ 1]; 380# 16: 2, 3, 9, 5, [ 1]; 381# 16: 2, 9, 5, 3, [ 1];
382# 16: 2, 3, 5, 9, [ 1]; 383# 16: 2, 9, 3, 5, [ 1]; 384# 16: 2, 5, 3, 9, [ 1];

```

[Count = 384] OK!

\*\* Calculated and Listed by Kanji Setsuda on  
Dec. 2, 2014 with MacOSX 10.10.1 & Xcode 6.1 \*\*

Just as you see above, every number 1, 2, 3, ..., 15,16 do exactly have their own fixed followers: 4 adjacent next to their master and 1 complementary pair of each.

And all of the 16 teams always respect and obey the same equation  $16 \times 24 = 384$ ;  
They certainly do that equally and simultaneously. How democratic it is!

And it is the next list below that everything is based on, in order to specify what are given from the set of Natural Numbers 1~16.

\*\* Re-arranged List of Essential Complementary Pairs led by their Masters \*\*

```

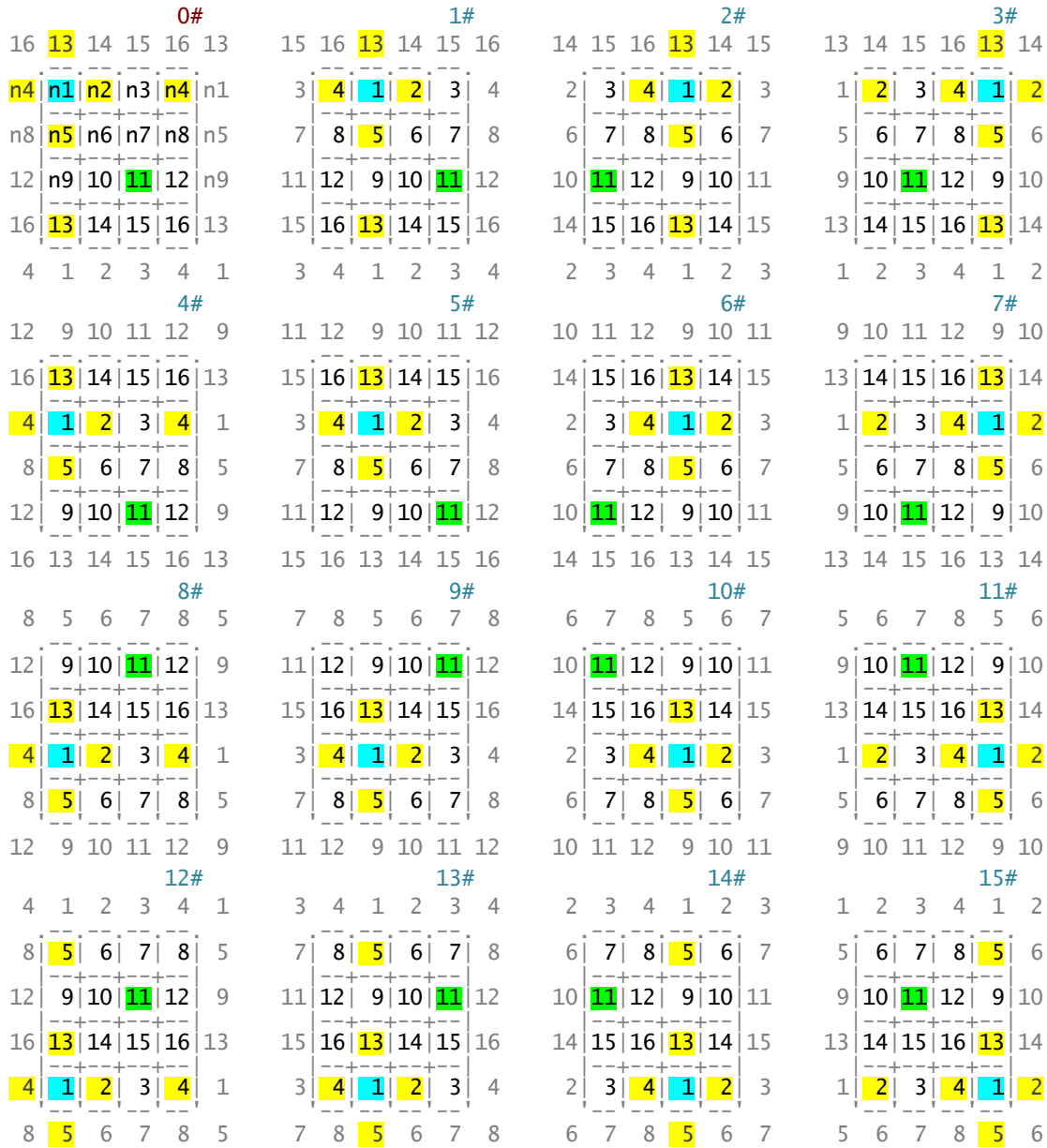
5 Groups: (S 9, S25) (S13, S21) (S15, S19) (S16, S18) (S17=S17)
( 1, X) = {( 1, 8), ( 1,12), ( 1,14), ( 1,15), ( 1,16)};
( 2, X) = {( 2, 7), ( 2,11), ( 2,13), ( 2,16), ( 2,15)};
( 3, X) = {( 3, 6), ( 3,10), ( 3,16), ( 3,13), ( 3,14)};
( 4, X) = {( 4, 5), ( 4, 9), ( 4,15), ( 4,14), ( 4,13)};
( 5, X) = {( 5, 4), ( 5,16), ( 5,10), ( 5,11), ( 5,12)};
( 6, X) = {( 6, 3), ( 6,15), ( 6, 9), ( 6,12), ( 6,11)};
( 7, X) = {( 7, 2), ( 7,14), ( 7,12), ( 7, 9), ( 7,10)};
( 8, X) = {( 8, 1), ( 8,13), ( 8,11), ( 8,10), ( 8, 9)};
( 9, X) = {( 9,16), ( 9, 4), ( 9, 6), ( 9, 7), ( 9, 8)};
(10, X) = {(10,15), (10, 3), (10, 5), (10, 8), (10, 7)};
(11, X) = {(11,14), (11, 2), (11, 8), (11, 5), (11, 6)};
(12, X) = {(12,13), (12, 1), (12, 7), (12, 6), (12, 5)};
(13, X) = {(13,12), (13, 8), (13, 2), (13, 3), (13, 4)};
(14, X) = {(14,11), (14, 7), (14, 1), (14, 4), (14, 3)};
(15, X) = {(15,10), (15, 6), (15, 4), (15, 1), (15, 2)};
(16, X) = {(16, 9), (16, 5), (16, 3), (16, 2), (16, 1)};

```

\*\* Calculated and Listed by Kanji Setsuda on  
Dec. 9, 2014 with MacOSX 10.10.1 & Xcode 6.1.1 \*\*

**#8.** Let me prove  $24 \times 16 = 384$  in another way here by making some experiment.  
 Let's reconstruct all the 384 solutions by some special Transformation Method making from the basic data 24 solutions when  $n1=1$ .  
 What does that transformation method look like, then?  
 Let me show you what is in my mind now with the conceptual diagrams as below.

**\*\* Basic Form of CCMS44 and the Concepts of 15 Transformations \*\***



I would like to move the number **1** regularly from the original position  $n1$  to  $n2$ ,  $n3$ ,  $n4$ ,  $n5$ , ...,  $n15$ ,  $n16$  in a consecutive order along with its fixed 5 'followers'.  
 Do we have any method of this kind for those transformations?  
 Please watch and check if the content of each row and column keeps same and unchanged or not. You can understand every 4 numbers are kept unchanged.  
 This method is exactly the same with the famous 'Pan-magic Shift' you know well.  
 Let's adopt this Pan-magic Shift for our transformation method now and reconstruct all the 384 solutions from the 24 basic data when  $n1=1$ , shall we?  
 Let me demonstrate all the results of my reconstruction as follows.

**\*\* Transform 24 by Pan-magic Shift and Reconstruct all the 384 \*\***  
**\*\* List of 24 Solutions when  $n1=1$  and the 15 Transformations of each \*\***

Group 1: (Top of each group is the Basic Data when n1=1 copied from the original list.)

1#	286#	129#	346#
13 8 10 3 13 8	3 13 8 10 3 13	10 3 13 8 10 3	8 10 3 13 8 10
12 1 15 6 12 1	6 12 1 15 6 12	15 6 12 1 15 6	1 15 6 12 1 15
7 14 4 9 7 14	9 7 14 4 9 7	4 9 7 14 4 9	14 4 9 7 14 4
2 11 5 16 2 11	16 2 11 5 16 2	5 16 2 11 5 16	11 5 16 2 11 5
13 8 10 3 13 8	3 13 8 10 3 13	10 3 13 8 10 3	8 10 3 13 8 10
12 1 15 6 12 1	6 12 1 15 6 12	15 6 12 1 15 6	1 15 6 12 1 15
185#	295#	60#	235#
2 11 5 16 2 11	16 2 11 5 16 2	5 16 2 11 5 16	11 5 16 2 11 5
13 8 10 3 13 8	3 13 8 10 3 13	10 3 13 8 10 3	8 10 3 13 8 10
12 1 15 6 12 1	6 12 1 15 6 12	15 6 12 1 15 6	1 15 6 12 1 15
7 14 4 9 7 14	9 7 14 4 9 7	4 9 7 14 4 9	14 4 9 7 14 4
2 11 5 16 2 11	16 2 11 5 16 2	5 16 2 11 5 16	11 5 16 2 11 5
13 8 10 3 13 8	3 13 8 10 3 13	10 3 13 8 10 3	8 10 3 13 8 10
256#	39#	384#	99#
7 14 4 9 7 14	9 7 14 4 9 7	4 9 7 14 4 9	14 4 9 7 14 4
2 11 5 16 2 11	16 2 11 5 16 2	5 16 2 11 5 16	11 5 16 2 11 5
13 8 10 3 13 8	3 13 8 10 3 13	10 3 13 8 10 3	8 10 3 13 8 10
12 1 15 6 12 1	6 12 1 15 6 12	15 6 12 1 15 6	1 15 6 12 1 15
7 14 4 9 7 14	9 7 14 4 9 7	4 9 7 14 4 9	14 4 9 7 14 4
2 11 5 16 2 11	16 2 11 5 16 2	5 16 2 11 5 16	11 5 16 2 11 5
325#	150#	200#	90#
12 1 15 6 12 1	6 12 1 15 6 12	15 6 12 1 15 6	1 15 6 12 1 15
7 14 4 9 7 14	9 7 14 4 9 7	4 9 7 14 4 9	14 4 9 7 14 4
2 11 5 16 2 11	16 2 11 5 16 2	5 16 2 11 5 16	11 5 16 2 11 5
13 8 10 3 13 8	3 13 8 10 3 13	10 3 13 8 10 3	8 10 3 13 8 10
12 1 15 6 12 1	6 12 1 15 6 12	15 6 12 1 15 6	1 15 6 12 1 15
7 14 4 9 7 14	9 7 14 4 9 7	4 9 7 14 4 9	14 4 9 7 14 4

Group 2: (Each Number# on the right shoulder is the Solution Number in the old list.)

2#	190#	225#	340#
13 12 6 3 13 12	3 13 12 6 3 13	6 3 13 12 6 3	12 6 3 13 12 6
8 1 15 10 8 1	10 8 1 15 10 8	15 10 8 1 15 10	1 15 10 8 1 15
11 14 4 5 11 14	5 11 14 4 5 11	4 5 11 14 4 5	14 4 5 11 14 4
2 7 9 16 2 7	16 2 7 9 16 2	9 16 2 7 9 16	7 9 16 2 7 9
13 12 6 3 13 12	3 13 12 6 3 13	6 3 13 12 6 3	12 6 3 13 12 6
8 1 15 10 8 1	10 8 1 15 10 8	15 10 8 1 15 10	1 15 10 8 1 15
281#	289#	58#	139#
2 7 9 16 2 7	16 2 7 9 16 2	9 16 2 7 9 16	7 9 16 2 7 9
13 12 6 3 13 12	3 13 12 6 3 13	6 3 13 12 6 3	12 6 3 13 12 6
8 1 15 10 8 1	10 8 1 15 10 8	15 10 8 1 15 10	1 15 10 8 1 15
11 14 4 5 11 14	5 11 14 4 5 11	4 5 11 14 4 5	14 4 5 11 14 4
2 7 9 16 2 7	16 2 7 9 16 2	9 16 2 7 9 16	7 9 16 2 7 9
13 12 6 3 13 12	3 13 12 6 3 13	6 3 13 12 6 3	12 6 3 13 12 6



<p>160#</p> <pre> 11 14 4 5 11 14  2   7   9   16   2   7  13   12   6   3   13   12  8   1   15   10   8   1  11   14   4   5   11   14  2   7   9   16   2   7 </pre>	<p>45#</p> <pre> 5 11 14 4 5 11 16   2   7   9   16   2  3   13   12   6   3   13 10   8   1   15   10   8  5   11   14   4   5   11 16   2   7   9   16   2 </pre>	<p>383#</p> <pre> 4 5 11 14 4 5  9   16   2   7   9   16  6   3   13   12   6   3 15   10   8   1   15   10  4   5   11   14   4   5  9   16   2   7   9   16 </pre>	<p>195#</p> <pre> 14 4 5 11 14 4  7   9   16   2   7   9 12   6   3   13   12   6  1   15   10   8   1   15 14   4   5   11   14   4  7   9   16   2   7   9 </pre>
<p>327#</p> <pre> 8 1 15 10 8 1 11   14   4   5   11   14  2   7   9   16   2   7 13   12   6   3   13   12  8   1   15   10   8   1 11   14   4   5   11   14 </pre>	<p>246#</p> <pre> 10 8 1 15 10 8  5   11   14   4   5   11 16   2   7   9   16   2  3   13   12   6   3   13 10   8   1   15   10   8  5   11   14   4   5   11 </pre>	<p>104#</p> <pre> 15 10 8 1 15 10  4   5   11   14   4   5  9   16   2   7   9   16  6   3   13   12   6   3 15   10   8   1   15   10  4   5   11   14   4   5 </pre>	<p>96#</p> <pre> 1 15 10 8 1 15 14   4   5   11   14   4  7   9   16   2   7   9 12   6   3   13   12   6  1   15   10   8   1   15 14   4   5   11   14   4 </pre>

Group 3: (Each Number# on the right shoulder is the Solution Number in the old list.)

<p>3#</p> <pre> 11 8 10 5 11 8 14   1   15   4   14   1  7   12   6   9   7   12  2   13   3   16   2   13 11 8 10 5 11 8 14 1 15 4 14 1 </pre>	<p>334#</p> <pre> 5 11 8 10 5 11  4   14   1   15   4   14  9   7   12   6   9   7 16   2   13   3   16   2  5   11   8   10   5   11  4   14   1   15   4   14 </pre>	<p>81#</p> <pre> 10 5 11 8 10 5 15   4   14   1   15   4  6   9   7   12   6   9  3   16   2   13   3   16 10 5 11 8 10 5 15 4 14 1 15 4 </pre>	<p>352#</p> <pre> 8 10 5 11 8 10  1   15   4   14   1   15 12   6   9   7   12   6 13   3   16   2   13   3  8   10   5   11   8   10  1   15   4   14   1   15 </pre>
<p>186#</p> <pre> 2 13 3 16 2 13 11   8   10   5   11   8 14   1   15   4   14   1  7   12   6   9   7   12  2   13   3   16   2   13 11 8 10 5 11 8 </pre>	<p>247#</p> <pre> 16 2 13 3 16 2  5   11   8   10   5   11  4   14   1   15   4   14  9   7   12   6   9   7 16   2   13   3   16   2  5 11 8 10 5 11 </pre>	<p>108#</p> <pre> 3 16 2 13 3 16 10   5   11   8   10   5 15   4   14   1   15   4  6   9   7   12   6   9  3   16   2   13   3   16 10 5 11 8 10 5 </pre>	<p>229#</p> <pre> 13 3 16 2 13 3  8   10   5   11   8   10  1   15   4   14   1   15 12   6   9   7   12   6 13   3   16   2   13   3  8 10 5 11 8 10 </pre>
<p>304#</p> <pre> 7 12 6 9 7 12  2   13   3   16   2   13 11   8   10   5   11   8 14   1   15   4   14   1  7   12   6   9   7   12  2   13   3   16   2   13 </pre>	<p>33#</p> <pre> 9 7 12 6 9 7 16   2   13   3   16   2  5   11   8   10   5   11  4   14   1   15   4   14  9   7   12   6   9   7 16 2 13 3 16 2 </pre>	<p>382#</p> <pre> 6 9 7 12 6 9  3   16   2   13   3   16 10   5   11   8   10   5 15   4   14   1   15   4  6   9   7   12   6   9  3   16   2   13   3   16 </pre>	<p>51#</p> <pre> 12 6 9 7 12 6 13   3   16   2   13   3  8   10   5   11   8   10  1   15   4   14   1   15 12   6   9   7   12   6 13 3 16 2 13 3 </pre>
<p>277#</p> <pre> 14 1 15 4 14 1  7   12   6   9   7   12  2   13   3   16   2   13 11   8   10   5   11   8 14   1   15   4   14   1  7   12   6   9   7   12 </pre>	<p>156#</p> <pre> 4 14 1 15 4 14  9   7   12   6   9   7 16   2   13   3   16   2  5   11   8   10   5   11  4   14   1   15   4   14  9 7 12 6 9 7 </pre>	<p>199#</p> <pre> 15 4 14 1 15 4  6   9   7   12   6   9  3   16   2   13   3   16 10   5   11   8   10   5 15   4   14   1   15   4  6 9 7 12 6 9 </pre>	<p>138#</p> <pre> 1 15 4 14 1 15 12   6   9   7   12   6 13   3   16   2   13   3  8   10   5   11   8   10  1   15   4   14   1   15 12 6 9 7 12 6 </pre>

Group 4:

<b>4#</b>	188#	227#	338#	329#	241#
1 15 10 8 12 6 3 13 7 9 16 2 14 4 5 11	8 1 15 10 13 12 6 3 2 7 9 16 11 14 4 5	10 8 1 15 3 13 12 6 16 2 7 9 5 11 14 4	15 10 8 1 6 3 13 12 9 16 2 7 4 5 11 14	14 4 5 11 1 15 10 8 12 6 3 13 7 9 16 2	11 14 4 5 8 1 15 10 13 12 6 3 2 7 9 16
106#	91#	158#	47#	381#	197#
5 11 14 4 10 8 1 15 3 13 12 6 16 2 7 9	4 5 11 14 15 10 8 1 6 3 13 12 9 16 2 7	7 9 16 2 14 4 5 11 1 15 10 8 12 6 3 13	2 7 9 16 11 14 4 5 8 1 15 10 13 12 6 3	16 2 7 9 5 11 14 4 10 8 1 15 3 13 12 6	9 16 2 7 4 5 11 14 15 10 8 1 6 3 13 12
279#	294#	56#	144#		
12 6 3 13 7 9 16 2 14 4 5 11 1 15 10 8	13 12 6 3 2 7 9 16 11 14 4 5 8 1 15 10	3 13 12 6 16 2 7 9 5 11 14 4 10 8 1 15	6 3 13 12 9 16 2 7 4 5 11 14 15 10 8 1		

Group 5:

<b>5#</b>	332#	83#	350#	282#	151#
1 15 4 14 8 10 5 11 13 3 16 2 12 6 9 7	14 1 15 4 11 8 10 5 2 13 3 16 7 12 6 9	4 14 1 15 5 11 8 10 16 2 13 3 9 7 12 6	15 4 14 1 10 5 11 8 3 16 2 13 6 9 7 12	12 6 9 7 1 15 4 14 8 10 5 11 13 3 16 2	7 12 6 9 14 1 15 4 11 8 10 5 2 13 3 16
204#	133#	302#	35#	380#	53#
9 7 12 6 4 14 1 15 5 11 8 10 16 2 13 3	6 9 7 12 15 4 14 1 10 5 11 8 3 16 2 13	13 3 16 2 12 6 9 7 1 15 4 14 8 10 5 11	2 13 3 16 7 12 6 9 14 1 15 4 11 8 10 5	16 2 13 3 9 7 12 6 4 14 1 15 5 11 8 10	3 16 2 13 6 9 7 12 15 4 14 1 10 5 11 8
181#	252#	103#	234#		
8 10 5 11 13 3 16 2 12 6 9 7 1 15 4 14	11 8 10 5 2 13 3 16 7 12 6 9 14 1 15 4	5 11 8 10 16 2 13 3 9 7 12 6 4 14 1 15	10 5 11 8 3 16 2 13 6 9 7 12 15 4 14 1		

Group 6:

<b>6#</b>	284#	131#	344#	330#	145#
1 15 6 12 8 10 3 13 11 5 16 2 14 4 9 7	12 1 15 6 13 8 10 3 2 11 5 16 7 14 4 9	6 12 1 15 3 13 8 10 16 2 11 5 9 7 14 4	15 6 12 1 10 3 13 8 5 16 2 11 4 9 7 14	14 4 9 7 1 15 6 12 8 10 3 13 11 5 16 2	7 14 4 9 12 1 15 6 13 8 10 3 2 11 5 16
202#	85#	254#	41#	379#	101#
9 7 14 4 6 12 1 15 3 13 8 10 16 2 11 5	4 9 7 14 15 6 12 1 10 3 13 8 5 16 2 11	11 5 16 2 14 4 9 7 1 15 6 12 8 10 3 13	2 11 5 16 7 14 4 9 12 1 15 6 13 8 10 3	16 2 11 5 9 7 14 4 6 12 1 15 3 13 8 10	5 16 2 11 4 9 7 14 15 6 12 1 10 3 13 8
183#	300#	55#	240#		
8 10 3 13 11 5 16 2 14 4 9 7 1 15 6 12	13 8 10 3 2 11 5 16 7 14 4 9 12 1 15 6	3 13 8 10 16 2 11 5 9 7 14 4 6 12 1 15	10 3 13 8 5 16 2 11 4 9 7 14 15 6 12 1		

Group 7:

<b>7#</b>	288#	153#	322#	179#	296#
1 14 7 12 15 4 9 6 10 5 16 3 8 11 2 13	12 1 14 7 6 15 4 9 3 10 5 16 13 8 11 2	7 12 1 14 9 6 15 4 16 3 10 5 2 13 8 11	14 7 12 1 4 9 6 15 5 16 3 10 11 2 13 8	8 11 2 13 1 14 7 12 15 4 9 6 10 5 16 3	13 8 11 2 12 1 14 7 6 15 4 9 3 10 5 16

36#	259#	232#	63#	378#	97#
2 13 8 11	11 2 13 8	10 5 16 3	3 10 5 16	16 3 10 5	5 16 3 10
7 12 1 14	14 7 12 1	8 11 2 13	13 8 11 2	2 13 8 11	11 2 13 8
9 6 15 4	4 9 6 15	1 14 7 12	12 1 14 7	7 12 1 14	14 7 12 1
16 3 10 5	5 16 3 10	15 4 9 6	6 15 4 9	9 6 15 4	4 9 6 15

349#	126#	206#	89#
15 4 9 6	6 15 4 9	9 6 15 4	4 9 6 15
10 5 16 3	3 10 5 16	16 3 10 5	5 16 3 10
8 11 2 13	13 8 11 2	2 13 8 11	11 2 13 8
1 14 7 12	12 1 14 7	7 12 1 14	14 7 12 1

**Group 8:**

8#	192#	249#	316#	275#	290#
1 14 11 8	8 1 14 11	11 8 1 14	14 11 8 1	12 7 2 13	13 12 7 2
15 4 5 10	10 15 4 5	5 10 15 4	4 5 10 15	1 14 11 8	8 1 14 11
6 9 16 3	3 6 9 16	16 3 6 9	9 16 3 6	15 4 5 10	10 15 4 5
12 7 2 13	13 12 7 2	2 13 12 7	7 2 13 12	6 9 16 3	3 6 9 16

34#	163#	136#	69#	377#	193#
2 13 12 7	7 2 13 12	6 9 16 3	3 6 9 16	16 3 6 9	9 16 3 6
11 8 1 14	14 11 8 1	12 7 2 13	13 12 7 2	2 13 12 7	7 2 13 12
5 10 15 4	4 5 10 15	1 14 11 8	8 1 14 11	11 8 1 14	14 11 8 1
16 3 6 9	9 16 3 6	15 4 5 10	10 15 4 5	5 10 15 4	4 5 10 15

351#	222#	110#	95#
15 4 5 10	10 15 4 5	5 10 15 4	4 5 10 15
6 9 16 3	3 6 9 16	16 3 6 9	9 16 3 6
12 7 2 13	13 12 7 2	2 13 12 7	7 2 13 12
1 14 11 8	8 1 14 11	11 8 1 14	14 11 8 1

**Group 9:**

9#	358#	75#	328#	180#	223#	114#	253#
1 14 4 15	15 1 14 4	4 15 1 14	14 4 15 1	8 11 5 10	10 8 11 5	5 10 8 11	11 5 10 8
12 7 9 6	6 12 7 9	9 6 12 7	7 9 6 12	1 14 4 15	15 1 14 4	4 15 1 14	14 4 15 1
13 2 16 3	3 13 2 16	16 3 13 2	2 16 3 13	12 7 9 6	6 12 7 9	9 6 12 7	7 9 6 12
8 11 5 10	10 8 11 5	5 10 8 11	11 5 10 8	13 2 16 3	3 13 2 16	16 3 13 2	2 16 3 13

310#	57#	376#	27#	271#	132#	205#	162#
13 2 16 3	3 13 2 16	16 3 13 2	2 16 3 13	12 7 9 6	6 12 7 9	9 6 12 7	7 9 6 12
8 11 5 10	10 8 11 5	5 10 8 11	11 5 10 8	13 2 16 3	3 13 2 16	16 3 13 2	2 16 3 13
1 14 4 15	15 1 14 4	4 15 1 14	14 4 15 1	8 11 5 10	10 8 11 5	5 10 8 11	11 5 10 8
12 7 9 6	6 12 7 9	9 6 12 7	7 9 6 12	1 14 4 15	15 1 14 4	4 15 1 14	14 4 15 1

**Group 10:**

10#	187#	251#	314#	353#	217#	112#	93#
1 14 11 8	8 1 14 11	11 8 1 14	14 11 8 1	15 4 5 10	10 15 4 5	5 10 15 4	4 5 10 15
12 7 2 13	13 12 7 2	2 13 12 7	7 2 13 12	1 14 11 8	8 1 14 11	11 8 1 14	14 11 8 1
6 9 16 3	3 6 9 16	16 3 6 9	9 16 3 6	12 7 2 13	13 12 7 2	2 13 12 7	7 2 13 12
15 4 5 10	10 15 4 5	5 10 15 4	4 5 10 15	6 9 16 3	3 6 9 16	16 3 6 9	9 16 3 6

134#	71#	375#	198#	273#	292#	32#	168#
6 9 16 3	3 6 9 16	16 3 6 9	9 16 3 6	12 7 2 13	13 12 7 2	2 13 12 7	7 2 13 12
15 4 5 10	10 15 4 5	5 10 15 4	4 5 10 15	6 9 16 3	3 6 9 16	16 3 6 9	9 16 3 6
1 14 11 8	8 1 14 11	11 8 1 14	14 11 8 1	15 4 5 10	10 15 4 5	5 10 15 4	4 5 10 15
12 7 2 13	13 12 7 2	2 13 12 7	7 2 13 12	1 14 11 8	8 1 14 11	11 8 1 14	14 11 8 1

**Group 11:**

11#	356#	77#	326#	276#	127#	210#	157#
1 14 4 15	15 1 14 4	4 15 1 14	14 4 15 1	12 7 9 6	6 12 7 9	9 6 12 7	7 9 6 12
8 11 5 10	10 8 11 5	5 10 8 11	11 5 10 8	1 14 4 15	15 1 14 4	4 15 1 14	14 4 15 1
13 2 16 3	3 13 2 16	16 3 13 2	2 16 3 13	8 11 5 10	10 8 11 5	5 10 8 11	11 5 10 8
12 7 9 6	6 12 7 9	9 6 12 7	7 9 6 12	13 2 16 3	3 13 2 16	16 3 13 2	2 16 3 13

308#	59#	374#	29#	175#	228#	109#	258#
13 2 16 3	3 13 2 16	16 3 13 2	2 16 3 13	8 11 5 10	10 8 11 5	5 10 8 11	11 5 10 8
12 7 9 6	6 12 7 9	9 6 12 7	7 9 6 12	13 2 16 3	3 13 2 16	16 3 13 2	2 16 3 13
1 14 4 15	15 1 14 4	4 15 1 14	14 4 15 1	12 7 9 6	6 12 7 9	9 6 12 7	7 9 6 12
8 11 5 10	10 8 11 5	5 10 8 11	11 5 10 8	1 14 4 15	15 1 14 4	4 15 1 14	14 4 15 1

**Group 12:**

12#	283#	155#	320#	354#	121#	208#	87#
1 14 7 12	12 1 14 7	7 12 1 14	14 7 12 1	15 4 9 6	6 15 4 9	9 6 15 4	4 9 6 15
8 11 2 13	13 8 11 2	2 13 8 11	11 2 13 8	1 14 7 12	12 1 14 7	7 12 1 14	14 7 12 1
10 5 16 3	3 10 5 16	16 3 10 5	5 16 3 10	8 11 2 13	13 8 11 2	2 13 8 11	11 2 13 8
15 4 9 6	6 15 4 9	9 6 15 4	4 9 6 15	10 5 16 3	3 10 5 16	16 3 10 5	5 16 3 10

230#	65#	373#	102#	177#	298#	31#	264#
10 5 16 3	3 10 5 16	16 3 10 5	5 16 3 10	8 11 2 13	13 8 11 2	2 13 8 11	11 2 13 8
15 4 9 6	6 15 4 9	9 6 15 4	4 9 6 15	10 5 16 3	3 10 5 16	16 3 10 5	5 16 3 10
1 14 7 12	12 1 14 7	7 12 1 14	14 7 12 1	15 4 9 6	6 15 4 9	9 6 15 4	4 9 6 15
8 11 2 13	13 8 11 2	2 13 8 11	11 2 13 8	1 14 7 12	12 1 14 7	7 12 1 14	14 7 12 1

**Group 13:**

13#	336#	147#	274#	173#	248#	42#	307#
1 12 7 14	14 1 12 7	7 14 1 12	12 7 14 1	8 13 2 11	11 8 13 2	2 11 8 13	13 2 11 8
15 6 9 4	4 15 6 9	9 4 15 6	6 9 4 15	1 12 7 14	14 1 12 7	7 14 1 12	12 7 14 1
10 3 16 5	5 10 3 16	16 5 10 3	3 16 5 10	15 6 9 4	4 15 6 9	9 4 15 6	6 9 4 15
8 13 2 11	11 8 13 2	2 11 8 13	13 2 11 8	10 3 16 5	5 10 3 16	16 5 10 3	3 16 5 10

238#	111#	372#	49#	343#	78#	212#	137#
10 3 16 5	5 10 3 16	16 5 10 3	3 16 5 10	15 6 9 4	4 15 6 9	9 4 15 6	6 9 4 15
8 13 2 11	11 8 13 2	2 11 8 13	13 2 11 8	10 3 16 5	5 10 3 16	16 5 10 3	3 16 5 10
1 12 7 14	14 1 12 7	7 14 1 12	12 7 14 1	8 13 2 11	11 8 13 2	2 11 8 13	13 2 11 8
15 6 9 4	4 15 6 9	9 4 15 6	6 9 4 15	1 12 7 14	14 1 12 7	7 14 1 12	12 7 14 1

**Group 14:**

14#	191#	297#	268#	323#	242#	40#	165#
1 12 13 8	8 1 12 13	13 8 1 12	12 13 8 1	14 7 2 11	11 14 7 2	2 11 14 7	7 2 11 14
15 6 3 10	10 15 6 3	3 10 15 6	6 3 10 15	1 12 13 8	8 1 12 13	13 8 1 12	12 13 8 1
4 9 16 5	5 4 9 16	16 5 4 9	9 16 5 4	15 6 3 10	10 15 6 3	3 10 15 6	6 3 10 15
14 7 2 11	11 14 7 2	2 11 14 7	7 2 11 14	4 9 16 5	5 4 9 16	16 5 4 9	9 16 5 4

88#	117#	371#	194#	345#	220#	62#	143#
4 9 16 5	5 4 9 16	16 5 4 9	9 16 5 4	15 6 3 10	10 15 6 3	3 10 15 6	6 3 10 15
14 7 2 11	11 14 7 2	2 11 14 7	7 2 11 14	4 9 16 5	5 4 9 16	16 5 4 9	9 16 5 4
1 12 13 8	8 1 12 13	13 8 1 12	12 13 8 1	14 7 2 11	11 14 7 2	2 11 14 7	7 2 11 14
15 6 3 10	10 15 6 3	3 10 15 6	6 3 10 15	1 12 13 8	8 1 12 13	13 8 1 12	12 13 8 1

**Group 15:**

15#	360#	123#	280#	174#	224#	66#	301#
1 12 6 15	15 1 12 6	6 15 1 12	12 6 15 1	8 13 3 10	10 8 13 3	3 10 8 13	13 3 10 8
14 7 9 4	4 14 7 9	9 4 14 7	7 9 4 14	1 12 6 15	15 1 12 6	6 15 1 12	12 6 15 1
11 2 16 5	5 11 2 16	16 5 11 2	2 16 5 11	14 7 9 4	4 14 7 9	9 4 14 7	7 9 4 14
8 13 3 10	10 8 13 3	3 10 8 13	13 3 10 8	11 2 16 5	5 11 2 16	16 5 11 2	2 16 5 11

262#	105#	370#	25#	319#	84#	211#	161#
11 2 16 5	5 11 2 16	16 5 11 2	2 16 5 11	14 7 9 4	4 14 7 9	9 4 14 7	7 9 4 14
8 13 3 10	10 8 13 3	3 10 8 13	13 3 10 8	11 2 16 5	5 11 2 16	16 5 11 2	2 16 5 11
1 12 6 15	15 1 12 6	6 15 1 12	12 6 15 1	8 13 3 10	10 8 13 3	3 10 8 13	13 3 10 8
14 7 9 4	4 14 7 9	9 4 14 7	7 9 4 14	1 12 6 15	15 1 12 6	6 15 1 12	12 6 15 1

**Group 16:**

16#	189#	299#	266#	347#	218#	64#	141#
1 12 13 8	8 1 12 13	13 8 1 12	12 13 8 1	15 6 3 10	10 15 6 3	3 10 15 6	6 3 10 15
14 7 2 11	11 14 7 2	2 11 14 7	7 2 11 14	1 12 13 8	8 1 12 13	13 8 1 12	12 13 8 1
4 9 16 5	5 4 9 16	16 5 4 9	9 16 5 4	14 7 2 11	11 14 7 2	2 11 14 7	7 2 11 14
15 6 3 10	10 15 6 3	3 10 15 6	6 3 10 15	4 9 16 5	5 4 9 16	16 5 4 9	9 16 5 4

86#	119#	369#	196#	321#	244#	38#	167#
4 9 16 5	5 4 9 16	16 5 4 9	9 16 5 4	14 7 2 11	11 14 7 2	2 11 14 7	7 2 11 14
15 6 3 10	10 15 6 3	3 10 15 6	6 3 10 15	4 9 16 5	5 4 9 16	16 5 4 9	9 16 5 4
1 12 13 8	8 1 12 13	13 8 1 12	12 13 8 1	15 6 3 10	10 15 6 3	3 10 15 6	6 3 10 15
14 7 2 11	11 14 7 2	2 11 14 7	7 2 11 14	1 12 13 8	8 1 12 13	13 8 1 12	12 13 8 1

**Group 17:**

17#	355#	125#	278#	324#	79#	216#	159#
1 12 6 15	15 1 12 6	6 15 1 12	12 6 15 1	14 7 9 4	4 14 7 9	9 4 14 7	7 9 4 14
8 13 3 10	10 8 13 3	3 10 8 13	13 3 10 8	1 12 6 15	15 1 12 6	6 15 1 12	12 6 15 1
11 2 16 5	5 11 2 16	16 5 11 2	2 16 5 11	8 13 3 10	10 8 13 3	3 10 8 13	13 3 10 8
14 7 9 4	4 14 7 9	9 4 14 7	7 9 4 14	11 2 16 5	5 11 2 16	16 5 11 2	2 16 5 11

260#	107#	368#	30#	169#	226#	61#	306#
11 2 16 5	5 11 2 16	16 5 11 2	2 16 5 11	8 13 3 10	10 8 13 3	3 10 8 13	13 3 10 8
14 7 9 4	4 14 7 9	9 4 14 7	7 9 4 14	11 2 16 5	5 11 2 16	16 5 11 2	2 16 5 11
1 12 6 15	15 1 12 6	6 15 1 12	12 6 15 1	14 7 9 4	4 14 7 9	9 4 14 7	7 9 4 14
8 13 3 10	10 8 13 3	3 10 8 13	13 3 10 8	1 12 6 15	15 1 12 6	6 15 1 12	12 6 15 1

**Group 18:**

18#	331#	149#	272#	348#	73#	214#	135#
1 12 7 14	14 1 12 7	7 14 1 12	12 7 14 1	15 6 9 4	4 15 6 9	9 4 15 6	6 9 4 15
8 13 2 11	11 8 13 2	2 11 8 13	13 2 11 8	1 12 7 14	14 1 12 7	7 14 1 12	12 7 14 1
10 3 16 5	5 10 3 16	16 5 10 3	3 16 5 10	8 13 2 11	11 8 13 2	2 11 8 13	13 2 11 8
15 6 9 4	4 15 6 9	9 4 15 6	6 9 4 15	10 3 16 5	5 10 3 16	16 5 10 3	3 16 5 10

236#	113#	367#	54#	171#	250#	37#	312#
10 3 16 5	5 10 3 16	16 5 10 3	3 16 5 10	8 13 2 11	11 8 13 2	2 11 8 13	13 2 11 8
15 6 9 4	4 15 6 9	9 4 15 6	6 9 4 15	10 3 16 5	5 10 3 16	16 5 10 3	3 16 5 10
1 12 7 14	14 1 12 7	7 14 1 12	12 7 14 1	15 6 9 4	4 15 6 9	9 4 15 6	6 9 4 15
8 13 2 11	11 8 13 2	2 11 8 13	13 2 11 8	1 12 7 14	14 1 12 7	7 14 1 12	12 7 14 1

**Group 19:**

19#	335#	243#	178#	269#	152#	48#	309#
1 8 11 14	14 1 8 11	11 14 1 8	8 11 14 1	12 13 2 7	7 12 13 2	2 7 12 13	13 2 7 12
15 10 5 4	4 15 10 5	5 4 15 10	10 5 4 15	1 8 11 14	14 1 8 11	11 14 1 8	8 11 14 1
6 3 16 9	9 6 3 16	16 9 6 3	3 16 9 6	15 10 5 4	4 15 10 5	5 4 15 10	10 5 4 15
12 13 2 7	7 12 13 2	2 7 12 13	13 2 7 12	6 3 16 9	9 6 3 16	16 9 6 3	3 16 9 6

142#	207#	366#	50#	337#	76#	116#	233#
6 3 16 9	9 6 3 16	16 9 6 3	3 16 9 6	15 10 5 4	4 15 10 5	5 4 15 10	10 5 4 15
12 13 2 7	7 12 13 2	2 7 12 13	13 2 7 12	6 3 16 9	9 6 3 16	16 9 6 3	3 16 9 6
1 8 11 14	14 1 8 11	11 14 1 8	8 11 14 1	12 13 2 7	7 12 13 2	2 7 12 13	13 2 7 12
15 10 5 4	4 15 10 5	5 4 15 10	10 5 4 15	1 8 11 14	14 1 8 11	11 14 1 8	8 11 14 1

**Group 20:**

20#	287#	291#	172#	317#	146#	46#	261#
1 8 13 12	12 1 8 13	13 12 1 8	8 13 12 1	14 11 2 7	7 14 11 2	2 7 14 11	11 2 7 14
15 10 3 6	6 15 10 3	3 6 15 10	10 3 6 15	1 8 13 12	12 1 8 13	13 12 1 8	8 13 12 1
4 5 16 9	9 4 5 16	16 9 4 5	5 16 9 4	15 10 3 6	6 15 10 3	3 6 15 10	10 3 6 15
14 11 2 7	7 14 11 2	2 7 14 11	11 2 7 14	4 5 16 9	9 4 5 16	16 9 4 5	5 16 9 4

94#	213#	365#	98#	339#	124#	68#	239#
4 5 16 9	9 4 5 16	16 9 4 5	5 16 9 4	15 10 3 6	6 15 10 3	3 6 15 10	10 3 6 15
14 11 2 7	7 14 11 2	2 7 14 11	11 2 7 14	4 5 16 9	9 4 5 16	16 9 4 5	5 16 9 4
1 8 13 12	12 1 8 13	13 12 1 8	8 13 12 1	14 11 2 7	7 14 11 2	2 7 14 11	11 2 7 14
15 10 3 6	6 15 10 3	3 6 15 10	10 3 6 15	1 8 13 12	12 1 8 13	13 12 1 8	8 13 12 1

**Group 21:**

21#	359#	219#	184#	270#	128#	72#	303#
1 8 10 15	15 1 8 10	10 15 1 8	8 10 15 1	12 13 3 6	6 12 13 3	3 6 12 13	13 3 6 12
14 11 5 4	4 14 11 5	5 4 14 11	11 5 4 14	1 8 10 15	15 1 8 10	10 15 1 8	8 10 15 1
7 2 16 9	9 7 2 16	16 9 7 2	2 16 9 7	14 11 5 4	4 14 11 5	5 4 14 11	11 5 4 14
12 13 3 6	6 12 13 3	3 6 12 13	13 3 6 12	7 2 16 9	9 7 2 16	16 9 7 2	2 16 9 7



## #9. How about the case of 24 basic data when $n_1=2$ ? Can we also reconstruct all?

\*\* List of 24 Solutions when  $n_1=2$  and the 15 Transformations of each \*\*

### Group 1:

25#	262#	105#	370#	161#	319#	84#	211#
2 16 5 11	11 2 16 5	5 11 2 16	16 5 11 2	7 9 4 14	14 7 9 4	4 14 7 9	9 4 14 7
13 3 10 8	8 13 3 10	10 8 13 3	3 10 8 13	2 16 5 11	11 2 16 5	5 11 2 16	16 5 11 2
12 6 15 1	1 12 6 15	15 1 12 6	6 15 1 12	13 3 10 8	8 13 3 10	10 8 13 3	3 10 8 13
7 9 4 14	14 7 9 4	4 14 7 9	9 4 14 7	12 6 15 1	1 12 6 15	15 1 12 6	6 15 1 12
280#	15#	360#	123#	301#	174#	224#	66#
12 6 15 1	1 12 6 15	15 1 12 6	6 15 1 12	13 3 10 8	8 13 3 10	10 8 13 3	3 10 8 13
7 9 4 14	14 7 9 4	4 14 7 9	9 4 14 7	12 6 15 1	1 12 6 15	15 1 12 6	6 15 1 12
2 16 5 11	11 2 16 5	5 11 2 16	16 5 11 2	7 9 4 14	14 7 9 4	4 14 7 9	9 4 14 7
13 3 10 8	8 13 3 10	10 8 13 3	3 10 8 13	2 16 5 11	11 2 16 5	5 11 2 16	16 5 11 2

### Group 2:

26#	166#	201#	364#	257#	313#	82#	115#
2 16 9 7	7 2 16 9	9 7 2 16	16 9 7 2	11 5 4 14	14 11 5 4	4 14 11 5	5 4 14 11
13 3 6 12	12 13 3 6	6 12 13 3	3 6 12 13	2 16 9 7	7 2 16 9	9 7 2 16	16 9 7 2
8 10 15 1	1 8 10 15	15 1 8 10	10 15 1 8	13 3 6 12	12 13 3 6	6 12 13 3	3 6 12 13
11 5 4 14	14 11 5 4	4 14 11 5	5 4 14 11	8 10 15 1	1 8 10 15	15 1 8 10	10 15 1 8
184#	21#	359#	219#	303#	270#	128#	72#
8 10 15 1	1 8 10 15	15 1 8 10	10 15 1 8	13 3 6 12	12 13 3 6	6 12 13 3	3 6 12 13
11 5 4 14	14 11 5 4	4 14 11 5	5 4 14 11	8 10 15 1	1 8 10 15	15 1 8 10	10 15 1 8
2 16 9 7	7 2 16 9	9 7 2 16	16 9 7 2	11 5 4 14	14 11 5 4	4 14 11 5	5 4 14 11
13 3 6 12	12 13 3 6	6 12 13 3	3 6 12 13	2 16 9 7	7 2 16 9	9 7 2 16	16 9 7 2

### Group 3:

27#	310#	57#	376#	162#	271#	132#	205#
2 16 3 13	13 2 16 3	3 13 2 16	16 3 13 2	7 9 6 12	12 7 9 6	6 12 7 9	9 6 12 7
11 5 10 8	8 11 5 10	10 8 11 5	5 10 8 11	2 16 3 13	13 2 16 3	3 13 2 16	16 3 13 2
14 4 15 1	1 14 4 15	15 1 14 4	4 15 1 14	11 5 10 8	8 11 5 10	10 8 11 5	5 10 8 11
7 9 6 12	12 7 9 6	6 12 7 9	9 6 12 7	14 4 15 1	1 14 4 15	15 1 14 4	4 15 1 14
328#	9#	358#	75#	253#	180#	223#	114#
14 4 15 1	1 14 4 15	15 1 14 4	4 15 1 14	11 5 10 8	8 11 5 10	10 8 11 5	5 10 8 11
7 9 6 12	12 7 9 6	6 12 7 9	9 6 12 7	14 4 15 1	1 14 4 15	15 1 14 4	4 15 1 14
2 16 3 13	13 2 16 3	3 13 2 16	16 3 13 2	7 9 6 12	12 7 9 6	6 12 7 9	9 6 12 7
11 5 10 8	8 11 5 10	10 8 11 5	5 10 8 11	2 16 3 13	13 2 16 3	3 13 2 16	16 3 13 2

### Group 4:

28#	164#	203#	362#	305#	265#	130#	67#
2 16 9 7	7 2 16 9	9 7 2 16	16 9 7 2	13 3 6 12	12 13 3 6	6 12 13 3	3 6 12 13
11 5 4 14	14 11 5 4	4 14 11 5	5 4 14 11	2 16 9 7	7 2 16 9	9 7 2 16	16 9 7 2
8 10 15 1	1 8 10 15	15 1 8 10	10 15 1 8	11 5 4 14	14 11 5 4	4 14 11 5	5 4 14 11
13 3 6 12	12 13 3 6	6 12 13 3	3 6 12 13	8 10 15 1	1 8 10 15	15 1 8 10	10 15 1 8
182#	23#	357#	221#	255#	318#	80#	120#
8 10 15 1	1 8 10 15	15 1 8 10	10 15 1 8	11 5 4 14	14 11 5 4	4 14 11 5	5 4 14 11
13 3 6 12	12 13 3 6	6 12 13 3	3 6 12 13	8 10 15 1	1 8 10 15	15 1 8 10	10 15 1 8
2 16 9 7	7 2 16 9	9 7 2 16	16 9 7 2	13 3 6 12	12 13 3 6	6 12 13 3	3 6 12 13
11 5 4 14	14 11 5 4	4 14 11 5	5 4 14 11	2 16 9 7	7 2 16 9	9 7 2 16	16 9 7 2

### Group 5:

29#	308#	59#	374#	258#	175#	228#	109#
2 16 3 13	13 2 16 3	3 13 2 16	16 3 13 2	11 5 10 8	8 11 5 10	10 8 11 5	5 10 8 11
7 9 6 12	12 7 9 6	6 12 7 9	9 6 12 7	2 16 3 13	13 2 16 3	3 13 2 16	16 3 13 2
14 4 15 1	1 14 4 15	15 1 14 4	4 15 1 14	7 9 6 12	12 7 9 6	6 12 7 9	9 6 12 7
11 5 10 8	8 11 5 10	10 8 11 5	5 10 8 11	14 4 15 1	1 14 4 15	15 1 14 4	4 15 1 14
326#	11#	356#	77#	157#	276#	127#	210#
14 4 15 1	1 14 4 15	15 1 14 4	4 15 1 14	7 9 6 12	12 7 9 6	6 12 7 9	9 6 12 7
11 5 10 8	8 11 5 10	10 8 11 5	5 10 8 11	14 4 15 1	1 14 4 15	15 1 14 4	4 15 1 14
2 16 3 13	13 2 16 3	3 13 2 16	16 3 13 2	11 5 10 8	8 11 5 10	10 8 11 5	5 10 8 11
7 9 6 12	12 7 9 6	6 12 7 9	9 6 12 7	2 16 3 13	13 2 16 3	3 13 2 16	16 3 13 2

### Group 6:

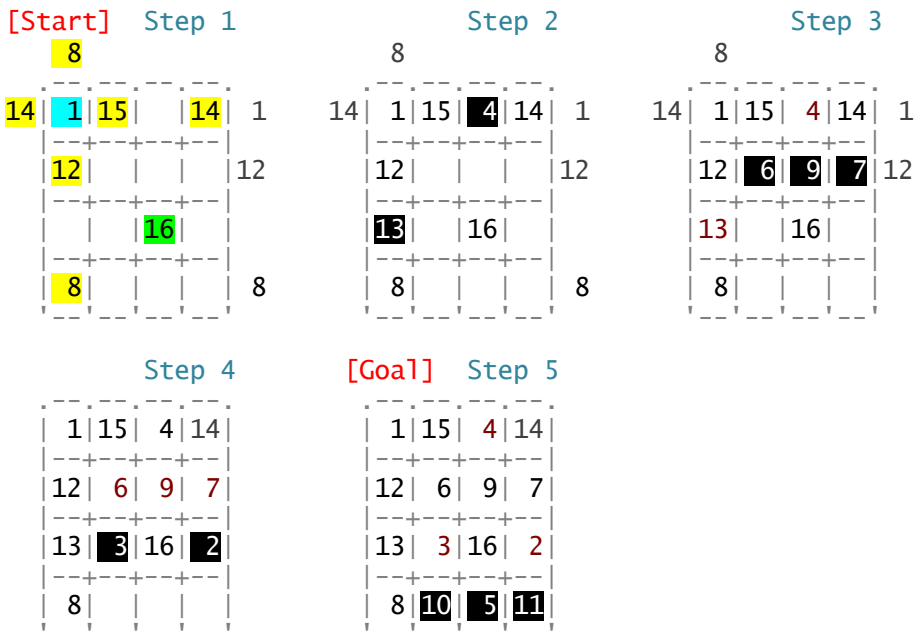




**#10.** Let's come back to the Standard Solutions of 'C&C' MS44 or Pan-diagonal MS44. Why do they have the 48 solutions? What does this 48 mean? If you factorize  $48 = 3 \times 2^4$ , can you understand what the factors mean? We had better step back to the 384 Primitive Solutions, and recall the factors of 384. We now know  $384 = {}_4P_4 \times 16 = 4 \times 3 \times 2 \times 1 \times 2^4$ ;  
 The Standard Solutions are collected from the set of Primitive Solutions under the Normalization, eliminating all 'Reflected Patterns' or 'Rotated Patterns', and actually chosen by the following Inequality conditions:  $n_1 < n_{16}$ ;  $n_1 < n_4 < n_{13}$ ; for instance.  
 You can finally have the  $48 (= 1/8 \times 384)$  solutions.  
 Then  $48 = 384/8 = (4 \times 3 \times 2 \times 1 \times 2^4)/8 = (8 \times 3 \times 16)/8 = 3 \times 16$ ;  
 The factor 3 means the 'inheritance' from  ${}_4P_4$  with a great loss.  
 And this 3 implies the count of 'Most Fundamental Solutions', I suppose.

**#11.** Let me introduce you something extra here at the end of this article. The Method: How to make the 'Composite & Complete' PMS44 by your own hand; You can make it only with your pen, paper and your brain, without using any PC.  
 (1) You must start everything with your great memory:  
 Put the number **1** on the Origin  $n_1$  as the Basic Diagram shows as follows.  
 Place the 4 numbers { **8**, **12**, **14** and **15** } around adjacent next to the number **1**.  
 You don't have to mind where they should sit among those 4 positions. They can sit freely as you like.  
 You can put the 2 same numbers on  $n_4$ (on the right top) and  $n_{13}$ (on the left bottom), can't you? You need to do so in the Extended Space.  
 Put the number **16**(Complementary Pair of 1) on  $n_{11}$ (2 steps left and 2 steps down from  $n_1$ ).

**\*\* How to make the Solution of 'C&C' MS44 by your own hand \*\***



- (2) From now on you have to do all only by your calculations. You can fill up the 2 numbers to the first 2 blank positions  $n_3$  and  $n_9$ . Use the definition of constant sums of rows and columns:  $C_{Sum}=34$ ;  
 $1+15+X+14=34$ ;  $X=34-1-15-14=4$ ; Put the number **4** on  $n_3$ .  
 $1+12+X+8=34$ ;  $X=34-1-12-8=13$ ; Put the number **13** on  $n_9$ .
- (3) You can fill the 3 numbers up to the second 3 blank positions  $n_6$ ,  $n_7$  and  $n_8$ . Use the definitions of 'Composite Conditions' of the 4 numbers:  $C_{Sum}=34$ ;

- $1+15+12+X=34$ ;  $X=34-1-15-12=6$ ; Put the Number **6** on n6.  
 $15+4+6+X=34$ ;  $X=34-15-4-6=9$ ; Put the Number **9** on n7.  
 $4+14+9+X=34$ ;  $X=34-4-14-9=7$ ; Put the Number **7** on n8.
- (4) Fill the next 2 blank positions using the Composite Conditions in the same way.
- $12+6+13+X=34$ ;  $X=34-12-6-13=3$ ; Put the Number **3** on n10.  
 $9+7+16+X=34$ ;  $X=34-9-7-16=2$ ; Put the Number **2** on n12.
- (5) You need no more explanation.
- $13+3+8+X=34$ ;  $X=34-13-3-8=10$ ; Put the Number **10** on n14.  
 $3+16+10+X=34$ ;  $X=34-3-16-10=5$ ; Put the Number **5** on n15.  
 $16+2+5+X=34$ ;  $X=34-16-2-5=11$ ; Put the Number **11** on n16.

At last you can get the beautiful piece of 'C&C' Pan-magic Square of Order 4.

You would be proud of your demonstration of composing such a 'treasure' as this, if you could show making it smart by your own hand.

If you change the sitting order of those 4 good fellows around 1, you can get your 24 different solutions in all. This is one of the strongest points of this method.

Written on May 21 and revised on Oct.29, 2007 with MacOSX and Xcode2;  
 Newest Revision on Dec.20, 2014 with MacOSX 10.10.1 and Xcode 6.1.1

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by Kanji Setsuda: jag12001@nifty.com: E-Mail Address

## << Appendix: New List of Program >>

```

/** 'Composite & Complete' Magic Squares of Order 4 **
/** Make the 24 when n1=1 trnsformed into the 384 **
/** File 'CCMS44PT.c': Dictated by Kanji Setsuda **
/** on Dec.18, 2004; May.16, 2007; Dec.20, 2014; **
/** working with MacOSX 10.10.1 and Xcode 6.1.1 **
//
/** Basic Form and a Model Solution **
//          0#          1#
// 16 13 14 15 16 13   13  8 10  3 13  8
//  .--.--.--.--.    .--.--.--.--.
//  4| 1| 2| 3| 4| 1   12| 1|15| 6|12| 1
//  |---+---+---+---|   |---+---+---+---|
//  8| 5| 6| 7| 8| 5   7|14| 4| 9| 7|14
//  |---+---+---+---|   |---+---+---+---|
// 12| 9|10|11|12| 9   2|11| 5|16| 2|11
//  |---+---+---+---|   |---+---+---+---|
// 16|13|14|15|16|13   13| 8|10| 3|13| 8
//  '---'---'---'---'   '---'---'---'---'
//  4  1  2  3  4  1   12  1 15  6 12  1
//
/** Basic Conditions: C=34; **
// n1+n2+n3+n4=C   ...rw1; | n1+n5+n9+n13=C   ...c1;
/** Composite Conditions: C=34; **
// n1+n2+n5+n6=C   ...c1; | n2+n3+n6+n7=C   ...c2;
// n3+n4+n7+n8=C   ...c3; | n4+n1+n8+n5=C   ...c4;
// n5+n6+n9+n10=C  ...c5; | n6+n7+n10+n11=C  ...c6;
// n7+n8+n11+n12=C ...c7; | n8+n5+n12+n9=C   ...c8;
// n9+n10+n13+n14=C ...c9; | n10+n11+n14+n15=C ...c10;
// n11+n12+n15+n16=C ...c11; | n12+n9+n16+n13=C ...c12;
// n13+n14+n1+n2=C  ...c13; | n14+n15+n2+n3=C  ...c14;
// n15+n16+n3+n4=C  ...c15; | n16+n13+n4+n1=C  ...c16;
/** Complete Conditions: CC=17; **
// n1+n11=n2+n12=n3+n9=n4+n10=n5+n15=n6+n16=n7+n13=n8+n14=CC...cc
//

```

```

#include <stdio.h>
//
short cnt, cnt3;
short LSM, PSM;
short nm[17], uflg[17];
short anm[9][17];
short snm[385][17], tnm[17][17];
short cnt01[17];
short refnmb[385];
//
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void svsol(void), ansprint(void), recprint(void);
void printsol(short x);
void fnd01(void);
void prcnt01(void);
void prans(short x), prsoln(short x);
void prtrns(void), prtrlst(void), trnsprc(void);
void trnsf01(short x, short y), trnsf02(short x, short y);
void prmon(void);
//
short fndon(short x);
//
/* Main Program *
int main(){
short n;
printf("\n");
printf("*** 'Composite & Complete' Magic Squares of Order 4 **\n");
printf("*** Basic Form **\n");
for(n=0;n<17;n++){anm[0][n]=n;}
printsol(1);
printf("*** Making the Primitive Solutions ... **\n");
for(n=0;n<17;n++){nm[n]=0; uflg[n]=0;}
LSM=34; PSM=17; cnt=0; cnt3=0;
stp01(); /* Begin the Calculations *
printf(" [Count = %d]\n",cnt);
printf("\n");
printf("*** Compact List the 384 Primitive Solutions in the Extended Space **\n");
ansprint();
if(cnt3>0){printsol(cnt3);}
printf(" [Solution Counts = %d]\n",cnt);
printf("\n");
printf("* Solution Counts according to the Positions of Value 1 [Position: Counts] *\n");
prcnt01();
printf("\n");
for(n=0;n<=cnt;n++){refnmb[n]=0;}
printf("*** Basic Form of CCMS44 and Concepts of 15 Transformations **\n");
cnt3=0; prtrns();
printf("*** Reconstruction of CCMS44 by Pan-magic Shifting Transformation: **\n");
printf("*** List of 24 Solutions when n1=1 and the 15 Transformations of each **\n");
cnt3=0; recprint();
printf(" ** Monitor List: Old Numbers of Solutions Reconstructed **\n");
prmon();
printf("\n");
printf(" OK!\n");
printf("*** Calculated and Listed by Kanji Setsuda on\n");
printf(" Dec.20, 2014 with MacOSX 10.10.1 & Xcode 6.1.1 **\n");
printf("\n");
return 0;
}
/* Begin the Calculations *
/* Search Level: 1 *
/* Set n1 & n11 *
void stp01(){
short a,b;

```

```

for(a=1;a<17;a++){b=PSM-a;
if((uflg[a]==0)&&(uflg[b]==0)){
uflg[a]=1; uflg[b]=1;
nm[1]=a; nm[11]=b;
stp02();
uflg[b]=0; uflg[a]=0;}
}
}
/* Set n2 & n12 *
void stp02(){
short a,b;
for(a=16;a>0;a--){b=PSM-a;
if((uflg[a]==0)&&(uflg[b]==0)){
uflg[a]=1; uflg[b]=1;
nm[2]=a; nm[12]=b;
stp03();
uflg[b]=0; uflg[a]=0;}
}
}
/* Set N5 & n15 *
void stp03(){
short a,b;
for(a=16;a>0;a--){b=PSM-a;
if((uflg[a]==0)&&(uflg[b]==0)){
uflg[a]=1; uflg[b]=1;
nm[5]=a; nm[15]=b;
stp04();
uflg[b]=0; uflg[a]=0;}
}
}
/* Set N4 & n10 *
void stp04(){
short a,b;
for(a=16;a>0;a--){b=PSM-a;
if((uflg[a]==0)&&(uflg[b]==0)){
uflg[a]=1; uflg[b]=1;
nm[4]=a; nm[10]=b;
stp05();
uflg[b]=0; uflg[a]=0;}
}
}
/* Set n3=LSM-n1-n2-n4 & n9 *
void stp05(){
short a,b;
a=LSM-nm[1]-nm[2]-nm[4];
if((0<a)&&(a<17)){
b=LSM-nm[10]-nm[11]-nm[12];
if(a+b==PSM){
if((uflg[a]==0)&&(uflg[b]==0)){
uflg[a]=1; uflg[b]=1;
nm[3]=a; nm[9]=b;
stp06();
uflg[b]=0; uflg[a]=0;}}}
}
/* Search Level: 2 *
/* Set n6=LSM-n1-n2-n5 & n16 *
void stp06(){
short a,b,c,d;
a=LSM-nm[1]-nm[2]-nm[5];
if((0<a)&&(a<17)){
b=LSM-nm[11]-nm[12]-nm[15];
if(a+b==PSM){
c=LSM-nm[5]-nm[9]-nm[10];
d=LSM-nm[15]-nm[3]-nm[4];
if((a==c)&&(b==d)){

```

```

        if((uflg[a]==0)&&(uflg[b]==0)){
            uflg[a]=1; uflg[b]=1;
            nm[6]=a; nm[16]=b;
            stp07();
            uflg[b]=0; uflg[a]=0;}}}}
    }
    /* Set n7=LSM-n2-n3-n6 & n13 *
void stp07(){
    short a,b,c,d,e;
    a=LSM-nm[2]-nm[3]-nm[6];
    if((0<a)&&(a<17)){
        b=LSM-nm[12]-nm[9]-nm[16];
        if(a+b==PSM){
            c=LSM-nm[6]-nm[10]-nm[11];
            d=LSM-nm[16]-nm[4]-nm[1];
            e=LSM-nm[1]-nm[5]-nm[9];
            if((a==c)&&(b==d)&&(b==e)){
                if((uflg[a]==0)&&(uflg[b]==0)){
                    uflg[a]=1; uflg[b]=1;
                    nm[7]=a; nm[13]=b;
                    stp08();
                    uflg[b]=0; uflg[a]=0;}}}}
        }
    /* Set n8=LSM-n3-n4-n7 & n14 *
void stp08(){
    short a,b,c,d,e,f,g,h;
    a=LSM-nm[3]-nm[4]-nm[7];
    if((0<a)&&(a<17)){
        b=LSM-nm[9]-nm[10]-nm[13];
        if(a+b==PSM){
            c=LSM-nm[4]-nm[1]-nm[5];
            d=LSM-nm[7]-nm[11]-nm[12];
            e=LSM-nm[5]-nm[12]-nm[9];
            if((a==c)&&(a==d)&&(a==e)){
                f=LSM-nm[10]-nm[11]-nm[15];
                g=LSM-nm[13]-nm[1]-nm[2];
                h=LSM-nm[15]-nm[2]-nm[3];
                if((b==f)&&(b==g)&&(b==h)){
                    if((uflg[a]==0)&&(uflg[b]==0)){
                        uflg[a]=1; uflg[b]=1;
                        nm[8]=a; nm[14]=b;
                        svsol();
                        uflg[b]=0; uflg[a]=0;}}}}}}
        }
    //
    /* Save the Solutions *
void svsol(){
    short n;
    cnt++; snm[cnt][0]=cnt;
    for(n=1;n<17;n++){snm[cnt][n]=nm[n];}
    n=0; /* Find 1 and Count solutions with that position *
    for(n=1;n<17;n++){
        if(nm[n]==1){cnt01[n]++;}
    }
}
//
/* Print the Answers *
void ansprint(){
    short m,n;
    for(m=1;m<=24;m++){
        anm[cnt3][0]=snm[m][0];
        for(n=1;n<17;n++){anm[cnt3][n]=snm[m][n];}
        cnt3++; if(cnt3==4){printsol(cnt3); cnt3=0;}
    }
    for(m=25;m<=cnt;m=m+24){

```

```

    anm[cnt3][0]=snm[m][0];
    for(n=1;n<17;n++){anm[cnt3][n]=snm[m][n];}
    cnt3++; if(cnt3==4){printsol(cnt3); cnt3=0;}
}
}
//
/** Print the Answers *
void printsol(short x){
short m;
for(m=0;m<x;m++){printf("%16d#   ",anm[m][0]);}; printf("\n");
for(m=0;m<x;m++){
    printf("   %2d %2d %2d %2d %2d %2d   ",
        anm[m][16],anm[m][13],anm[m][14],anm[m][15],anm[m][16],anm[m][13]);}
printf("\n");
for(m=0;m<x;m++){printf("   .--.--.--.--   ");}; printf("\n");
for(m=0;m<x;m++){
    printf("   %2d|%2d|%2d|%2d|%2d|%2d   ",
        anm[m][4],anm[m][1],anm[m][2],anm[m][3],anm[m][4],anm[m][1]);}
printf("\n");
for(m=0;m<x;m++){printf("   |-----|   ");}; printf("\n");
for(m=0;m<x;m++){
    printf("   %2d|%2d|%2d|%2d|%2d|%2d   ",
        anm[m][8],anm[m][5],anm[m][6],anm[m][7],anm[m][8],anm[m][5]);}
printf("\n");
for(m=0;m<x;m++){printf("   |-----|   ");}; printf("\n");
for(m=0;m<x;m++){
    printf("   %2d|%2d|%2d|%2d|%2d|%2d   ",
        anm[m][12],anm[m][9],anm[m][10],anm[m][11],anm[m][12],anm[m][9]);}
printf("\n");
for(m=0;m<x;m++){printf("   |-----|   ");}; printf("\n");
for(m=0;m<x;m++){
    printf("   %2d|%2d|%2d|%2d|%2d|%2d   ",
        anm[m][16],anm[m][13],anm[m][14],anm[m][15],anm[m][16],anm[m][13]);}
printf("\n");
for(m=0;m<x;m++){printf("   '---'---'---'   ");}; printf("\n");
for(m=0;m<x;m++){
    printf("   %2d %2d %2d %2d %2d %2d   ",
        anm[m][4],anm[m][1],anm[m][2],anm[m][3],anm[m][4],anm[m][1]);}
printf("\n\n");
}
//
/** Print the Solutions N by N *
void prsoln(short x){
short l,l4,n,p;
for(n=0;n<x;n++){
    printf("%11d#",anm[n][0]);
    if(n+1<x){printf(" ");};}
printf("\n");
for(l=0;l<4;l++){l4=l*4;
    for(p=0;p<x;p++){
        for(n=1;n<5;n++){printf("%3d",anm[p][l4+n]);}
        if(p+1<x){printf(" ");}
    }
    printf("\n");}
printf("\n");
}
//
/** Solution Counts according to the Positions of Value 1 *
void prcnt01(){
short m,n;
for(m=1;m<17;m++){n=cnt01[m];
    printf("%4d:%3d",m,n);
    if((m%8)==0){printf("\n");}
}
printf("\n");

```

```

}
//
/** Print the Reconstruction *
void reprint(){
    short m,n;
    for(m=1;m<=24;m++){
        printf(" Group %d:\n",snm[m][0]);
        for(n=1;n<17;n++){nm[n]=snm[m][n]; tnm[0][n]=snm[m][n];};
        trnsprc();
        prtrlst();
    }
}
//
/** Conceptual Diagrams of Transformations by Pan-magic Plane Shifting *
void ptrtrns(){
    short m,n;
    for(n=0;n<17;n++){nm[n]=n; tnm[0][n]=n;};
    trnsprc();
    cnt3=0; for(n=0;n<17;n++){tnm[n][0]=n;}
    for(m=0;m<16;m++){
        for(n=0;n<17;n++){anm[cnt3][n]=tnm[m][n];}
        cnt3++; if(cnt3==4){printsol(cnt3); cnt3=0;}
    }
}
//
/** Transform them by the Pan-magic Line Shifting *
void trnsprc(){
    trnsf01( 0, 1); trnsf01( 1, 2); trnsf01( 2, 3);
    trnsf02( 0, 4); trnsf01( 4, 5); trnsf01( 5, 6); trnsf01( 6, 7);
    trnsf02( 4, 8); trnsf01( 8, 9); trnsf01( 9,10); trnsf01(10,11);
    trnsf02( 8,12); trnsf01(12,13); trnsf01(13,14); trnsf01(14,15);
}
//
/** Fundamental types of Transformation *
void trnsf01(short x, short y){
    short n;
    short mm[17], nn[17];
    for(n=1;n<17;n++){nn[n]=tnm[x][n];}
    mm[ 1]=nn[ 4]; mm[ 2]=nn[ 1]; mm[ 3]=nn[ 2]; mm[ 4]=nn[ 3];
    mm[ 5]=nn[ 8]; mm[ 6]=nn[ 5]; mm[ 7]=nn[ 6]; mm[ 8]=nn[ 7];
    mm[ 9]=nn[12]; mm[10]=nn[ 9]; mm[11]=nn[10]; mm[12]=nn[11];
    mm[13]=nn[16]; mm[14]=nn[13]; mm[15]=nn[14]; mm[16]=nn[15];
    for(n=1;n<17;n++){tnm[y][n]=mm[n];}
};
void trnsf02(short x, short y){
    short n;
    short mm[17], nn[17];
    for(n=1;n<17;n++){nn[n]=tnm[x][n];}
    mm[ 1]=nn[13]; mm[ 2]=nn[14]; mm[ 3]=nn[15]; mm[ 4]=nn[16];
    mm[ 5]=nn[ 1]; mm[ 6]=nn[ 2]; mm[ 7]=nn[ 3]; mm[ 8]=nn[ 4];
    mm[ 9]=nn[ 5]; mm[10]=nn[ 6]; mm[11]=nn[ 7]; mm[12]=nn[ 8];
    mm[13]=nn[ 9]; mm[14]=nn[10]; mm[15]=nn[11]; mm[16]=nn[12];
    for(n=1;n<17;n++){tnm[y][n]=mm[n];}
};
//
/** Print the Result List of Transformations *
void prtrlst(){
    short m,n,oldn;
    for(m=0;m<16;m++){
        for(n=0;n<17;n++){anm[cnt3][n]=tnm[m][n];}
        oldn=fndon(cnt3); anm[cnt3][0]=oldn; refnmb[oldn]++;
        cnt3++; if(cnt3==6){prsoln(cnt3); cnt3=0;}
    }
    if(cnt3>0){prsoln(cnt3); cnt3=0;}
}

```

```

//
/** Find the Relative Solution Number in the Old List */
short fndon(short x){
short d,m;
short n,f;
d=0;
for(m=1;m<=cnt;m++){f=0;
for(n=1;n<17;n++){
if(anm[x][n]==snm[m][n]){f++;}
else{break;}
}
if(f==16){d=snm[m][0]; break;}
}
return d;
}
//
/** Monitor of the Old Numbers Reffered to the Solution **
void prmon(){
short n;
printf(" ??: %d",refnmb[0]);
for(n=1;n<385;n++){
if(n%32==1){printf("\n%4d:",n);}
printf("%2d",refnmb[n]);
}
printf("\n");
}
//

```