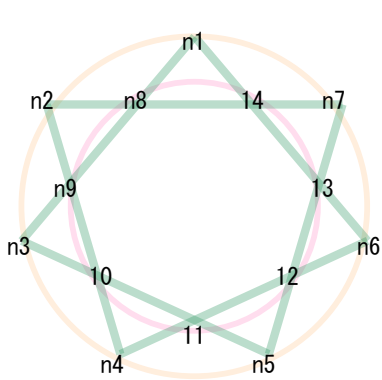
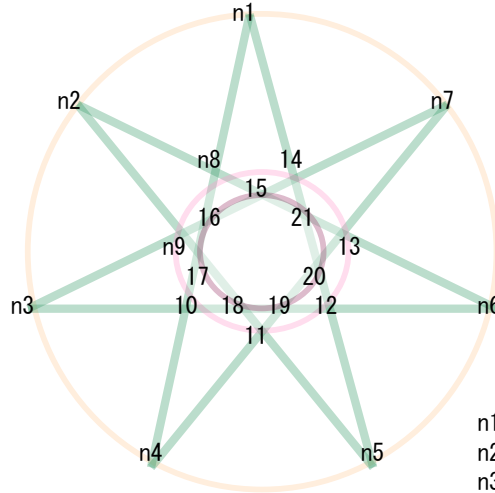


## 第2部：『星陣の基礎的研究』（改訂版）：撰田 寛二

### 第1章：序：星陣とは何か



$$\begin{aligned} n1+n8+n9+n3 &= K \\ n2+n9+n10+n4 &= K \\ n3+n10+n11+n5 &= K \\ n4+n11+n12+n6 &= K \\ n5+n12+n13+n7 &= K \\ n6+n13+n14+n1 &= K \\ n7+n14+n8+n2 &= K \end{aligned}$$

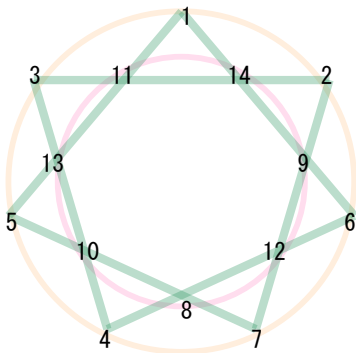


$$\begin{aligned} n1+n8+n16+n17+n10+n4 &= K \\ n2+n9+n17+n18+n11+n5 &= K \\ n3+n10+n18+n19+n12+n6 &= K \\ n4+n11+n19+n20+n13+n7 &= K \\ n5+n12+n20+n21+n14+n1 &= K \\ n6+n13+n21+n15+n8+n2 &= K \\ n7+n14+n15+n16+n9+n3 &= K \end{aligned}$$

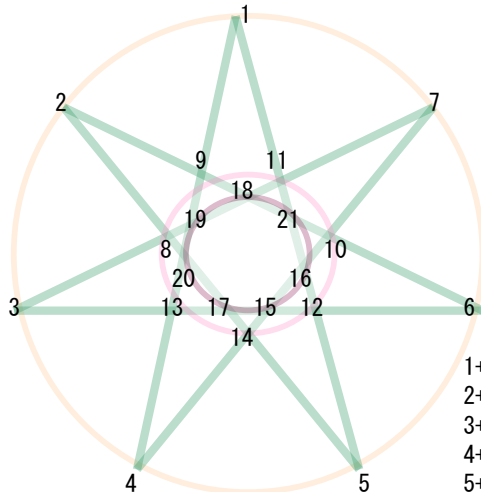
星陣には、古来幾つかのタイプのもものが知られているが、代表は上図のような二周星陣と三周星陣だ。上の例は、七星陣だが、二周タイプは $2 \times 7$ 個の数字を用い、三周タイプは $3 \times 7$ 個の数字を用いて構成する。どの場合も1から始まる連続する自然数列を、各1度ずつ使い、対角線の交点上に配置する。そして、対角線上の4数、6数の和が常に等しくなるようにする。この線和は、タイプと次数によって固有の値に決まる。試みに上の定義式の辺々を全部加えてみられよ。 $K$ の値が計算で求まる。ゆえに $K$ を「定和値」という。

これらの約束の仕方が「魔方陣」の場合と良く似ているので、古来魔方陣の一種と見なされてきた。比較的簡単なパズルなので、初心者にも長く親しまれて来た歴史がある。

#### \* 解の実例 \*



$$\begin{aligned} 1+11+13+5 &= 30; & 3+13+10+4 &= 30; \\ 5+10+8+7 &= 30; & 4+8+12+6 &= 30; \\ 7+12+9+2 &= 30; & 6+9+14+1 &= 30; \\ 2+14+11+3 &= 30; \end{aligned}$$



$$\begin{aligned} 1+9+19+20+13+4 &= 66; \\ 2+8+20+17+14+5 &= 66; \\ 3+13+17+15+12+6 &= 66; \\ 4+14+15+16+10+7 &= 66; \\ 5+12+16+21+11+1 &= 66; \\ 6+10+21+18+9+2 &= 66; \\ 7+11+18+19+8+3 &= 66; \end{aligned}$$

今後しばらくは、左側の二周星陣の諸タイプを研究することにしよう。

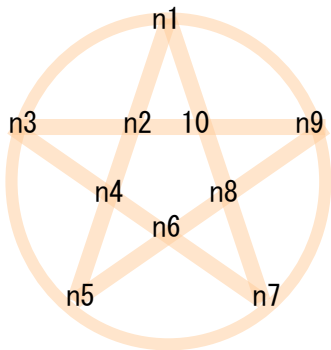
次ページの図のように円周上の $n$ 等分点を1つおきに結び対角線を $n$ 本作る。そして対角線の交点に $1 \sim 2 \times n$ の自然数を全個重複なく置いて、各対角線上の4数の和を等しくする。下図のように式で書く方がわりやすいかも知れない。

4数の定和は、各星陣固有の値に決まる。 $n$ の関数形( $C=4 \times n + 2$ )で表せる。

常に4数の定和が求められる点と、数要素が必ず2回ずつ用いられる点とが、ユニークである。はたして解はどのようなもので、幾つあるであろうか。

平面方陣と多少は似ているが、研究するにつれて少しく違った世界が開けて来る。それを楽しみに考察を一步ずつ進めて行こう。

**\* 各次の二周星陣の基本図と基本定義式一覧 \***

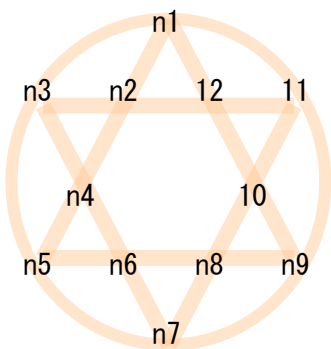


**\* 二周五星陣の定義式 \***

$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & 11 \\ n3+n4+n6+n7 &= C & \dots & 12 \\ n5+n6+n8+n9 &= C & \dots & 13 \\ n7+n8+n10+n1 &= C & \dots & 14 \\ n9+n10+n2+n3 &= C & \dots & 15 \end{aligned}$$

$$\begin{aligned} 2*(n1+n2+n3+\dots+n9+n10) &= 5*C \\ 2*55 &= 5*C \end{aligned}$$

ゆえに  $C=22 \quad \dots \quad r0$

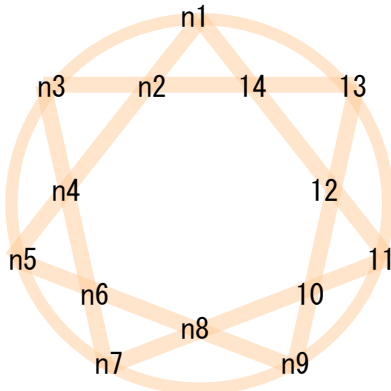


**\* 二周六星陣の定義式 \***

$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & 11 \\ n3+n4+n6+n7 &= C & \dots & 12 \\ n5+n6+n8+n9 &= C & \dots & 13 \\ n7+n8+n10+n11 &= C & \dots & 14 \\ n9+n10+n12+n1 &= C & \dots & 15 \\ n11+n12+n2+n3 &= C & \dots & 16 \end{aligned}$$

$$\begin{aligned} 2*(n1+n2+n3+\dots+n11+n12) &= 6*C \\ 2*78 &= 6*C \end{aligned}$$

ゆえに  $C=26 \quad \dots \quad r0$

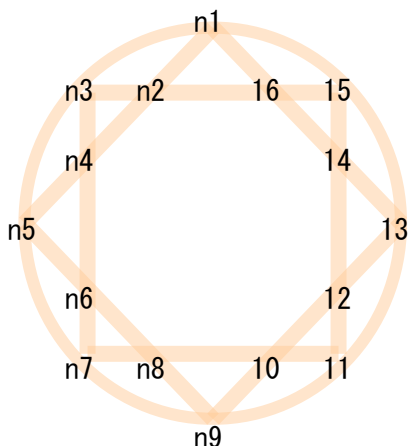


**\* 二周七星陣の定義式 \***

$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & 11 \\ n3+n4+n6+n7 &= C & \dots & 12 \\ n5+n6+n8+n9 &= C & \dots & 13 \\ n7+n8+n10+n11 &= C & \dots & 14 \\ n9+n10+n12+n13 &= C & \dots & 15 \\ n11+n12+n14+n1 &= C & \dots & 16 \\ n13+n14+n2+n3 &= C & \dots & 17 \end{aligned}$$

$$\begin{aligned} 2*(n1+n2+n3+\dots+n13+n14) &= 7*C \\ 2*105 &= 7*C \end{aligned}$$

ゆえに  $C=30 \quad \dots \quad r0$



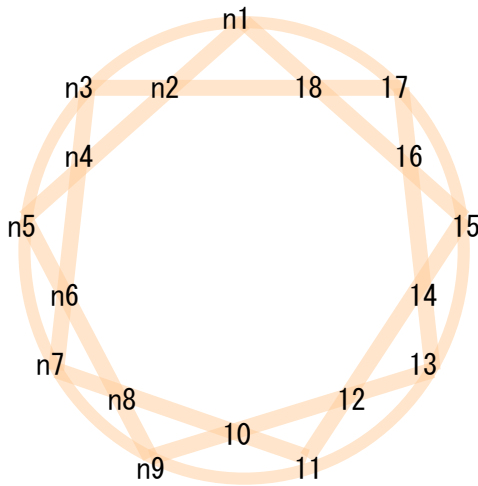
**\* 二周八星陣の定義式 \***

$$\begin{aligned} n1+n2+n4+n5 &= C & \dots & 11 \\ n3+n4+n6+n7 &= C & \dots & 12 \\ n5+n6+n8+n9 &= C & \dots & 13 \\ n7+n8+n10+n11 &= C & \dots & 14 \\ n9+n10+n12+n13 &= C & \dots & 15 \\ n11+n12+n14+n15 &= C & \dots & 16 \\ n13+n14+n16+n1 &= C & \dots & 17 \\ n15+n16+n2+n3 &= C & \dots & 18 \end{aligned}$$

$$\begin{aligned} 2*(n1+n2+n3+\dots+n15+n16) &= 8*C \\ 2*136 &= 8*C \end{aligned}$$

ゆえに  $C=34 \quad \dots \quad r0$

こうした数の並び方を漠然と眺めると、外円を除けば、凹凸が星形に見える。ゆえに古来『星陣』と名付けられて来たが、高次になるとだんだん別の形に近づく。

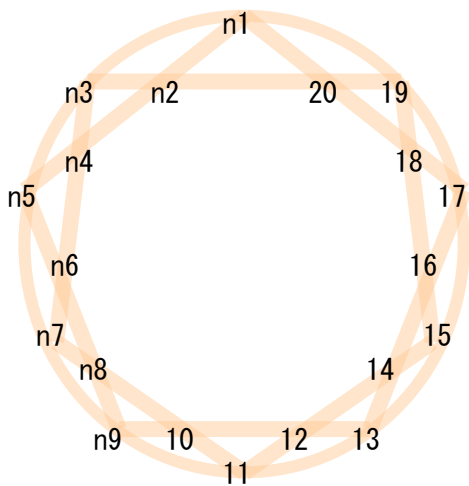


\* 二周九星陣の定義式 \*

$n1+n2+n4+n5=C$	...	11
$n3+n4+n6+n7=C$	...	12
$n5+n6+n8+n9=C$	...	13
$n7+n8+n10+n11=C$	...	14
$n9+n10+n12+n13=C$	...	15
$n11+n12+n14+n15=C$	...	16
$n13+n14+n16+n17=C$	...	17
$n15+n16+n18+n1=C$	...	18
$n17+n18+n2+n3=C$	...	19

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$2*(n1+n2+n3+\dots+n17+n18)=9*C$   
 $2*171=9*C$   
 ゆえに  $C=38$  ... r0

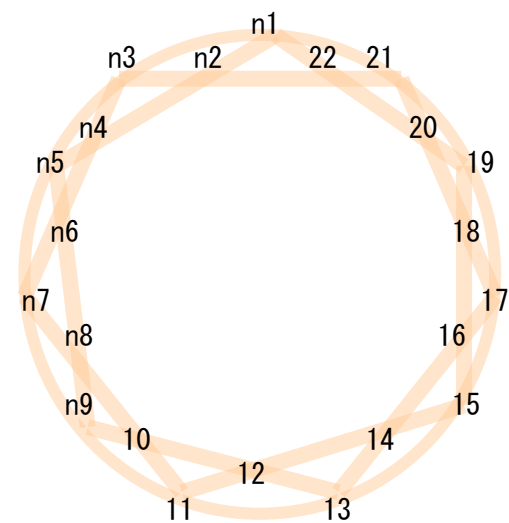


\* 二周十星陣の定義式 \*

$n1+n2+n4+n5=C$	...	11
$n3+n4+n6+n7=C$	...	12
$n5+n6+n8+n9=C$	...	13
$n7+n8+n10+n11=C$	...	14
$n9+n10+n12+n13=C$	...	15
$n11+n12+n14+n15=C$	...	16
$n13+n14+n16+n17=C$	...	17
$n15+n16+n18+n19=C$	...	18
$n17+n18+n20+n1=C$	...	19
$n19+n20+n2+n3=C$	...	110

---

$2*(n1+n2+n3+\dots+n19+n20)=10*C$   
 $2*210=10*C$   
 ゆえに  $C=42$  ... r0



\* 二周十一星陣の定義式 \*

$n1+n2+n4+n5=C$	...	11
$n3+n4+n6+n7=C$	...	12
$n5+n6+n8+n9=C$	...	13
$n7+n8+n10+n11=C$	...	14
$n9+n10+n12+n13=C$	...	15
$n11+n12+n14+n15=C$	...	16
$n13+n14+n16+n17=C$	...	17
$n15+n16+n18+n19=C$	...	18
$n17+n18+n20+n21=C$	...	19
$n19+n20+n22+n1=C$	...	110
$n21+n22+n2+n3=C$	...	111

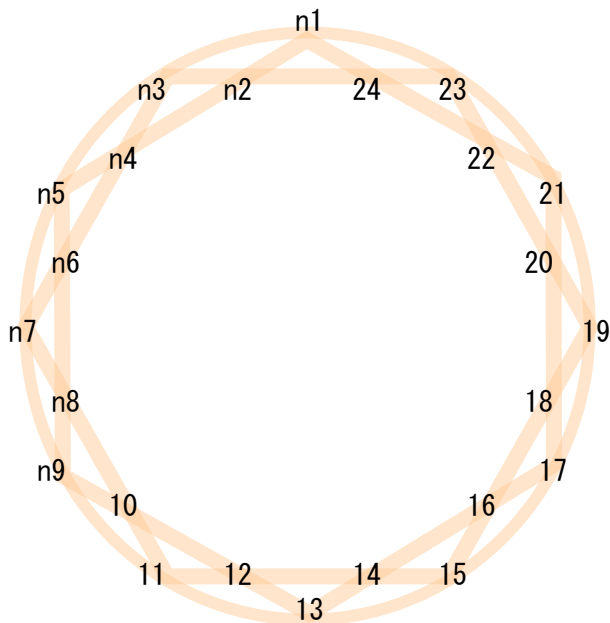
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$2*(n1+n2+n3+\dots+n21+n22)=11*C$   
 $2*253=11*C$   
 ゆえに  $C=46$  ... r0

高次になると、星形の凹凸が浅くなって、全てが「環状に」並ぶ印象が強まる。この形は、腕輪のようにも見えるし、また王冠の如く見えなくもない。

その本質的な特徴は、二周する輪環と網構造だ。高次になればなるほど、全てが外周に向かって近づいて行く。内周の方は図に描き込んではいないが、すぐに見て取れよう。いつも外周上には奇数番号の変数が並び、内周の上には偶数名の変数が並ぶ。

外周と内周とを緊密に結び付けているのは、網構造だ。対角線上の4数を、外周・内周のそれぞれが2数ずつ分有するという形で、編み目（網目）を作っている。



**\* 二周十二星陣の定義式 \***

$n1+n2+n4+n5=C$	...	l1
$n3+n4+n6+n7=C$	...	l2
$n5+n6+n8+n9=C$	...	l3
$n7+n8+n10+n11=C$	...	l4
$n9+n10+n12+n13=C$	...	l5
$n11+n12+n14+n15=C$	...	l6
$n13+n14+n16+n17=C$	...	l7
$n15+n16+n18+n19=C$	...	l8
$n17+n18+n20+n21=C$	...	l9
$n19+n20+n22+n23=C$	...	l10
$n21+n22+n24+n1=C$	...	l11
$n23+n24+n2+n3=C$	...	l12

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$$2*(n1+n2+n3+...+n23+n24)=12*C$$

$$2*300=12*C$$

ゆえに  $C=50$  ... r0

## 第2章：二周五星陣の基礎的研究

さて次は、各論に入る。次数を1つずつ上げながら各個別に研究して行こう。

三次・四次の星陣は、構成が不可能である。二周する輪環も網構造も作ることができないからだ。よって、最小の星陣として考え得るのは、次の五星陣ということになる。

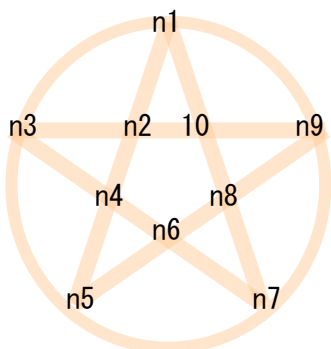
では、**五星陣**をどう具体的に作ることができるか。

これが、どうやっても、何度試みても、出来ないのである。1から10までの連続する自然数を各1回ずつ使って構成しようとする限り、いつも「解無し」となってしまう。

理論的に不可能なのか、それとも実際上不可能なのか。それを是が非でも見極めなければならぬ。

しばらく代数的計算を進めて、何が問題なのかを調べて見よう。

### \* 二周五星陣の基本図と基本定義式 \*



**\* 定義式 \***

$n1+n2+n4+n5=C$	...	l1
$n3+n4+n6+n7=C$	...	l2
$n5+n6+n8+n9=C$	...	l3
$n7+n8+n10+n1=C$	...	l4
$n9+n10+n2+n3=C$	...	l5

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$$2*(n1+n2+n3+...+n9+n10)=5*C$$

$$2*55=5*C$$

ゆえに  $C=22$  ... r1

もちろん使う数要素は、{1, 2, 3, 4, 5, 6, 7, 8, 9, 10} の自然数列であり、重複無く欠落無く各1回ずつ使うこととする。ゆえに4数の定和は、必ず22となる。

$$11+14+15$$

$$\begin{aligned} n1+n2+n4+n5+n7+n8+n10+n1+n9+n10+n2+n3 &= 3 * C \\ (n1+n2+n10) + (n1+n2+n3+n4+n5+n7+n8+n9+n10) &= 3 * 22 \end{aligned}$$

両辺に  $n6$  を加える。

$$\begin{aligned} (n1+n2+n10) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10) &= 66+n6 \\ (n1+n2+n10) + 55 &= 66+n6 \end{aligned}$$

$$\text{ゆえに } n1+n2+n10=11+n6 \quad \text{あるいは} \quad n1=11-n2+n6-n10 \quad \dots r2$$

$$11+12+15$$

$$\begin{aligned} n1+n2+n4+n5+n3+n4+n6+n7+n9+n10+n2+n3 &= 3 * C \\ (n2+n3+n4) + (n1+n2+n3+n4+n5+n6+n7+n9+n10) &= 66 \end{aligned}$$

両辺に  $n8$  を加える。

$$(n2+n3+n4) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10) = 66+n8$$

$$\text{ゆえに } n2+n3+n4=11+n8 \quad \text{あるいは} \quad n3=11-n2-n4+n8 \quad \dots r3$$

$$11+12+13$$

$$\begin{aligned} n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9 &= 3 * C \\ (n4+n5+n6) + (n1+n2+n3+n4+n5+n6+n7+n8+n9) &= 66 \end{aligned}$$

両辺に  $n10$  を加える。

$$(n4+n5+n6) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10) = 66+n10$$

$$\text{ゆえに } n4+n5+n6=11+n10 \quad \text{あるいは} \quad n5=11-n4-n6+n10 \quad \dots r4$$

$$\text{同様に } n6+n7+n8=11+n2 \quad \text{あるいは} \quad n7=11+n2-n6-n8 \quad \dots r5$$

$$n8+n9+n10=11+n4 \quad \text{あるいは} \quad n9=11+n4-n8-n10 \quad \dots r6$$

$$11+13+15$$

$$\begin{aligned} n1+n2+n4+n5+n5+n6+n8+n9+n9+n10+n2+n3 &= 3 * C \\ (n2+n5+n9) + (n1+n2+n3+n4+n5+n6+n8+n9+n10) &= 66 \end{aligned}$$

両辺に  $n7$  を加える。

$$(n2+n5+n9) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10) = 66+n7$$

$$\text{ゆえに } n2+n5+n9=11+n7 \quad \text{あるいは} \quad n2=11-n5+n7-n9 \quad \dots r7$$

$$11+12+14$$

$$\begin{aligned} n1+n2+n4+n5+n3+n4+n6+n7+n7+n8+n10+n1 &= 3 * C \\ (n1+n4+n7) + (n1+n2+n3+n4+n5+n6+n7+n8+n10) &= 66 \end{aligned}$$

両辺に  $n9$  を加える。

$$(n1+n4+n7) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10) = 66+n9$$

$$\text{ゆえに } n1+n4+n7=11+n9 \quad \text{あるいは} \quad n4=11-n1-n7+n9 \quad \dots r8$$

$$\text{同様に } n3+n6+n9=11+n1 \quad \text{あるいは} \quad n6=11+n1-n3-n9 \quad \dots r9$$

$$n1+n5+n8=11+n3 \quad \text{あるいは} \quad n8=11+n3-n5-n1 \quad \dots r10$$

$$n3+n7+n10=11+n5 \quad \text{あるいは} \quad n10=11-n3+n5-n7 \quad \dots r11$$

$$11+12+13+14-15$$

$$\begin{aligned} n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9+n7+n8+n10+n1-n9-n10-n2-n3 &= 3 * C \\ 2 * (n1+n4+n5+n6+n7+n8) &= 66 \end{aligned}$$

$$\text{ゆえに } n1+n4+n5+n6+n7+n8=33 \quad \text{あるいは} \quad n1=33-n4-n5-n6-n7-n8 \quad \dots r12$$

$$-11+12+13+14+15$$

$$\begin{aligned} -n1-n2-n4-n5+n3+n4+n6+n7+n5+n6+n8+n9+n7+n8+n10+n1+n9+n10+n2+n3 &= 3 * C \\ 2 * (n3+n6+n7+n8+n9+n10) &= 66 \end{aligned}$$

$$\text{ゆえに } n3+n6+n7+n8+n9+n10=33 \quad \text{あるいは} \quad n3=33-n6-n7-n8-n9-n10 \quad \dots r13$$

$$11-12+13+14+15$$

$$\begin{aligned} n1+n2+n4+n5-n3-n4-n6-n7+n5+n6+n8+n9+n7+n8+n10+n1+n9+n10+n2+n3 &= 3 * C \\ 2 * (n1+n2+n5+n8+n9+n10) &= 66 \end{aligned}$$

$$\text{ゆえに } n1+n2+n5+n8+n9+n10=33 \quad \text{あるいは} \quad n5=33-n1-n2-n8-n9-n10 \quad \dots r14$$

$$11+12-13+14+15$$

$$\begin{aligned} n1+n2+n4+n5+n3+n4+n6+n7-n5-n6-n8-n9+n7+n8+n10+n1+n9+n10+n2+n3 &= 3 * C \\ 2 * (n1+n2+n3+n4+n7+n10) &= 66 \end{aligned}$$

ゆえに  $n1+n2+n3+n4+n7+n10=33$  あるいは  $n7=33-n1-n2-n3-n4-n10 \dots r15$

$11+12+13-14+15$

$n1+n2+n4+n5+n3+n4+n6+n7+n5+n6+n8+n9-n7-n8-n10-n1+n9+n10+n2+n3=3*C$

$2*(n2+n3+n4+n5+n6+n9)=66$

ゆえに  $n2+n3+n4+n5+n6+n9=33$  あるいは  $n9=33-n2-n3-n4-n5-n6 \dots r16$

\* 得られた関係式をまとめた表 \*

$n1+n2+n10=11+n6$	あるいは	$n1=11-n2+n6-n10$	$\dots r2$
$n2+n3+n4=11+n8$	あるいは	$n3=11-n2-n4+n8$	$\dots r3$
$n4+n5+n6=11+n10$	あるいは	$n5=11-n4-n6+n10$	$\dots r4$
$n6+n7+n8=11+n2$	あるいは	$n7=11+n2-n6-n8$	$\dots r5$
$n8+n9+n10=11+n4$	あるいは	$n9=11+n4-n8-n10$	$\dots r6$
$n2+n5+n9=11+n7$	あるいは	$n2=11-n5+n7-n9$	$\dots r7$
$n1+n4+n7=11+n9$	あるいは	$n4=11-n1-n7+n9$	$\dots r8$
$n3+n6+n9=11+n1$	あるいは	$n6=11+n1-n3-n9$	$\dots r9$
$n1+n5+n8=11+n3$	あるいは	$n8=11+n3-n5-n1$	$\dots r10$
$n3+n7+n10=11+n5$	あるいは	$n10=11-n3+n5-n7$	$\dots r11$
$n1+n4+n5+n6+n7+n8=33$	あるいは	$n1=33-n4-n5-n6-n7-n8$	$\dots r12$
$n3+n6+n7+n8+n9+n10=33$	あるいは	$n3=33-n6-n7-n8-n9-n10$	$\dots r13$
$n1+n2+n5+n8+n9+n10=33$	あるいは	$n5=33-n1-n2-n8-n9-n10$	$\dots r14$
$n1+n2+n3+n4+n7+n10=33$	あるいは	$n7=33-n1-n2-n3-n4-n10$	$\dots r15$
$n2+n3+n4+n5+n6+n9=33$	あるいは	$n9=33-n2-n3-n4-n5-n6$	$\dots r16$

これらの関係式は、全部が最初の定義式から導き出されたものである。ゆえに五星陣では、これらの全てが同時に成り立たなければならない。

式相互の間に明らかな論理的矛盾というようなものは無い。しかし、変数が 10 個しか無いのに、式がたくさんあり過ぎる。これではたして実際に成り立つものかどうか。

ここは、もう数値テストを実行する以外には無い。実際に各変数に数値を入れてみて、それで成り立つかどうかを逐一調べる以外には無いと思われる。

早速にも、 $n1$  に具体的数値である 1 を入れてみよう。 $n1=1$  のタイプは、基本形として扱える重要なものだ。それで解が求まるなら、 $n3=1$  のタイプも、 $n5=1$ 、 $n7=1$ 、 $n9=1$  のタイプにも、解があると期待することができる。なぜなら、元の解に対して 72 度ずつ回転移動を繰り返すだけで、それらのタイプを簡単に作れるからである。

次に  $n2=1$  のタイプが可能かどうかを調べよう。そうすれば、 $n4=1$ 、 $n6=1$ 、 $n8=1$ 、そして  $n10=1$  のタイプの可能性を調べたのと同じ意味を持つ。

代入する式には、 $r2$ 、 $r8$ 、 $r10$  そして  $r9$  の 4 式を選ぶ。そして  $n1=1$  を消去しよう。

$n2=1$  を消去する場合には、 $r2$ 、 $r3$ 、 $r7$  そして  $r5$  の 4 式を選ぶことにしよう。

そうした上で、他の変数にどのような数値が代入可能かを、一々調べ尽くすのである。

\* 数値代入テスト \*

もし  $n1=1$  ならば  $\{n2+n10-n6=10; n4+n7-n9=10; n5+n8-n3=10; n3+n6+n9=12;\}$

辺々を加えると  $(n2+n10-n6)+(n4+n7-n9)+(n5+n8-n3)+(n3+n6+n9)=42;$

ゆえに  $n2+n4+n5+n7+n8+n10=42;$

もし  $n2=1$  ならば  $\{n1+n10-n6=10; n3+n4-n8=10; n5+n9-n7=10; n6+n7+n8=12;\}$

辺々を加えると  $n1+n10-n6+n3+n4-n8+n5+n9-n7+n6+n7+n8=42;$

ゆえに  $n1+n3+n4+n5+n9+n10=42;$

\* 五星陣の不可能性を調べるテスト \*

\* 各変数を取り得る数値のリスト: \*

\*  $n1=1$  の場合: \*

$$n_2+n_4+n_5+n_7+n_8+n_{10}=42; n_3+n_6+n_9=10;$$

$$n_2+n_{10}-n_6=10; n_4+n_7-n_9=10; n_5+n_8-n_3=10;$$

- [ 1 ] (2, 6, 7, 8, 9, 10); (3, 4, 5);  
 { (6, 7, 3), (6, 8, 4), (6, 9, 5), (7, 8, 5), () }
- [ 2 ] (3, 5, 7, 8, 9, 10); (2, 4, 6);  
 { (3, 9, 2), (5, 7, 2), (5, 9, 4), (7, 9, 6), () }
- [ 3 ] (4, 5, 6, 8, 9, 10); (2, 3, 7);  
 { (4, 8, 2), (4, 9, 3), (5, 8, 3), (8, 9, 7), () }

\* n2=1 の場合: \*

$$n_1+n_3+n_4+n_5+n_9+n_{10}=42; n_6+n_7+n_8=12;$$

$$n_1+n_{10}-n_6=10; n_3+n_4-n_8=10; n_5+n_9-n_7=10;$$

- [ 1 ] (2, 6, 7, 8, 9, 10); (3, 4, 5);  
 { (6, 7, 3), (6, 8, 4), (6, 9, 5), (7, 8, 5), () }
- [ 2 ] (3, 5, 7, 8, 9, 10); (2, 4, 6);  
 { (3, 9, 2), (5, 7, 2), (5, 9, 4), (7, 9, 6), () }
- [ 3 ] (4, 5, 6, 8, 9, 10); (2, 3, 7);  
 { (4, 8, 2), (4, 9, 3), (5, 8, 3), (8, 9, 7), () }

どの場合でも、全ての変数の値を違えながら、全ての式を満たすことは不可能であることがわかる。全4式を是非成り立たせようとする、ある数の重複・欠落が避けられない。自然数列 {1, 2, 3, 4, 5, ..., 9} として 10} を全個重複無く欠落無く使おうとすると、どうしても成り立たない式が出て来てしまう。

n1=1でも n2=1でも成り立たないのだから、n3=1でも、n4=1, n5=1, n6=1, n7=1, n8=1, n9=1, n10=1でも成り立たない。つまり、全ての場合で成り立たず、実際上五星陣は構成が不可能であることがわかる。予想は、かくして証明された。

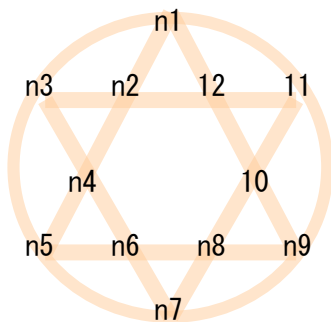
では、どんな場合に五星陣の構成が可能か。読者は、そう考えるかも知れない。

これはもう、「自然数列を重複無く欠落無く使う」という最初の約束を放棄するしかない。だが正直のところ、どんな数列をどう使ったらよいのか、小生には良くわからない。

またそれを研究したいとも、小生は思わない。たとえ頑固と非難されようとも、小生は古典的な約束を忠実に守る立場に立つ。古典主義者は、だから、五星陣の不可能性を潔く受け入れる。この構成不可能性は「知性の敗北」を意味するのでは決してなく、あくまで五星陣の重要な特性の一つであると、小生は考えている。

### 第3章：二周六星陣の基礎的研究（改定増補版）

次は、六星陣を研究しよう。



\* 二周六星陣の定義式 \*

$$n_1+n_2+n_4+n_5=C \quad \dots \quad l_1$$

$$n_3+n_4+n_6+n_7=C \quad \dots \quad l_2$$

$$n_5+n_6+n_8+n_9=C \quad \dots \quad l_3$$

$$n_7+n_8+n_{10}+n_{11}=C \quad \dots \quad l_4$$

$$n_9+n_{10}+n_{12}+n_1=C \quad \dots \quad l_5$$

$$n_{11}+n_{12}+n_2+n_3=C \quad \dots \quad l_6$$

$$2*(n_1+n_2+n_3+\dots+n_{11}+n_{12})=6*C$$

$$2*78=6*C$$

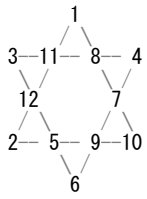
$$\text{ゆえに } C=26 \quad \dots \quad r_0$$

いったい全体に幾つの解があるのだろうか？

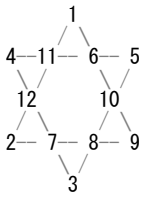
最初から長大なリストで恐縮するが、回転解や反転解を含む「原始解」を全部示そう。

**\*\* 六星陣の原始解 960 個のリスト (全) \*\***

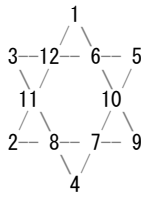
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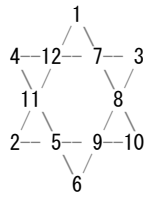
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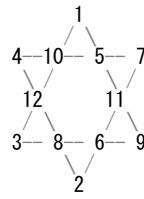
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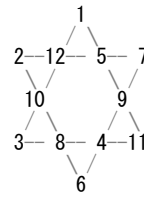
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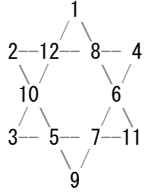
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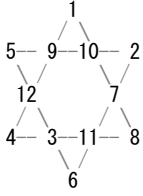
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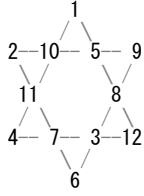
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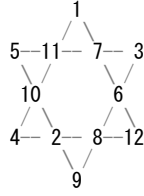
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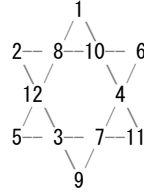
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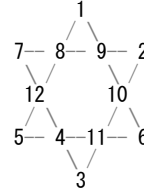
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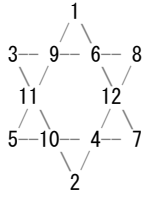
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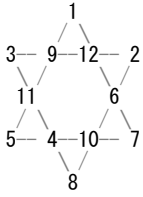
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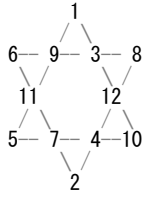
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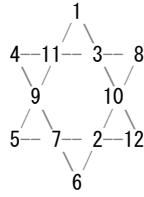
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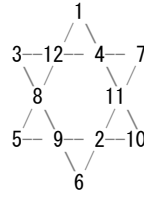
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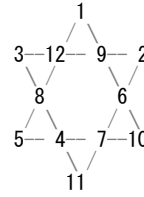
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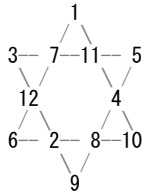
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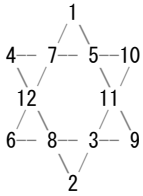
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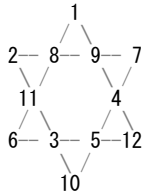
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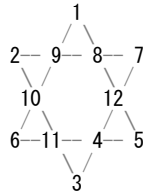
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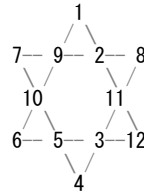
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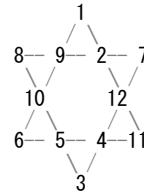
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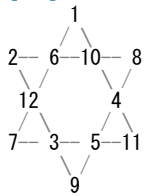
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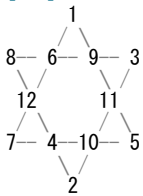
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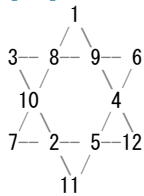
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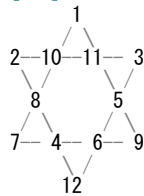
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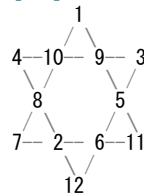
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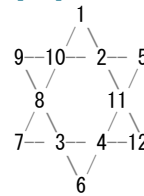
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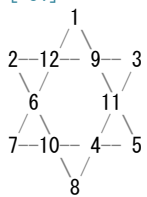
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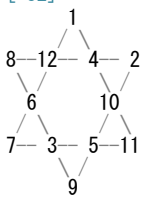
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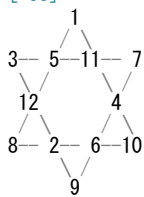
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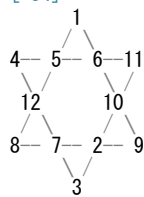
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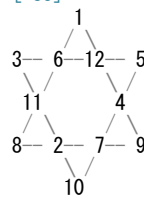
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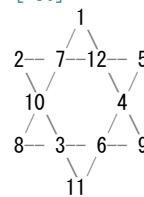
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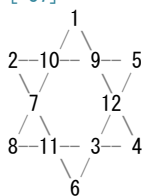
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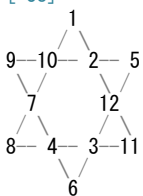
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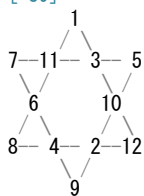
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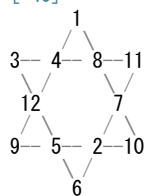
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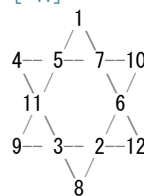
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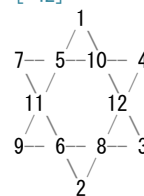
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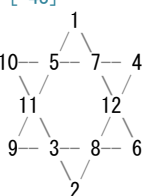
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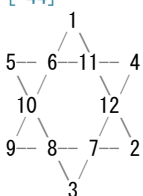
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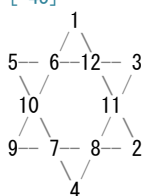
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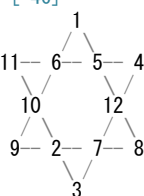
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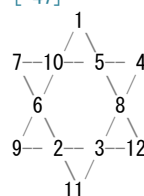
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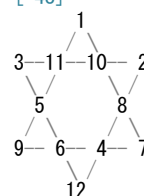
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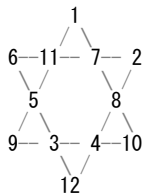


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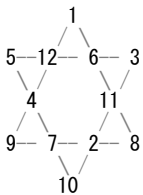




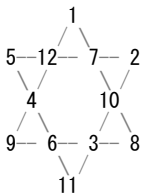
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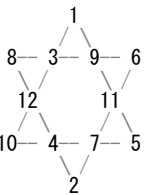
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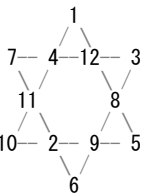
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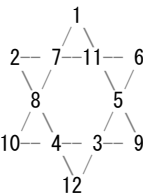
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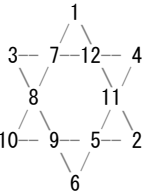
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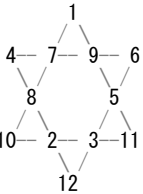
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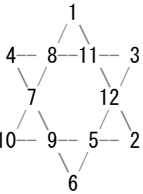
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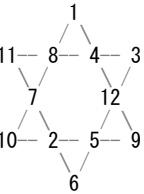
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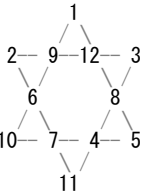
[ 57]



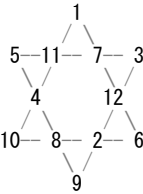
[ 58]



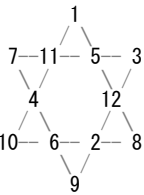
[ 59]



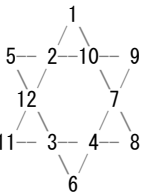
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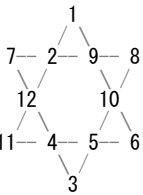
[ 61]



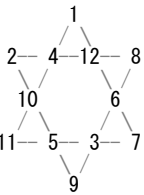
[ 62]



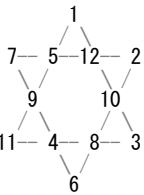
[ 63]



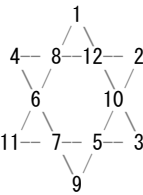
[ 64]



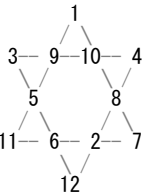
[ 65]



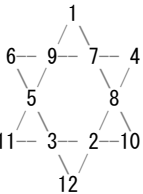
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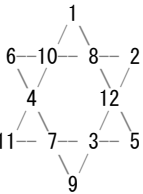
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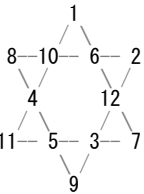
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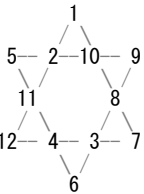
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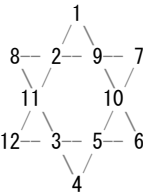
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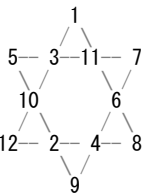
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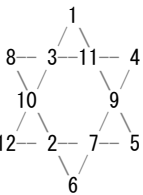
[ 72]



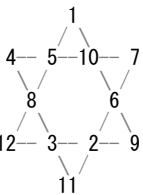
[ 73]



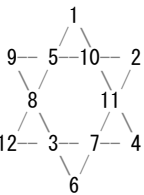
[ 74]



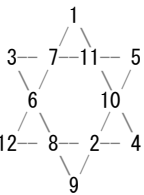
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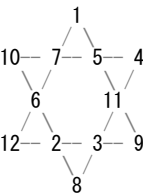
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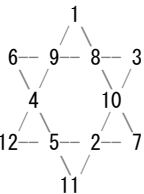
[ 77]



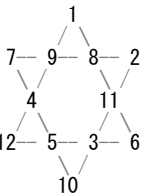
[ 78]



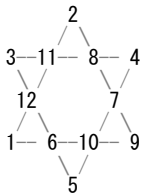
[ 79]



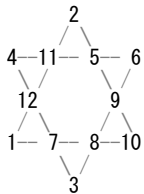
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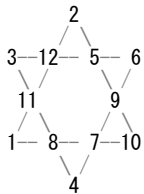
[ 81]



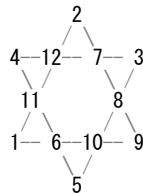
[ 82]



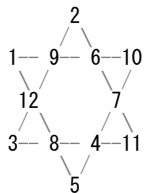
[ 83]



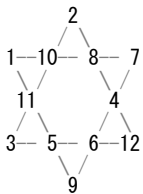
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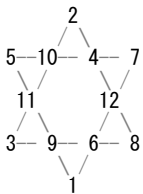
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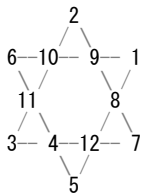
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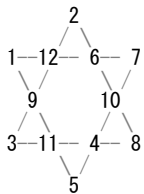
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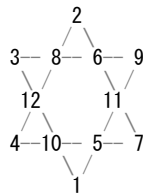
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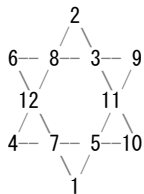
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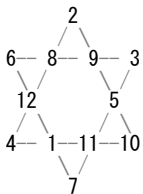
[ 90]



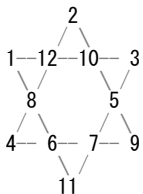
[ 91]



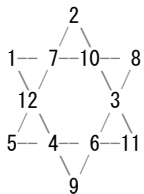
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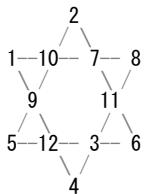
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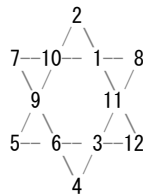
[ 94]



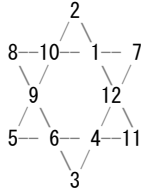
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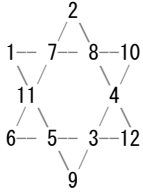
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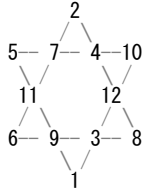
[ 97]



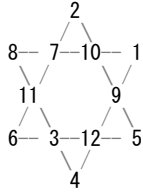
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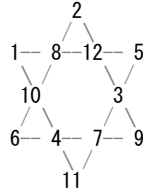
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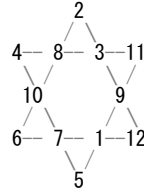
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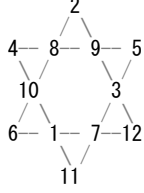
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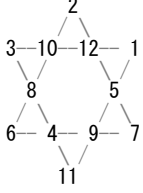
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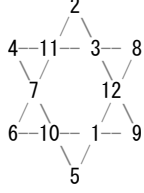
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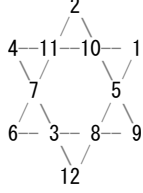
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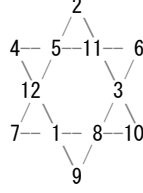
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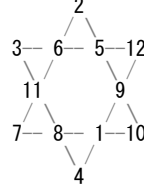
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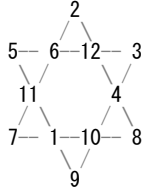
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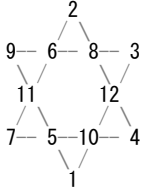
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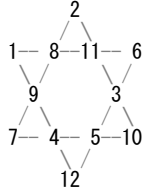
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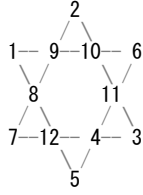
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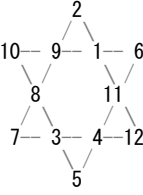
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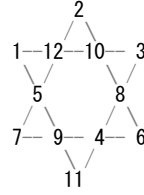
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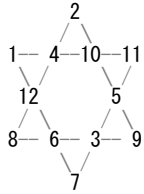
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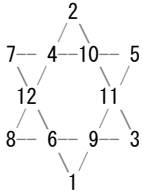
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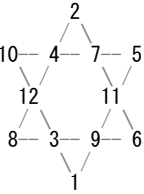
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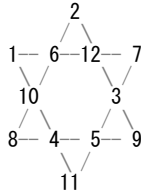
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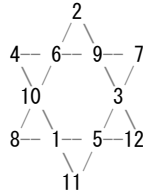
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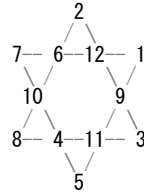
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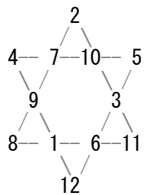
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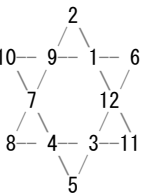
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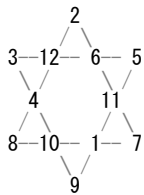
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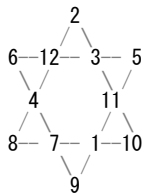
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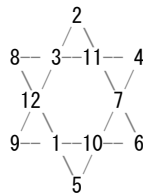
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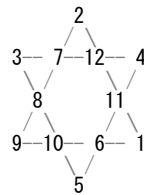
[124]



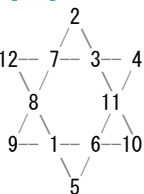
[125]



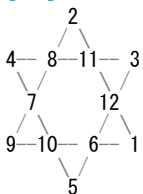
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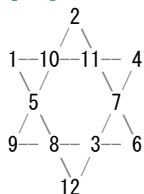
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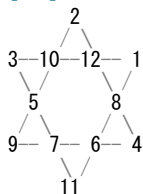
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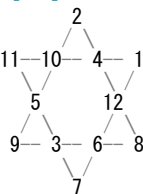
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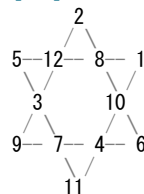
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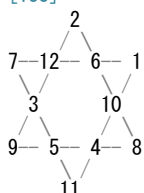
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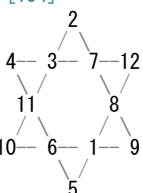
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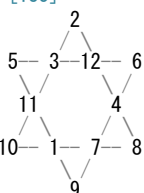
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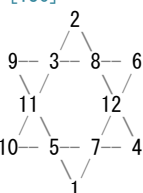
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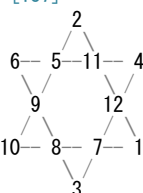
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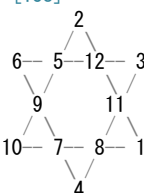
[136]



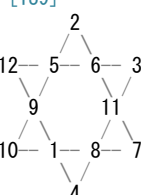
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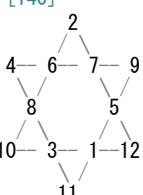
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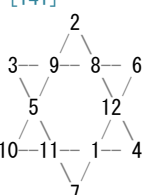
[139]



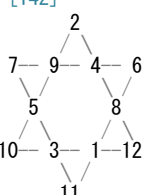
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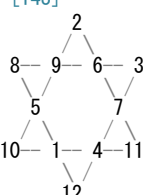
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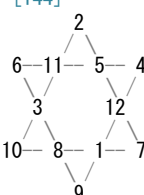
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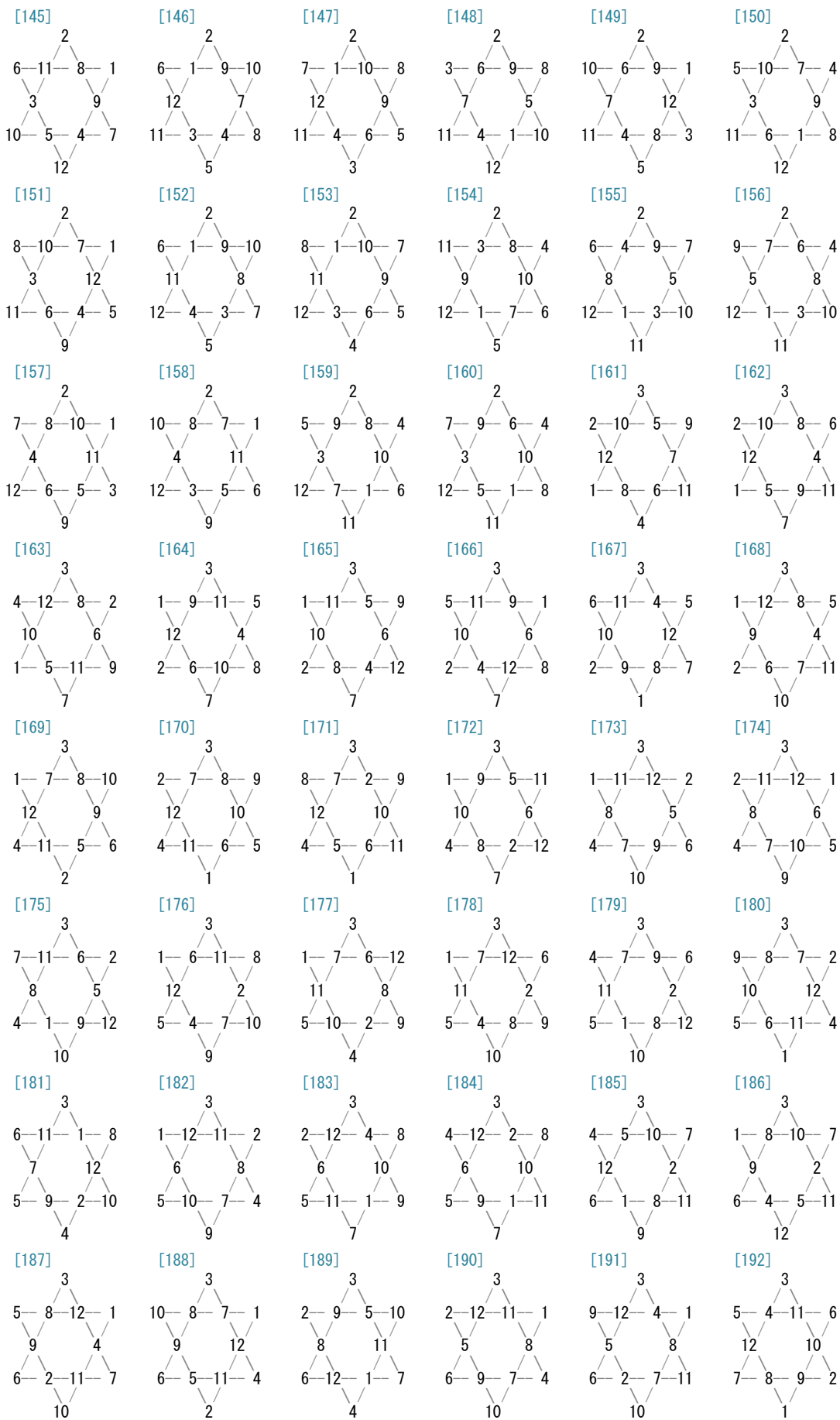


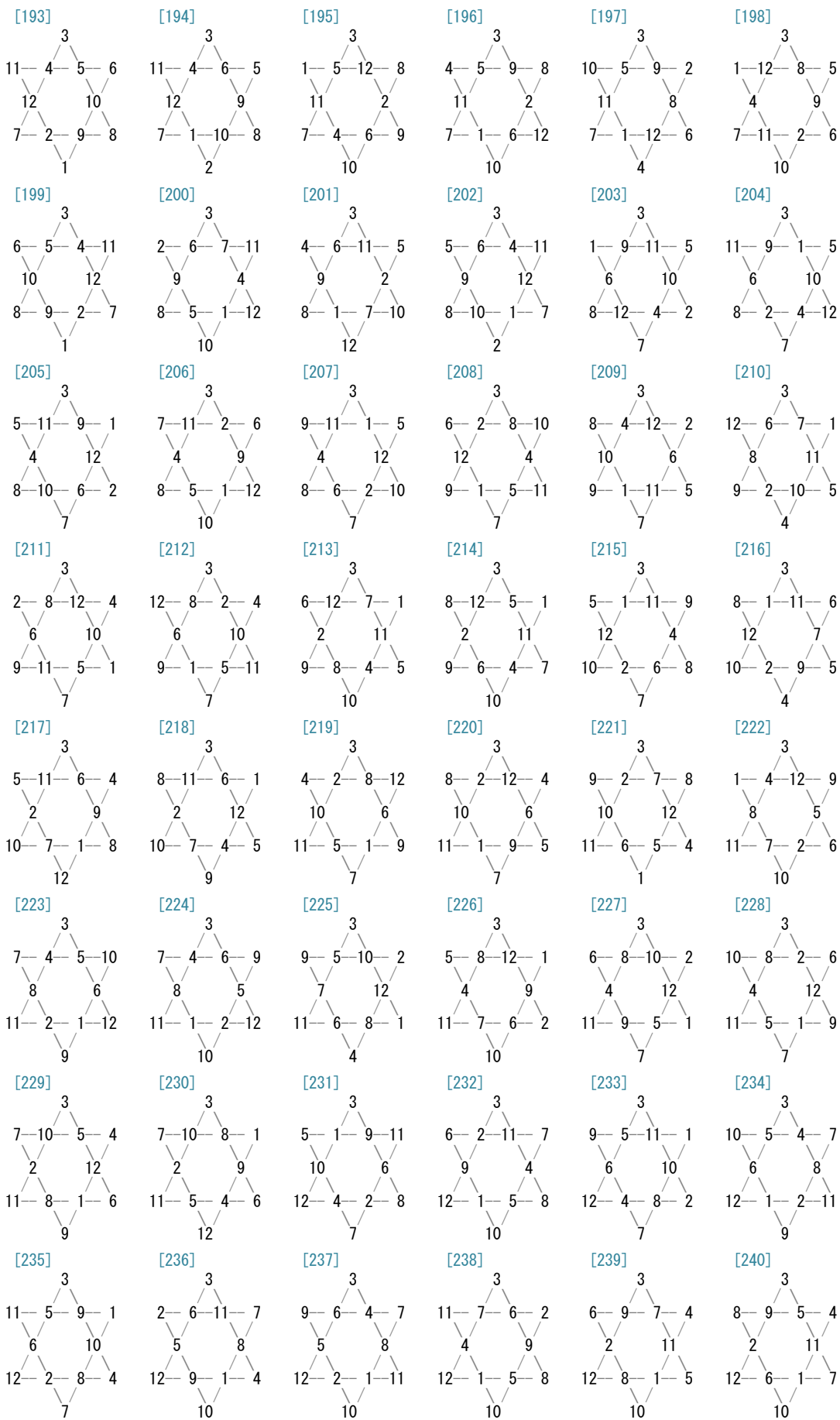
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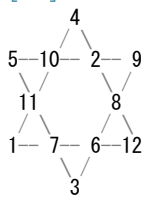
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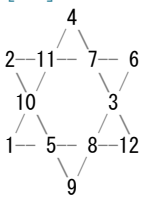




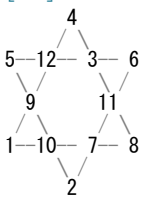
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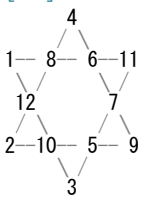
[242]



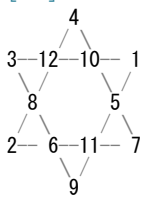
[243]



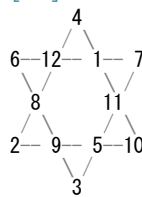
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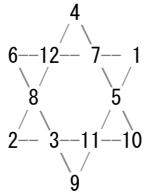
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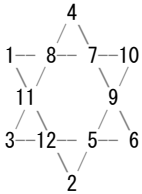
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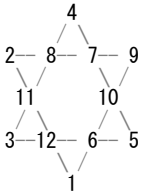
[247]



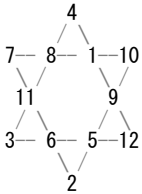
[248]



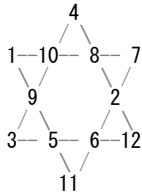
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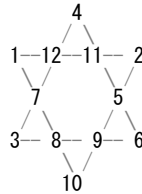
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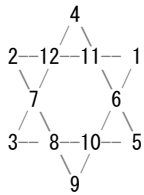
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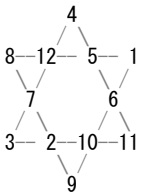
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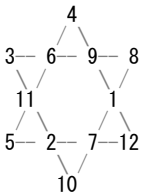
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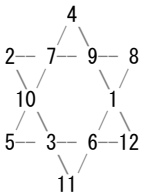
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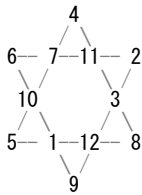
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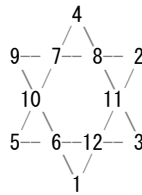
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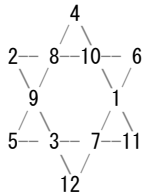
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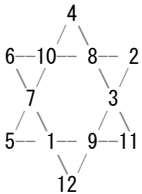
[258]



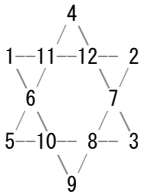
[259]



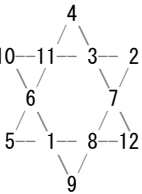
[260]



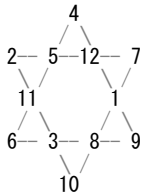
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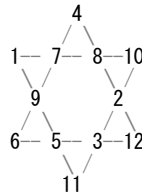
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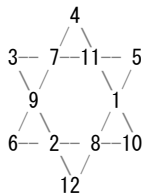
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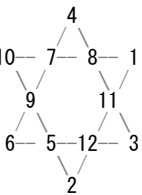
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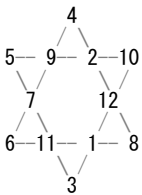
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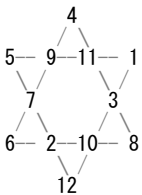
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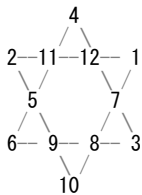
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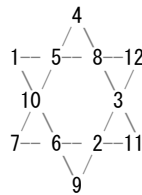
[268]



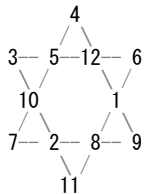
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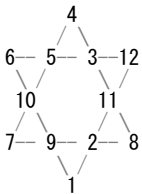
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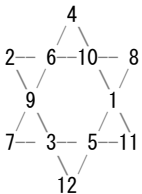
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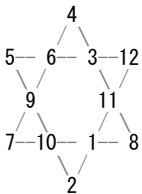
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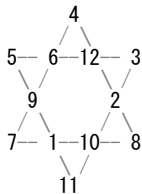
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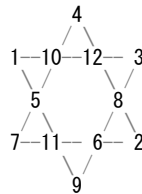
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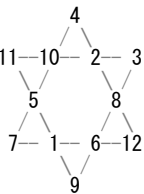
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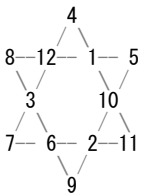
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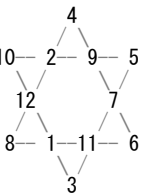
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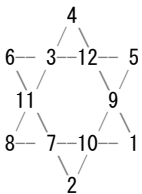
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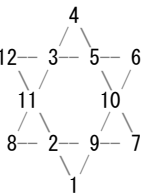
[279]



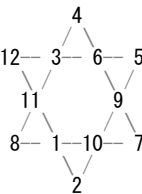
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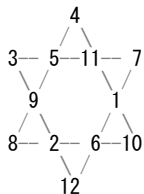
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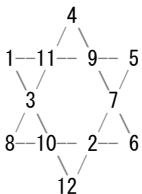
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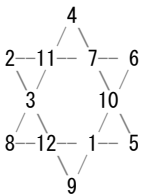
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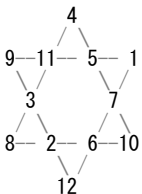
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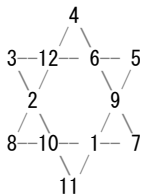
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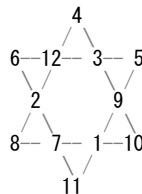
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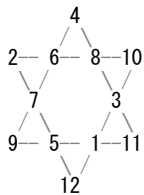
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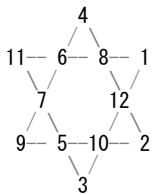
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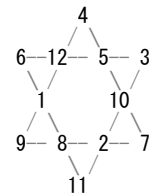
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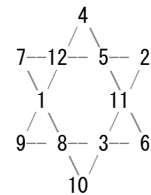
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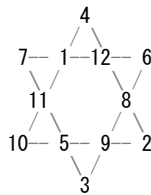
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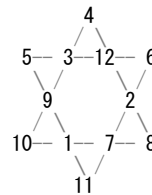
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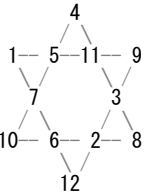
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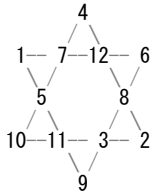
[294]



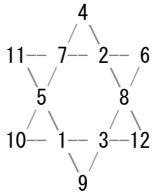
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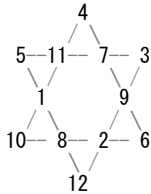
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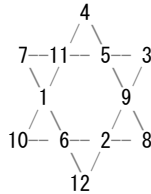
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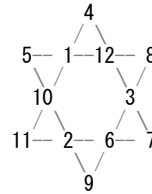
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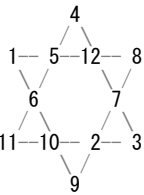
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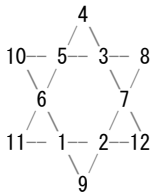
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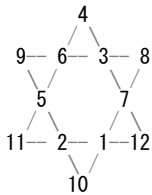
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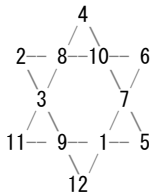
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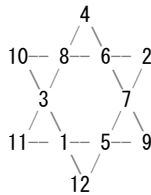
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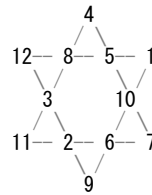
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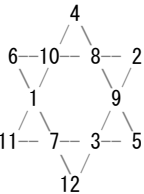
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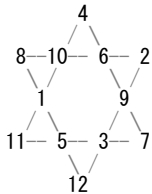
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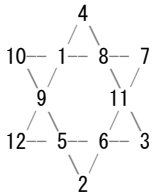
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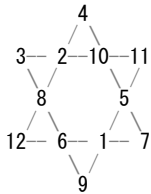
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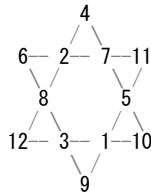
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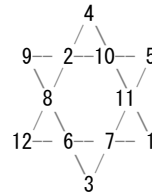
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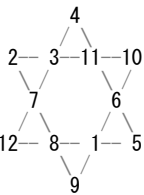
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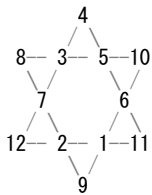
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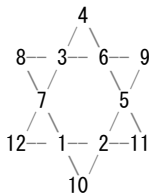
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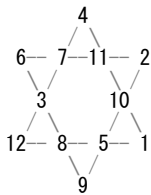
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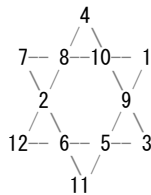
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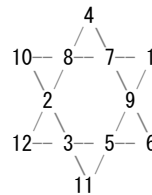
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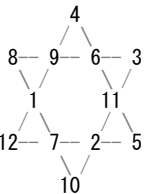
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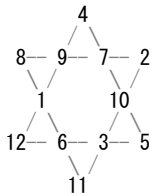
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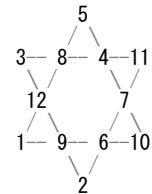
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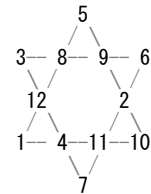
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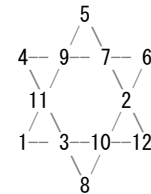
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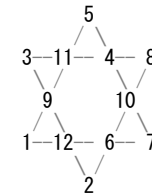
[322]



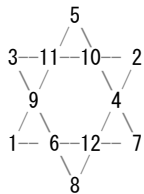
[323]



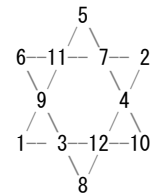
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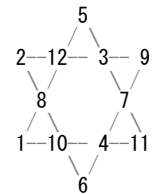
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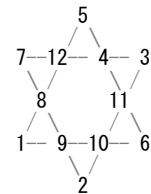
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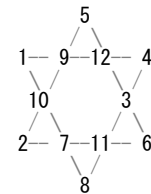
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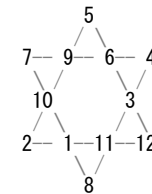
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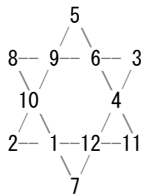
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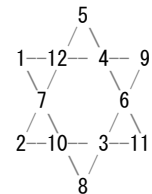
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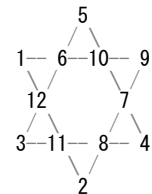
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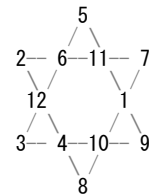
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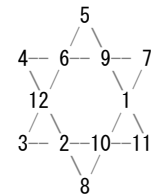
[333]



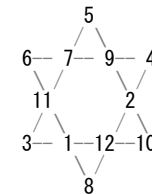
[334]



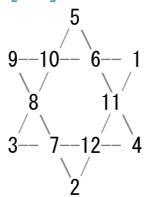
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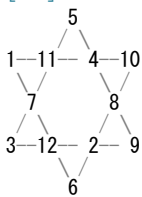
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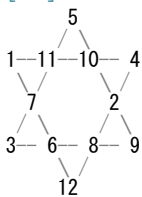
[337]



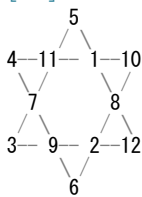
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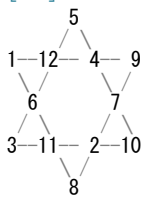
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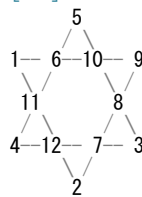
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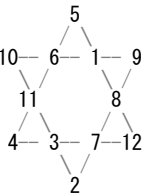
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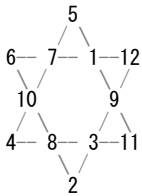
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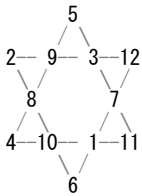
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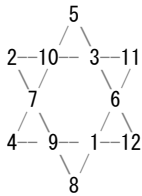
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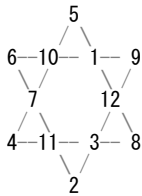
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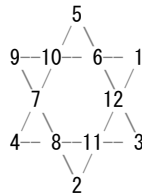
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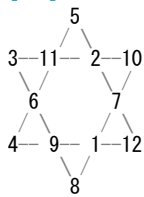
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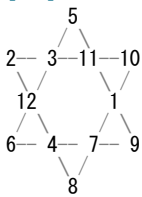
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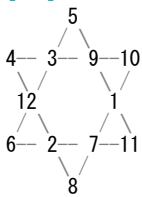
[349]



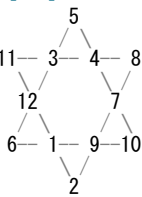
[350]



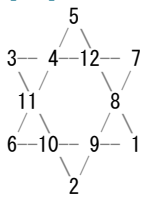
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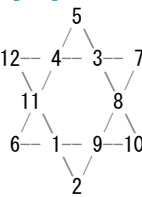
[352]



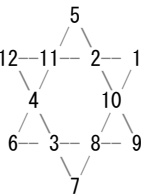
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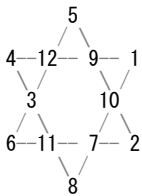
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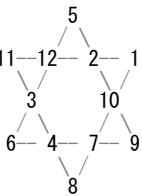
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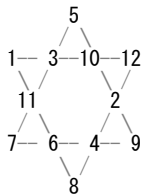
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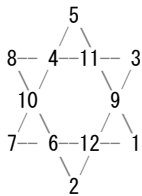
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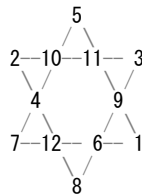
[358]



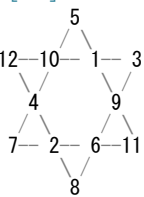
[359]



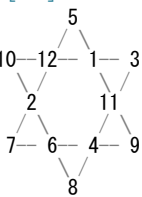
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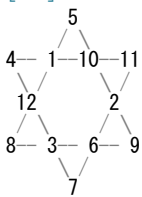
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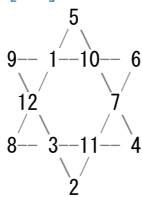
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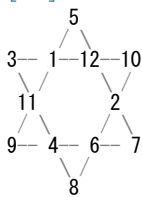
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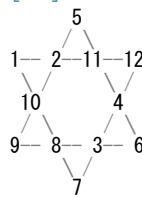
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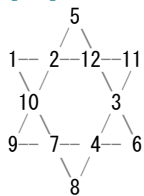
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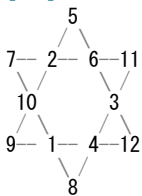
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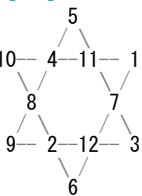
[367]



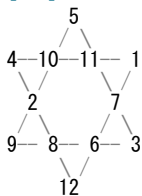
[368]



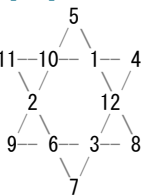
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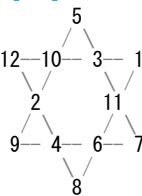
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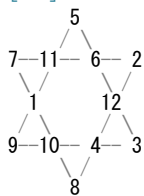
[371]



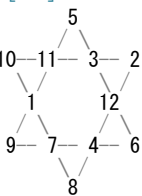
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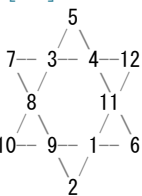
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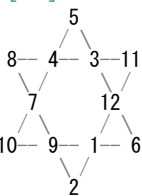
[374]



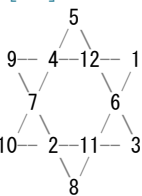
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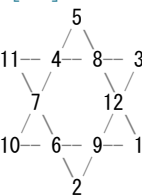
[376]



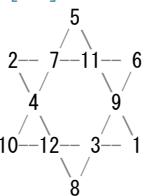
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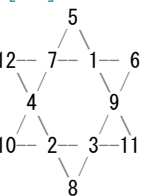
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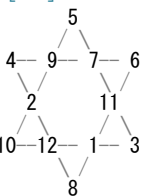
[379]



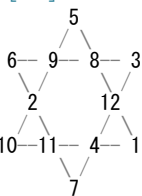
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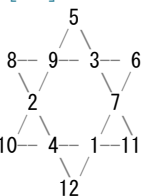
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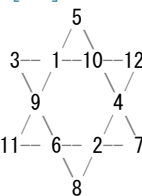
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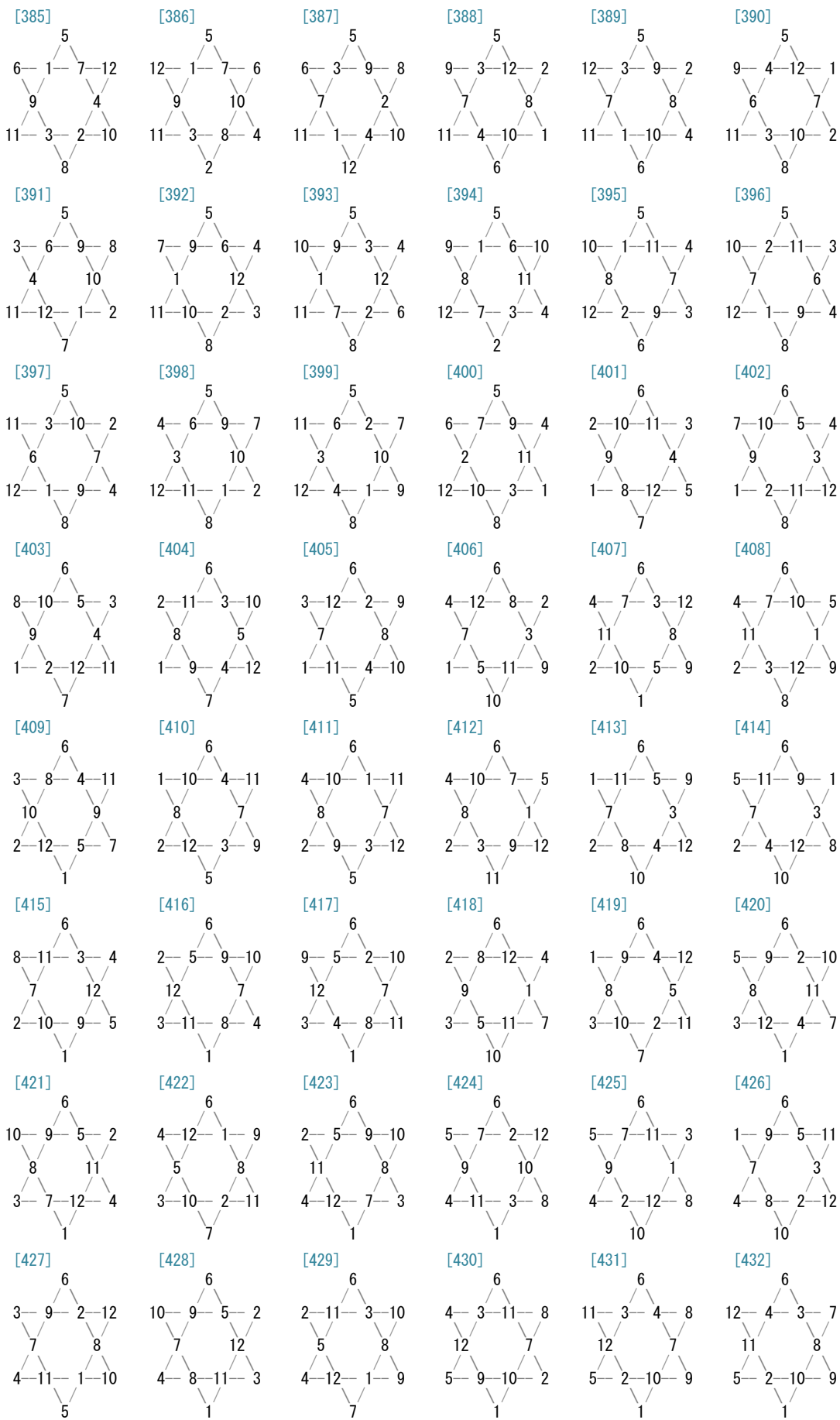


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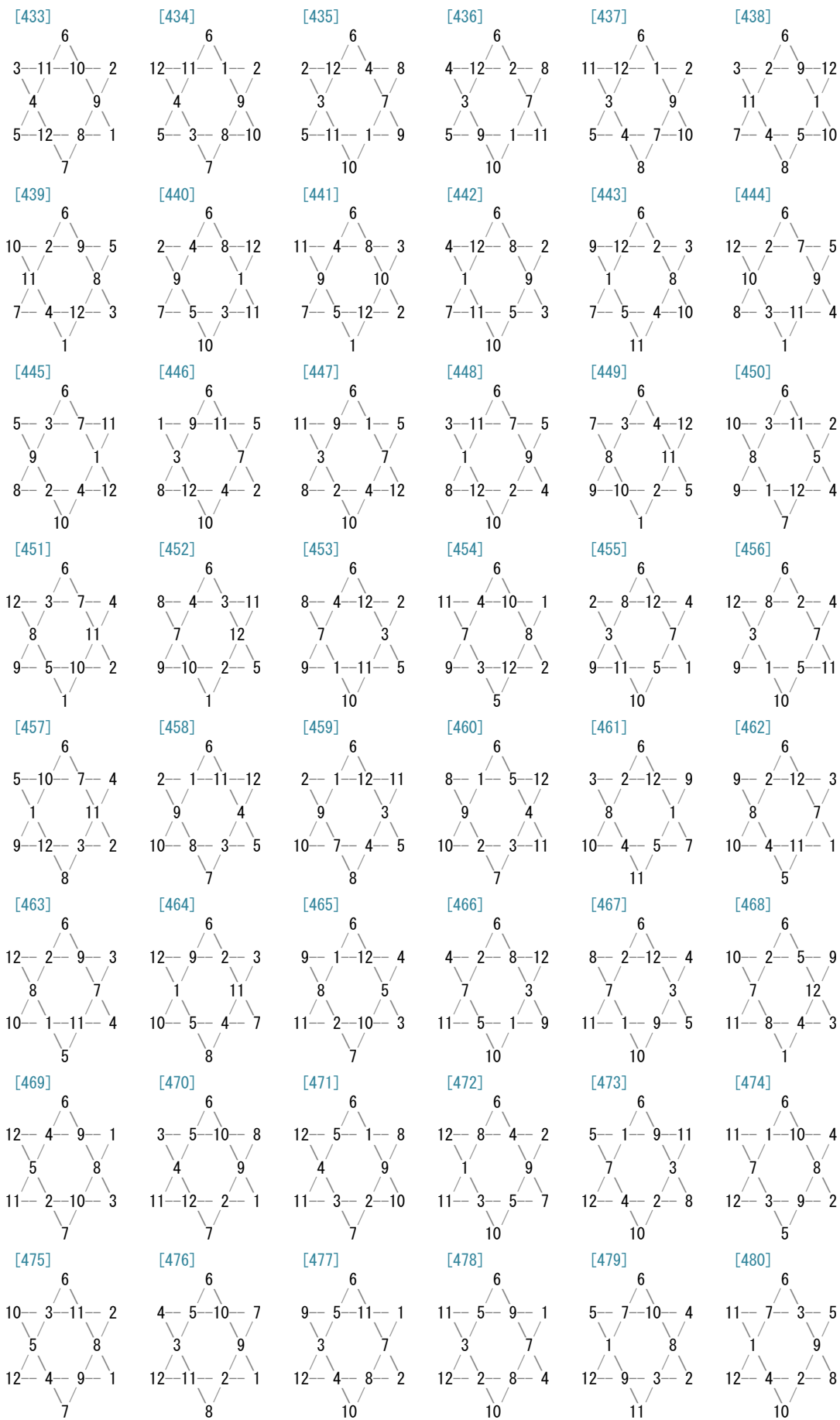


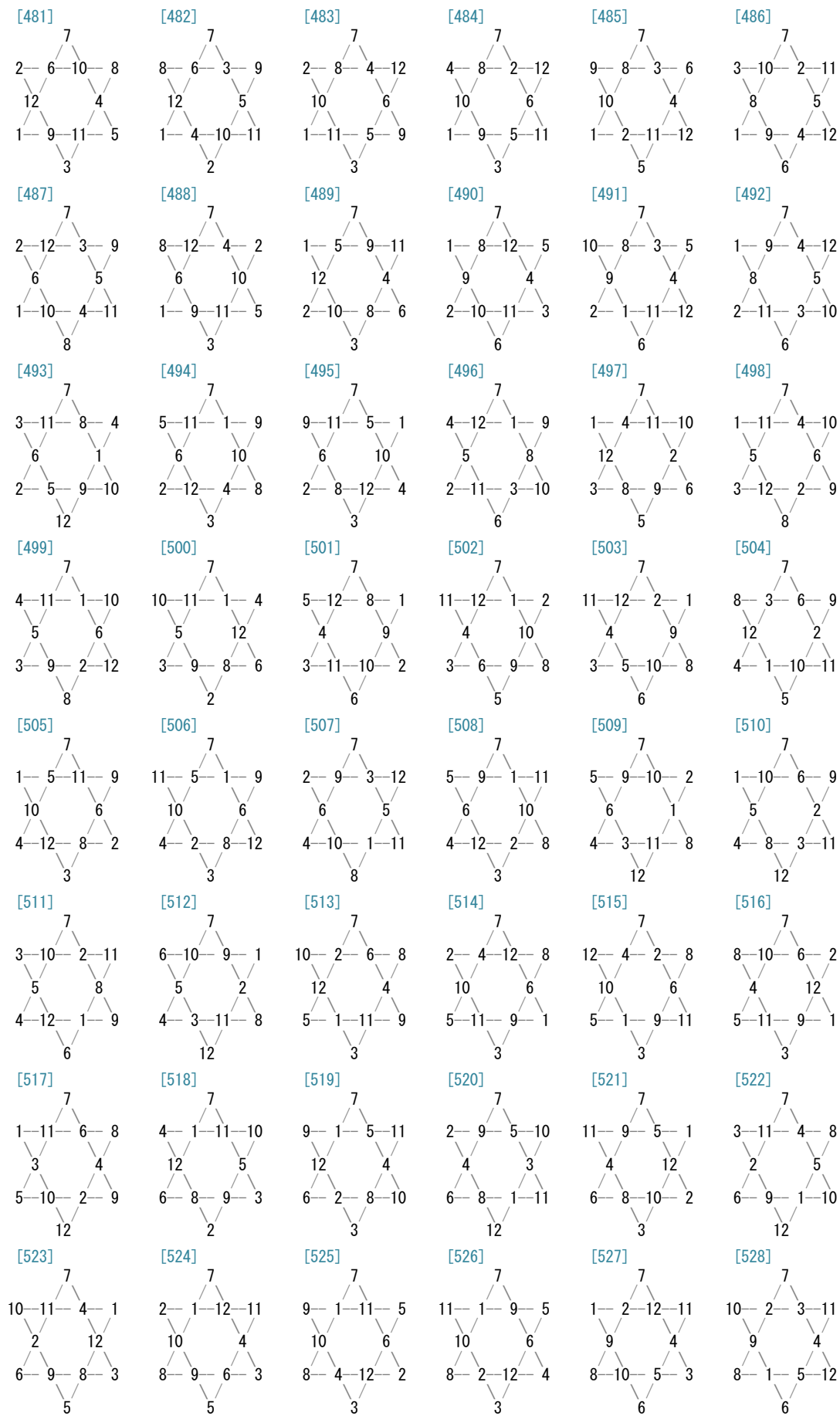
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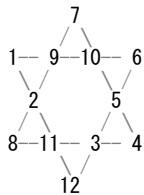




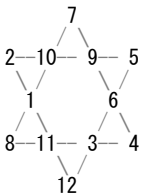




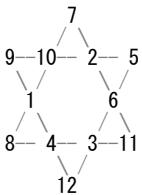
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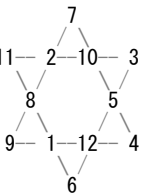
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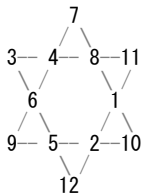
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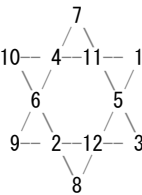
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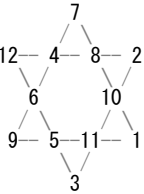
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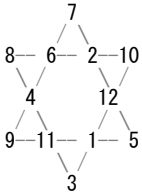
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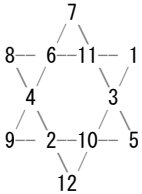
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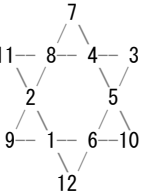
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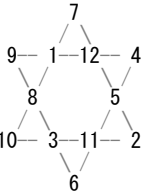
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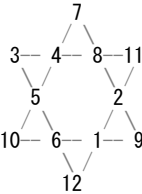
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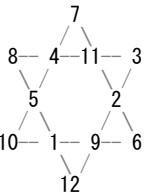
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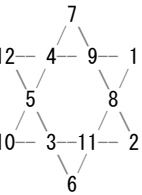
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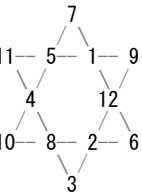
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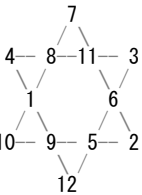
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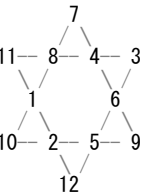
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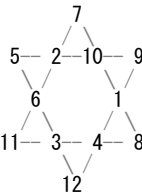
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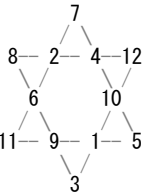
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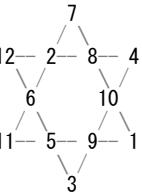
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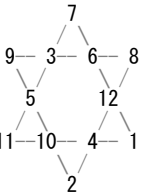
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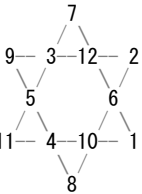
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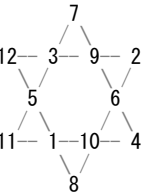
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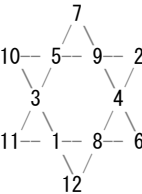
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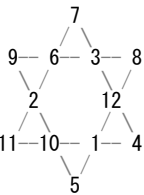
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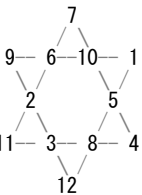
[552]



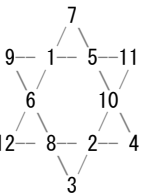
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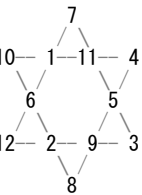
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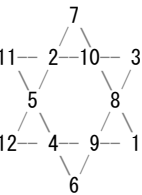
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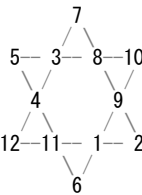
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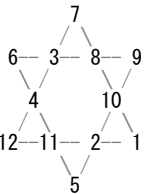
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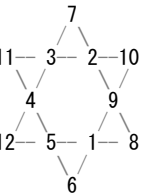
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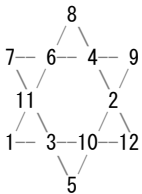
[559]



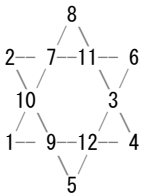
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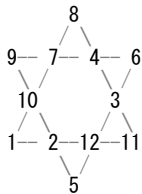
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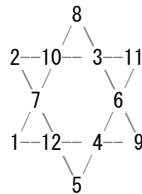
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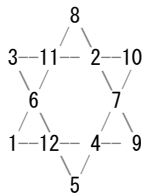
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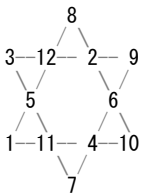
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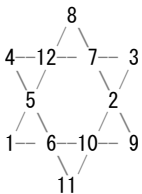
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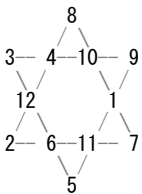
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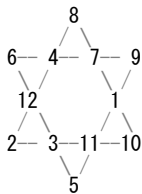
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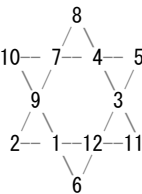
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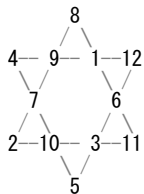
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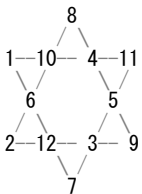
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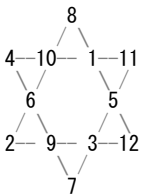
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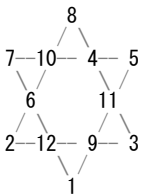
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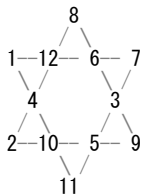
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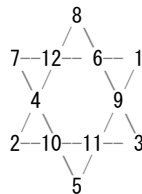
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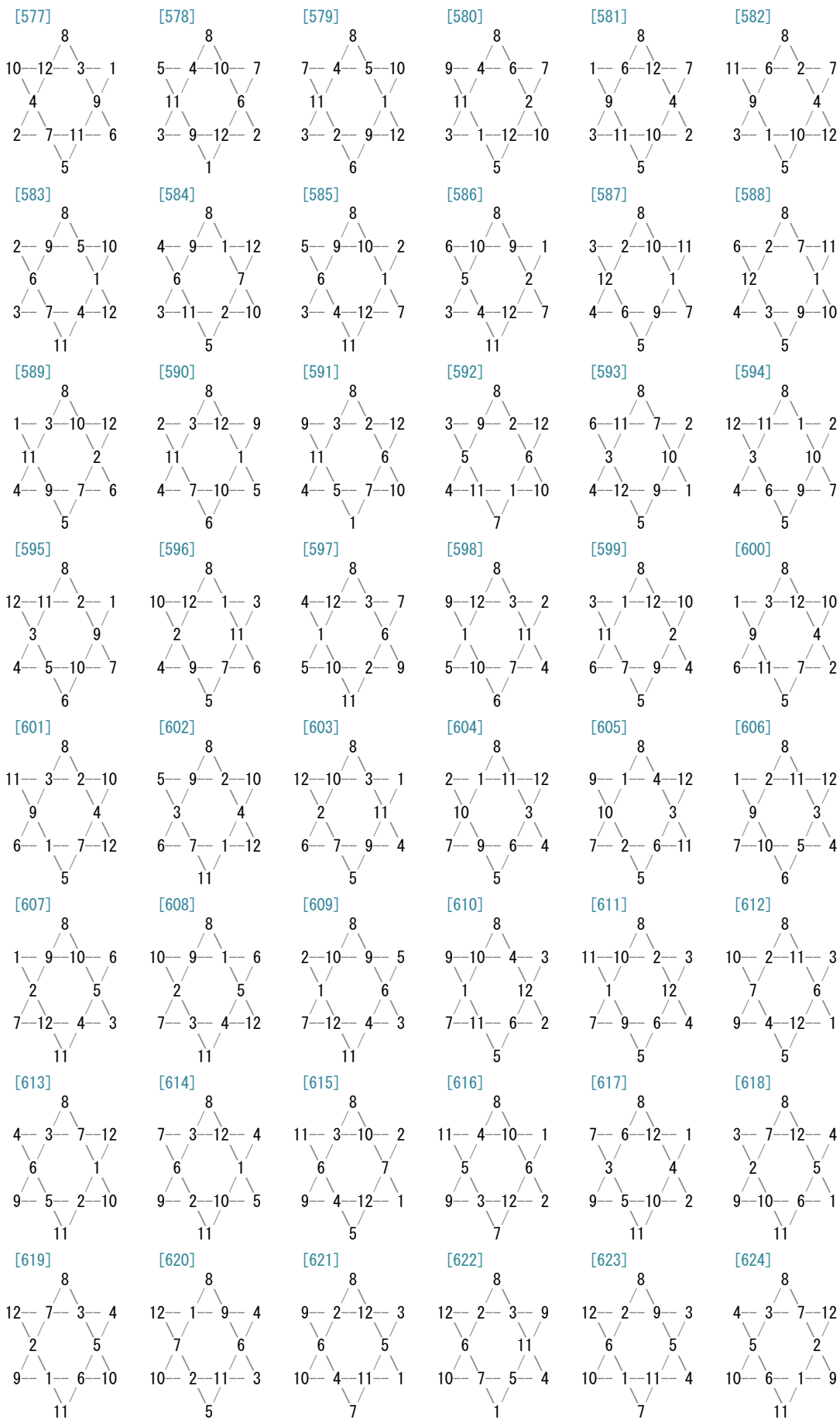


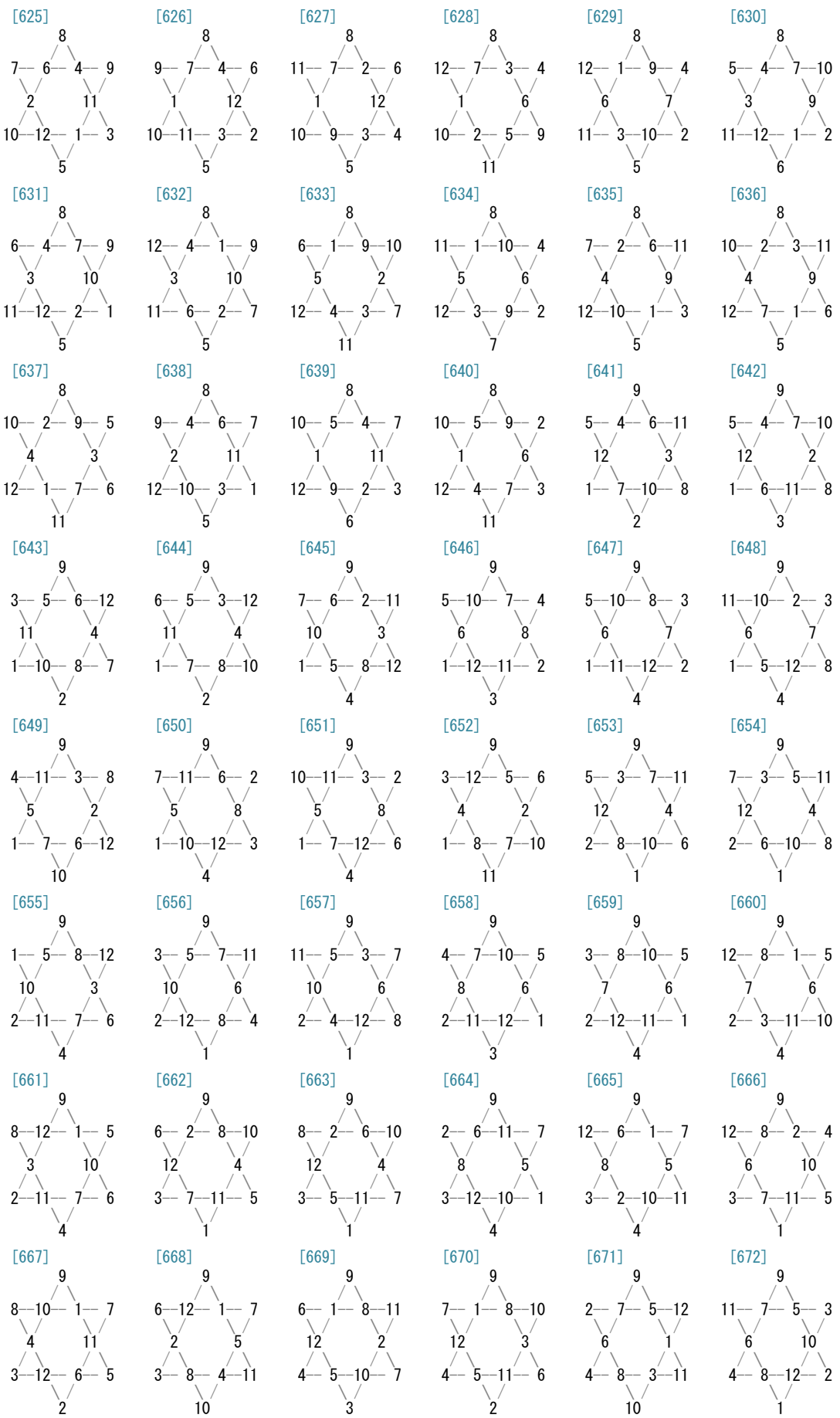
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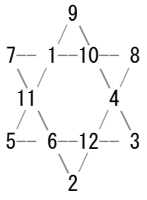
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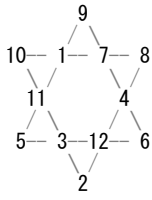




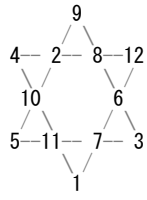
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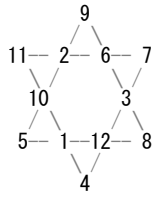
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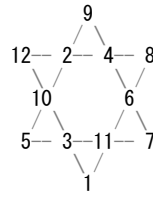
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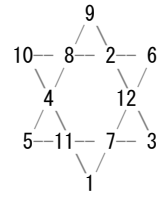
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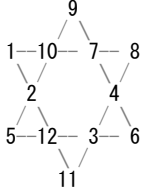
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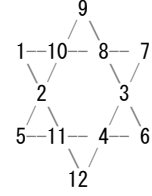
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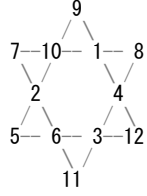
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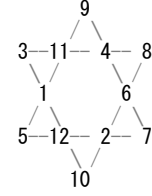
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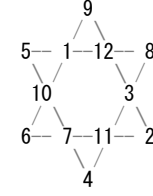
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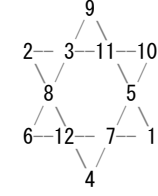
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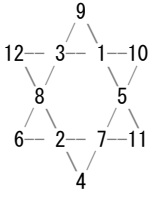
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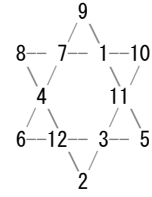
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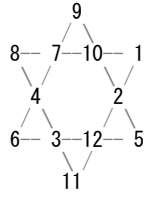
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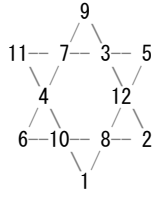
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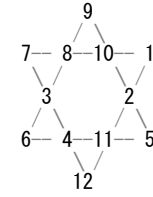
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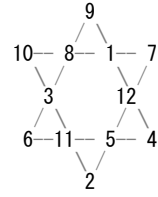
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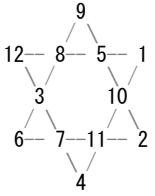
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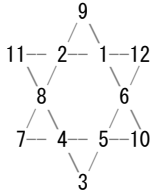
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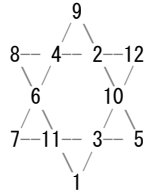
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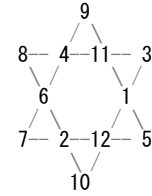
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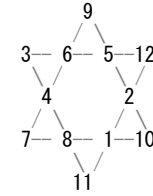
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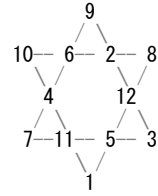
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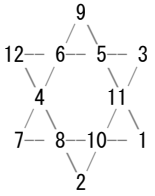
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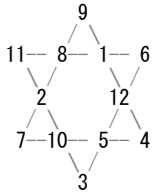
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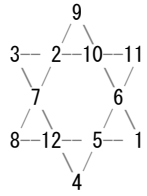
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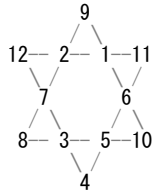
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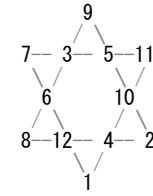
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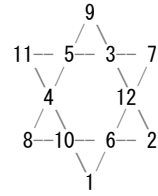
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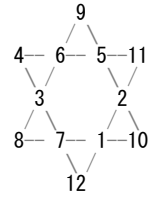
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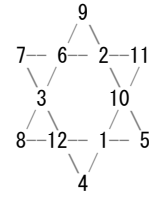
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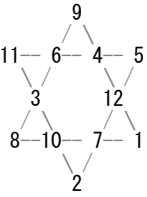
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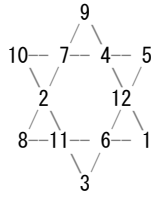
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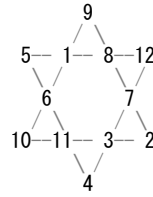
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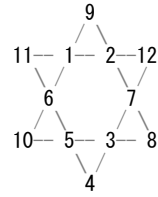
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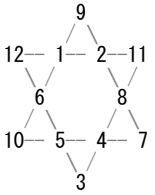
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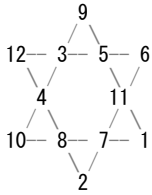
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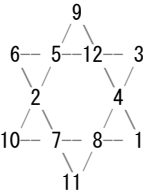
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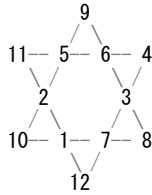
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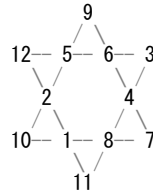
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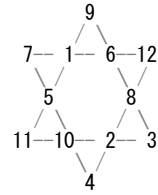
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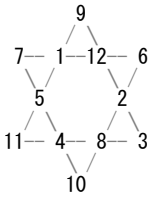
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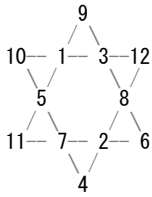
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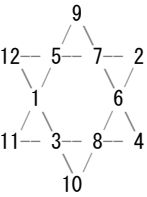
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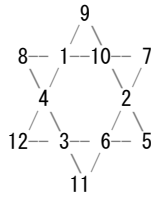
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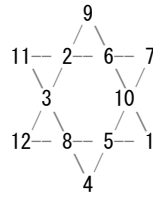
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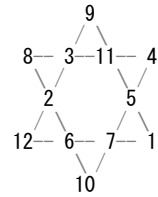
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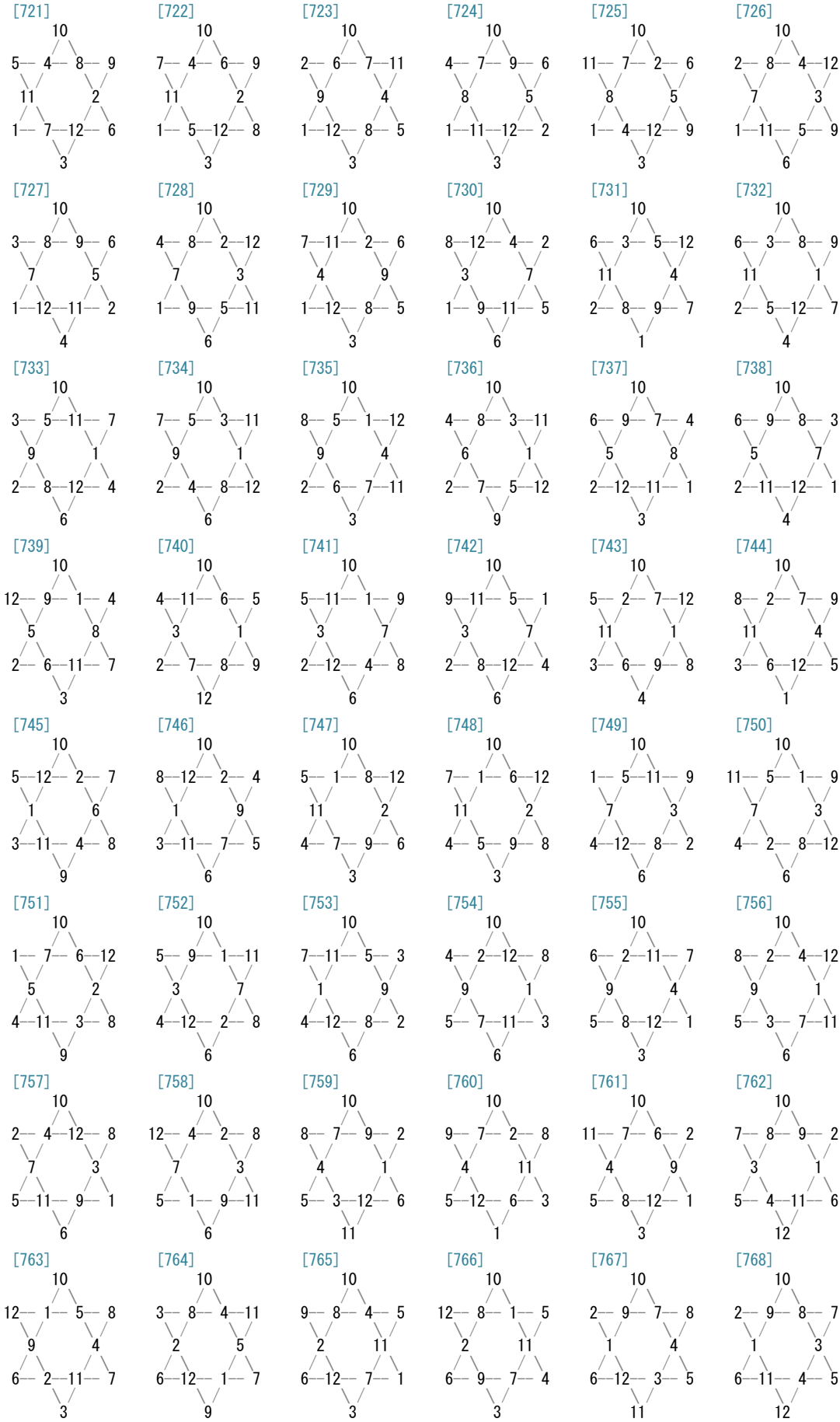


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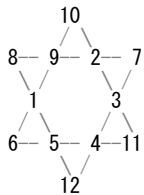


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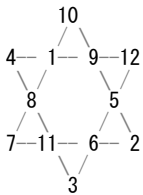




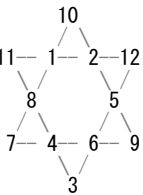
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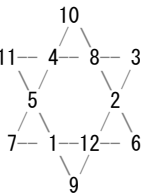
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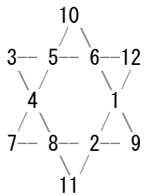
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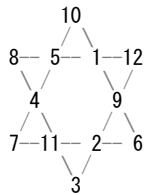
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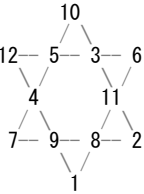
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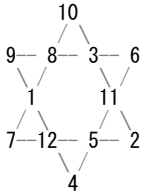
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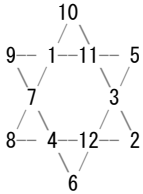
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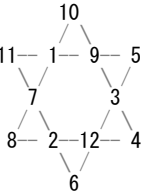
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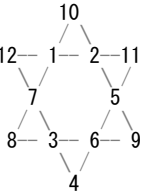
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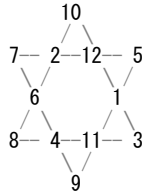
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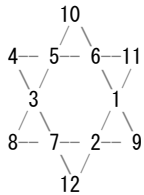
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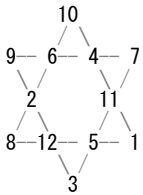
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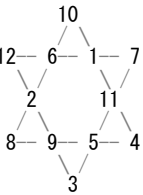
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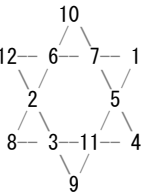
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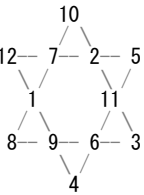
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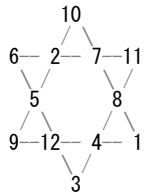
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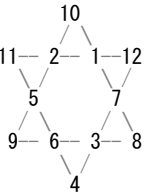
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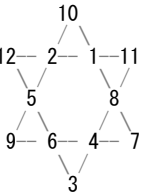
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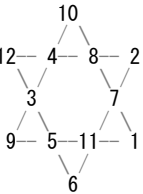
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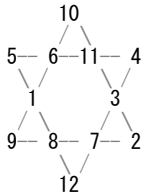
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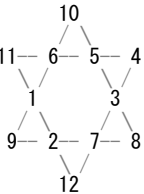
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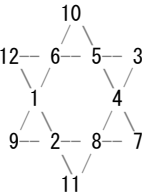
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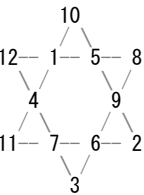
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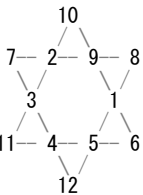
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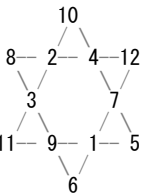
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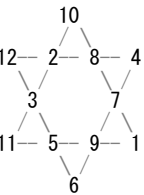
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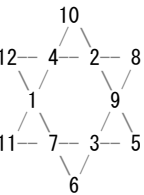
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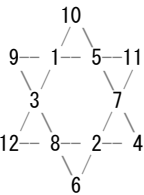
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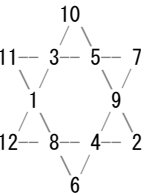
[797]



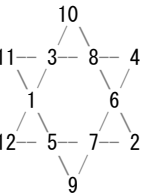
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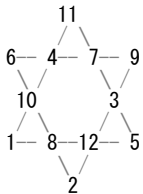
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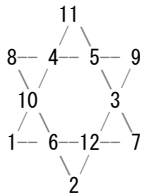
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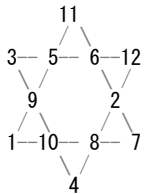
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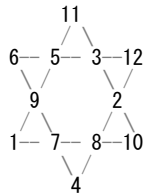
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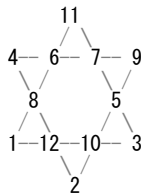
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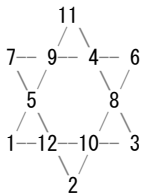
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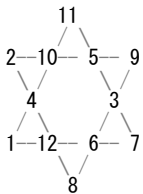
[805]



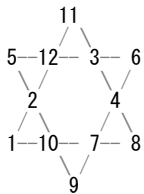
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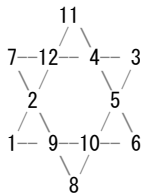
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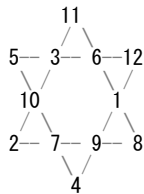
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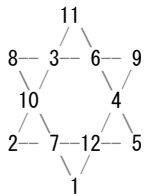
[809]



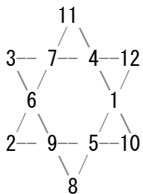
[810]



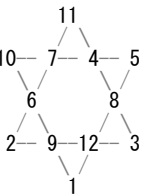
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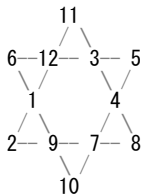
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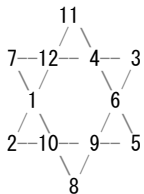
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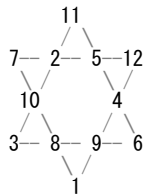
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[815]



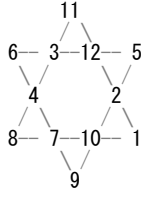
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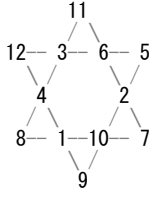




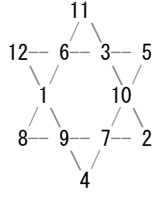
[865]



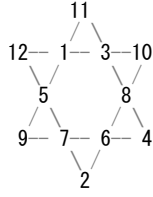
[866]



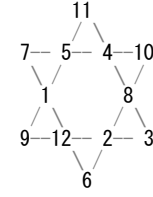
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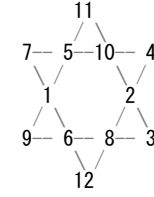
[868]



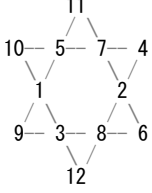
[869]



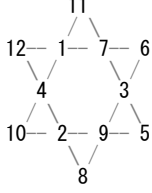
[870]



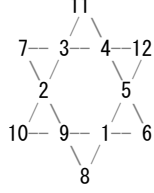
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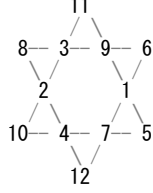
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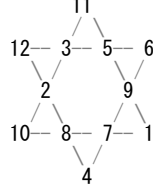
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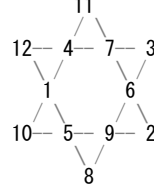
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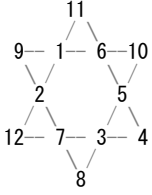
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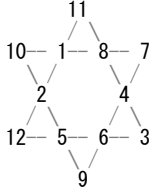
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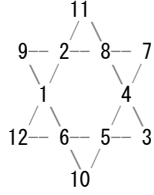
[877]



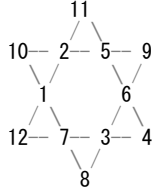
[878]



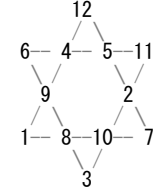
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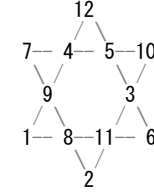
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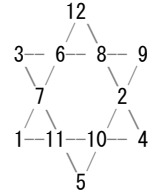
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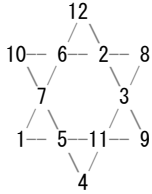
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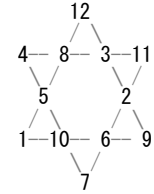
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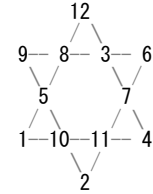
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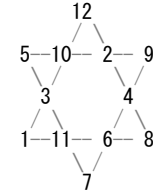
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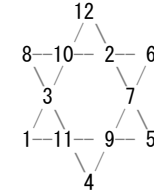
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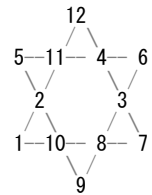
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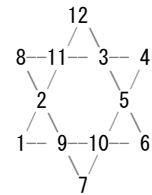
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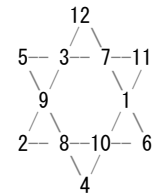
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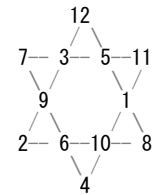
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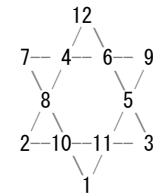
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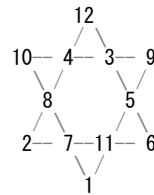
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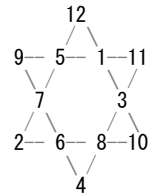
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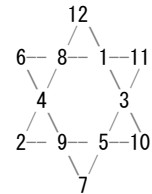
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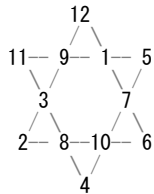
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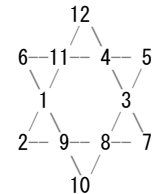
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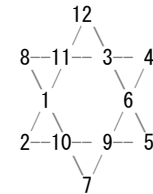
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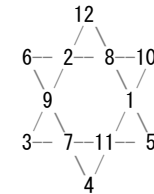
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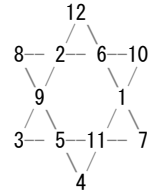
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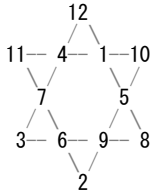
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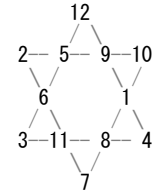
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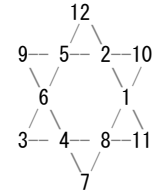
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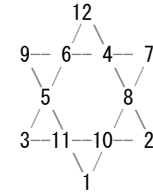
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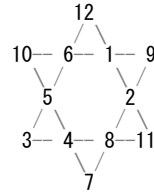
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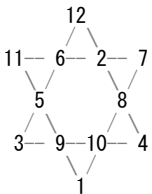
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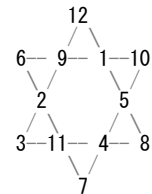
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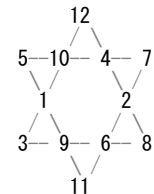
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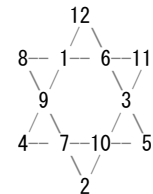
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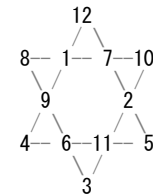
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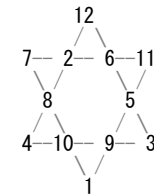
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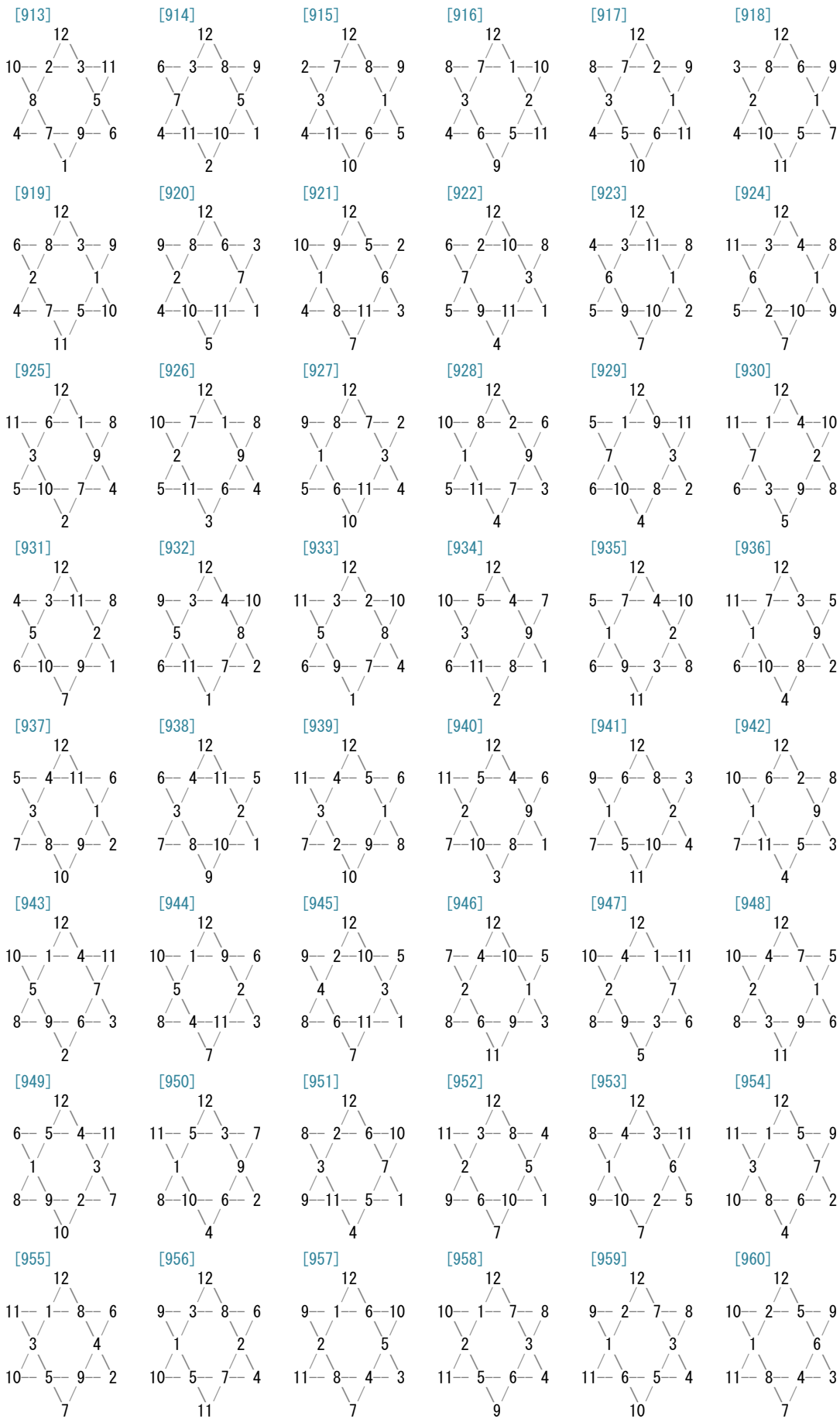


[911]



[912]





[原始解の総数 = 960] OK!

\*\* Calculated and listed by Kanji Setsuda on Apr.19, 2017 with MacOSX 10.12.4 & Xcode 7.3 \*\*

次に、回転形や反転形を省いた「標準解」のリストを作ろう。

頭初に示した基本図と定義式を前提条件にして、次のようなプログラムを書いてみた。  
もう今となっては古い言語になった‘C’を用いて、最新のシステム下で書いている。

```
/** Magic Stars of Order 6: List of Standard Solutions **
/** File 'MStar6.c': Dictated by Kanji Setsuda **
/** in 2003 and 2005; Recompiled on May 21, 2017 **
/** with macOS Sierra 10.12.5 and Xcode 7.3 **
//
#include <stdio.h>
//
short cnt, cnt2;
short LSM;
short nm[13], uflg[13];
short anm[5][13];
//
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void stp09(void), stp10(void), stp11(void), stp12(void);
void ansprint(void), pr4ans(void);
//
/** Main Program: Reset all and Go! *
int main(){
short n;
printf("//\n");
printf("/** Magic Stars of Order 6: List out all the Standard Solutions **\n");
printf("//\n");
for(n=0;n<13;n++){nm[n]=0; uflg[n]=0;}
LSM=26;
cnt=0; cnt2=0;
stp01(); /** Begin the Calculations *
printf("//\n");
printf("// [Solutions Count = %d] OK!\n",cnt);
printf("//\n");
printf("/** Recompiled by Kanji Setsuda on May 21, 2017\n");
printf("// with macOS Sierra 10.12.5 and Xcode 7.3 **\n");
printf("//\n");
return 0;
}
//
/** Begin the Calculations *
/** Set N1 *
void stp01(){
short a;
for(a=1;a<8;a++){
if(uflg[a]==0){
nm[1]=a; uflg[a]=1;
stp02();
uflg[a]=0;
}
}
}
/** Set N5(>n1) *
void stp02(){
short a;
for(a=nm[1]+1;a<13;a++){
if(uflg[a]==0){
nm[5]=a; uflg[a]=1;
stp03();
uflg[a]=0;
}
}
}
}
/** Set N2 *
void stp03(){
short a;
for(a=12;a>0;a--){
if(uflg[a]==0){
nm[2]=a; uflg[a]=1;
stp04();
uflg[a]=0;
}
}
}
/** Set n4=30-n1-n2-n5 *
void stp04(){
short a;
a=LSM-nm[1]-nm[2]-nm[5];
if((0<a)&&(a<13)){
if(uflg[a]==0){
nm[4]=a; uflg[a]=1;
stp05();
uflg[a]=0;
}
}
}
}
```

```

}
/** Set N3(>n1) *
void stp05(){
    short a;
    for(a=nm[1]+1;a<13;a++){
        if(uflg[a]==0){
            nm[3]=a; uflg[a]=1;
            stp06();
            uflg[a]=0;
        }
    }
}
/** Set N7(>n1) *
void stp06(){
    short a;
    for(a=nm[1]+1;a<13;a++){
        if(uflg[a]==0){
            nm[7]=a; uflg[a]=1;
            stp07();
            uflg[a]=0;
        }
    }
}
/** Set n6=30-n3-n4-n7 *
void stp07(){
    short a;
    a=LSM-nm[3]-nm[4]-nm[7];
    if((0<a)&&(a<13)){
        if(uflg[a]==0){
            nm[6]=a; uflg[a]=1;
            stp08();
            uflg[a]=0;
        }
    }
}
/** Set N9(>n5) *
void stp08(){
    short a;
    for(a=nm[5]+1;a<13;a++){
        if(uflg[a]==0){
            nm[9]=a; uflg[a]=1;
            stp09();
            uflg[a]=0;
        }
    }
}
/** Set n8=30-n5-n6-n9 *
}

void stp09(){
    short a;
    a=LSM-nm[5]-nm[6]-nm[9];
    if((0<a)&&(a<13)){
        if(uflg[a]==0){
            nm[8]=a; uflg[a]=1;
            stp10();
            uflg[a]=0;
        }
    }
}
/** Set N11(>n1) *
void stp10(){
    short a;
    for(a=nm[1]+1;a<13;a++){
        if(uflg[a]==0){
            nm[11]=a; uflg[a]=1;
            stp11();
            uflg[a]=0;
        }
    }
}
/** Set n10=30-n7-n8-n11 *
void stp11(){
    short a;
    a=LSM-nm[7]-nm[8]-nm[11];
    if((0<a)&&(a<13)){
        if(uflg[a]==0){
            nm[10]=a; uflg[a]=1;
            stp12();
            uflg[a]=0;
        }
    }
}
/** Set n12=30-n9-n10-n11 *
void stp12(){
    short a,b;
    a=LSM-nm[9]-nm[10]-nm[11];
    b=LSM-nm[11]-nm[2]-nm[3];
    if((a==b)&&(0<a)&&(a<13)){
        if(uflg[a]==0){
            nm[12]=a; uflg[a]=1;
            ansprint();
            uflg[a]=0;
        }
    }
}

}

/** Print The Answers *
void ansprint(){
    short n;
    cnt++;
    anm[cnt2][0]=cnt; anm[cnt2+1][0]=0;
    for(n=1;n<13;n++){anm[cnt2][n]=nm[n]; anm[cnt2+1][n]=0;}
    cnt2++;
    if(cnt2==4){pr4ans(); cnt2=0;}
}
/** Print 4 Answers *
void pr4ans(){
    short m;
    printf("//");
    for(m=0;m<4;m++){
        printf(" [%2d] ",anm[m][0]);
    }
    printf("\n//");
    for(m=0;m<4;m++){
        printf("%9d ",anm[m][1]);
    }
    printf("\n//");
    printf(" / \\\n//");
}

```

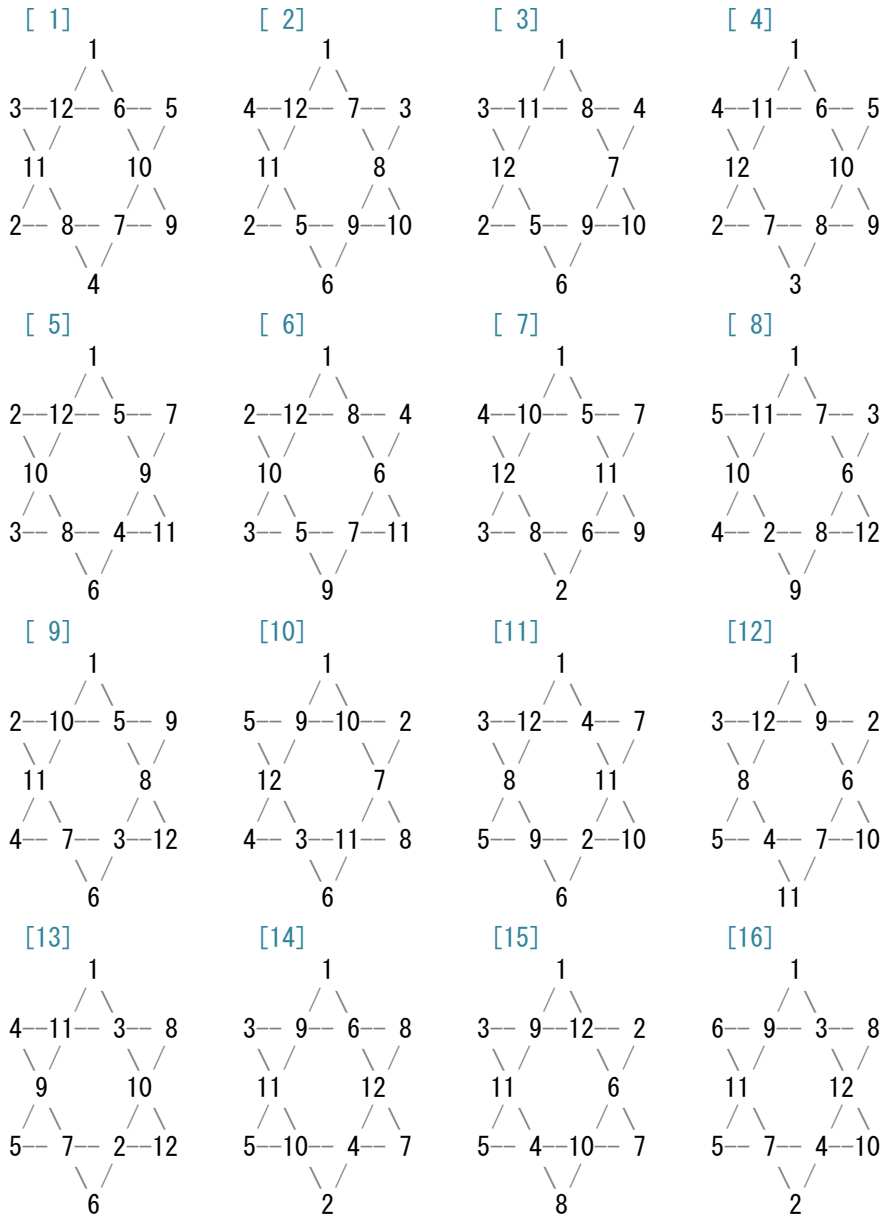
```

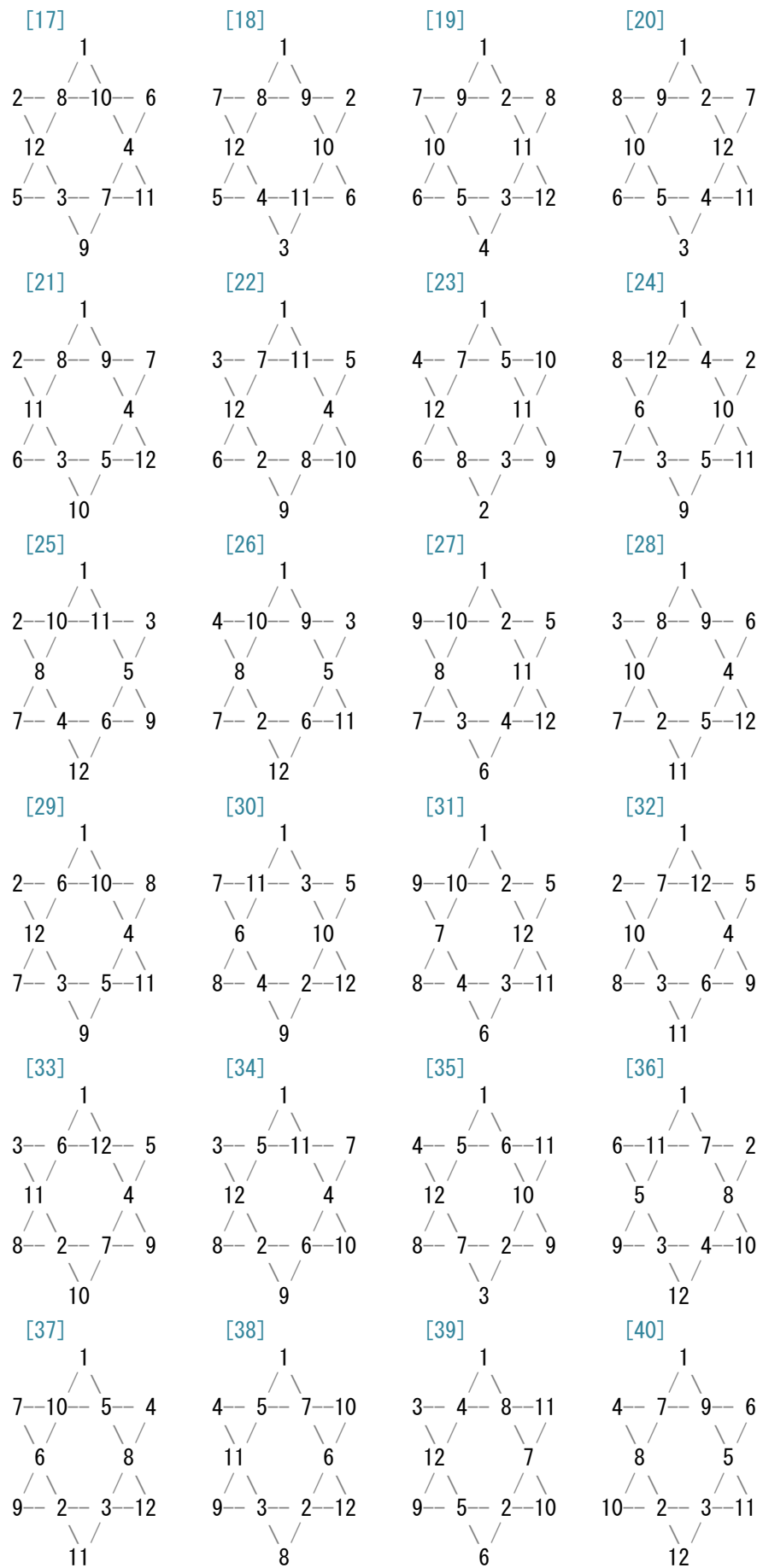
for(m=0;m<4;m++){
    printf("%3d--%2d--%2d--%2d  ",anm[m][3],anm[m][2],anm[m][12],anm[m][11]);}
printf("\n//");
printf("  \\ /      \\ /      \\ /      \\ /      \\ /      \\ /      \\ /      \\ \n//");
for(m=0;m<4;m++){
    printf("%5d%8d      ",anm[m][4],anm[m][10]);}
printf("\n//");
printf("  / \\      / \\      / \\      / \\      / \\      / \\      / \\      / \\ \n//");
for(m=0;m<4;m++){
    printf("%3d--%2d--%2d--%2d  ",anm[m][5],anm[m][6],anm[m][8],anm[m][9]);}
printf("\n//");
printf("      \\ /      \\ /      \\ /      \\ /      \\ \n//");
for(m=0;m<4;m++){
    printf("%9d      ",anm[m][7]);}
printf("\n");
}
//

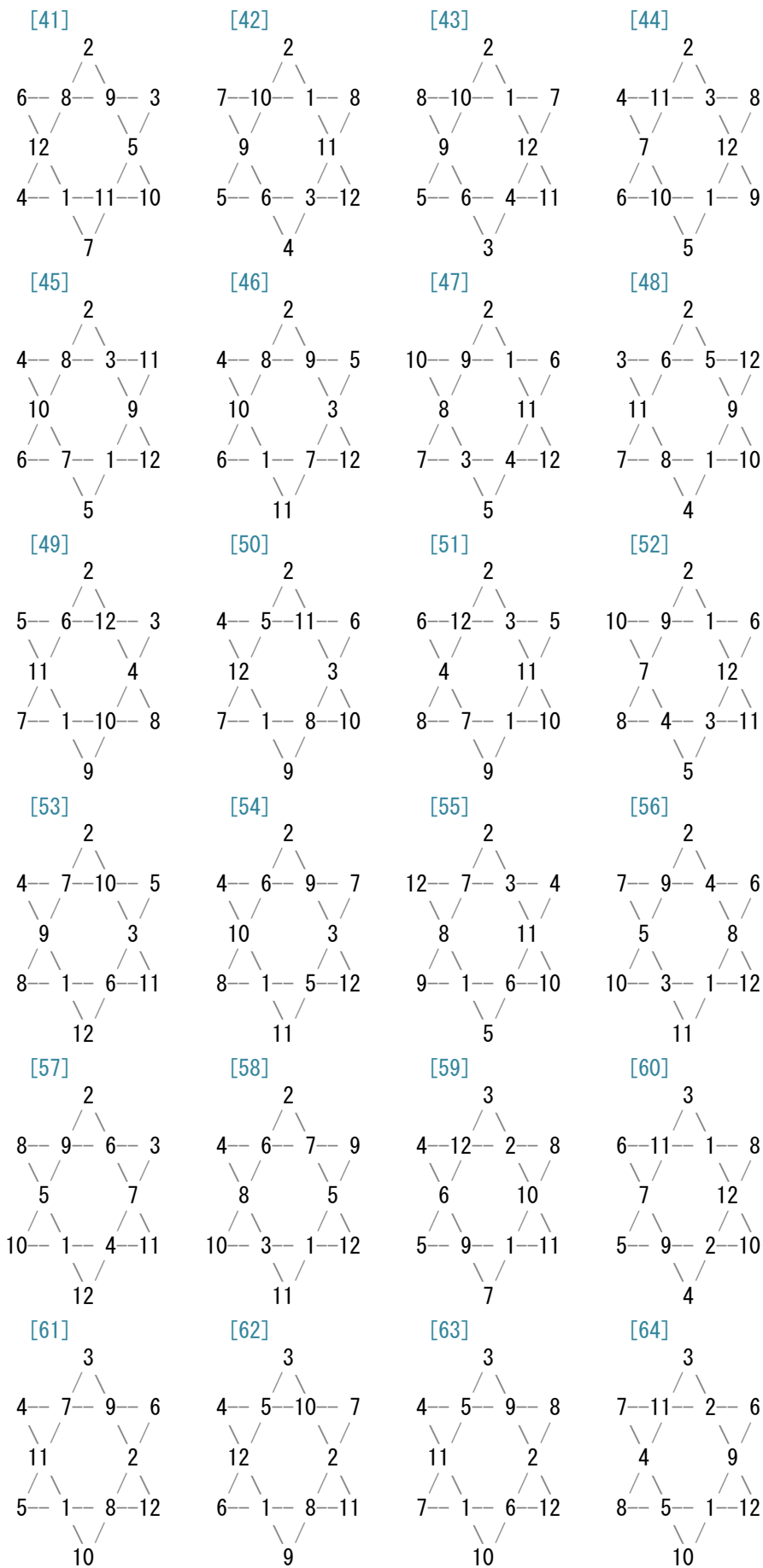
```

**\*\* 六星陣の「標準解」80個のリスト \*\***

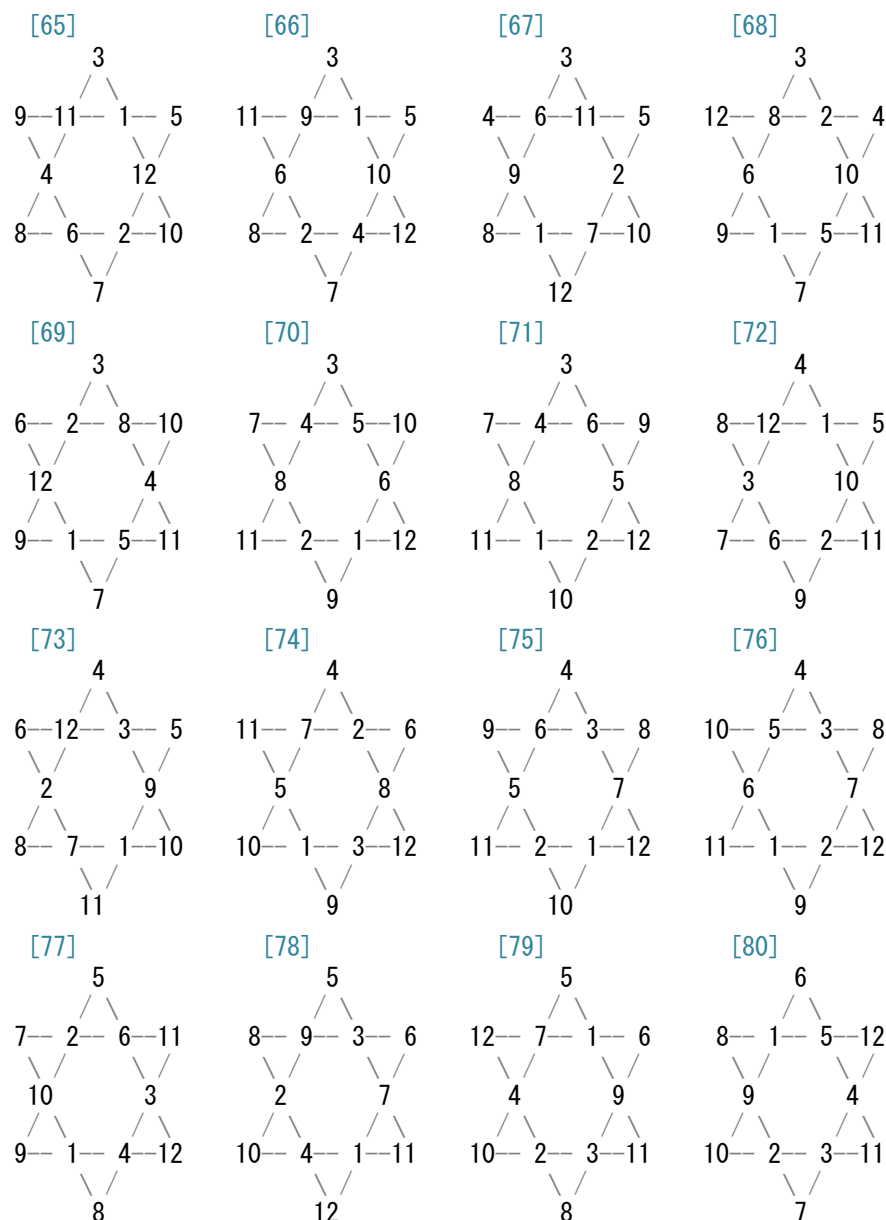
**\*\* Magic Stars of Order 6: List out all the Standard Solutions \*\***











[Solutions Count = 80]      Recompiled by Kanji Setsuda on May 21, 2017

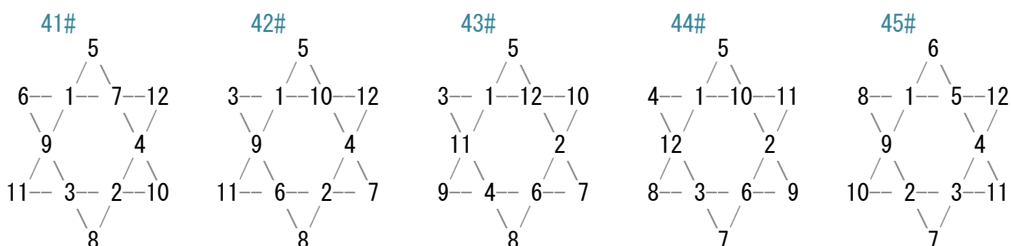
上のリストが、実行結果である。解の鏡映反転や回転を次の不等式群で封じている。

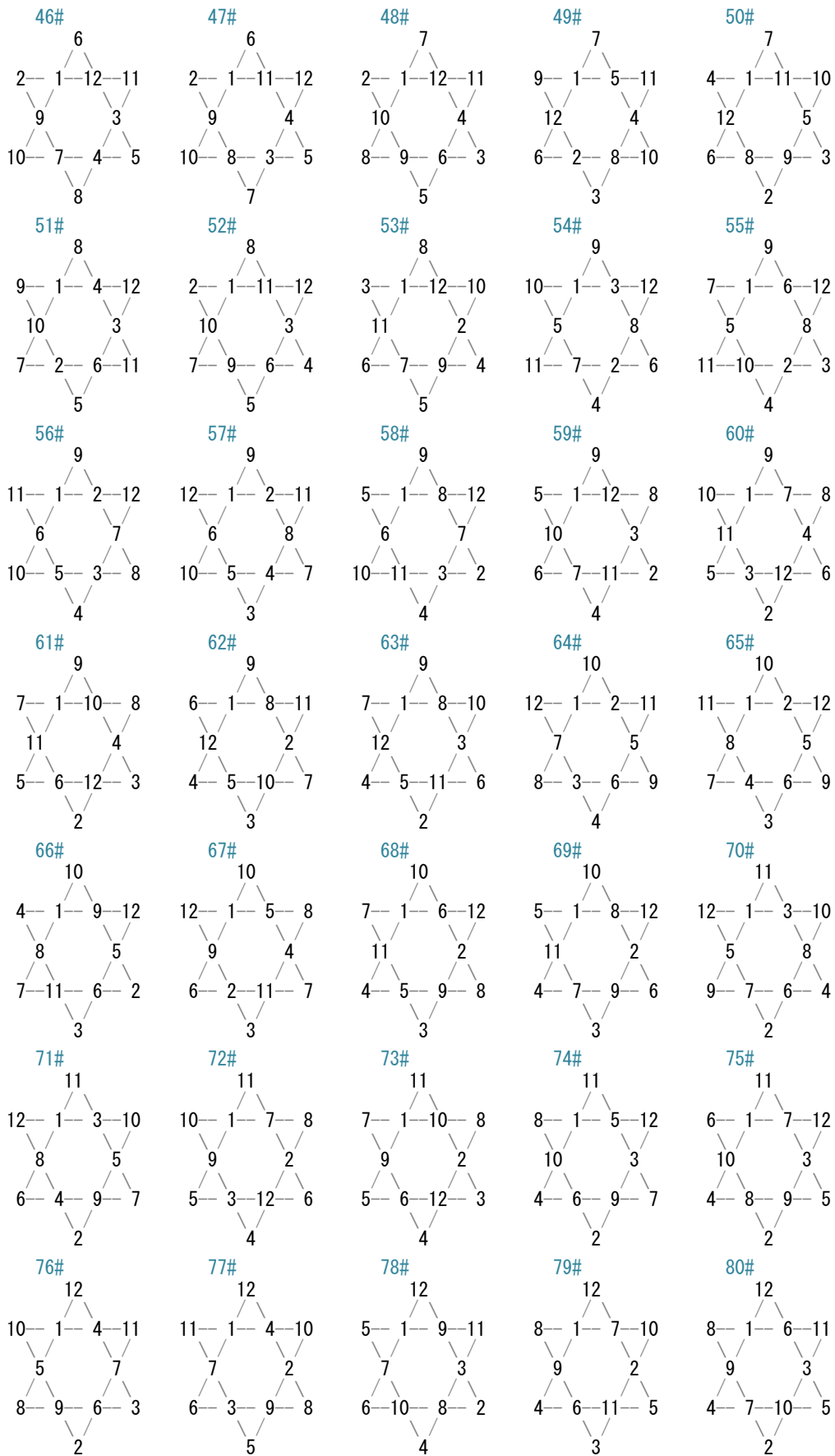
$n1 < n3$ ;  $n1 < n5$ ;  $n1 < n7$ ;  $n1 < n9$ ;  $n1 < n11$ ; そして  $n5 < n9$ ;

めでたく六星陣の標準解 80 個を出力した。その内訳は、 $n1 = 1$  のタイプが 40 個、他のタイプが 40 個である。後者の 40 個は、どれもが  $n2 = 1$ ;  $n4 = 1$ ;  $n6 = 1$ ;  $n8 = 1$ ;  $n10 = 1$ ;  $n12 = 1$ ; のいずれかである。つまり内周上に数値 1 があるタイプになっている。

この 40 個は、回転移動と反転操作によって、 $n2 = 1$  のタイプにまとめることができる。

**\*\* Double-Ringed Magic Stars of Order 6: List of Standard Solutions when  $n2=1$ ; \*\***





[Solutions Count(when n1=1 and n2=1) = 40 + 40 = 80] OK!

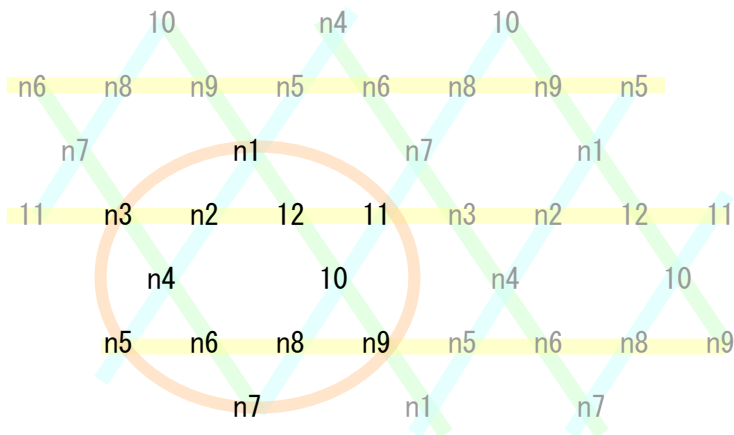
\*\* Dictated by Kanji Setsuda on June 18, 2017 with macOS Sierra 10.12.5 & Xcode 7.3; \*\*

要するに、全ての解は、外周上に1があるものと内周上に1があるものの2つのタイプに分類することができる。この特質は、次数に限らず全ての星陣に共通に存在する。ただし、この2つのタイプの解の総数が一致するのは、六星陣だけの特質だ。他の高次星陣では、 $n1=1$ の解と $n2=1$ の解の総数が一致せずに相異なる。何故そうなるのかが不思議なのだが、逆に六星陣の希少価値がいよいよ高貴に映る。

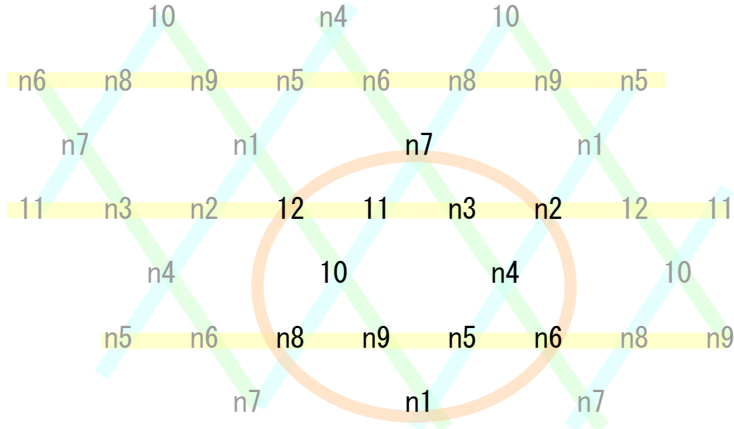
そのためなのか、六星陣においては珍しい性質がいろいろ成り立つようだ。

試みに星陣空間を外へ拡張してみよう。かつて汎対角線型魔方陣で成功した方陣空間の拡張を、星陣でも試してみよう。対角線そのものを下図のように一つおきにコピーする。拡張した結果(下図)を良く見てほしい。はたしてそこから何が読み取れるであろうか。

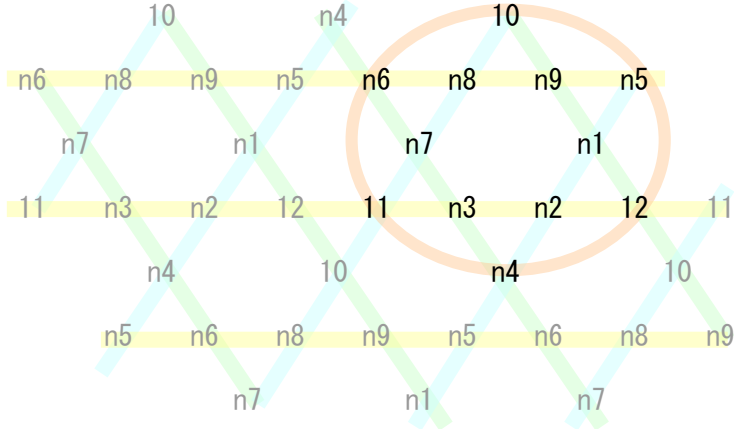
\* 六星陣拡張空間 [原形と星陣拡張空間] \*



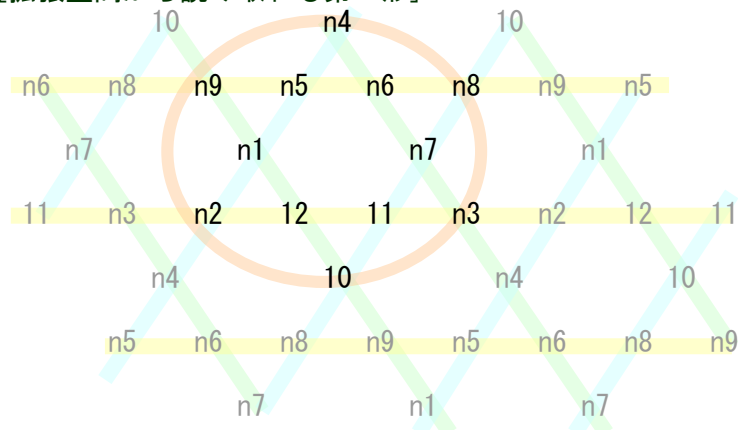
[拡張空間から読み取れる第2形]



[拡張空間から読み取れる第3形]



[拡張空間から読み取れる第4形]

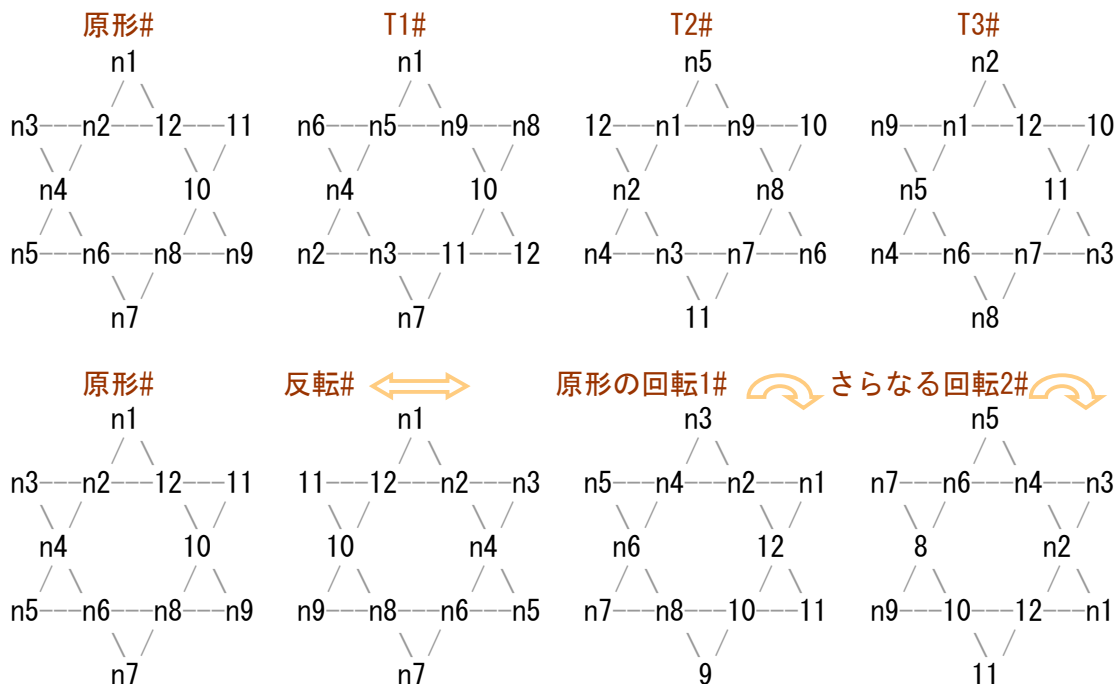


原形も含めて何と4つの解の形が、この拡張空間には含まれている。我々は、それをただ読み出せば良かったのである。これは、汎魔方陣の場合と全く同様の現象だ。

これら4形は、相互に変換が可能な「兄弟解」と考えられる。今後は、いつもまとめて1つのグループに扱う。そして、それらの代表解として「原形」を指名することにしよう。

そして、下図のような6種類の変換法を、あらためて仮定する。

\* 変換の概念図 \*



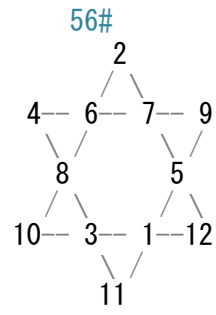
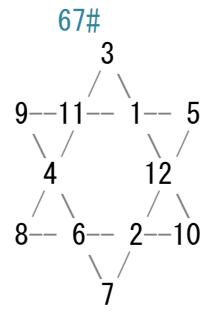
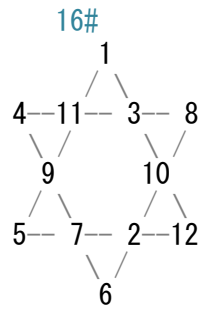
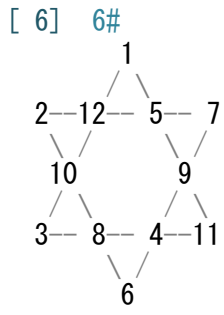
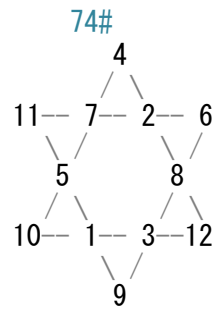
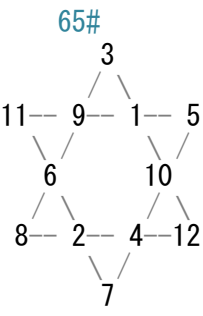
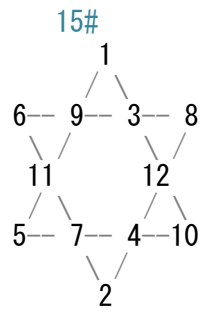
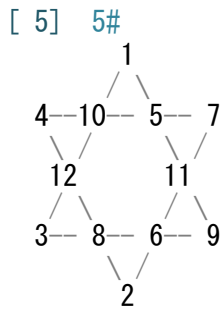
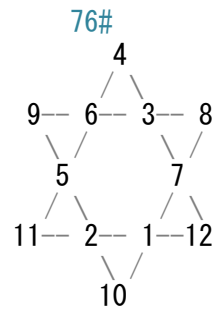
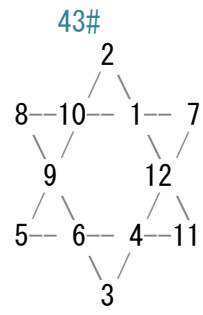
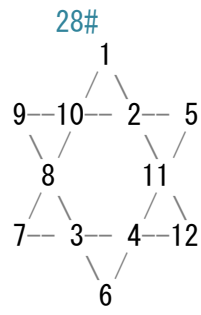
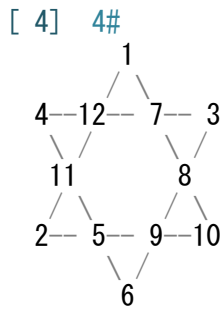
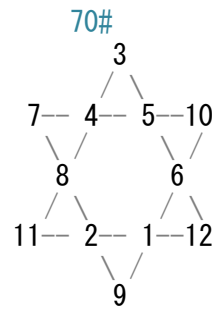
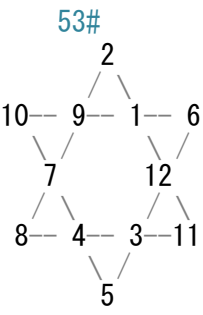
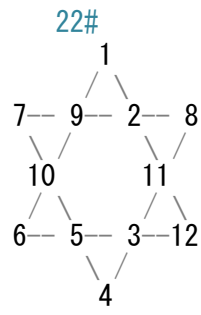
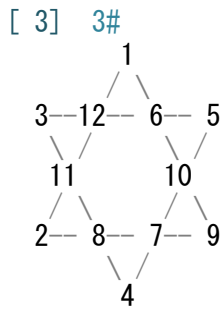
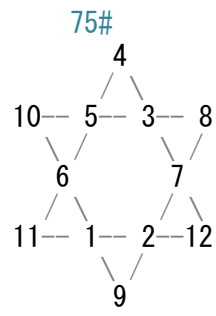
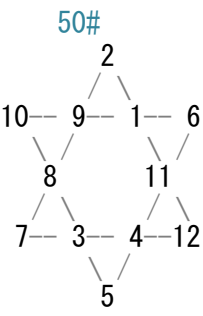
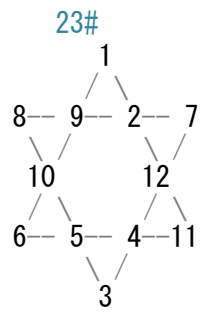
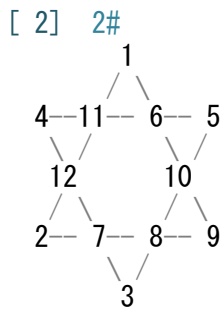
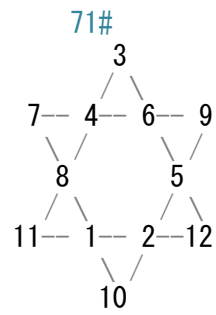
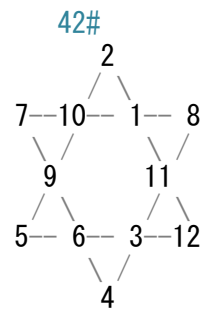
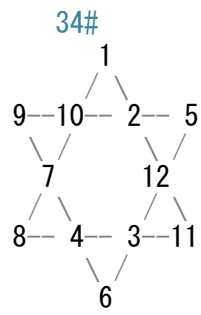
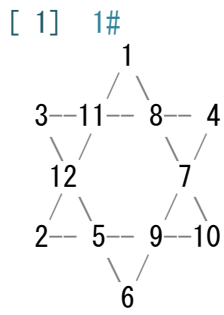
そうすれば、次のようなことが可能になるからだ。

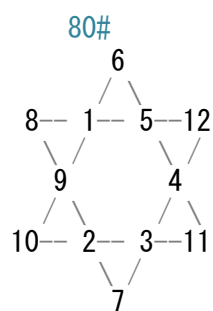
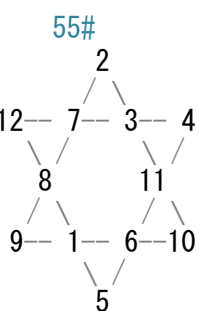
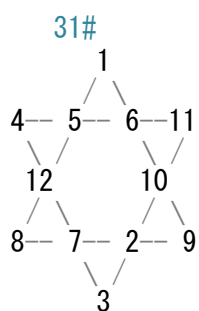
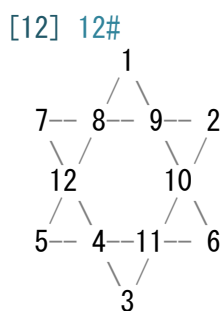
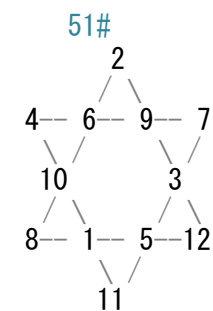
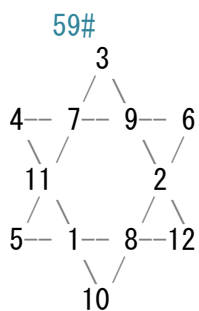
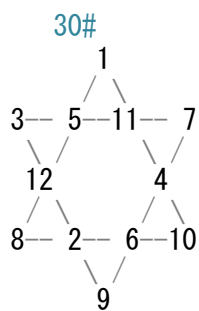
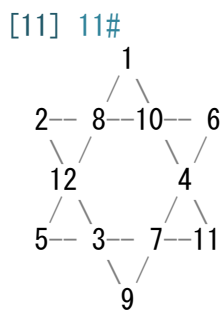
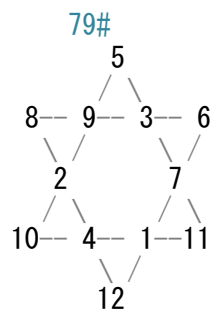
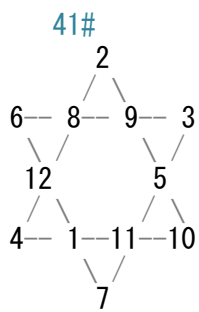
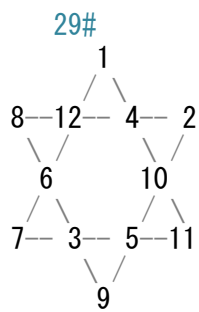
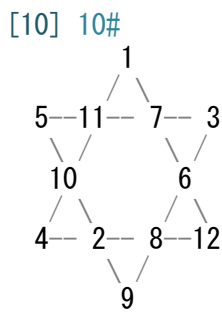
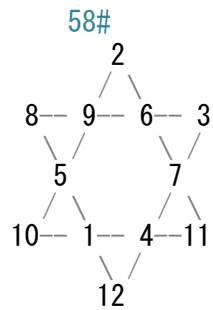
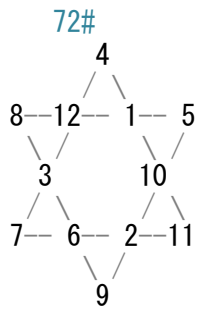
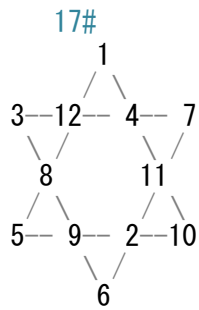
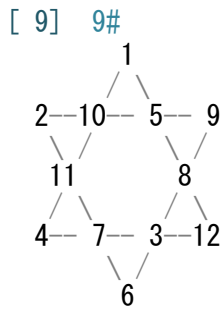
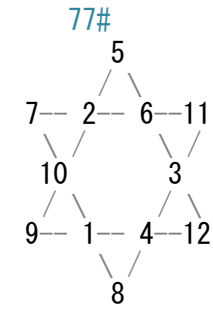
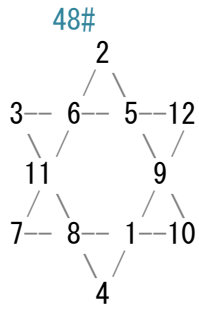
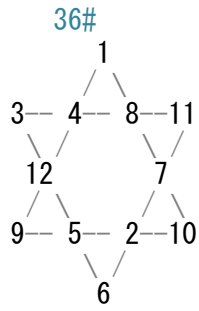
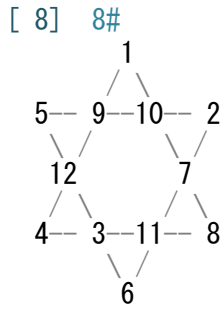
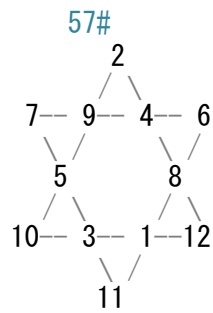
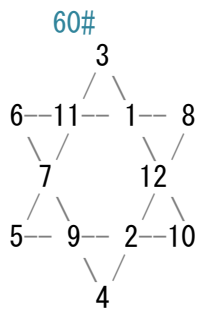
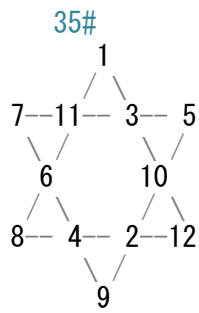
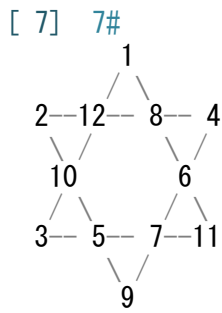
- (1) 先ず総数が20個の「基本解」を求めて用意する。
- (2) 上の変換法によって、基本解20個から標準解80個を再構成できる。必要があれば、反転・回転操作を加える。
- (3) 古い標準解のリストと比較検討して、事の成否を確かめる。

次のリストが、我がPCに計算させた20×4個の実行結果だ。

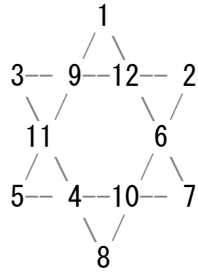
基本解を求めるには、条件式： $n1=1$ ； $n2>n5$ ；そして $n4<n10$ ；を用いた。

\* 二周六星陣の解の分類：最新リスト；  
基本解20個(左端)とその変換解各3個 \*

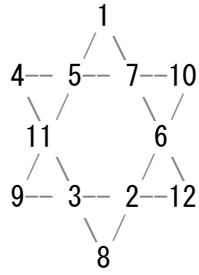




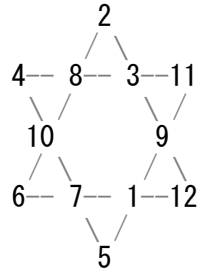
[13] 14#



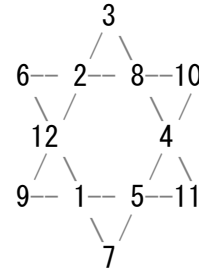
37#



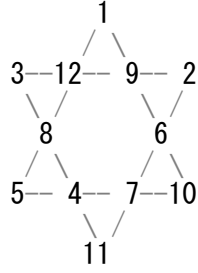
44#



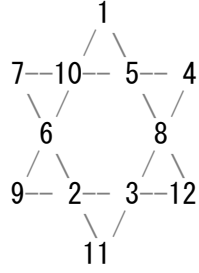
68#



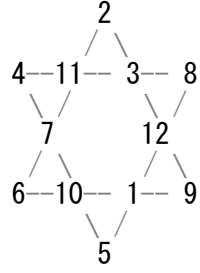
[14] 18#



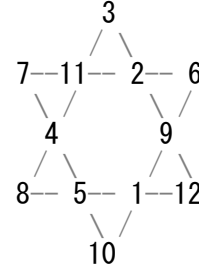
38#



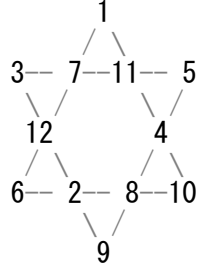
46#



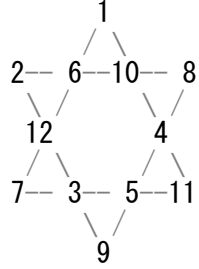
66#



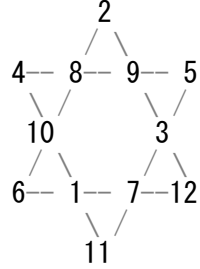
[15] 19#



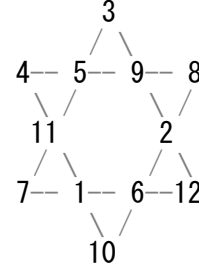
24#



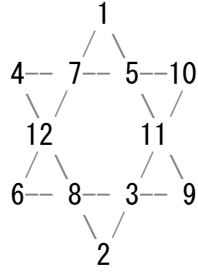
45#



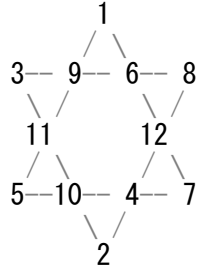
63#



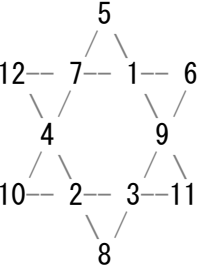
[16] 20#



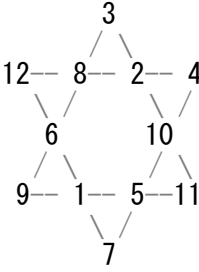
13#



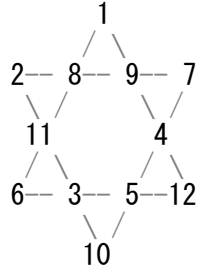
78#



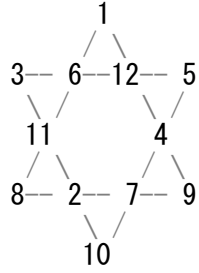
69#



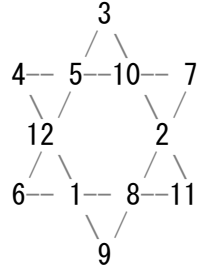
[17] 21#



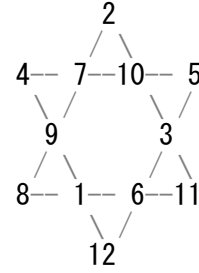
32#



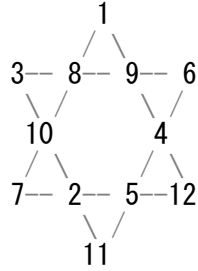
62#



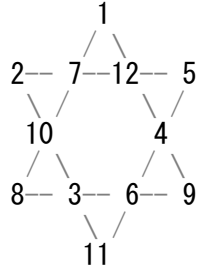
52#



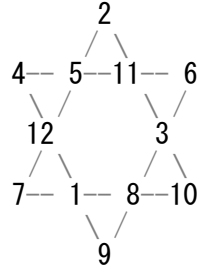
[18] 25#



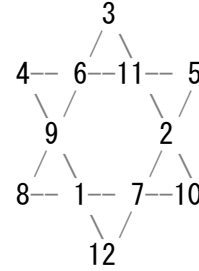
33#

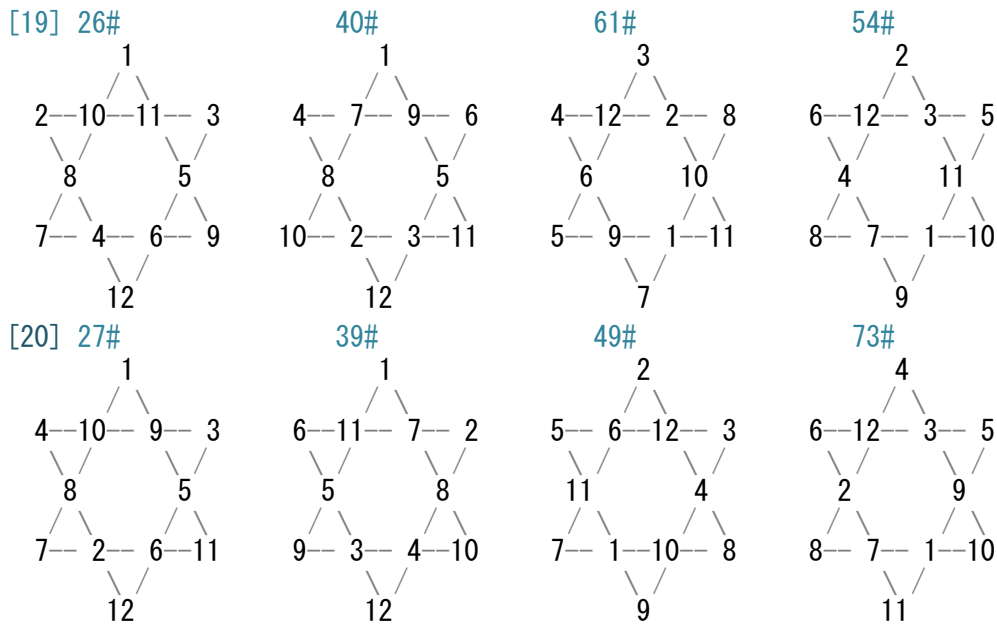


47#



64#





[Count = 20 x 4 = 80]

上表20段の各4個ずつの解の構成を一つ一つ良く見て欲しい。各変換解の6本の対角線の内容が、出現する場所と順序こそ違え、左端の基本解のそれと全く同じ構成になっている。これこそがまさに、各4解を同一の拡張空間から読み取ったことの証拠になっている！

新旧のリストの間で対応関係を調べたところ、下表の如く標準解の全数を重複無く欠落無く再構成できていることがわかった。

**\* 新旧の解のリストの間で比較対照した結果 \***

?: 0

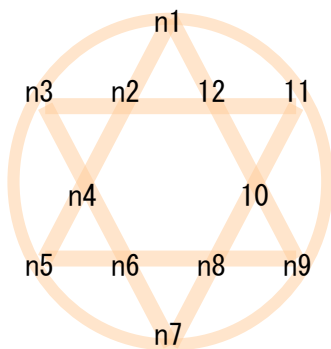
1:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
41:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
61:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

何とも素晴らしい成果が得られた。

このような「奇跡」が可能なのは、六星陣の場合だけである。残念ながら他の高次星陣では、拡張空間を作れない。全体に通用するような変換法も、発見することができない。

いったい何故なのだろう。六星陣では、何故に可能だったのだろうか。

ここで、代数的研究を少ししておこう。



**\* 二周六星陣の定義式 \***

- $n1+n2+n4+n5=C \quad \dots \quad l1$
- $n3+n4+n6+n7=C \quad \dots \quad l2$
- $n5+n6+n8+n9=C \quad \dots \quad l3$
- $n7+n8+n10+n11=C \quad \dots \quad l4$
- $n9+n10+n12+n1=C \quad \dots \quad l5$
- $n11+n12+n2+n3=C \quad \dots \quad l6$

---


$$2*(n1+n2+n3+\dots+n11+n12)=6*C$$

$$2*78=6*C$$

ゆえに  $C=26 \quad \dots \quad r0$

$l1+l3+l5$

$$n1+n2+n4+n5+n5+n6+n8+n9+n9+n10+n12+n1=3*26$$

$$2*(n1+n5+n9) + (n2+n4+n6+n8+n10+n12) = 78 \quad \dots \quad r1$$



I2+I4+I6

$$n^3+n^4+n^6+n^7+n^7+n^8+n^{10}+n^{11}+n^{11}+n^{12}+n^2+n^3=3*26$$

$$2*(n^3+n^7+n^{11})+(n^2+n^4+n^6+n^8+n^{10}+n^{12})=78 \quad \dots r2$$

r1-r2

$$n^1+n^5+n^9=n^3+n^7+n^{11} \quad \dots r3$$

I1+I2+I3

$$n^1+n^2+n^4+n^5+n^3+n^4+n^6+n^7+n^5+n^6+n^8+n^9=3*26$$

$$(n^4+n^5+n^6)+n^1+n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9=78$$

両辺に  $(n^{10}+n^{11}+n^{12})$  を加える。

$$(n^4+n^5+n^6)+n^1+n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12}=78+n^{10}+n^{11}+n^{12}$$

$$\text{ゆえに} \quad n^4+n^5+n^6=n^{10}+n^{11}+n^{12} \quad \dots r4$$

I2+I3+I4

$$n^3+n^4+n^6+n^7+n^5+n^6+n^8+n^9+n^7+n^8+n^{10}+n^{11}=3*26$$

$$(n^6+n^7+n^8)+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}=78$$

両辺に  $(n^1+n^2+n^{12})$  を加える。

$$(n^6+n^7+n^8)+n^1+n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12}=78+n^1+n^2+n^{12}$$

$$\text{ゆえに} \quad n^6+n^7+n^8=n^1+n^2+n^{12} \quad \dots r5$$

I3+I4+I5 そして 両辺に  $(n^2+n^3+n^4)$  を加える。

$$\text{同様に} \quad n^8+n^9+n^{10}=n^2+n^3+n^4 \quad \dots r6$$

I1+I2+I4+I5

$$n^1+n^2+n^4+n^5+n^3+n^4+n^6+n^7+n^7+n^8+n^{10}+n^{11}+n^9+n^{10}+n^{12}+n^1=4*C$$

$$(n^1+n^4+n^7+n^{10})+(n^1+n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12})=104$$

$$\text{ゆえに} \quad n^1+n^4+n^7+n^{10}=26 \quad \dots r7$$

I2+I3+I5+I6

$$n^3+n^4+n^6+n^7+n^5+n^6+n^8+n^9+n^9+n^{10}+n^{12}+n^1+n^{11}+n^{12}+n^2+n^3=4*C$$

$$(n^3+n^6+n^9+n^{12})+(n^1+n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12})=104$$

$$\text{ゆえに} \quad n^3+n^6+n^9+n^{12}=26 \quad \dots r8$$

I1+I3+I4+I6

$$n^1+n^2+n^4+n^5+n^5+n^6+n^8+n^9+n^7+n^8+n^{10}+n^{11}+n^{11}+n^{12}+n^2+n^3=4*C$$

$$(n^2+n^5+n^8+n^{11})+(n^1+n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12})=104$$

$$\text{ゆえに} \quad n^2+n^5+n^8+n^{11}=26 \quad \dots r9$$

I2+I3+I4+I5

$$n^1+n^3+n^4+n^5+n^6+n^6+n^7+n^7+n^8+n^8+n^9+n^9+n^{10}+n^{10}+n^{11}+n^{12}=4*C$$

$$(n^6+n^7+n^8+n^9+n^{10})+(n^1+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12})=104$$

両辺に  $n^2$  を加える。

$$(n^6+n^7+n^8+n^9+n^{10})+(n^1+n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12})=104+n^2$$

$$\text{ゆえに} \quad n^2=n^6+n^7+n^8+n^9+n^{10}-26 \quad \dots r10$$

I3+I4+I5+I6

$$n^5+n^6+n^8+n^9+n^7+n^8+n^{10}+n^{11}+n^9+n^{10}+n^{12}+n^1+n^{11}+n^{12}+n^2+n^3=4*C$$

$$(n^8+n^9+n^{10}+n^{11}+n^{12})+(n^1+n^2+n^3+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12})=104$$

両辺に  $n^4$  を加える。

$$(n^8+n^9+n^{10}+n^{11}+n^{12})+(n^1+n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12})=104+n^4$$

$$\text{ゆえに} \quad n^4=n^8+n^9+n^{10}+n^{11}+n^{12}-26 \quad \dots r11$$

$$\text{同様に} \quad n^6=n^1+n^2+n^{10}+n^{11}+n^{12}-26 \quad \dots r12$$

$$n^8=n^1+n^2+n^3+n^4+n^{12}-26 \quad \dots r13$$

$$n^{10}=n^2+n^3+n^4+n^5+n^6-26 \quad \dots r14$$

$$n^{12}=n^4+n^5+n^6+n^7+n^8-26 \quad \dots r15$$

I2+I3+I4+I6

$$n^3+n^4+n^6+n^7+n^5+n^6+n^8+n^9+n^7+n^8+n^{10}+n^{11}+n^{11}+n^{12}+n^2+n^3=4*C$$

$$(n^3+n^6+n^7+n^8+n^{11})+(n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12})=104$$

両辺に  $n^1$  を加える。

$$(n^3+n^6+n^7+n^8+n^{11})+(n^1+n^2+n^3+n^4+n^5+n^6+n^7+n^8+n^9+n^{10}+n^{11}+n^{12})=104+n^1$$

$$\text{ゆえに} \quad n^1=n^3+n^6+n^7+n^8+n^{11}-26 \quad \dots r16$$

I1+I3+I4+I5

$$n^1+n^2+n^4+n^5+n^5+n^6+n^8+n^9+n^7+n^8+n^{10}+n^{11}+n^9+n^{10}+n^{12}+n^1=4*C$$

$$(n1+n5+n8+n9+n10) + (n1+n2+n4+n5+n6+n7+n8+n9+n10+n11+n12) = 104$$

両辺に  $n3$  を加える。

$$(n1+n5+n8+n9+n10) + (n1+n2+n3+n4+n5+n6+n7+n8+n9+n10+n11+n12) = 104+n3$$

ゆえに	$n3 = n1+n5+n8+n9+n10-26$	... r17
同様に	$n5 = n3+n7+n10+n11+n12-26$	... r18
	$n7 = n1+n2+n5+n9+n12-26$	... r19
	$n9 = n2+n3+n4+n7+n11-26$	... r20
	$n11 = n1+n4+n5+n6+n9-26$	... r21

\* 得られた関係式をまとめた表 \*

$n1+n5+n9 = n3+n7+n11$	... r3
$n4+n5+n6 = n10+n11+n12$	... r4
$n6+n7+n8 = n1+n2+n12$	... r5
$n8+n9+n10 = n2+n3+n4$	... r6
$n1+n4+n7+n10 = 26$	... r7
$n3+n6+n9+n12 = 26$	... r8
$n2+n5+n8+n11 = 26$	... r9
$n2 = n6+n7+n8+n9+n10-26$	... r10
$n4 = n8+n9+n10+n11+n12-26$	... r11
$n6 = n1+n2+n10+n11+n12-26$	... r12
$n8 = n1+n2+n3+n4+n12-26$	... r13
$n10 = n2+n3+n4+n5+n6-26$	... r14
$n12 = n4+n5+n6+n7+n8-26$	... r15
$n1 = n3+n6+n7+n8+n11-26$	... r16
$n3 = n1+n5+n8+n9+n10-26$	... r17
$n5 = n3+n7+n10+n11+n12-26$	... r18
$n7 = n1+n2+n5+n9+n12-26$	... r19
$n9 = n2+n3+n4+n7+n11-26$	... r20
$n11 = n1+n4+n5+n6+n9-26$	... r21

これらの関係式全てが、六星陣の希有の奇跡を可能にしているのである。特に  $r3 \sim r9$  式の存在が重要である。これあればこそ、大幅な書き換えが可能なのだと思う。

七星陣以上の高次星陣では、中々こううまくは行かない。

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by 撰田 寛二 (Kanji Setsuda) with macOS Sierra 10.12.5 and Xcode 7.3:)

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