

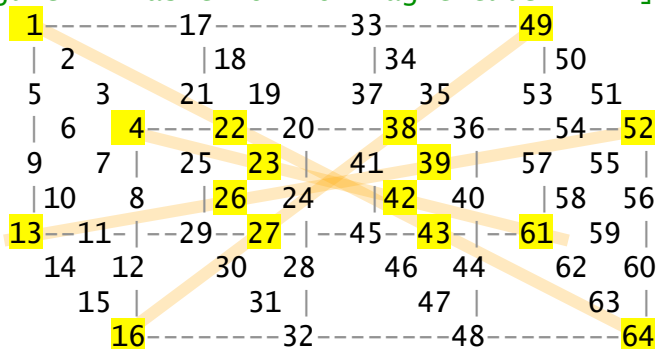
Part 1 : "Basic Studies of Magic Squares and Cubes"  
 Chapter 4 : "Advanced Algebraic Study of Magic Cubes"  
 Section 1 : **Pan-Magic Cubes of Order 4**: Kanji Setsuda

#0. **Classical Concept of "Pan-Magic Cubes" of Order 4**

Let's study about the Pan-magic Cubes of Order 4 now.

Any ordinary type of Magic Cubes must be built in the Basic Form as you see below. It must be made up of the natural numbers 1 to 64, using each one strictly once, and they must be placed in the 4x4x4 regular array. The Basic Form shows their positions as below and let's call the variables "n1, n2, n3, n4, ..., n63 and n64."

[Figure 1: Basic Form of Magic Cube 4x4x4]



The Four Primary Triagonals:

- $n1+n22+n43+n64=K \quad \dots Pt1$
- $n4+n23+n42+n61=K \quad \dots Pt2$
- $n13+n26+n39+n52=K \quad \dots Pt3$
- $n16+n27+n38+n49=K \quad \dots Pt4$

When we make an ordinary type, we usually assume all four entries on every row, every column and every pillar in the three directions add up to the magic constant.

Even all four entries on every four Primary Triagonals above should add up to the same constant sum. This is one of the most basic rules we should always observe.

But, what do they mean by "Pan-magic" and "Pan-triagonals"?

I will try to explain about them here simply according to our classical concept.

They have come from the 'Pan-magic Shifts' just as illustrated as below.

We want to make these kinds of transformation always possible for our object.

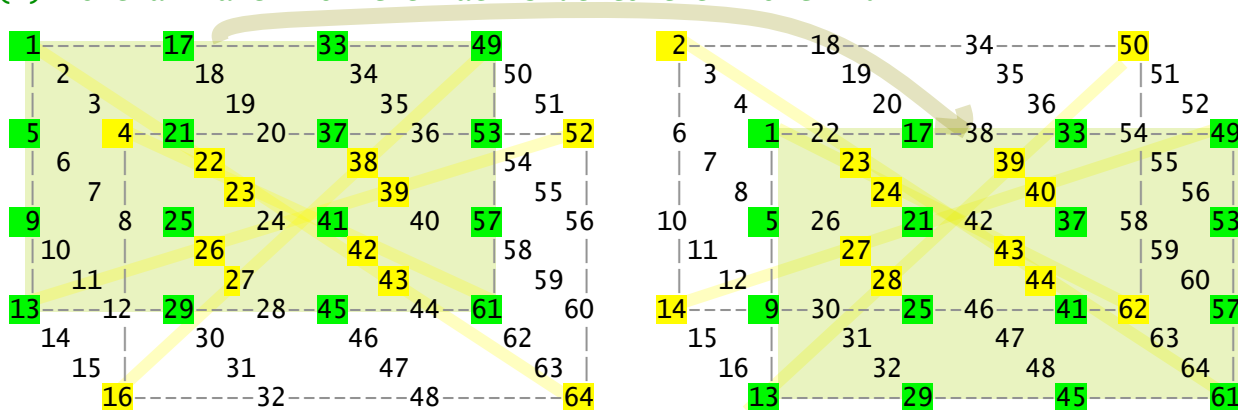
For that purpose, what conditions do we have to assume before all?

(1) Watch the first set of diagrams below to know what we want to do, please.

We want to move the whole plane with 16 numbers at the back side far beyond, put it on to the front end and make it for the face of our new magic cube.

**\*\* Three Types of 'Pan-magic Shift' \*\***

(1) Move a Plane from the Back Side to the Front End

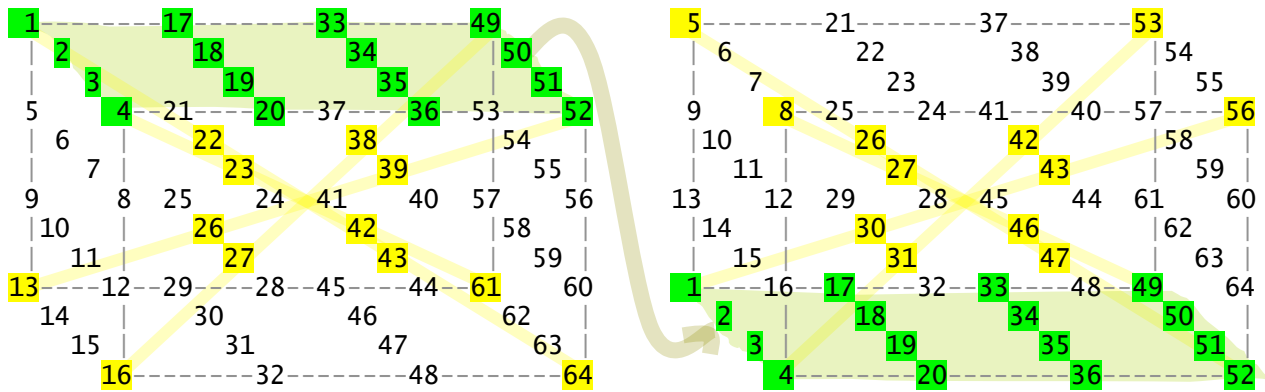


(2) Watch the second set of diagrams below to know what we are trying to do, too.

Yes. We are moving the whole plane with 16 numbers on the top down far to the

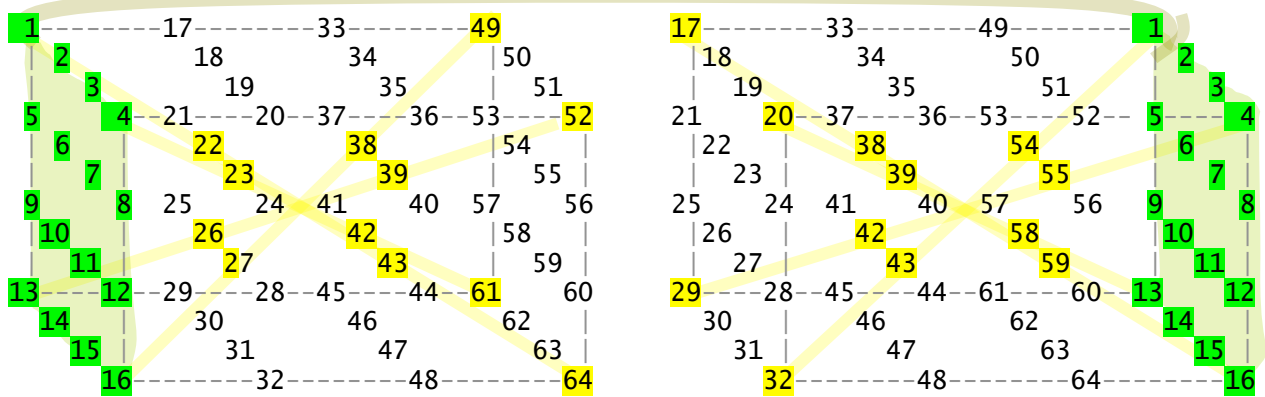
bottom end and making it for the base of our new object.

(2) Move a Plane from the Top Roof to the Bottom End



(3) As you see below, the third set illustrates our movement of the plane at the left side far to the right end and make it for the opposite wall of our new object.

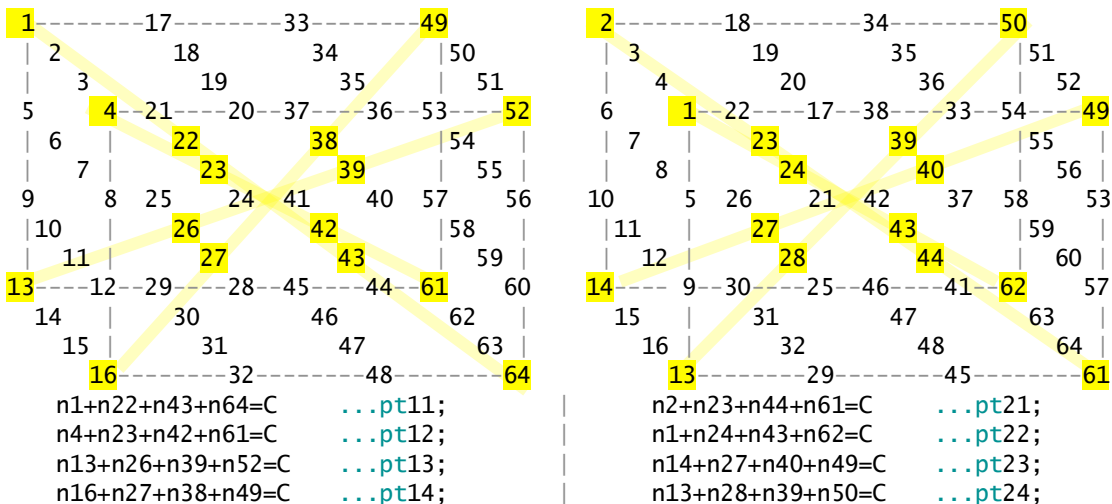
(3) Move a Plane from the Left Side to the Right End



For all the results of those types of transformations, we want to have the same kind of Magic Cubes of Order 4 here. All of them are requested to have the constant sums of any row, column, pillar and any new Primary Triagonals.

What conditions do we have to assume at the first definition stage?

\*\* The List of New Conditions for the Primary Triagonals of MC444: \*\*  
 \*\* Made by Kanji Setsuda on July 1, 2010 with MacOSX & Xcode 3.1.2 \*\*



3	-----	19	-----	35	-----	51							
4		20		36		52							
1		17		33		49							
7	2--23	-----	18--39	-----	34--55	-----	50						
8		24		40		56							
5		21		37		53							
11	6	27	22	43	38	59	54						
12		28		44		60							
9		25		41		57							
15	-----	10	-----	31	-----	26	-----	47	-----	42	-----	63	58
16		32		48		64							
13		29		45		61							
14	-----	30	-----	46	-----	62							
n3+n24+n41+n62=C	...	pt31;											
n2+n21+n44+n63=C	...	pt32;											
n15+n28+n37+n50=C	...	pt33;											
n14+n25+n40+n51=C	...	pt34;											

4	-----	20	-----	36	-----	52							
1		17		33		49							
2		18		34		50							
8	3--24	-----	19--40	-----	35--56	-----	51						
5		21		37		53							
6		22		38		54							
12	7	28	23	44	39	60	55						
9		25		41		57							
10		26		42		58							
16	-----	11	-----	32	-----	27	-----	48	-----	43	-----	64	59
13		29		45		61							
14		30		46		62							
15	-----	31	-----	47	-----	63							
n4+n21+n42+n63=C	...	pt41;											
n3+n22+n41+n64=C	...	pt42;											
n16+n25+n38+n51=C	...	pt43;											
n15+n26+n37+n52=C	...	pt44;											

This long list above and below shows the whole possibility of making new magic objects by the applications of those three types of Pan-magic Shift.

No. 2 is made of No. 1 by the first type of pan-magic shift. No. 3 is made of No. 2, and No. 4 is made of No. 3 by repeating applications of the same type of shift.

If you add the new conditions for Primary Triagonals listed above to the definitions at first, you can always make those new cubes realized true.

No. 5 below is made of No. 1 by the **second** type of pan-magic shift. No. 6~No. 8 are made of No. 5~No. 7 one after another by applying the **first** type of shift.

5	-----	21	-----	37	-----	53							
6		22		38		54							
7		23		39		55							
9	8--25	-----	24--41	-----	40--57	-----	56						
10		26		42		58							
11		27		43		59							
13	12	29	28	45	44	61	60						
14		30		46		62							
15		31		47		63							
1	-----	16	-----	17	-----	32	-----	33	-----	48	-----	49	64
2		18		34		50							
3		19		35		51							
4	-----	20	-----	36	-----	52							
n5+n26+n47+n52=C	...	pt51;											
n8+n27+n46+n49=C	...	pt52;											
n1+n30+n43+n56=C	...	pt53;											
n4+n31+n42+n53=C	...	pt54;											

6	-----	22	-----	38	-----	54							
7		23		39		55							
8		24		40		56							
10	5--26	-----	21--42	-----	37--58	-----	53						
11		27		43		59							
12		28		44		60							
14	9	30	25	46	41	62	57						
15		31		47		63							
16		32		48		64							
2	-----	13	-----	18	-----	29	-----	34	-----	45	-----	50	61
3		19		35		51							
4		20		36		52							
1	-----	17	-----	33	-----	49							
n6+n27+n48+n49=C	...	pt61;											
n5+n28+n47+n50=C	...	pt62;											
n2+n31+n44+n53=C	...	pt63;											
n1+n32+n43+n54=C	...	pt64;											

7	-----	23	-----	39	-----	55							
8		24		40		56							
5		21		37		53							
11	6--27	-----	22--43	-----	38--59	-----	54						
12		28		44		60							
9		25		41		57							
15	10	31	26	47	42	63	58						
16		32		48		64							
13		29		45		61							
3	-----	14	-----	19	-----	30	-----	35	-----	46	-----	51	62
4		20		36		52							
1		17		33		49							
2	-----	18	-----	34	-----	50							
n7+n28+n45+n50=C	...	pt71;											
n6+n25+n48+n51=C	...	pt72;											
n3+n32+n41+n54=C	...	pt73;											
n2+n29+n44+n55=C	...	pt74;											

8	-----	24	-----	40	-----	56							
5		21		37		53							
6		22		38		54							
12	7--28	-----	23--44	-----	39--60	-----	55						
9		25		41		57							
10		26		42		58							
16	11	32	27	48	43	64	59						
13		29		45		61							
14		30		46		62							
4	-----	15	-----	20	-----	31	-----	36	-----	47	-----	52	63
1		17		33		49							
2		18		34		50							
3	-----	19	-----	35	-----	51							
n8+n25+n46+n51=C	...	pt81;											
n7+n26+n45+n52=C	...	pt82;											
n4+n29+n42+n55=C	...	pt83;											
n3+n30+n41+n56=C	...	pt84;											

```

9-----25-----41-----57
|10      26      42      58
| 11      27      43      59
13 12--29--28--45--44--61--60
|14      30      46      62
| 15      31      47      63
1 16 17 32 33 48 49 64
| 2      18      34      50
| 3      19      35      51
5---4--21---20--37---36--53 52
| 6      22      38      54
| 7      23      39      55
| 8-----24-----40-----56
n9+n30+n35+n56=C ...pt91;
n12+n31+n34+n53=C ...pt92;
n5+n18+n47+n60=C ...pt93;
n8+n19+n46+n57=C ...pt94;

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10-----26-----42-----58
|11      27      43      59
| 12      28      44      60
14 9--30---25---46---41--62---57
|15      31      47      63
| 16      32      48      64
2 13 18 29 34 45 50 61
| 3      19      35      51
| 4      20      36      52
6---1--22---17--38---33--54 49
| 7      23      39      55
| 8      24      40      56
| 5-----21-----37-----53
n10+n31+n36+n53=C ...pt101;
n9+n32+n35+n54=C ...pt102;
n6+n19+n48+n57=C ...pt103;
n5+n20+n47+n58=C ...pt104;

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11-----27-----43-----59
|12      28      44      60
| 9      25      41      57
15 10-31---26--47---42--63---58
|16      32      48      64
| 13      29      45      61
3 14 19 30 35 46 51 62
| 4      20      36      52
| 1      17      33      49
7---2--23---18--39---34--55 50
| 8      24      40      56
| 5      21      37      53
| 6-----22-----38-----54
n11+n32+n33+n54=C ...pt111;
n10+n29+n36+n55=C ...pt112;
n7+n20+n45+n58=C ...pt113;
n6+n17+n48+n59=C ...pt114;

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12-----28-----44-----60
| 9      25      41      57
| 10     26      42      58
16 11-32---27--48---43--64---59
|13      29      45      61
| 14      30      46      62
4 15 20 31 36 47 52 63
| 1      17      33      49
| 2      18      34      50
8---3--24---19--40---35--56 51
| 5      21      37      53
| 6      22      38      54
| 7-----23-----39-----55
n12+n29+n34+n55=C ...pt121;
n11+n30+n33+n56=C ...pt122;
n8+n17+n46+n59=C ...pt123;
n7+n18+n45+n60=C ...pt124;

```

No.9 is made of No.5 by the **second** type. No.10~No.12 are made of No.9~No.11 by the **first** type one after another. If you want all the result always true, you have to add all the conditions of new primary triagonals to the definition list at the beginning.

No.13 is made of No.9 by the **second** type. No.14~No.16 are made of No.13~No.15 by the **first** type one after another. If you want all the result always true, you have to add all the new conditions to the definition list at first.

```

13-----29-----45-----61
|14      30      46      62
| 15      31      47      63
1 16-17--32--33--48--49--64
| 2      18      34      50
| 3      19      35      51
5 4 21 20 37 36 53 52
| 6      22      38      54
| 7      23      39      55
9---8--25---24--41---40--57 56
|10     26      42      58
| 11     27      43      59
|12-----28-----44-----60
n13+n18+n39+n60=C ...pt131;
n16+n19+n38+n57=C ...pt132;
n9+n22+n35+n64=C ...pt133;
n12+n23+n34+n61=C ...pt134;

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14-----30-----46-----62
|15      31      47      63
| 16      32      48      64
2 13-18--29--34--45--50--61
| 3      19      35      51
| 4      20      36      52
6 1 22 17 38 33 54 49
| 7      23      39      55
| 8      24      40      56
10---5--26---21--42---37--58 53
|11     27      43      59
| 12     28      44      60
| 9-----25-----41-----57
n14+n19+n40+n57=C ...pt141;
n13+n20+n39+n58=C ...pt142;
n10+n23+n36+n61=C ...pt143;
n9+n24+n35+n62=C ...pt144;

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15-----31-----47-----63
|16      32      48      64
| 13      29      45      61

```

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16-----32-----48-----64
|13      29      45      61
| 14      30      46      62

```

3 14--19---30--35---46--51---62  
 | 4 | 20 | 36 | 52 |  
 | 1 | 17 | 33 | 49 |  
 7 2 23 18 39 34 55 50  
 | 8 | 24 | 40 | 56 |  
 | 5 | 21 | 37 | 53 |  
 11---6--27---22--43---38--59 54  
 | 12 | 28 | 44 | 60 |  
 | 9 | 25 | 41 | 57 |  
 10-----26-----42-----58  
 n15+n20+n37+n58=C ...pt151;  
 n14+n17+n40+n59=C ...pt152;  
 n11+n24+n33+n62=C ...pt153;  
 n10+n21+n36+n63=C ...pt154;

4 15--20---31--36---47--52---63  
 | 1 | 17 | 33 | 49 |  
 | 2 | 18 | 34 | 50 |  
 8 3 24 19 40 35 56 51  
 | 5 | 21 | 37 | 53 |  
 | 6 | 22 | 38 | 54 |  
 12---7--28---23--44---39--60 55  
 | 9 | 25 | 41 | 57 |  
 | 10 | 26 | 42 | 58 |  
 11-----27-----43-----59  
 n16+n17+n38+n59=C ...pt161;  
 n15+n18+n37+n60=C ...pt162;  
 n12+n21+n34+n63=C ...pt163;  
 n11+n22+n33+n64=C ...pt164;

17-----33-----49-----1  
 | 18 | 34 | 50 | 2 |  
 | 19 | 35 | 51 | 3 |  
 21 20--37---36--53---52--5---4  
 | 22 | 38 | 54 | 6 |  
 | 23 | 39 | 55 | 7 |  
 25 24 41 40 57 56 9 8  
 | 26 | 42 | 58 | 10 |  
 | 27 | 43 | 59 | 11 |  
 29---28--45---44--61---60--13 12  
 | 30 | 46 | 62 | 14 |  
 | 31 | 47 | 63 | 15 |  
 32-----48-----64-----16  
 n17+n38+n59+n16=C ...pt171(=pt161);  
 n20+n39+n58+n13=C ...pt172(=pt142);  
 n29+n42+n55+n4=C ...pt173(=pt83);  
 n32+n43+n54+n1=C ...pt174(=pt64);

18-----34-----50-----2  
 | 19 | 35 | 51 | 3 |  
 | 20 | 36 | 52 | 4 |  
 22 17--38---33--54---49--6---1  
 | 23 | 39 | 55 | 7 |  
 | 24 | 40 | 56 | 8 |  
 26 21 42 37 58 53 10 5  
 | 27 | 43 | 59 | 11 |  
 | 28 | 44 | 60 | 12 |  
 30---25--46---41--62---57--14 9  
 | 31 | 47 | 63 | 15 |  
 | 32 | 48 | 64 | 16 |  
 29-----45-----61-----13  
 n18+n39+n60+n13=C ...pt181(=pt131);  
 n17+n40+n59+n14=C ...pt182(=pt152);  
 n30+n43+n56+n1=C ...pt183(=pt53);  
 n29+n44+n55+n2=C ...pt184(=pt74);

33-----49-----1-----17  
 | 34 | 50 | 2 | 18 |  
 | 35 | 51 | 3 | 19 |  
 37 36--53---52--5---4--21---20  
 | 38 | 54 | 6 | 22 |  
 | 39 | 55 | 7 | 23 |  
 41 40 57 56 9 8 25 24  
 | 42 | 58 | 10 | 26 |  
 | 43 | 59 | 11 | 27 |  
 45---44--61---60--13---12--29 28  
 | 46 | 62 | 14 | 30 |  
 | 47 | 63 | 15 | 31 |  
 48-----64-----16-----32  
 n33+n54+n11+n32=C ...pt331(=pt111);  
 n36+n55+n10+n29=C ...pt332(=pt112);  
 n45+n58+n7+n20=C ...pt333(=pt113);  
 n48+n59+n6+n17=C ...pt334(=pt114);

34-----50-----2-----18  
 | 35 | 51 | 3 | 19 |  
 | 36 | 52 | 4 | 20 |  
 38 33--54---49--6---1--22---17  
 | 39 | 55 | 7 | 23 |  
 | 40 | 56 | 8 | 24 |  
 42 37 58 53 10 5 26 21  
 | 43 | 59 | 11 | 27 |  
 | 44 | 60 | 12 | 28 |  
 46---41--62---57--14---9--30 25  
 | 47 | 63 | 15 | 31 |  
 | 48 | 64 | 16 | 32 |  
 45-----61-----13-----29  
 n34+n55+n12+n29=C ...pt341(=pt121);  
 n33+n56+n11+n30=C ...pt342(=pt122);  
 n46+n59+n8+n17=C ...pt343(=pt123);  
 n45+n60+n7+n18=C ...pt344(=pt124);

49-----1-----17-----33  
 | 50 | 2 | 18 | 34 |  
 | 51 | 3 | 19 | 35 |  
 53 52--5---4--21---20--37---36  
 | 54 | 6 | 22 | 38 |  
 | 55 | 7 | 23 | 39 |  
 57 56 9 8 25 24 41 40  
 | 58 | 10 | 26 | 42 |  
 | 59 | 11 | 27 | 43 |  
 61---60--13---12--29---28--45 44  
 | 62 | 14 | 30 | 46 |  
 | 63 | 15 | 31 | 47 |  
 64-----16-----32-----48  
 n49+n6+n27+n48=C ...pt491(=pt61);

50-----2-----18-----34  
 | 51 | 3 | 19 | 35 |  
 | 52 | 4 | 20 | 36 |  
 54 49--6---1--22---17--38---33  
 | 55 | 7 | 23 | 39 |  
 | 56 | 8 | 24 | 40 |  
 58 53 10 5 26 21 42 37  
 | 59 | 11 | 27 | 43 |  
 | 60 | 12 | 28 | 44 |  
 62---57--14---9--30---25--46 41  
 | 63 | 15 | 31 | 47 |  
 | 64 | 16 | 32 | 48 |  
 61-----13-----29-----45  
 n50+n7+n28+n45=C ...pt501(=pt71);

n52+n7+n26+n45=C ...pt492(=pt82);		n49+n8+n27+n46=C ...pt502(=pt52);
n61+n10+n23+n36=C ...pt493(=pt143);		n62+n11+n24+n33=C ...pt503(=pt153);
n64+n11+n22+n33=C ...pt494(=pt164);		n61+n12+n23+n34=C ...pt504(=pt134);

No.17 is made of No.1 by the **third** type of Pan-magic Shift. No.18 is made of No.17 by the **first** type again.

No.33 is made of No.17 by the **third** type. No.49 is made of No.33 by the same type.

Do you also have to add all the new conditions of primary triagonals of No.17~No.64 to the first definition list this time?

No, you don't have to.

Just as the list shows after No.17, you don't have any more new conditions about the primary triagonals, but you always have repetitions of the old conditions we have already got. The Orders of equations and variables are actually changed, as you see, but the content of each condition is certainly the same as old and kept unchanged.

All the conditions we need at first do not count so many. We need only as many as 64(=16x4) conditions below. We need no more or no less than these 64.

Let's call these triagonals as the "**Pan-triagonals**" from now on. Let's call our new magic cubes as the "**Pan-triagonal Magic Cubes**" of Order 4, too.

You don't have to worry about the new conditions we need for the constant sums of rows, columns and pillars. All we have got are totally the same as the old, first ones.

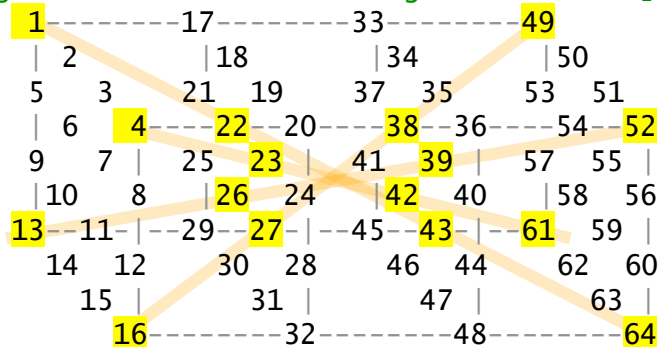
**\*\* The 64 Pan-triagonal Conditions for Pan-magic Cubes of Order 4: \*\***  
**\*\* Listed by Kanji Setsuda on July 1, 2010 with MacOSX & Xcode 3.1 \*\***

n1+n22+n43+n64=C ...pt11;		n4+n23+n42+n61=C ...pt12;
n13+n26+n39+n52=C ...pt13;		n16+n27+n38+n49=C ...pt14;
n2+n23+n44+n61=C ...pt21;		n1+n24+n43+n62=C ...pt22;
n14+n27+n40+n49=C ...pt23;		n13+n28+n39+n50=C ...pt24;
n3+n24+n41+n62=C ...pt31;		n2+n21+n44+n63=C ...pt32;
n15+n28+n37+n50=C ...pt33;		n14+n25+n40+n51=C ...pt34;
n4+n21+n42+n63=C ...pt41;		n3+n22+n41+n64=C ...pt42;
n16+n25+n38+n51=C ...pt43;		n15+n26+n37+n52=C ...pt44;
n5+n26+n47+n52=C ...pt51;		n8+n27+n46+n49=C ...pt52;
n1+n30+n43+n56=C ...pt53;		n4+n31+n42+n53=C ...pt54;
n6+n27+n48+n49=C ...pt61;		n5+n28+n47+n50=C ...pt62;
n2+n31+n44+n53=C ...pt63;		n1+n32+n43+n54=C ...pt64;
n7+n28+n45+n50=C ...pt71;		n6+n25+n48+n51=C ...pt72;
n3+n32+n41+n54=C ...pt73;		n2+n29+n44+n55=C ...pt74;
n8+n25+n46+n51=C ...pt81;		n7+n26+n45+n52=C ...pt82;
n4+n29+n42+n55=C ...pt83;		n3+n30+n41+n56=C ...pt84;
n9+n30+n35+n56=C ...pt91;		n12+n31+n34+n53=C ...pt92;
n5+n18+n47+n60=C ...pt93;		n8+n19+n46+n57=C ...pt94;
n10+n31+n36+n53=C ...pt101;		n9+n32+n35+n54=C ...pt102;
n6+n19+n48+n57=C ...pt103;		n5+n20+n47+n58=C ...pt104;
n11+n32+n33+n54=C ...pt111;		n10+n29+n36+n55=C ...pt112;
n7+n20+n45+n58=C ...pt113;		n6+n17+n48+n59=C ...pt114;
n12+n29+n34+n55=C ...pt121;		n11+n30+n33+n56=C ...pt122;
n8+n17+n46+n59=C ...pt123;		n7+n18+n45+n60=C ...pt124;
n13+n18+n39+n60=C ...pt131;		n16+n19+n38+n57=C ...pt132;
n9+n22+n35+n64=C ...pt133;		n12+n23+n34+n61=C ...pt134;
n14+n19+n40+n57=C ...pt141;		n13+n20+n39+n58=C ...pt142;
n10+n23+n36+n61=C ...pt143;		n9+n24+n35+n62=C ...pt144;
n15+n20+n37+n58=C ...pt151;		n14+n17+n40+n59=C ...pt152;
n11+n24+n33+n62=C ...pt153;		n10+n21+n36+n63=C ...pt154;
n16+n17+n38+n59=C ...pt161;		n15+n18+n37+n60=C ...pt162;
n12+n21+n34+n63=C ...pt163;		n11+n22+n33+n64=C ...pt164;

## #1. New Definitions for the Basic Conditions and Extended Space

We must assume the constant sums of all four entries on every row, every column and every pillar in the three directions, and all four entries on the 64 pan-triagonals must add up to the same sum  $K=130$ .

[Figure 1: Basic Form of Magic Cube 4x4x4]



\*\* Basic Conditions for Rows, Columns and Pillars:  $K=130$  \*\*

$$\begin{array}{lll}
 n_1+n_2+n_3+n_4=K; & n_1+n_5+n_9+n_{13}=K; & n_1+n_{17}+n_{33}+n_{49}=K; \\
 n_5+n_6+n_7+n_8=K; & n_2+n_6+n_{10}+n_{14}=K; & n_2+n_{18}+n_{34}+n_{50}=K; \\
 n_9+n_{10}+n_{11}+n_{12}=K; & n_3+n_7+n_{11}+n_{15}=K; & n_3+n_{19}+n_{35}+n_{51}=K; \\
 n_{13}+n_{14}+n_{15}+n_{16}=K; & n_4+n_8+n_{12}+n_{16}=K; & n_4+n_{20}+n_{36}+n_{52}=K; \\
 n_{17}+n_{18}+n_{19}+n_{20}=K; & n_{17}+n_{21}+n_{25}+n_{29}=K; & n_5+n_{21}+n_{37}+n_{53}=K; \\
 n_{21}+n_{22}+n_{23}+n_{24}=K; & n_{18}+n_{22}+n_{26}+n_{30}=K; & n_6+n_{22}+n_{38}+n_{54}=K; \\
 n_{25}+n_{26}+n_{27}+n_{28}=K; & n_{19}+n_{23}+n_{27}+n_{31}=K; & n_7+n_{23}+n_{39}+n_{55}=K; \\
 n_{29}+n_{30}+n_{31}+n_{32}=K; & n_{20}+n_{24}+n_{28}+n_{32}=K; & n_8+n_{24}+n_{40}+n_{56}=K; \\
 n_{33}+n_{34}+n_{35}+n_{36}=K; & n_{33}+n_{37}+n_{41}+n_{45}=K; & n_9+n_{25}+n_{41}+n_{57}=K; \\
 n_{37}+n_{38}+n_{39}+n_{40}=K; & n_{34}+n_{38}+n_{42}+n_{46}=K; & n_{10}+n_{26}+n_{42}+n_{58}=K; \\
 n_{41}+n_{42}+n_{43}+n_{44}=K; & n_{35}+n_{39}+n_{43}+n_{47}=K; & n_{11}+n_{27}+n_{43}+n_{59}=K; \\
 n_{45}+n_{46}+n_{47}+n_{48}=K; & n_{36}+n_{40}+n_{44}+n_{48}=K; & n_{12}+n_{28}+n_{44}+n_{60}=K; \\
 n_{49}+n_{50}+n_{51}+n_{52}=K; & n_{49}+n_{53}+n_{57}+n_{61}=K; & n_{13}+n_{29}+n_{45}+n_{61}=K; \\
 n_{53}+n_{54}+n_{55}+n_{56}=K; & n_{50}+n_{54}+n_{58}+n_{62}=K; & n_{14}+n_{30}+n_{46}+n_{62}=K; \\
 n_{57}+n_{58}+n_{59}+n_{60}=K; & n_{51}+n_{55}+n_{59}+n_{63}=K; & n_{15}+n_{31}+n_{47}+n_{63}=K; \\
 n_{61}+n_{62}+n_{63}+n_{64}=K; & n_{52}+n_{56}+n_{60}+n_{64}=K; & n_{16}+n_{32}+n_{48}+n_{64}=K
 \end{array}$$

\*\* For Pan-Triagonals:  $K=130$  \*\*

$$\begin{array}{llll}
 n_1+n_{22}+n_{43}+n_{64}=K; & n_1+n_{24}+n_{43}+n_{62}=K; & n_1+n_{30}+n_{43}+n_{56}=K; & n_1+n_{32}+n_{43}+n_{54}=K; \\
 n_2+n_{23}+n_{44}+n_{61}=K; & n_2+n_{21}+n_{44}+n_{63}=K; & n_2+n_{31}+n_{44}+n_{53}=K; & n_2+n_{29}+n_{44}+n_{55}=K; \\
 n_3+n_{24}+n_{41}+n_{62}=K; & n_3+n_{22}+n_{41}+n_{64}=K; & n_3+n_{32}+n_{41}+n_{54}=K; & n_3+n_{30}+n_{41}+n_{56}=K; \\
 n_4+n_{21}+n_{42}+n_{63}=K; & n_4+n_{23}+n_{42}+n_{61}=K; & n_4+n_{29}+n_{42}+n_{55}=K; & n_4+n_{31}+n_{42}+n_{53}=K; \\
 n_5+n_{26}+n_{47}+n_{52}=K; & n_5+n_{28}+n_{47}+n_{50}=K; & n_5+n_{18}+n_{47}+n_{60}=K; & n_5+n_{20}+n_{47}+n_{58}=K; \\
 n_6+n_{27}+n_{48}+n_{49}=K; & n_6+n_{25}+n_{48}+n_{51}=K; & n_6+n_{19}+n_{48}+n_{57}=K; & n_6+n_{17}+n_{48}+n_{59}=K; \\
 n_7+n_{28}+n_{45}+n_{50}=K; & n_7+n_{26}+n_{45}+n_{52}=K; & n_7+n_{20}+n_{45}+n_{58}=K; & n_7+n_{18}+n_{45}+n_{60}=K; \\
 n_8+n_{25}+n_{46}+n_{51}=K; & n_8+n_{27}+n_{46}+n_{49}=K; & n_8+n_{17}+n_{46}+n_{59}=K; & n_8+n_{19}+n_{46}+n_{57}=K; \\
 n_9+n_{30}+n_{35}+n_{56}=K; & n_9+n_{32}+n_{35}+n_{54}=K; & n_9+n_{22}+n_{35}+n_{64}=K; & n_9+n_{24}+n_{35}+n_{62}=K; \\
 n_{10}+n_{31}+n_{36}+n_{53}=K; & n_{10}+n_{29}+n_{36}+n_{55}=K; & n_{10}+n_{23}+n_{36}+n_{61}=K; & n_{10}+n_{21}+n_{36}+n_{63}=K; \\
 n_{11}+n_{32}+n_{33}+n_{54}=K; & n_{11}+n_{30}+n_{33}+n_{56}=K; & n_{11}+n_{24}+n_{33}+n_{62}=K; & n_{11}+n_{22}+n_{33}+n_{64}=K; \\
 n_{12}+n_{29}+n_{34}+n_{55}=K; & n_{12}+n_{31}+n_{34}+n_{53}=K; & n_{12}+n_{21}+n_{34}+n_{63}=K; & n_{12}+n_{23}+n_{34}+n_{61}=K; \\
 n_{13}+n_{18}+n_{39}+n_{60}=K; & n_{13}+n_{20}+n_{39}+n_{58}=K; & n_{13}+n_{26}+n_{39}+n_{52}=K; & n_{13}+n_{28}+n_{39}+n_{50}=K; \\
 n_{14}+n_{19}+n_{40}+n_{57}=K; & n_{14}+n_{17}+n_{40}+n_{59}=K; & n_{14}+n_{27}+n_{40}+n_{49}=K; & n_{14}+n_{25}+n_{40}+n_{51}=K; \\
 n_{15}+n_{20}+n_{37}+n_{58}=K; & n_{15}+n_{18}+n_{37}+n_{60}=K; & n_{15}+n_{28}+n_{37}+n_{50}=K; & n_{15}+n_{26}+n_{37}+n_{52}=K; \\
 n_{16}+n_{17}+n_{38}+n_{59}=K; & n_{16}+n_{19}+n_{38}+n_{57}=K; & n_{16}+n_{25}+n_{38}+n_{51}=K; & n_{16}+n_{27}+n_{38}+n_{49}=K
 \end{array}$$

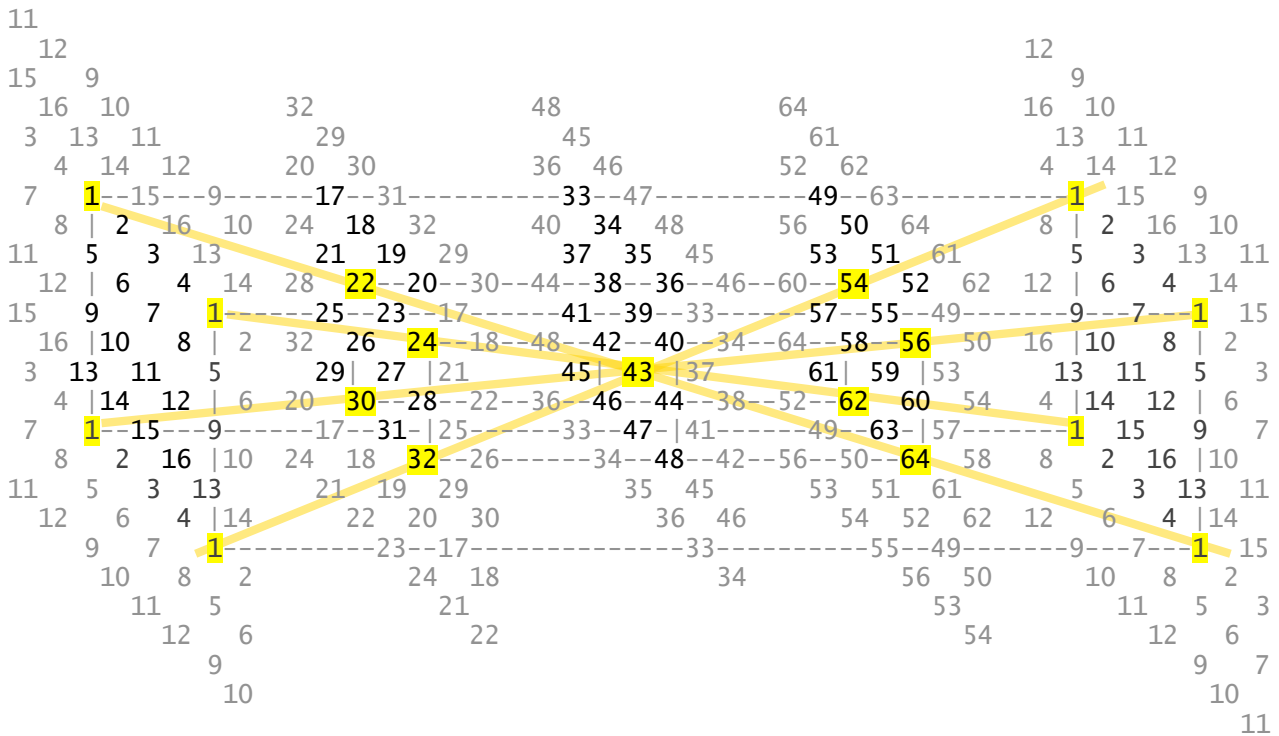
We must assume such the simultaneous equations listed as above, so many as 112 equations in all ( $112=48+64$ ).

The next diagram is drawn for the 'Extended Space' of Pan-magic Cube of Order 4.

I invented it so that you could easily see any pan-triagonal in a 'straight' line and

find some other cubes from your different viewpoints.

[Fig.2: Extended Space for Pan-Triagonals of Magic Cube 4\*4\*4]



## #2. Algebraic Study: Calculate and Find some Interesting Properties

Let's pick up some equations of pan-triagonals and compare and calculate them.

Compare those equations and take out complementary pairs as many as possible.

$$n1+n22+n43+n64=K; \quad n1+n24+n43+n62=K; \quad n1+n30+n43+n56=K; \quad n1+n32+n43+n54=K; \\ (n1+n43)=C1; \quad n22+n64=n24+n62=n30+n56=n32+n54=K-C1=D1; \quad C1+D1=K \quad \dots (130)$$

$$n2+n23+n44+n61=K; \quad n2+n21+n44+n63=K; \quad n2+n31+n44+n53=K; \quad n2+n29+n44+n55=K; \\ (n2+n44)=C2; \quad n23+n61=n21+n63=n31+n53=n29+n55=K-C2=D2; \quad C2+D2=K \quad \dots (131)$$

$$n3+n24+n41+n62=K; \quad n3+n22+n41+n64=K; \quad n3+n32+n41+n54=K; \quad n3+n30+n41+n56=K; \\ (n3+n41)=C3; \quad n24+n62=n22+n64=n32+n54=n30+n56=K-C3=D3; \quad C3+D3=K \quad \dots (132)$$

$$n4+n21+n42+n63=K; \quad n4+n23+n42+n61=K; \quad n4+n29+n42+n55=K; \quad n4+n31+n42+n53=K; \\ (n4+n42)=C4; \quad n21+n63=n23+n61=n29+n55=n31+n53=K-C4=D4; \quad C4+D4=K \quad \dots (133)$$

Compare (130)~(133), and you will find  $D3=D1$ ;  $D4=D2$ ;  $C3=C1$ ; and  $C4=C2$ ;

$$n5+n26+n47+n52=K; \quad n5+n28+n47+n50=K; \quad n5+n18+n47+n60=K; \quad n5+n20+n47+n58=K; \\ (n5+n47)=C5; \quad n26+n52=n28+n50=n18+n60=n20+n58=K-C5=D5; \quad C5+D5=K \quad \dots (134)$$

$$n6+n27+n48+n49=K; \quad n6+n25+n48+n51=K; \quad n6+n19+n48+n57=K; \quad n6+n17+n48+n59=K; \\ (n6+n48)=C6; \quad n27+n49=n25+n51=n19+n57=n17+n59=K-C6=D6; \quad C6+D6=K \quad \dots (135)$$

$$n7+n28+n45+n50=K; \quad n7+n26+n45+n52=K; \quad n7+n20+n45+n58=K; \quad n7+n18+n45+n60=K; \\ (n7+n45)=C7; \quad n28+n50=n26+n52=n20+n58=n18+n60=K-C7=D7; \quad C7+D7=K \quad \dots (136)$$

$$n8+n25+n46+n51=K; \quad n8+n27+n46+n49=K; \quad n8+n17+n46+n59=K; \quad n8+n19+n46+n57=K; \\ (n8+n46)=C8; \quad n25+n51=n27+n49=n17+n59=n19+n57=K-C8=D8; \quad C8+D8=K \quad \dots (137)$$

Compare (134)~(137), and you will find  $D7=D5$ ;  $D8=D6$ ;  $C7=C5$ ; and  $C8=C6$ ;

$$(n9+n35)=C9; \quad n30+n56=n32+n54=n22+n64=n24+n62=K-C9=D9; \quad C9+D9=K \quad \dots (138)$$

$$(n10+n36)=C10; \quad n31+n53=n29+n55=n23+n61=n21+n63=K-C10=D10; \quad C10+D10=K \quad \dots (139)$$

$$(n11+n33)=C11; \quad n32+n54=n30+n56=n24+n62=n22+n64=K-C11=D11; \quad C11+D11=K \quad \dots (140)$$

$$(n12+n34)=C12; \quad n29+n55=n31+n53=n21+n63=n23+n61=K-C12=D12; \quad C12+D12=K \quad \dots (141)$$

Compare (138)~(141) with (130)~(133), you will find

$$D9=D1; \quad D10=D2; \quad D11=D1; \quad D12=D2; \quad C9=C1; \quad C10=C2; \quad C11=C1; \quad \text{and} \quad C12=C2$$

$$(n13+n39)=C13; \quad n18+n60=n20+n58=n26+n52=n28+n50=K-C13=D13; \quad C13+D13=K \quad \dots (142)$$



$$(n_{14}+n_{40})=C_{14}; n_{19}+n_{57}=n_{17}+n_{59}=n_{27}+n_{49}=n_{25}+n_{51}=K-C_{14}=D_{14}; C_{14}+D_{14}=K \dots (143)$$

$$(n_{15}+n_{37})=C_{15}; n_{20}+n_{58}=n_{18}+n_{60}=n_{28}+n_{50}=n_{26}+n_{52}=K-C_{15}=D_{15}; C_{15}+D_{15}=K \dots (144)$$

$$(n_{16}+n_{38})=C_{16}; n_{17}+n_{59}=n_{19}+n_{57}=n_{25}+n_{51}=n_{27}+n_{49}=K-C_{16}=D_{16}; C_{16}+D_{16}=K \dots (145)$$

Compare (142)~(145) with (134)~(137), you will find

$$D_{13}=D_5; D_{14}=D_6; D_{15}=D_5; D_{16}=D_6; C_{13}=C_5; C_{14}=C_6; C_{15}=C_5; \text{ and } C_{16}=C_6$$

$$n_6+n_{27}+n_{48}+n_{49}=K; n_8+n_{27}+n_{46}+n_{49}=K; n_{14}+n_{27}+n_{40}+n_{49}=K; n_{16}+n_{27}+n_{38}+n_{49}=K;$$

$$(n_{27}+n_{49})=D_6; n_6+n_{48}=n_8+n_{46}=n_{14}+n_{40}=n_{16}+n_{38}=K-D_6=C_6$$

$$D_6 \quad C_6 = C_8 = C_{14} = C_{16}$$

$$n_1+n_{22}+n_{43}+n_{64}=K; n_3+n_{22}+n_{41}+n_{64}=K; n_9+n_{22}+n_{35}+n_{64}=K; n_{11}+n_{22}+n_{33}+n_{64}=K;$$

$$(n_{22}+n_{64})=D_1; n_1+n_{43}=n_3+n_{41}=n_9+n_{35}=n_{11}+n_{33}=K-D_1=C_1$$

$$C_1 = C_3 = C_9 = C_{11}$$

Make 4 groups according to its values:

$$C_1+D_1 \text{ Group } [C_1+D_1, C_3+D_3, C_9+D_9, C_{11}+D_{11}]$$

$$C_2+D_2 \text{ Group } [C_2+D_2, C_4+D_4, C_{10}+D_{10}, C_{12}+D_{12}]$$

$$C_5+D_5 \text{ Group } [C_5+D_5, C_7+D_7, C_{13}+D_{13}, C_{15}+D_{15}]$$

$$C_6+D_6 \text{ Group } [C_6+D_6, C_8+D_8, C_{14}+D_{14}, C_{16}+D_{16}]$$

$$C_1+C_2+C_3+C_4$$

$$=(n_1+n_{43})+(n_2+n_{44})+(n_3+n_{41})+(n_4+n_{42})=(n_1+n_2+n_3+n_4)+(n_{41}+n_{42}+n_{43}+n_{44})=2*K$$

$$C_3=C_1; C_4=C_2; \rightarrow C_1+C_2+C_3+C_4=C_1+C_2+C_1+C_2=2*(C_1+C_2)=2*K; C_1+C_2=K;$$

$$\text{But } C_1+D_1=K; C_2+D_2=K; \text{ Consequently } C_2=D_1; C_1=D_2$$

$$C_5+C_6+C_7+C_8$$

$$=(n_5+n_{47})+(n_6+n_{48})+(n_7+n_{45})+(n_8+n_{46})=(n_5+n_6+n_7+n_8)+(n_{45}+n_{46}+n_{47}+n_{48})=2*K$$

$$C_7=C_5; C_8=C_6; \rightarrow C_5+C_6+C_7+C_8=C_5+C_6+C_5+C_6=2*(C_5+C_6)=2*K; C_5+C_6=K;$$

$$\text{But } C_5+D_5=K; C_6+D_6=K; \text{ As a consequence } C_6=D_5; C_5=D_6$$

$$C_1+C_5+C_9+C_{13}$$

$$=(n_1+n_{43})+(n_5+n_{47})+(n_9+n_{35})+(n_{13}+n_{39})=(n_1+n_5+n_9+n_{13})+(n_{35}+n_{39}+n_{43}+n_{47})=2*K$$

$$C_9=C_1; C_{13}=C_5; \rightarrow C_1+C_5+C_9+C_{13}=C_1+C_5+C_1+C_5=2*(C_1+C_5)=2*K; C_1+C_5=K;$$

$$\text{But } C_1+D_1=K; C_5+D_5=K; \text{ Therefore } C_1=D_5; C_5=D_1$$

$$C_2+C_6+C_{10}+C_{14}$$

$$=(n_2+n_{44})+(n_6+n_{48})+(n_{10}+n_{36})+(n_{14}+n_{40})=(n_2+n_6+n_{10}+n_{14})+(n_{36}+n_{40}+n_{44}+n_{48})=2*K$$

$$C_{10}=C_2; C_{14}=C_6; \rightarrow C_2+C_6+C_{10}+C_{14}=C_2+C_6+C_2+C_6=2*(C_2+C_6)=2*K; C_2+C_6=K;$$

$$\text{But } C_2+D_2=K; C_6+D_6=K; \text{ As a result } C_2=D_6; C_6=D_2$$

Remake the groups according to their values:

$$CC \text{ Group } [C_1, C_3, C_6, C_8, C_9, C_{11}, C_{14}, C_{16}, D_2, D_4, D_5, D_7, D_{10}, D_{12}, D_{13}, D_{15}]$$

$$DC \text{ Group } [C_2, C_4, C_5, C_7, C_{10}, C_{12}, C_{13}, C_{15}, D_1, D_3, D_6, D_8, D_9, D_{11}, D_{14}, D_{16}]$$

That means:

$$n_1+n_{43}=n_3+n_{41}=n_6+n_{48}=n_8+n_{46}=n_9+n_{35}=n_{11}+n_{33}=n_{14}+n_{40}=n_{16}+n_{38}$$

$$=n_{18}+n_{60}=n_{20}+n_{58}=n_{21}+n_{63}=n_{23}+n_{61}=n_{26}+n_{52}=n_{28}+n_{50}=n_{29}+n_{55}=n_{31}+n_{53}=CC \dots (146)$$

$$n_2+n_{44}=n_4+n_{42}=n_5+n_{47}=n_7+n_{45}=n_{10}+n_{36}=n_{12}+n_{34}=n_{13}+n_{39}=n_{15}+n_{37}$$

$$=n_{17}+n_{59}=n_{19}+n_{57}=n_{22}+n_{64}=n_{24}+n_{62}=n_{25}+n_{51}=n_{27}+n_{49}=n_{30}+n_{56}=n_{32}+n_{54}=DD \dots (147)$$

$$(CC+DD=K)$$

All complementary pairs on pan-triagonals are classified into two groups according to their values. One group has 16 pairs of the same sum CC and the other has 16 pairs of DD. Let's call CC, DD simply as 'C, D' from now on.

All pan-triagonals are made of complementary pairs of C and of D.

Take some of the primary triagonals for instance:

$$n_1+n_{22}+n_{43}+n_{64}=(n_1+n_{43})+(n_{22}+n_{64})=C+D=K$$

$$n_4+n_{23}+n_{42}+n_{61}=(n_4+n_{42})+(n_{23}+n_{61})=D+C=K$$

$$n_{13}+n_{26}+n_{39}+n_{52}=(n_{13}+n_{39})+(n_{26}+n_{52})=D+C=K$$

$$n_{16}+n_{27}+n_{38}+n_{49}=(n_{16}+n_{38})+(n_{27}+n_{49})=C+D=K$$

If you put the value 1 to  $n_1$  and 32 to  $n_{43}$ ,  
then  $n_1+n_{43}=1+32=33=C$ ;  $n_{22}+n_{64}=D=K-C=130-33=97$ .

If you put  $n_1=1$  and  $n_{43}=64$ ,  
then  $C=n_1+n_{43}=1+64=65$ ; and  $n_{22}+n_{64}=D=K-C=130-65=65$ ; That means  $C=D$ .

What values could any pair of  $\{C, D\}$  take, then? Of course,  $C+D=K$ . Are there so many pairs of  $\{C, D\}$  as 16?

### #3. Algebraic Study-2: How many Pairs of C and D could we find?

For instance, suppose  $n_1=1$  and  $n_{43}=16$ ; and you will know  $C=n_1+n_{43}=1+16=17$ ;  
 $D=K-C=130-17=113$

How many pairs could we make then?

$C_1(1, 16), D_1(64, 49)$ ;  $C_2(2, 15), D_2(63, 50)$ ;  $C_3(3, 14), D_3(62, 51)$ ;  
 $C_4(4, 13), D_4(61, 52)$ ;  $C_5(5, 12), D_5(60, 53)$ ;  $C_6(6, 11), D_6(59, 54)$ ;  
 $C_7(7, 10), D_7(58, 55)$ ;  $C_8(8, 9), D_8(57, 56)$ ;  $C_9(9, 8), D_9(56, 57)$ ? ...

But  $C_9=C_8, D_9=D_8$ . This means the end of making new pairs.

We could only make 8 pairs of C and also 8 pairs of D.

With so few pairs we cannot make any Pan-triagonal Magic Cube of Order 4 at all.

The next list below is the result of making pairs according to the values of C and D.  
I found we could only make less than 16 pairs when  $C < 33$ .

#### \*\* Find Possible Complementary Pairs for Magic Cube 4x4x4 \*\*

$CC=32$ :  $C_1(1, 31), C_2(2, 30), C_3(3, 29), C_4(4, 28), C_5(5, 27), C_6(6, 26)$ ,  
 $C_7(7, 25), C_8(8, 24), C_9(9, 23), C_{10}(10, 22), C_{11}(11, 21), C_{12}(12, 20)$ ,  
 $C_{13}(13, 19), C_{14}(14, 18), C_{15}(15, 17)$ ,

$DD=98$ :  $D_1(64, 34), D_2(63, 35), D_3(62, 36), D_4(61, 37), D_5(60, 38), D_6(59, 39)$ ,  
 $D_7(58, 40), D_8(57, 41), D_9(56, 42), D_{10}(55, 43), D_{11}(54, 44), D_{12}(53, 45)$ ,  
 $D_{13}(52, 46), D_{14}(51, 47), D_{15}(50, 48)$ ,

$CC=33$ :  $C_1(1, 32), C_2(2, 31), C_3(3, 30), C_4(4, 29), C_5(5, 28), C_6(6, 27)$ ,  
 $C_7(7, 26), C_8(8, 25), C_9(9, 24), C_{10}(10, 23), C_{11}(11, 22), C_{12}(12, 21)$ ,  
 $C_{13}(13, 20), C_{14}(14, 19), C_{15}(15, 18), C_{16}(16, 17)$ ,

$DD=97$ :  $D_1(64, 33), D_2(63, 34), D_3(62, 35), D_4(61, 36), D_5(60, 37), D_6(59, 38)$ ,  
 $D_7(58, 39), D_8(57, 40), D_9(56, 41), D_{10}(55, 42), D_{11}(54, 43), D_{12}(53, 44)$ ,  
 $D_{13}(52, 45), D_{14}(51, 46), D_{15}(50, 47), D_{16}(49, 48)$  ..... [OK!]

$CC=34$ :  $C_1(1, 33), C_2(3, 31), C_3(4, 30), C_4(5, 29), C_5(6, 28), C_6(7, 27)$ ,  
 $C_7(8, 26), C_8(9, 25), C_9(10, 24), C_{10}(11, 23), C_{11}(12, 22), C_{12}(13, 21)$ ,  
 $C_{13}(14, 20), C_{14}(15, 19), C_{15}(16, 18)$ ,

$DD=96$ :  $D_1(64, 32), D_2(62, 34), D_3(61, 35), D_4(60, 36), D_5(59, 37), D_6(58, 38)$ ,  
 $D_7(57, 39), D_8(56, 40), D_9(55, 41), D_{10}(54, 42), D_{11}(53, 43), D_{12}(52, 44)$ ,  
 $D_{13}(51, 45), D_{14}(50, 46), D_{15}(49, 47)$ ,

.....

$CC=48$ :  $C_1(1, 47), C_2(2, 46), C_3(3, 45), C_4(4, 44), C_5(5, 43), C_6(6, 42)$ ,  
 $C_7(7, 41), C_8(8, 40), C_9(9, 39), C_{10}(10, 38), C_{11}(11, 37), C_{12}(12, 36)$ ,  
 $C_{13}(13, 35), C_{14}(14, 34), C_{15}(15, 33)$ ,

$DD=82$ :  $D_1(64, 18), D_2(63, 19), D_3(62, 20), D_4(61, 21), D_5(60, 22), D_6(59, 23)$ ,  
 $D_7(58, 24), D_8(57, 25), D_9(56, 26), D_{10}(55, 27), D_{11}(54, 28), D_{12}(53, 29)$ ,  
 $D_{13}(52, 30), D_{14}(51, 31), D_{15}(50, 32)$ ,

$CC=49$ :  $C_1(1, 48), C_2(2, 47), C_3(3, 46), C_4(4, 45), C_5(5, 44), C_6(6, 43)$ ,  
 $C_7(7, 42), C_8(8, 41), C_9(9, 40), C_{10}(10, 39), C_{11}(11, 38), C_{12}(12, 37)$ ,  
 $C_{13}(13, 36), C_{14}(14, 35), C_{15}(15, 34), C_{16}(16, 33)$ ,

DD=81: D1(64, 17), D2(63, 18), D3(62, 19), D4(61, 20), D5(60, 21), D6(59, 22),  
D7(58, 23), D8(57, 24), D9(56, 25), D10(55, 26), D11(54, 27), D12(53, 28),  
D13(52, 29), D14(51, 30), D15(50, 31), D16(49, 32) . . . . . [OK!]

CC=50: C1(1, 49), C2(2, 48), C3(3, 47), C4(4, 46), C5(5, 45), C6(6, 44),  
C7(7, 43), C8(8, 42), C9(9, 41), C10(10, 40), C11(11, 39), C12(12, 38),  
C13(13, 37), C14(14, 36), C15(15, 35),

DD=80: D1(64, 16), D2(63, 17), D3(62, 18), D4(61, 19), D5(60, 20), D6(59, 21),  
D7(58, 22), D8(57, 23), D9(56, 24), D10(55, 25), D11(54, 26), D12(53, 27),  
D13(52, 28), D14(51, 29), D15(50, 30),

. . . . .

CC=56: C1(1, 55), C2(2, 54), C3(3, 53), C4(4, 52), C5(5, 51), C6(6, 50),  
C7(7, 49), C8(8, 48), C9(9, 47), C10(19, 37), C11(20, 36), C12(21, 35),  
C13(22, 34), C14(23, 33),

DD=74: D1(64, 10), D2(63, 11), D3(62, 12), D4(61, 13), D5(60, 14), D6(59, 15),  
D7(58, 16), D8(57, 17), D9(56, 18), D10(46, 28), D11(45, 29), D12(44, 30),  
D13(43, 31), D14(42, 32),

CC=57: C1(1, 56), C2(2, 55), C3(3, 54), C4(4, 53), C5(5, 52), C6(6, 51),  
C7(7, 50), C8(8, 49), C9(17, 40), C10(18, 39), C11(19, 38), C12(20, 37),  
C13(21, 36), C14(22, 35), C15(23, 34), C16(24, 33),

DD=73: D1(64, 9), D2(63, 10), D3(62, 11), D4(61, 12), D5(60, 13), D6(59, 14),  
D7(58, 15), D8(57, 16), D9(48, 25), D10(47, 26), D11(46, 27), D12(45, 28),  
D13(44, 29), D14(43, 30), D15(42, 31), D16(41, 32) . . . . . [OK!]

CC=58: C1(1, 57), C2(2, 56), C3(3, 55), C4(4, 54), C5(5, 53), C6(6, 52),  
C7(7, 51), C8(15, 43), C9(16, 42), C10(17, 41), C11(18, 40), C12(19, 39),  
C13(20, 38), C14(21, 37),

DD=72: D1(64, 8), D2(63, 9), D3(62, 10), D4(61, 11), D5(60, 12), D6(59, 13),  
D7(58, 14), D8(50, 22), D9(49, 23), D10(48, 24), D11(47, 25), D12(46, 26),  
D13(45, 27), D14(44, 28),

CC=59: C1(1, 58), C2(2, 57), C3(3, 56), C4(4, 55), C5(5, 54), C6(6, 53),  
C7(13, 46), C8(14, 45), C9(15, 44), C10(16, 43), C11(17, 42), C12(18, 41),  
C13(25, 34), C14(26, 33), C15(29, 30),

DD=71: D1(64, 7), D2(63, 8), D3(62, 9), D4(61, 10), D5(60, 11), D6(59, 12),  
D7(52, 19), D8(51, 20), D9(50, 21), D10(49, 22), D11(48, 23), D12(47, 24),  
D13(40, 31), D14(39, 32), D15(36, 35),

CC=60: C1(1, 59), C2(2, 58), C3(3, 57), C4(4, 56), C5(5, 55), C6(11, 49),  
C7(12, 48), C8(13, 47), C9(14, 46), C10(15, 45), C11(21, 39), C12(22, 38),  
C13(23, 37), C14(24, 36), C15(25, 35),

DD=70: D1(64, 6), D2(63, 7), D3(62, 8), D4(61, 9), D5(60, 10), D6(54, 16),  
D7(53, 17), D8(52, 18), D9(51, 19), D10(50, 20), D11(44, 26), D12(43, 27),  
D13(42, 28), D14(41, 29), D15(40, 30),

CC=61: C1(1, 60), C2(2, 59), C3(3, 58), C4(4, 57), C5(9, 52), C6(10, 51),  
C7(11, 50), C8(12, 49), C9(17, 44), C10(18, 43), C11(19, 42), C12(20, 41),  
C13(25, 36), C14(26, 35), C15(27, 34), C16(28, 33),

DD=69: D1(64, 5), D2(63, 6), D3(62, 7), D4(61, 8), D5(56, 13), D6(55, 14),  
D7(54, 15), D8(53, 16), D9(48, 21), D10(47, 22), D11(46, 23), D12(45, 24),  
D13(40, 29), D14(39, 30), D15(38, 31), D16(37, 32) . . . . . [OK!]

CC=62: C1(1, 61), C2(2, 60), C3(3, 59), C4(7, 55), C5(8, 54), C6(9, 53),  
C7(13, 49), C8(14, 48), C9(15, 47), C10(19, 43), C11(20, 42), C12(21, 41),  
C13(25, 37), C14(26, 36), C15(27, 35),

DD=68: D1(64, 4), D2(63, 5), D3(62, 6), D4(58, 10), D5(57, 11), D6(56, 12),  
D7(52, 16), D8(51, 17), D9(50, 18), D10(46, 22), D11(45, 23), D12(44, 24),  
D13(40, 28), D14(39, 29), D15(38, 30),

CC=63: C1(1, 62), C2(2, 61), C3(5, 58), C4(6, 57), C5(9, 54), C6(10, 53),  
 C7(13, 50), C8(14, 49), C9(17, 46), C10(18, 45), C11(21, 42), C12(22, 41),  
 C13(25, 38), C14(26, 37), C15(29, 34), C16(30, 33),  
 DD=67: D1(64, 3), D2(63, 4), D3(60, 7), D4(59, 8), D5(56, 11), D6(55, 12),  
 D7(52, 15), D8(51, 16), D9(48, 19), D10(47, 20), D11(44, 23), D12(43, 24),  
 D13(40, 27), D14(39, 28), D15(36, 31), D16(35, 32) ..... [OK!]

CC=64: C1(1, 63), C2(3, 61), C3(5, 59), C4(7, 57), C5(9, 55), C6(11, 53),  
 C7(13, 51), C8(15, 49), C9(17, 47), C10(19, 45), C11(21, 43), C12(23, 41),  
 C13(25, 39), C14(27, 37), C15(29, 35), C16(31, 33),  
 DD=66: D1(64, 2), D2(62, 4), D3(60, 6), D4(58, 8), D5(56, 10), D6(54, 12),  
 D7(52, 14), D8(50, 16), D9(48, 18), D10(46, 20), D11(44, 22), D12(42, 24),  
 D13(40, 26), D14(38, 28), D15(36, 30), D16(34, 32) ..... [OK!]

CC=DD=65: 1(1, 64), 2(2, 63), 3(3, 62), 4(4, 61), 5(5, 60), 6(6, 59), 7(7, 58), 8(8, 57),  
 9(9, 56), 10(10, 55), 11(11, 54), 12(12, 53), 13(13, 52), 14(14, 51),  
 15(15, 50), 16(16, 49), 17(17, 48), 18(18, 47), 19(19, 46), 20(20, 45),  
 21(21, 44), 22(22, 43), 23(23, 42), 24(24, 41), 25(25, 40), 26(26, 39),  
 27(27, 38), 28(28, 37), 29(29, 36), 30(30, 35), 31(31, 34), 32(32, 33) .... [OK!]

I found that the next seven cases could only satisfy our request:

(C,D)=(33, 97), (49, 81), (57, 73), (61, 69), (63, 67), (64, 66) and (65, 65)

The last case (C=D=65) needs not be classified into two groups of C and D, because we can find no difference between them. We must accept it has 32 pairs of 65 in all, and all pairs should be placed anywhere only on pan-triagonals but no one elsewhere.

I would like to call this type as "Complete Pan-magic" from now on.

Notice every case has the pair C1(1,N) on the list head.

Each of (1, 32), (1, 48), (1, 56), (1, 60), (1, 62), (1, 63) and (1, 64) is supposed to be the index pair of the group of which it must be a representative member.

If you put the value 1 to n1, then you must put any of {32, 48, 56, 60, 62, 63 or 64} to n43, and you must not put anything else there.

However surprising it might look, it is surely one of the destinies of our Pan-magic Cube of Order 4.

#### #4. How many Solutions could we find?

Take the first Basic Form and the 112 equations of our definition stage, and assume the 'standard form' with n1=1 and put some inequality conditions to it:

$n_2 > n_5 > n_{17}$ ;  $n_2 > n_4$ ;  $n_5 > n_{13}$ ; and  $n_{17} > n_{49}$

It is because you can exchange the 2nd and 4th planes with each other in all three directions, and you can identify the patterns of which the 2nd-4th planes were exchanged with the original ones.

We want to count only the 'fundamental solutions' and prevent us from double counting of any simple reflections or rotations.

(n1, n43)=(1, 32): 469,921 solutions are found.  
 (n1, n43)=(1, 48): 377,803 ...  
 (n1, n43)=(1, 56): 355,173 ...  
 (n1, n43)=(1, 60): 355,173 ...  
 (n1, n43)=(1, 62): 377,803 ...  
 (n1, n43)=(1, 63): 469,921 ...  
 (n1, n43)=(1, 64): ???

In the last case (C=D=65) I could not yet count up through, because it took too much time for me to calculate and count up. But the counts seem to exceed far over

the sum of all the former six cases.

**\*\* Pan-triagonal Type of Magic Cubes of Order 4: \*\***

**\*\* List of Fundamental Solutions with n1=1 & n43=32 \*\***

1/

```

1-----60-----10-----59
|64      5      52      | 9
62  2   13  53   39  27   16  48
| 3  63---55-12---29-41---43-14
6  47 | 49  22   31  36   44  25 |
|56  18 | 19  40   34  26   |21  46
61--23-|-- 8--38---50--32---11  37 |
7  45 | 51  24   15  33   57  28
58 |      17      35      20 |
4-----54-----30-----42
  
```

2715/

```

1-----60-----13-----56
|64      18     39      | 9
63  3    8  45   49  28   10  54
| 2  62---44- 7---29-50---55-11
5  59 | 43  21   30  36   52  14 |
|47  6 | 22  57   35  16   |26  51
61--20-|--19-41---38--32---12  37 |
17  58 | 46  24   27  33   40  15
48 |      23      34      25 |
4-----42-----31-----53
  
```

5295/

```

1-----62-----10-----57
|64      3      55      | 8
63  4    7  54   44  27   16  45
| 2  61---58-11---21-38---49-20
6  50 | 52  19   29  37   43  24 |
|59  15 | 13  46   36  28   |22  41
60--23-|-- 9--40---47--32---14  35 |
5  42 | 56  25   18  33   51  30
53 |      17      34      26 |
12-----48-----31-----39
  
```

11263/

```

1-----61-----12-----56
|64      19     38      | 9
63  5    7  44   50  29   10  52
| 2  60---43- 6---30-51---55-13
4  57 | 45  22   28  35   53  16 |
|46  8 | 20  58   37  15   |27  49
62--21-|--17-41---40--32---11  36 |
18  59 | 48  24   25  33   39  14
47 |      23      34      26 |
3-----42-----31-----54
  
```

13726/

```

1-----61-----10-----58
|64      9      43      |14
63  6    3  44   56  29   8  51
| 2  59---55-16---21-48---52- 7
4  60 | 46  22   27  35   53  13 |
|49  5 | 26  50   38  18   |17  57
62--23-|--20-39---37-32---11  36 |
15  54 | 40  19   28  33   47  24
41 |      25      34      30 |
12-----45-----31-----42
  
```

19268/

```

1-----61----- 9-----59
|64      10     44      |12
63  7    20  43   45  29   2  51
| 6  58---55-16---28-48---41- 8
4  47 | 46  18   26  35   54  30 |
|49  14 | 25  37   39  22   |17  57
62--24-|-- 3--38---50--32---15  36 |
11  53 | 40  21   19  33   60  23
52 |      31      34      13 |
5-----56-----27-----42
  
```

24658/

```

1-----61-----12-----56
|64      6      51      | 9
63  8    3  44   54  29   10  49
| 2  57---60-19---13-38---55-16
4  58 | 48  22   25  35   53  15 |
|59  7 | 17  45   40  28   |14  50
62--21-|--18-41---39-32---11  36 |
5  46 | 47  24   26  33   52  27
43 |      23      34      30 |
20-----42-----31-----37
  
```

37428/

```

1-----61-----13-----55
|64      6      38      |22
63  9    18  44   39  29   10  48
|16  56---51-19---26-50---37- 5
4  43 | 49  21   24  35   53  31 |
|47  8 | 28  40   41  30   |14  52
62--20-|-- 2--42---54--32---12  36 |
3  59 | 45  11   25  33   57  27
58 |      23      34      15 |
7-----60-----17-----46
  
```

40201/

```

1-----61----- 8-----60
|64      12     49      | 5
63  10   3  40   58  29   6  51
| 2  55---54-17---15-44---59-14
4  56 | 46  26   23  35   57  13 |
|53  9 | 19  47   42  22   |16  52
62--25-|--20-37---41-32--- 7  36 |
11  48 | 45  28   24  33   50  21
39 |      27      34      30 |
18-----38-----31-----43
  
```

46086/

```

1-----61----- 8-----60
|64      12     49      | 5
63  11   2  40   55  29   10  50
| 6  54---59-17---14-44---51-15
4  52 | 47  30   22  35   57  13 |
|53  9 | 18  39   43  26   |16  56
62--25-|--20-37---45-32--- 3  36 |
7  48 | 41  28   24  33   58  21
42 |      23      34      31 |
19-----46-----27-----38
  
```

51506/

```

1-----61----- 8-----60
|64      18     43      | 5
  
```

63877/

```

1-----61-----18-----50
|64      3      40      |23
  
```

```

63 12 2 40 59 29 6 49
| 9 53---56--11---13--50---+52--16
4 55 | 48 26 21 35 57 14 |
|47 3 | 17 46 44 23 |22 58
62--25-|--19--37---42--32--- 7 36 |
10 54 39 28 30 33 51 15
38 | 27 34 31 |
20-----45-----24-----41

```

```

63 13 6 42 53 29 8 46
| 7 52---49--24---14--43---60--11
4 58 | 51 16 20 35 55 21 |
|54 2 | 22 59 45 28 | 9 41
62--15-|--12--47---39--32---17 36 |
5 57 56 10 31 33 38 30
44 | 25 34 27 |
19-----37-----26-----48

```

69519/

```

1-----61-----10-----58
|64 22 37 | 7
63 14 3 38 56 29 8 49
| 2 51---44-- 9---27--54---+57--16
4 52 | 48 28 19 35 59 15 |
|43 13 17 55 46 12 |24 50
62--23-|--18--39---45--32--- 5 36 |
21 60 47 26 20 33 42 11
41 | 25 34 30 |
6-----40-----31-----53

```

81654/

```

1-----61-----24-----44
|64 7 38 |21
63 15 16 40 45 29 6 46
| 5 50---49--22---20--39---+56--19
4 54 | 51 10 18 35 57 31 |
|58 8 14 55 47 30 |11 37
62-- 9-|-- 2--53---43--32---23 36 |
3 59 60 12 25 33 42 26
52 | 27 34 17 |
13-----41-----28-----48

```

.....

\*\* List of Fundamental Solutions with n1=1 & n43=48 \*\*

1/

```

1-----59-----38-----32
|64 10 23 |33
62 2 9 54 24 43 35 31
| 3 63---52-- 7---45--26---+30--34
6 60 | 50 13 47 20 27 37 |
|55 5 15 56 18 41 |42 28
61--11-|--12--49---21--48---36 22 |
8 58 53 16 44 17 25 39
57 | 14 19 40 |
4-----51-----46-----29

```

1357/

```

1-----60-----13-----56
|64 34 23 | 9
63 3 8 29 49 44 10 54
| 2 62---28-- 7---45--50---+55--11
5 59 | 27 37 46 20 52 14 |
|31 6 38 57 19 16 |42 51
61--36-|--35--25---22--48---12 21 |
33 58 30 40 43 17 24 15
32 | 39 18 41 |
4-----26-----47-----53

```

2819/

```

1-----62-----11-----56
|64 3 54 | 9
63 4 7 30 50 43 10 53
| 2 61---58--35---15--22---+55--12
6 57 | 28 36 45 21 51 16 |
|59 8 37 29 20 44 |14 49
60--38-|--33--25---24--48---13 19 |
5 27 32 40 41 17 52 46
31 | 39 18 42 |
34-----26-----47-----23

```

7094/

```

1-----61-----12-----56
|64 35 22 | 9
63 5 7 28 50 45 10 52
| 2 60---27-- 6---46--51---+55--13
4 57 | 29 38 44 19 53 16 |
|30 8 36 58 21 15 |43 49
62--37-|--33--25---24--48---11 20 |
34 59 32 40 41 17 23 14
31 | 39 18 42 |
3-----26-----47-----54

```

8745/

```

1-----61-----12-----56
|64 8 49 | 9
63 6 3 50 54 45 10 29
| 2 59---58--11---15--24---+55--36
4 60 | 52 16 43 19 31 35 |
|57 5 13 53 22 42 |38 30
62--37-|--14--25---21--48---33 20 |
7 32 51 40 44 17 28 41
27 | 39 18 46 |
34-----26-----47-----23

```

12776/

```

1-----61-----12-----56
|64 8 49 | 9
63 7 36 28 21 45 10 50
| 6 58---29--33---44--24---+51--15
4 26 | 31 38 42 19 53 47 |
|57 35 34 27 23 46 |16 22
62--37-|-- 2--25---55--48---11 20 |
3 32 59 40 14 17 54 41
60 | 39 18 13 |
5-----30-----43-----52

```

16798/

```

1-----61-----12-----56
|64 6 51 | 9
63 8 3 28 54 45 10 49
| 2 57---60--35---13--22---+55--16
4 58 | 32 38 41 19 53 15 |
|59 7 33 29 24 44 |14 50
62--37-|--34--25---23--48---11 20 |

```

28082/

```

1-----61-----36-----32
|64 12 21 |33
63 9 2 52 31 45 34 24
| 6 56---59-- 5---38--28---+27--41
4 58 | 57 14 40 19 29 39 |
|53 3 8 55 25 42 |44 30
62--13-|--10--49---23--48---35 20 |

```

```

5 30 31 40 42 17 52 43
27 | 39 18 46 |
36-----26-----47-----21

```

```

7 60 51 16 46 17 26 37
50 | 15 18 47 |
11-----54-----43-----22

```

29891/

```

1-----61-----8-----60
|64 12 49 | 5
63 10 3 24 58 45 6 51
| 2 55---54-33---15-28---59-14
4 56 | 30 42 39 19 57 13 |
|53 9 35 31 26 38 |16 52
62--41-|--36-21---25-48--- 7 20 |
11 32 29 44 40 17 50 37
23 | 43 18 46 |
34-----22-----47-----27

```

34520/

```

1-----61-----8-----60
|64 12 49 | 5
63 11 2 24 55 45 10 50
| 6 54---59-33---14-28---51-15
4 52 | 31 46 38 19 57 13 |
|53 9 34 23 27 42 |16 56
62--41-|--36-21---29-48--- 3 20 |
7 32 25 44 40 17 58 37
26 | 39 18 47 |
35-----30-----43-----22

```

38747/

```

1-----61-----8-----60
|64 10 51 | 5
63 12 3 24 58 45 6 49
| 2 53---56-35---13-26---59-16
4 54 | 32 42 37 19 57 15 |
|55 11 33 29 28 40 |14 50
62--41-|--34-21---27-48--- 7 20 |
9 30 31 44 38 17 52 39
23 | 43 18 46 |
36-----22-----47-----25

```

49332/

```

1-----60-----11-----58
|64 39 20 | 7
63 13 6 22 53 45 8 50
| 2 52---25-9---46-54---57-15
4 51 | 31 44 36 19 59 16 |
|27 14 34 55 29 12 |40 49
62--38-|--33-23---30-48--- 5 21 |
37 61 32 42 35 17 26 10
28 | 41 18 43 |
3-----24-----47-----56

```

53880/

```

1-----61-----10-----58
|64 38 21 | 7
63 14 3 22 56 45 8 49
| 2 51---28-9---43-54---57-16
4 52 | 32 44 35 19 59 15 |
|27 13 33 55 30 12 |40 50
62--39-|--34-23---29-48--- 5 20 |
37 60 31 42 36 17 26 11
25 | 41 18 46 |
6-----24-----47-----53

```

64471/

```

1-----62-----9-----58
|64 37 22 | 7
63 15 4 20 55 43 8 52
| 2 50---27-11---44-56---57-13
6 49 | 29 46 34 21 61 14 |
|25 16 36 53 31 10 |38 51
60--40-|--35-23---32-48--- 3 19 |
39 59 30 42 33 17 28 12
26 | 41 18 45 |
5-----24-----47-----54

```

76049/

```

1-----59-----12-----58
|64 40 19 | 7
63 16 5 20 54 43 8 51
| 2 49---26-11---45-56---57-14
6 50 | 30 46 33 21 61 13 |
|25 15 35 53 32 10 |38 52
60--37-|--36-23---31-48--- 3 22 |
39 62 29 42 34 17 28 9
27 | 41 18 44 |
4-----24-----47-----55

```

91889/

```

1-----61-----8-----60
|64 9 52 | 5
63 33 10 24 55 45 2 28
| 3 32---59-36---14-25---54-37
4 30 | 53 38 16 19 57 43 |
|56 34 12 23 49 42 |13 31
62--41-|--6-21---51-48---11 20 |
7 29 50 44 15 17 58 40
26 | 47 18 39 |
35-----27-----46-----22

```

\*\* List of Fundamental Solutions with n1=1 & n43=56 \*\*

1/

```

1-----60-----21-----48
|64 5 44 |17
62 2 35 31 26 51 7 46
| 3 63---30-34---39-14---58-19
6 45 | 27 24 55 12 42 49 |
|59 20 38 41 10 53 |23 16
61--36-|--8-25---28-56---33 13 |
4 29 57 40 37 9 32 52
47 | 50 11 22 |
18-----15-----54-----43

```

977/

```

1-----60-----21-----48
|64 34 15 |17
63 3 8 29 41 52 18 46
| 2 62---28-7---53-42---47-19
5 59 | 27 37 54 12 44 22 |
|31 6 38 57 11 24 |50 43
61--36-|--35-25---14-56---20 13 |
33 58 30 40 51 9 16 23
32 | 39 10 49 |
4-----26-----55-----45

```

2318/

```

1-----62-----19-----48

```

6223/

```

1-----61-----20-----48

```

```

|64      3      46      |17
63  4      7 30      42 51      18 45
| 2 61---58-35---23-14---+47-20
 6 57 | 28 36      53 13      43 24 |
|59  8      37 29      12 52      |22 41
60--38-|--33-25---16--56---21 11 |
  5 27      32 40      49  9      44 54
  31 |      39      10      50 |
  34-----26-----55-----15

```

```

|64      35      14      |17
63  5      7 28      42 53      18 44
| 2 60---27- 6---54-43---+47-21
 4 57 | 29 38      52 11      45 24 |
|30  8      36 58      13 23      |51 41
62--37-|--33-25---16--56---19 12 |
  34 59      32 40      49  9      15 22
  31 |      39      10      50 |
  3-----26-----55-----46

```

7711/

```

1-----61-----20-----48
|64      8      41      |17
63  6      3 42      46 53      18 29
| 2 59---58-19---23-16---+47-36
 4 60 | 44 24      51 11      31 35 |
|57  5      21 45      14 50      |38 30
62--37-|--22-25---13-56---33 12 |
  7 32      43 40      52  9      28 49
  27 |      39      10      54 |
  34-----26-----55-----15

```

11673/

```

1-----61-----20-----48
|64      8      41      |17
63  7      36 28      13 53      18 42
| 6 58---29-33---52-16---+43-23
 4 26 | 31 38      50 11      45 55 |
|57 35      34 27      15 54      |24 14
62--37-|-- 2-25---47-56---19 12 |
  3 32      59 40      22  9      46 49
  60 |      39      10      21 |
  5-----30-----51-----44

```

15823/

```

1-----61-----20-----48
|64      6      43      |17
63  8      3 28      46 53      18 41
| 2 57---60-35---21-14---+47-24
 4 58 | 32 38      49 11      45 23 |
|59  7      33 29      16 52      |22 42
62--37-|--34-25---15-56---19 12 |
  5 30      31 40      50  9      44 51
  27 |      39      10      54 |
  36-----26-----55-----13

```

27343/

```

1-----61-----36-----32
|64      19      14      |33
63 17      7 44      26 53      34 16
| 2 48---43- 6---54-27---+31-49
 4 45 | 57 22      40 11      29 52 |
|46 20      8 58      25 39      |51 13
62--21-|-- 5-41---28-56---35 12 |
  18 59      60 24      37  9      15 38
  47 |      23      10      50 |
  3-----42-----55-----30

```

28470/

```

1-----61----- 8-----60
|64      20      41      | 5
63 18      3 16      58 53      6 43
| 2 47---46-33---23-28---+59-22
 4 48 | 30 50      39 11      57 21 |
|45 17      35 31      26 38      |24 44
62--49-|--36-13---25-56--- 7 12 |
  19 32      29 52      40  9      42 37
  15 |      51      10      54 |
  34-----14-----55-----27

```

32586/

```

1-----61----- 8-----60
|64      36      25      | 5
63 19      18 16      43 53      6 42
| 2 46---15-17---54-44---+59-23
 4 32 | 31 50      38 11      57 37 |
|29 33      34 47      27 22      |40 28
62--49-|--20-13---41-56--- 7 12 |
  35 48      45 52      24  9      26 21
  30 |      51      10      39 |
  3-----14-----55-----58

```

36461/

```

1-----61----- 8-----60
|64      34      27      | 5
63 20      2 16      59 53      6 41
|17 45---48-19---21-42---+44-24
 4 47 | 32 50      37 11      57 22 |
|31  3      33 30      28 39      |38 58
62--49-|--35-13---26-56--- 7 12 |
  18 46      15 52      54  9      43 23
  14 |      51      10      55 |
  36-----29-----40-----25

```

46500/

```

1-----48-----49-----32
|64      17      12      |37
63 21      18 14      15 53      34 42
| 2 44---47-51---54-16---+27-19
 4 43 | 31 52      36 11      59 24 |
|57 22      38 13      29 50      | 6 45
62-- 8-|--33-41---30-56--- 5 25 |
  7 61      28 20      35  9      60 40
  58 |      23      10      39 |
  3-----46-----55-----26

```

50529/

```

1-----61-----20-----48
|64      8      41      |17
63 22      3 42      46 53      18 13
| 2 43---58-19---23-16---+47-52
 4 44 | 60 24      35 11      31 51 |
|57 21      5 45      30 50      |38 14
62--37-|-- 6-25---29-56---33 12 |
  7 32      59 40      36  9      28 49
  27 |      39      10      54 |

```

59742/

```

1-----60-----21-----48
|64      5      12      |49
63 23      6 28      43 53      18 26
| 2 42---59-37---54-44---+15- 7
 4 41 | 47 38      34 11      45 40 |
|29 24      50 27      31 22      |20 57
62--36-|--17-25---32-56---19 13 |
  35 61      16  8      33  9      46 52
  30 |      39      10      51 |

```



34-----26-----55-----15  
 69835/  
 1-----61-----8-----60  
 |64 5 25 |36  
 63 24 3 26 58 53 6 27  
 | 2 41---59-38---39-44---30-- 7  
 4 42 | 46 40 33 11 47 37 |  
 |29 23 50 28 32 22 |19 57  
 62--49-|--20-13---31--56---17 12 |  
 35 48 16 21 34 9 45 52  
 15 | 51 10 54 |  
 18-----43-----55-----14

3-----58-----55-----14  
 83496/  
 1-----61-----20-----48  
 |64 35 14 |17  
 63 33 7 28 42 53 18 16  
 | 2 32---27-- 6---54-43---47--49  
 4 29 | 57 38 24 11 45 52 |  
 |30 36 8 58 41 23 |51 13  
 62--37-|-- 5--25---44--56---19 12 |  
 34 59 60 40 21 9 15 22  
 31 | 39 10 50 |  
 3-----26-----55-----46

84986/  
 1-----61-----8-----60  
 |64 18 43 | 5  
 63 34 3 16 58 53 6 27  
 | 2 31---48-35---21--26---59--38  
 4 32 | 46 50 23 11 57 37 |  
 |47 33 19 29 42 40 |22 28  
 62--49-|--20-13---41--56--- 7 12 |  
 17 30 45 52 24 9 44 39  
 15 | 51 10 54 |  
 36-----14-----55-----25

89099/  
 1-----61-----8-----60  
 |64 19 42 | 5  
 63 35 20 16 41 53 6 26  
 |17 30---45-34---24--27---44--39  
 4 14 | 47 50 22 11 57 55 |  
 |46 36 18 15 43 54 |23 25  
 62--49-|-- 2--13---59--56--- 7 12 |  
 3 31 48 52 21 9 58 38  
 32 | 51 10 37 |  
 33-----29-----40-----28

92981/  
 1-----61-----8-----60  
 |64 20 41 | 5  
 63 36 3 16 58 53 6 25  
 | 2 29---46-33---23--28---59--40  
 4 30 | 48 50 21 11 57 39 |  
 |45 35 17 31 44 38 |24 26  
 62--49-|--18-13---43--56--- 7 12 |  
 19 32 47 52 22 9 42 37  
 15 | 51 10 54 |  
 34-----14-----55-----27

102993/  
 1-----48-----49-----32  
 |64 21 12 |33  
 63 37 18 26 15 53 34 14  
 | 2 28---43-35---54--16---31--51  
 4 27 | 59 40 20 11 47 52 |  
 |57 38 6 29 45 50 |22 13  
 62-- 8-|-- 5--41---46--56---17 25 |  
 7 61 60 24 19 9 44 36  
 58 | 23 10 39 |  
 3-----42-----55-----30

.....

\*\* List of Fundamental Solutions with n1=1 & n43=60 \*\*

1/  
 1-----61-----20-----48  
 |64 4 45 |17  
 62 2 12 31 37 50 19 47  
 | 3 63---53-34---28--15---46--18  
 11 55 | 22 36 59 13 38 26 |  
 |54 10 43 29 6 52 |27 39  
 56--41-|--35-21---14--60---25 8 |  
 9 24 30 44 51 5 40 57  
 32 | 42 7 49 |  
 33-----23-----58-----16

1128/  
 1-----61-----20-----48  
 |64 4 45 |17  
 63 3 12 30 37 51 18 46  
 | 2 62---53-35---28--14---47--19  
 10 54 | 23 36 58 13 39 27 |  
 |55 11 42 29 7 52 |26 38  
 56--41-|--34-21---15--60---25 8 |  
 9 24 31 44 50 5 40 57  
 32 | 43 6 49 |  
 33-----22-----59-----16

2618/  
 1-----62-----19-----48  
 |64 3 46 |17  
 63 4 11 30 38 51 18 45  
 | 2 61---54-35---27--14---47--20  
 10 53 | 24 36 57 13 39 28 |  
 |55 12 41 29 8 52 |26 37  
 56--42-|--33-21---16--60---25 7 |  
 9 23 32 44 49 5 40 58  
 31 | 43 6 50 |  
 34-----22-----59-----15

7208/  
 1-----55-----26-----48  
 |64 10 39 |17  
 63 9 12 24 37 57 18 40  
 | 2 56---53-41---28-- 8---47--25  
 4 54 | 29 42 52 7 45 27 |  
 |61 11 36 23 13 58 |20 38  
 62--35-|--34-21---15--60---19 14 |  
 3 30 31 44 50 5 46 51  
 32 | 43 6 49 |  
 33-----22-----59-----16

8184/  
 1-----61-----20-----48  
 |64 12 37 |17  
 63 10 3 24 46 57 18 39

12300/  
 1-----61-----20-----48  
 |64 12 37 |17  
 63 11 36 24 13 57 18 38

| 2 55---54-33---27-16---+47--26  
 4 56 | 30 42 51 7 45 25 |  
 |53 9 35 31 14 50 |28 40  
 62--41-|--36--21---13--60---19 8 |  
 11 32 29 44 52 5 38 49  
 23 | 43 6 58 |  
 34-----22-----59-----15

|10 54---29-33---52-16---+39-27  
 4 22 | 31 42 50 7 45 59 |  
 |53 35 34 23 15 58 |28 14  
 62--41-|--2--21---47--60---19 8 |  
 3 32 55 44 26 5 46 49  
 56 | 43 6 25 |  
 9-----30-----51-----40

16413/

1-----61-----20-----48  
 |64 10 39 |17  
 63 12 3 24 46 57 18 37  
 | 2 53---56-35---25-14---+47--28  
 4 54 | 32 42 49 7 45 27 |  
 |55 11 33 29 16 52 |26 38  
 62--41-|--34--21---15--60---19 8 |  
 9 30 31 44 50 5 40 51  
 23 | 43 6 58 |  
 36-----22-----59-----13

26896/

1-----56-----18-----55  
 |64 25 8 |33  
 63 17 10 40 23 57 34 16  
 |41 48---32--9---42-47---+15-26  
 4 24 | 53 49 44 7 29 50 |  
 |22 2 35 39 21 58 |52 31  
 62--43-|--11-14---45--60---12 13 |  
 3 61 38 28 59 5 30 36  
 46 | 27 6 51 |  
 19-----54-----20-----37

28237/

1-----61-----12-----56  
 |64 20 37 | 9  
 63 18 3 16 54 57 10 39  
 | 2 47---46-33---27-24---+55-26  
 4 48 | 30 50 43 7 53 25 |  
 |45 17 35 31 22 42 |28 40  
 62--49-|--36--13---21--60---11 8 |  
 19 32 29 52 44 5 38 41  
 15 | 51 6 58 |  
 34-----14-----59-----23

32112/

1-----61-----12-----56  
 |64 20 37 | 9  
 63 19 2 16 47 57 18 38  
 |10 46---55-33---26-24---+39-27  
 4 40 | 31 58 42 7 53 25 |  
 |45 17 34 15 23 50 |28 48  
 62--49-|--36--13---29--60---3 8 |  
 11 32 21 52 44 5 54 41  
 22 | 43 6 59 |  
 35-----30-----51-----14

35994/

1-----61-----12-----56  
 |64 34 23 | 9  
 63 20 2 16 55 57 10 37  
 |17 45---48-19---25-38---+40-28  
 4 47 | 32 50 41 7 53 26 |  
 |31 3 33 30 24 43 |42 54  
 62--49-|--35--13---22--60---11 8 |  
 18 46 15 52 58 5 39 27  
 14 | 51 6 59 |  
 36-----29-----44-----21

46262/

1-----55-----50-----24  
 |64 17 8 |41  
 63 25 26 16 31 57 10 32  
 | 9 40---39-42---34-15---48-33  
 4 56 | 37 18 36 7 53 49 |  
 |54 2 28 47 29 58 |19 23  
 62--11-|--12-45---13--60---43 14 |  
 3 61 46 20 59 5 22 44  
 38 | 51 6 35 |  
 27-----21-----52-----30

50167/

1-----61-----44-----24  
 |64 10 15 |41  
 63 26 3 16 22 57 42 31  
 | 2 39---56-43---49-14---+23-34  
 4 40 | 38 50 35 7 53 33 |  
 |55 25 27 21 30 52 |18 32  
 62--17-|--28-45---29--60---11 8 |  
 9 54 37 20 36 5 48 51  
 47 | 19 6 58 |  
 12-----46-----59-----13

60206/

1-----61-----44-----24  
 |64 11 14 |41  
 63 27 28 16 29 57 10 30  
 | 9 38---37-42---36-15---48-35  
 4 46 | 39 18 34 7 53 59 |  
 |54 12 26 47 31 58 |19 13  
 62--17-|--2-45---23--60---43 8 |  
 3 55 56 20 49 5 22 50  
 40 | 51 6 33 |  
 25-----21-----52-----32

70218/

1-----61-----12-----56  
 |64 36 21 | 9  
 63 28 3 22 54 57 10 23  
 | 2 37---30-11---43-40---+55-42  
 4 38 | 46 44 33 7 47 41 |  
 |29 27 19 53 32 26 |50 24  
 62--49-|--20--13---31--60---17 8 |  
 35 48 45 52 34 5 16 25  
 15 | 51 6 58 |  
 18-----14-----59-----39

83761/

1-----56-----25-----48  
 |64 9 8 |49  
 63 33 10 24 54 57 3 16  
 | 2 32---55-41---43-40---+30-17  
 4 46 | 53 27 28 7 45 50 |  
 |29 19 44 38 37 26 |20 47  
 62--36-|--11-21---23--60---34 13 |  
 35 61 22 12 42 5 31 52  
 15 | 58 6 51 |  
 18-----39-----59-----14

.....

\*\* List of Fundamental Solutions with n1=1 & n43=62 \*\*

1/

1-----56-----25-----48  
 |64 9 40 |17  
 60 2 37 31 20 53 13 44  
 | 5 63---28-34---45-12---+52--21  
 10 43 | 23 30 61 8 36 49 |  
 |55 22 42 35 4 57 |29 16  
 59--38-|--14-19---24--62---33 11 |  
 6 27 51 46 41 3 32 54  
 47 | 50 7 26 |  
 18-----15-----58-----39

1854/

1-----59-----22-----48  
 |64 6 43 |17  
 63 5 14 28 35 53 18 44  
 | 2 60---51-37---30-12---+47--21  
 10 52 | 23 38 58 11 39 29 |  
 |55 13 42 27 7 54 |26 36  
 56--41-|--34-19---15--62---25 8 |  
 9 24 31 46 50 3 40 57  
 32 | 45 4 49 |  
 33-----20-----61-----16

3210/

1-----60-----18-----51  
 |64 5 47 |14  
 63 6 13 44 24 53 30 27  
 | 2 59---52-21---41-12---+35--38  
 10 36 | 40 37 57 11 23 46 |  
 |55 29 25 28 8 54 |42 19  
 56--45-|--17-16---31--62---26 7 |  
 9 20 48 49 34 3 39 58  
 43 | 33 4 50 |  
 22-----32-----61-----15

7988/

1-----59-----22-----48  
 |64 14 35 |17  
 63 9 34 24 15 57 18 40  
 | 2 56---23-33---58-16---+47--25  
 6 28 | 27 42 54 7 43 53 |  
 |51 37 38 31 11 50 |30 12  
 60--41-|--10-19---39--62---21 8 |  
 13 32 55 46 26 3 36 49  
 52 | 45 4 29 |  
 5-----20-----61-----44

9450/

1-----59-----18-----52  
 |64 17 23 |26  
 63 10 5 24 48 57 14 39  
 | 2 55---47-30---41-32---+40--13  
 6 56 | 28 42 53 7 43 25 |  
 |35 9 50 36 12 34 |33 51  
 60--45-|--38-15---11--62---21 8 |  
 29 44 16 37 54 3 31 46  
 19 | 49 4 58 |  
 22-----27-----61-----20

14196/

1-----59-----22-----48  
 |64 14 35 |17  
 63 13 38 24 11 57 18 36  
 |10 52---27-33---54-16---+39--29  
 6 20 | 31 42 50 7 43 61 |  
 |51 37 34 23 15 58 |30 12  
 60--41-|--2-19---47--62---21 8 |  
 5 32 55 46 26 3 44 49  
 56 | 45 4 25 |  
 9-----28-----53-----40

18471/

1-----59-----22-----48  
 |64 10 39 |17  
 63 14 5 24 44 57 18 35  
 | 2 51---56-37---25-12---+47--30  
 6 52 | 32 42 49 7 43 29 |  
 |55 13 33 27 16 54 |26 36  
 60--41-|--34-19---15--62---21 8 |  
 9 28 31 46 50 3 40 53  
 23 | 45 4 58 |  
 38-----20-----61-----11

29240/

1-----59-----38-----32  
 |64 22 11 |33  
 63 17 2 40 31 57 34 16  
 |10 48---55-9---42--24---+23--49  
 6 52 | 51 26 46 7 27 45 |  
 |43 5 14 47 19 50 |54 28  
 60--25-|--18-35---15--62---37 8 |  
 13 56 39 30 58 3 20 41  
 36 | 29 4 61 |  
 21-----44-----53-----12

30891/

1-----59-----14-----56  
 |64 22 35 | 9  
 63 18 5 16 52 57 10 39  
 | 2 47---44-33---29-24---+55--26  
 6 48 | 28 50 45 7 51 25 |  
 |43 17 37 31 20 42 |30 40  
 60--49-|--38-11---19--62---13 8 |  
 21 32 27 54 46 3 36 41  
 15 | 53 4 58 |  
 34-----12-----61-----23

35497/

1-----59-----14-----56  
 |64 22 35 | 9  
 63 21 2 16 47 57 18 36  
 |10 44---55-33---26-24---+39--29  
 6 40 | 31 58 42 7 51 25 |  
 |43 17 34 15 23 50 |30 48  
 60--49-|--38-11---27--62---5 8 |  
 13 32 19 54 46 3 52 41  
 20 | 45 4 61 |  
 37-----28-----53-----12

39528/

1-----59-----14-----56  
 |64 18 39 | 9  
 63 22 5 16 52 57 10 35  
 | 2 43---48-37---25-20---+55--30  
 6 44 | 32 50 41 7 51 29 |

49350/

1-----59-----34-----36  
 |64 5 16 |45  
 63 25 10 20 43 57 14 28  
 |13 40---35-46---26-23---+56--21  
 6 52 | 39 30 38 7 47 41 |

```

|47 21 33 27 24 46 |26 36
60--49-|--34--11---23--62---13 8 |
17 28 | 31 54 42 3 40 45
15 | 53 4 58 |
38-----12-----61-----19

```

```

|44 2 42 55 27 54 |17 19
60--29-|--22--31---15--62---33 8 |
9 51 | 48 18 61 3 12 58
24 | 49 4 53 |
37-----11-----50-----32

```

53372/

```

1-----59-----18-----52
|64 42 11 |13
63 26 5 12 48 57 14 35
| 2 39---24--17---53--44---+51--30
6 40 | 32 54 37 7 55 29 |
|23 25 33 47 28 22 |46 36
60--45-|--34--15---27--62---9 8 |
41 56 31 50 38 3 20 21
19 | 49 4 58 |
10-----16-----61-----43

```

63937/

```

1-----59-----46-----24
|64 13 12 |41
63 29 30 16 27 57 10 28
| 9 36---35--42---38--15---+48--37
6 44 | 39 18 34 7 51 61 |
|52 14 26 47 31 58 |21 11
60--17-|--2--43---23--62---45 8 |
5 55 56 22 49 3 20 50
40 | 53 4 33 |
25-----19-----54-----32

```

.....

\*\* List of Fundamental Solutions with n1=1 & n43=63 \*\*

1/

```

1-----56-----19-----54
|64 9 40 |17
60 3 25 42 14 53 31 32
| 5 62---46--23---57--18---+22--27
11 30 | 34 43 61 8 24 49 |
|48 35 37 16 4 51 |41 28
58--45-|--15--12---36--63---21 10 |
13 26 38 47 29 2 50 55
52 | 33 6 39 |
7-----44-----59-----20

```

2715/

```

1-----56-----25-----48
|64 35 14 |17
62 5 15 26 34 55 19 44
| 3 60---24--13---57--36---+46--21
9 54 | 22 41 59 8 40 27 |
|30 11 43 50 6 31 |51 38
58--39-|--37--18---12--63---23 10 |
33 52 28 47 53 2 16 29
32 | 45 4 49 |
7-----20-----61-----42

```

5295/

```

1-----60-----19-----50
|64 5 46 |15
62 7 13 44 24 53 31 26
| 3 58---52--21---41--12---+34--39
11 36 | 40 37 57 10 22 47 |
|54 29 25 28 8 55 |43 18
56--45-|--17--16---30--63---27 6 |
9 20 48 49 35 2 38 59
42 | 33 4 51 |
23-----32-----61-----14

```

11263/

```

1-----58-----23-----48
|64 37 12 |17
62 9 13 24 36 57 19 40
| 3 56---22--11---59--38---+46--25
7 50 | 26 43 55 6 42 31 |
|28 15 39 52 10 29 |53 34
60--41-|--33--18---16--63---21 8 |
35 54 32 47 49 2 14 27
30 | 45 4 51 |
5-----20-----61-----44

```

13726/

```

1-----58-----19-----52
|64 17 22 |27
62 11 5 24 48 57 15 38
| 3 54---46--31---41--32---+40--13
7 56 | 28 43 53 6 42 25 |
|34 9 51 36 12 35 |33 50
60--45-|--39--14---10--63---21 8 |
29 44 16 37 55 2 30 47
18 | 49 4 59 |
23-----26-----61-----20

```

19268/

```

1-----58-----17-----54
|64 19 24 |23
62 13 39 22 26 57 3 38
|11 52---46--31---55--32---+18--15
7 30 | 28 35 51 6 44 59 |
|34 27 49 10 14 43 |33 50
60--47-|--5--12---36--63---29 8 |
21 42 16 41 37 2 56 45
40 | 61 4 25 |
9-----48-----53-----20

```

24658/

```

1-----58-----23-----48
|64 11 38 |17
62 15 5 24 44 57 19 34
| 3 50---56--37---25--12---+46--31
7 52 | 32 43 49 6 42 29 |
|54 13 33 26 16 55 |27 36
60--41-|--35--18---14--63---21 8 |
9 28 30 47 51 2 40 53
22 | 45 4 59 |
39-----20-----61-----10

```

37428/

```

1-----58-----25-----46
|64 11 12 |43
62 17 35 24 14 57 19 32
|31 48---38--37---51--36---+10--9
7 22 | 34 41 47 6 42 61 |
|30 15 55 16 18 59 |27 40
60--39-|--3--20---44--63---23 8 |
5 54 26 21 49 2 50 53
52 | 45 4 29 |
13-----56-----33-----28

```

40201/

```

1-----58-----15-----56
|64      23      34      | 9
62 19    5 16    52 57    11 38
| 3 46---44-33---29-24---+54-27
7 48 | 28 51    45 6    50 25 |
|42 17 | 37 30    20 43 |31 40
60--49-|-39-10---18-63---13 8 |
21 32  26 55   47 2    36 41
14 |      53      4      59 |
35-----12-----61-----22

```

46086/

```

1-----58-----15-----56
|64      23      34      | 9
62 21    3 16    46 57    19 36
|11 44---54-33---27-24---+38-29
7 40 | 30 59    43 6    50 25 |
|42 17 | 35 14    22 51 |31 48
60--49-|-39-10---26-63---5 8 |
13 32  18 55   47 2    52 41
20 |      45      4      61 |
37-----28-----53-----12

```

51506/

```

1-----58-----15-----56
|64      35      22      | 9
62 23    3 16    54 57    11 34
|17 42---48-21---25-36---+40-31
7 46 | 32 51    41 6    50 27 |
|30 5   33 28    24 45 |43 52
60--49-|-37-10---20-63---13 8 |
19 44  14 55   59 2    38 29
12 |      53      4      61 |
39-----26-----47-----18

```

63877/

```

1-----58-----35-----36
|64      5      16      |45
62 25    11 20    42 57    15 28
|13 40---34-47---27-22---+56-21
7 52 | 38 31    39 6    46 41 |
|44 3   43 54    26 55 |17 18
60--29-|-23-30---14-63---33 8 |
9 50   48 19   61 2    12 59
24 |      49      4      53 |
37-----10-----51-----32

```

69519/

```

1-----58-----19-----52
|64      43      10      |13
62 27    5 12    48 57    15 34
| 3 38---24-17---53-44---+50-31
7 40 | 32 55    37 6    54 29 |
|22 25  33 46    28 23 |47 36
60--45-|-35-14---26-63---9 8 |
41 56  30 51   39 2    20 21
18 |      49      4      59 |
11-----16-----61-----42

```

81654/

```

1-----58-----47-----24
|64      13      12      |41
62 29    31 16    26 57    11 28
| 9 36---34-43---39-14---+48-37
7 44 | 38 19    35 6    50 61 |
|52 15  27 46    30 59 |21 10
60--17-|-3-42---22-63---45 8 |
5 54   56 23   49 2    20 51
40 |      53      4      33 |
25-----18-----55-----32

```

94633/

```

1-----58-----15-----56
|64      9      18      |39
62 31    5 20    52 57    11 22
| 3 34---54-43---45-40---+28-13
7 36 | 44 47    33 6    46 41 |
|26 29  51 24    32 27 |21 50
60--49-|-23-10---30-63---17 8 |
37 48  16 25   35 2    42 55
14 |      53      4      59 |
19-----38-----61-----12

```

111564/

```

1-----58-----15-----56
|64      17      40      | 9
62 33    19 16    46 57    3 24
| 5 32---54-39---27-18---+44-41
7 28 | 42 43    31 6    50 53 |
|48 35  23 14    34 51 |25 30
60--49-|-11-10---38-63---21 8 |
13 26  36 55   29 2    52 47
20 |      61      4      45 |
37-----22-----59-----12

```

\*\* List of Fundamental Solutions with n1=1 & n43=64 \*\*

1/

```

1-----58-----14-----57
|63      20      21      |26
60 4    16 33    37 55    17 38
|12 62---23-19---52-40---+43-9
10 47 | 27 41    61 6    32 36 |
|25 11  56 50    3 35 |46 34
59--51-|-29-8---18-64---24 7 |
30 44  31 39   54 2    15 45
28 |      48      5      49 |
13-----22-----53-----42

```

36523/

```

1-----57-----16-----56
|63      5      42      |20
59 4    21 32    35 53    15 41
|10 62---48-36---34-19---+38-13
12 47 | 24 39    61 7    33 37 |
|46 14  52 22    3 54 |29 40
58--49-|-28-9---18-64---26 8 |
11 23  25 45   51 2    43 60
30 |      50      6      44 |
31-----27-----55-----17

```

64754/

```

1-----56-----18-----55
|63      32      20      |15
58 4    28 31    39 51    5 44
|13 62---27-11---48-41---+42-16

```

90289/

```

1-----55-----20-----54
|63      18      22      |27
57 4    14 40    44 49    15 37
| 7 62---35-17---46-39---+42-12

```

14 53 | 21 29 61 8 34 40 |  
 |24 6 49 46 3 35 |54 43  
 57--47-|--25--10--12--64--36 9 |  
 30 45 | 22 50 59 2 19 33  
 26 | 60 7 37 |  
 17-----23-----52-----38

16 60 | 28 29 61 9 25 32 |  
 |26 6 53 52 3 31 |48 41  
 56--45-|--33--11--5--64--36 10 |  
 34 43 | 24 38 59 2 13 47  
 21 | 50 8 51 |  
 19-----23-----58-----30

110678/

1-----54-----22-----53  
 |63 29 33 | 5  
 56 4 19 21 30 47 25 58  
 | 6 62--23-26--49-28--52-14  
 18 48 | 7 57 61 10 44 15 |  
 |37 20 51 31 3 41 |39 38  
 55--43-|--50--12--17--64--8 11 |  
 24 32 27 60 45 2 34 36  
 35 | 40 9 46 |  
 16-----13-----59-----42

126454/

1-----53-----24-----52  
 |63 27 26 |14  
 55 4 25 32 28 45 22 49  
 | 6 62--7-18--57-35--60-15  
 20 48 | 16 42 61 11 33 29 |  
 |30 21 50 56 3 34 |47 19  
 54--41-|--36--13--17--64--23 12 |  
 31 39 46 51 44 2 9 38  
 37 | 43 10 40 |  
 8-----5-----59-----58

137519/

1-----52-----26-----51  
 |63 25 34 | 8  
 54 4 16 33 37 43 23 50  
 |10 62--56-20--35-27--29-21  
 22 59 | 15 41 61 12 32 18 |  
 |38 7 44 17 3 46 |45 60  
 53--39-|--47--14--6--64--24 13 |  
 19 31 5 57 58 2 48 40  
 28 | 42 11 49 |  
 30-----36-----55-----9

145908/

1-----51-----28-----50  
 |63 11 22 |34  
 53 4 35 58 25 41 17 27  
 | 8 62--44-10--60-39--18-19  
 24 49 | 38 9 61 13 7 59 |  
 |26 20 46 42 3 32 |55 36  
 52--37-|--6-15--16--64--56 14 |  
 33 43 29 31 45 2 23 54  
 40 | 48 12 30 |  
 5-----47-----57-----21

152392/

1-----50-----30-----49  
 |63 27 32 | 8  
 52 4 53 31 20 39 5 56  
 |25 62--44-22--37-29--24-17  
 26 46 | 9 23 61 14 34 47 |  
 |36 7 48 10 3 59 |43 54  
 51--35-|--18--16--19--64--42 15 |  
 6 33 11 57 58 2 55 38  
 45 | 60 13 12 |  
 28-----41-----40-----21

156265/

1-----49-----34-----46  
 |63 40 21 | 6  
 51 4 20 33 12 37 47 56  
 |30 62--11-8--48-38--41-22  
 28 42 | 9 60 61 15 32 13 |  
 |27 7 43 39 3 55 |57 29  
 50--31-|--52--19--23--64--5 16 |  
 10 44 36 59 58 2 26 25  
 53 | 18 14 45 |  
 17-----24-----35-----54

159000/

1-----48-----38-----43  
 |63 7 39 |21  
 50 4 46 34 11 35 23 57  
 | 6 62--12-41--52-18--60--9  
 30 45 | 8 32 61 16 31 37 |  
 |47 29 56 40 3 51 |24 10  
 49--27-|--28--22--20--64--33 17 |  
 14 26 55 44 36 2 25 58  
 54 | 42 15 19 |  
 13-----5-----59-----53

160565/

1-----47-----36-----46  
 |63 34 24 | 9  
 49 4 50 39 8 33 23 54  
 |27 62--45-10--52-37--6-21  
 32 40 | 11 30 61 17 26 43 |  
 |28 14 44 5 3 53 |55 58  
 48--29-|--22--19--25--64--35 18 |  
 12 41 7 56 51 2 60 31  
 57 | 42 16 15 |  
 13-----59-----38-----20

161379/

1-----46-----44-----39  
 |63 38 20 | 9  
 48 4 59 41 13 31 10 54  
 |22 62--23-5--49-35--36-28  
 34 53 | 11 8 61 18 24 51 |  
 |30 7 37 40 3 50 |60 33  
 47--21-|--14--26--12--64--57 19 |  
 15 45 32 56 58 2 25 27  
 52 | 55 17 6 |  
 16-----29-----43-----42

162227/

1-----44-----51-----34  
 |63 6 33 |28  
 47 4 22 55 5 29 56 42  
 |11 62--58-25--49-17--12-26  
 36 52 | 23 35 61 19 10 24 |  
 |48 20 39 15 3 57 |40 38  
 46--14-|--41--31--13--64--30 21 |  
 8 32 27 37 45 2 50 59  
 60 | 9 18 43 |  
 16-----53-----54-----7

.....

In autumn 2005 I tried again to count up the last type: Complete Pan-magic Cubes of Order 4 through, but only all of 'Complete Euler Type' by my newest method.

I could successfully have known the total count of standard solutions with  $n_1=1$ : 133856760. How big the count was! But I was more surprised with the fact itself I could count them all through during my life. I really had no idea about that.

But, I could not yet count up the total count of all Complete Pan-magic Cubes including 'Non-Euler' type. I can not yet imagine how big it might be, either.

(Originally Written in English on Sep.10, 2001; Revised on Feb.27, 2007;  
Newest Revision on Jul.13, 2010 with MacOSX & Xcode3.1 by Kanji Setsuda)

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## Section 2: Self-Complementary Magic Cubes of Order 4

### #0. Basic Form and Basic Conditions

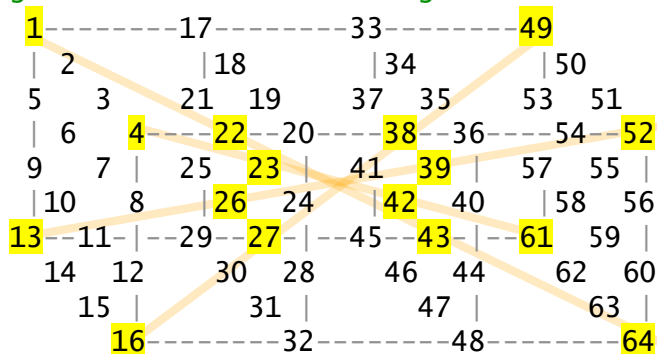
Let's study about the Self-complementary type of Magic Cube 4x4x4 next.

I prepared such the Basic Form and Basic Conditions as follows for our discussion.

\*\* Basic Conditions for Rows, Columns and Pillars:  $K=130$  \*\*

$n_1+n_2+n_3+n_4=K$ ;	$n_1+n_5+n_9+n_{13}=K$ ;	$n_1+n_{17}+n_{33}+n_{49}=K$ ;
$n_5+n_6+n_7+n_8=K$ ;	$n_2+n_6+n_{10}+n_{14}=K$ ;	$n_2+n_{18}+n_{34}+n_{50}=K$ ;
$n_9+n_{10}+n_{11}+n_{12}=K$ ;	$n_3+n_7+n_{11}+n_{15}=K$ ;	$n_3+n_{19}+n_{35}+n_{51}=K$ ;
$n_{13}+n_{14}+n_{15}+n_{16}=K$ ;	$n_4+n_8+n_{12}+n_{16}=K$ ;	$n_4+n_{20}+n_{36}+n_{52}=K$ ;
$n_{17}+n_{18}+n_{19}+n_{20}=K$ ;	$n_{17}+n_{21}+n_{25}+n_{29}=K$ ;	$n_5+n_{21}+n_{37}+n_{53}=K$ ;
$n_{21}+n_{22}+n_{23}+n_{24}=K$ ;	$n_{18}+n_{22}+n_{26}+n_{30}=K$ ;	$n_6+n_{22}+n_{38}+n_{54}=K$ ;
$n_{25}+n_{26}+n_{27}+n_{28}=K$ ;	$n_{19}+n_{23}+n_{27}+n_{31}=K$ ;	$n_7+n_{23}+n_{39}+n_{55}=K$ ;
$n_{29}+n_{30}+n_{31}+n_{32}=K$ ;	$n_{20}+n_{24}+n_{28}+n_{32}=K$ ;	$n_8+n_{24}+n_{40}+n_{56}=K$ ;
$n_{33}+n_{34}+n_{35}+n_{36}=K$ ;	$n_{33}+n_{37}+n_{41}+n_{45}=K$ ;	$n_9+n_{25}+n_{41}+n_{57}=K$ ;
$n_{37}+n_{38}+n_{39}+n_{40}=K$ ;	$n_{34}+n_{38}+n_{42}+n_{46}=K$ ;	$n_{10}+n_{26}+n_{42}+n_{58}=K$ ;
$n_{41}+n_{42}+n_{43}+n_{44}=K$ ;	$n_{35}+n_{39}+n_{43}+n_{47}=K$ ;	$n_{11}+n_{27}+n_{43}+n_{59}=K$ ;
$n_{45}+n_{46}+n_{47}+n_{48}=K$ ;	$n_{36}+n_{40}+n_{44}+n_{48}=K$ ;	$n_{12}+n_{28}+n_{44}+n_{60}=K$ ;
$n_{49}+n_{50}+n_{51}+n_{52}=K$ ;	$n_{49}+n_{53}+n_{57}+n_{61}=K$ ;	$n_{13}+n_{29}+n_{45}+n_{61}=K$ ;
$n_{53}+n_{54}+n_{55}+n_{56}=K$ ;	$n_{50}+n_{54}+n_{58}+n_{62}=K$ ;	$n_{14}+n_{30}+n_{46}+n_{62}=K$ ;
$n_{57}+n_{58}+n_{59}+n_{60}=K$ ;	$n_{51}+n_{55}+n_{59}+n_{63}=K$ ;	$n_{15}+n_{31}+n_{47}+n_{63}=K$ ;
$n_{61}+n_{62}+n_{63}+n_{64}=K$ ;	$n_{52}+n_{56}+n_{60}+n_{64}=K$ ;	$n_{16}+n_{32}+n_{48}+n_{64}=K$ ;

[Figure 1: Basic Form of Magic Cube 4x4x4]



[Symmetrical Self-Complementary Pairs:]

$n_1+n_{64}=n_2+n_{63}=n_3+n_{62}=n_4+n_{61}=n_5+n_{60}=n_6+n_{59}=n_7+n_{58}=n_8+n_{57}$   
 $=n_9+n_{56}=n_{10}+n_{55}=n_{11}+n_{54}=n_{12}+n_{53}=n_{13}+n_{52}=n_{14}+n_{51}=n_{15}+n_{50}=n_{16}+n_{49}$   
 $=n_{17}+n_{48}=n_{18}+n_{47}=n_{19}+n_{46}=n_{20}+n_{45}=n_{21}+n_{44}=n_{22}+n_{43}=n_{23}+n_{42}=n_{24}+n_{41}$   
 $=n_{25}+n_{40}=n_{26}+n_{39}=n_{27}+n_{38}=n_{28}+n_{37}=n_{29}+n_{36}=n_{30}+n_{35}=n_{31}+n_{34}=n_{32}+n_{33}=C=65$

Each of the Complementary Pairs of 65 above should be placed symmetrically with

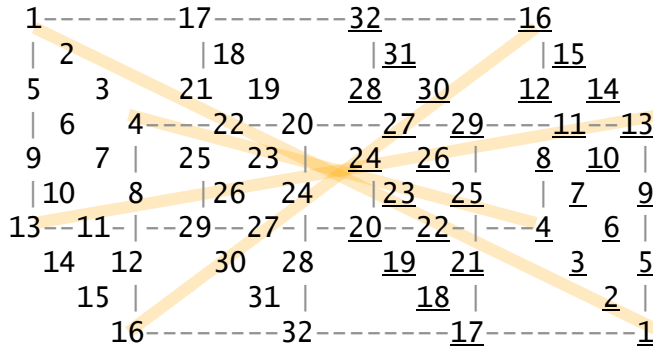
respect to the geometric center of our cube.

The next expression was invented to explain 'complementary relations' explicitly so that you could easily find where any Complementary Pair exists.

**New Notation for Self-Complementary Pairs:**

$$\begin{aligned}
 n64=C-n1=\underline{n1}; & \quad n63=C-n2=\underline{n2}; \quad n62=C-n3=\underline{n3}; \quad n61=C-n4=\underline{n4}; \\
 n60=C-n5=\underline{n5}; & \quad n59=C-n6=\underline{n6}; \quad n58=C-n7=\underline{n7}; \quad n57=C-n8=\underline{n8}; \\
 & \quad \dots\dots\dots \\
 n36=C-n29=\underline{n29}; & \quad n35=C-n30=\underline{n30}; \quad n34=C-n31=\underline{n31}; \quad n33=C-n32=\underline{n32};
 \end{aligned}$$

[Figure 2: Basic Diagram according to the New Notation]



$$\begin{aligned}
 n1+n1=n2+n2=n3+n3=n4+n4=n5+n5=n6+n6=n7+n7=n8+n8 \\
 =n9+n9=n10+n10=\dots=n29+n29=n30+n30=n31+n31=n32+n32=C=65
 \end{aligned}$$

Look at the diagram above, and see if all Complementary Pairs of 65 are located symmetrically with respect to the geometric center of the cube.

$$\begin{aligned}
 n1+n22+n43+n64=(n1+n64)+(n22+n43)=(n1+n\underline{n1})+(n22+n\underline{n22})=C+C=K; \\
 n4+n23+n42+n61=(n4+n61)+(n23+n42)=(n4+n\underline{n4})+(n23+n\underline{n23})=C+C=K; \\
 n13+n26+n39+n52=(n13+n52)+(n26+n39)=(n13+n\underline{n13})+(n26+n\underline{n26})=C+C=K; \\
 n16+n27+n38+n49=(n16+n49)+(n27+n38)=(n16+n\underline{n16})+(n27+n\underline{n27})=C+C=K;
 \end{aligned}$$

Just as you see, each of the 4 Primary Triagonals above necessarily adds up to the magic constant K(=130), even if you did not assume them for the first definitions.

**#1. What do the Solutions actually look like?**

Take those basic forms and equations at the definition stage, and assume the 'Standard Form' with n1=1 on the top left back and add some inequalities such as:

$$n2 > n5 > n17; \quad n2 > n3; \quad n5 > n9; \quad n17 > n33$$

It is because you can exchange the 2nd and 3rd planes with each other in all three directions, and you can identify the patterns of which the 2nd-3rd planes were exchanged or not. I want to show you only the 'fundamental solutions' and prevent us from double counting of any simple reflections or rotations.

**\*\* Examples of Self-Complementary Magic Cubes of Order 4: \*\***

<p>1/          1-----58-----57-----14           63        21        18         28          60 62    38 22    17 13    15 33           26  4----16--29----35--42----+53--55          59 19   11 31   20 24   40 56              9 25   41 45   34 54   46 6            10--12- --23--30----36--49----61 39              32 50   52 48   43 27   3 5            37      47        44        2              51----- 8----- 7-----64</p>	<p>27581/          1-----60-----56-----13           63        17        27         23          59 62    11 8     39 19    21 41           25  4----36--45----14--28----+55--53          58 16   22 33   15 34   35 47             18 30   31 50   32 43   49 7            12--10- --37--51----20--29----61 40              24 44   46 26   57 54   3 6            42      38        48        2              52----- 9----- 5-----64</p>
--	--



49922/

1-----60-----53-----16  
 |63        25        20        |22  
 58 62    32 18    29 9     11 41  
 |30 4----15-27---26-48---+59-51  
 57 19 | 21 28    10 31    42 52 |  
 |13 23    34 55    37 44    |46 8  
 14--6-|--17-39---38-50---61 35 |  
 24 54    56 36    47 33    3 7  
 43 |        45        40        2 |  
 49-----12-----5-----64

71495/

1-----60-----52-----17  
 |63        18        10        |39  
 57 62    20 28    30 15    23 25  
 | 6 4----19-24---51-53---+54-49  
 56 31 | 38 33    7 22    29 44 |  
 |21 36    43 58    32 27    |34 9  
 16--11-|--12-14---41-46---61 59 |  
 40 42    50 35    37 45    3 8  
 26 |        55        47        2 |  
 48-----13-----5-----64

89941/

1-----60-----50-----19  
 |63        11        32        |24  
 56 62    17 22    45 6     12 40  
 |34 4----16-37---29-42---+51-47  
 55 13 | 30 39    7 21    38 57 |  
 | 8 27    44 58    26 35    |52 10  
 18--14-|--23-36---28-49---61 31 |  
 25 53    59 20    43 48    3 9  
 41 |        33        54        2 |  
 46-----15-----5-----64

106417/

1-----60-----47-----22  
 |63        6        17        |44  
 55 62    35 41    26 8     14 19  
 | 9 4----29-23---40-58---+52-45  
 54 34 | 28 16    15 27    33 53 |  
 |12 32    38 50    49 37    |31 11  
 20--13-|--7-25---42-36---61 56 |  
 46 51    57 39    24 30    3 10  
 21 |        48        59        2 |  
 43-----18-----5-----64

119377/

1-----60-----44-----25  
 |63        6        13        |48  
 54 62    42 38    19 16    15 14  
 | 7 4----41-26---35-57---+47-43  
 53 33 | 20 10    28 31    29 56 |  
 | 9 36    34 37    55 45    |32 12  
 22--18-|--8-30---39-24---61 58 |  
 51 50    49 46    27 23    3 11  
 17 |        52        59        2 |  
 40-----21-----5-----64

130294/

1-----60-----43-----26  
 |63        6        21        |40  
 53 62    19 27    50 18    8 23  
 |11 4----45-37---16-48---+58-41  
 52 36 | 34 10    9 33    35 51 |  
 |14 30    32 56    55 31    |29 13  
 24--7-|--17-49---28-20---61 54 |  
 42 57    47 15    38 46    3 12  
 25 |        44        59        2 |  
 39-----22-----5-----64

139835/

1-----60-----41-----28  
 |63        19        23        |25  
 52 62    50 21    18 9     10 38  
 |33 4----6-30---43-57---+48-39  
 51 11 | 12 45    36 16    31 58 |  
 | 7 34    49 29    20 53    |54 14  
 26--17-|--8-22---35-59---61 32 |  
 27 55    56 47    44 15    3 13  
 40 |        42        46        2 |  
 37-----24-----5-----64

146268/

1-----60-----35-----34  
 |63        6        29        |32  
 51 62    9 19    58 22    12 27  
 |13 4----55-45---8-44---+54-37  
 50 24 | 40 18    17 39    23 49 |  
 |16 42    26 48    47 25    |41 15  
 28--11-|--21-57---20-10---61 52 |  
 38 53    43 7     46 56    3 14  
 33 |        36        59        2 |  
 31-----30-----5-----64

151646/

1-----60-----37-----32  
 |63        7        41        |19  
 50 62    12 6     42 18    26 44  
 |17 4----51-57---10-34---+52-35  
 49 9 | 27 45    43 40    11 36 |  
 |29 54    25 22    20 38    |56 16  
 30--13-|--31-55---8-14---61 48 |  
 21 39    47 23    59 53    3 15  
 46 |        24        58        2 |  
 33-----28-----5-----64

155814/

1-----60-----28-----41  
 |63        7        34        |26  
 49 62    52 27    23 11    6 30  
 |18 4----44-36---12-57---+56-33  
 48 20 | 10 19    50 40    22 51 |  
 |14 43    25 15    46 55    |45 17  
 32--9-|--8-53---29-21---61 47 |  
 35 59    54 42    38 13    3 16  
 39 |        31        58        2 |  
 24-----37-----5-----64

159094/

1-----60-----33-----36  
 |63        23        16        |28  
 48 62    10 9     58 24    14 35  
 |11 4----45-38---15-57---+59-31  
 47 25 | 52 22    12 44    19 39 |  
 |26 46    21 53    43 13    |40 18  
 34--6-|--8-50---27-20---61 54 |

160735/

1-----60-----32-----37  
 |63        20        23        |24  
 47 62    50 7     27 21    6 40  
 |34 4----13-43---35-54---+48-29  
 46 10 | 9 51    49 12    26 57 |  
 | 8 39    53 16    14 56    |55 19  
 36--17-|--11-30---22-52---61 31 |

30 51 41 7 56 55 3 17  
 37 | 49 42 2 |  
 29-----32-----5-----64

25 59 44 38 58 15 3 18  
 41 | 42 45 2 |  
 28-----33-----5-----64

161865/

1-----60-----17-----52  
 |63 22 12 |33  
 46 62 35 8 39 42 10 18  
 |11 4---51--40---24--59---+44--27  
 45 15 | 29 28 49 31 7 56 |  
 | 9 58 34 16 37 36 |50 20  
 38--21--| 6--41---25--14---61 54 |  
 47 55 23 26 57 30 3 19  
 32 | 53 43 2 |  
 13-----48-----5-----64

162449/

1-----59-----22-----48  
 |63 11 38 |18  
 45 62 33 10 36 19 16 39  
 |13 4---31--50---30--51---+56--25  
 44 12 | 24 58 57 23 5 37 |  
 |28 60 42 8 7 41 |53 21  
 40--9--| 14--35---15--34---61 52 |  
 26 49 46 29 55 32 3 20  
 47 | 27 54 2 |  
 17-----43-----6-----64

162859/

1-----59-----24-----46  
 |63 7 34 |26  
 44 62 15 8 60 25 11 35  
 |17 4---51--56---10--47---+52--23  
 43 16 | 38 36 37 33 12 45 |  
 |20 53 32 28 29 27 |49 22  
 42--13--| 18--55---9--14---61 48 |  
 30 54 40 5 57 50 3 21  
 39 | 31 58 2 |  
 19-----41-----6-----64

163175/

1-----58-----25-----46  
 |63 10 12 |45  
 43 62 24 13 57 37 6 18  
 | 5 4---38--49---36--56---+51--21  
 42 34 | 39 35 32 11 17 50 |  
 |15 48 54 33 30 26 |31 23  
 44--14--| 9--29---16--27---61 60 |  
 47 59 28 8 52 41 3 22  
 20 | 53 55 2 |  
 19-----40-----7-----64

163519/

1-----58-----22-----49  
 |63 14 9 |44  
 42 62 34 10 39 40 15 18  
 | 8 4---54--48---30--59---+38--19  
 41 20 | 32 29 52 28 5 53 |  
 |12 60 37 13 36 33 |45 24  
 46--27--| 6--35---17--11---61 57 |  
 47 50 25 26 55 31 3 23  
 21 | 56 51 2 |  
 16-----43-----7-----64

163805/

1-----60-----20-----49  
 |63 6 30 |31  
 41 62 36 8 46 27 7 33  
 |14 4---47--56---15--53---+54--17  
 40 23 | 22 37 55 26 13 44 |  
 |21 52 39 10 28 43 |42 25  
 48--11--| 12--50---9--18---61 51 |  
 32 58 38 19 57 29 3 24  
 34 | 35 59 2 |  
 16-----45-----5-----64

164061/

1-----53-----22-----54  
 |63 16 31 |20  
 40 62 30 6 52 21 8 41  
 |14 4---33--55---23--56---+60--15  
 39 18 | 38 48 46 28 7 36 |  
 |29 58 37 19 17 27 |47 26  
 50--5--| 9--42---10--32---61 51 |  
 24 57 44 13 59 35 3 25  
 45 | 34 49 2 |  
 11-----43-----12-----64

164087/

1-----48-----24-----57  
 |63 9 33 |25  
 39 62 29 14 55 19 7 35  
 |15 4---44--59---18--54---+53--13  
 38 16 | 42 37 45 34 5 43 |  
 |22 60 31 20 28 23 |49 27  
 52--12--| 11--47---6--21---61 50 |  
 30 58 46 10 51 36 3 26  
 40 | 32 56 2 |  
 8-----41-----17-----64

164142/

1-----41-----31-----57  
 |63 10 25 |32  
 38 62 36 23 51 15 5 30  
 |12 4---49--56---17--59---+52--11  
 37 22 | 47 19 39 44 7 45 |  
 |20 58 21 26 46 18 |43 28  
 54--13--| 6--48---9--16---61 53 |  
 35 60 50 14 42 29 3 27  
 33 | 40 55 2 |  
 8-----34-----24-----64

164144/

1-----58-----56-----15  
 |63 12 33 |22  
 62 61 14 20 23 13 31 36  
 |11 5---47--40---17--28---+55--57  
 59 16 | 21 30 26 46 24 38 |  
 |27 41 19 39 35 44 |49 6  
 8--10--| 37--48---25--18---60 54 |  
 29 34 52 42 45 51 4 3  
 43 | 32 53 2 |  
 50-----9-----7-----64

.....

#2. Comparison between Self-complementary type and Complete Pan-magic

The next comparative list shows some extract examples of the Self-complementary

type and the Complete Pan-magic type that are corresponding to each other. They look quite similar to each other, don't they?

**\*\* Compare Self-Complementary Type with Complete Pan-magic: \*\***

**\*\* Solution List of Magic Cubes of Order 4 with N1=1: by K. Setsuda \*\***

<p>1/C <span style="margin-left: 200px;">&lt;=&gt;</span></p> <pre> 1-----61-----32-----36  63      6      8       53 62 42    7 14    35 43    26 31   5 24----19-49---54-47---+52--10 22 25   34 48    23 20    51 37    18 38    55 56    41 21     16 15 45--33- --28--29---40--64---17  4     44 57    50 12    27  2      9 59   30        39      3       58     11-----13-----60-----46 </pre>	<p>1/S</p> <pre> 1-----61-----36-----32  63      6      53       8 62 24    7 49    26 10    35 47   5 42----19-14---52--31---+54--43 45 38   28 56    17 15    40 21    44 25    50 48     9 37     27 20 22--11- --34--13---51--46---23 60     18 30    55 39    16 58    41  3   57        12      59        2     33-----29-----4-----64 </pre>
--	---

<p>2/C <span style="margin-left: 200px;">&lt;=&gt;</span></p> <pre> 1-----61-----21-----47  63      6      35      26 62 32    7 36    19 22    42 40   5 34----20-27---54--52---+51--17 43  8   25 53    33 41    29 28    13 55    48 50    31 16     38  9 24--44- --37--18---57--64---12  4     49 30    56 39    10  2      15 59   46        23      3       58     11-----14-----60-----45 </pre>	<p>2/S</p> <pre> 1-----61-----47-----21  63      6      26      35 62 34    7 27    42 17    19 52   5 32----20-36---51--40---+54--22 24 55   37 50    12  9     57 16    49  8    56 53    15 28     10 41 43--11- --25--14---29--45---33 60     13 46    48 23    38 58    31  3   30        39      59        2     44-----18-----4-----64 </pre>
--	--

<p>3/C <span style="margin-left: 200px;">&lt;=&gt;</span></p> <pre> 1-----61-----48-----20  63      6      18      43 62 32    7  8    19 49    42 41   5 34----29-55---44--15---+52--26 16 35   24 54    33 14    57 27    50 28    39 40    31 53     10  9 51--17- --38--45---30--64---11  4     12 47    56 22    37  2      25 59   46        23      3       58     21-----13-----60-----36 </pre>	<p>3/S</p> <pre> 1-----61-----20-----48  63      6      43      18 62 34    7 55    42 26    19 15   5 32----29-- 8---52--41---+44--49 51 28   38 40    11  9     30 53    12 35    56 54    25 27     37 14 16--21- --24--13---57--36---33 60     50 46    39 23    10 58    31  3   47        22      59        2     17-----45-----4-----64 </pre>
--	---

<p>4/C <span style="margin-left: 200px;">&lt;=&gt;</span></p> <pre> 1-----61-----52-----16  63      6      8       53 62 22    7 24    15 33    46 51   5 44----29-39---54--37---+42--10 32 45   14 38    43 30    41 17    28 18    55 56    21 31     26 25 35--13- --48--49---20--64---27  4     34 57    40 12    47  2      9 59   50        19      3       58     11-----23-----60-----36 </pre>	<p>4/S</p> <pre> 1-----61-----16-----52  63      6      53       8 62 44    7 39    46 10    15 37   5 22----29-24---42--51---+54--33 35 18   48 56    27 25    20 31    34 45    40 38     9 17     47 30 32--11- --14--23---41--36---43 60     28 50    55 19    26 58    21  3   57        12      59        2     13-----49-----4-----64 </pre>
---	---

<p>5/C <span style="margin-left: 200px;">&lt;=&gt;</span></p> <pre> 1-----61-----48-----20  63      6      18      43 62 16    7 24    19 33    42 57   5 50----29-39---44--31---+52--10 32 51    8 38    49 30    41 11    34 12    55 56    15 37     26 25 35--17- --54--45---14--64---27  4     28 47    40 22    53  2      9 59   46        23      3       58     21-----13-----60-----36 </pre>	<p>5/S</p> <pre> 1-----61-----20-----48  63      6      43      18 62 50    7 39    42 10    19 31   5 16----29-24---52--57---+44--33 35 12   54 56    27 25    14 37    28 51    40 38     9 11     53 30 32--21- -- 8--13---41--36---49 60     34 46    55 23    26 58    15  3   47        22      59        2     17-----45-----4-----64 </pre>
---	---

. . . . .

Yes. There is surely the 'One-to-one Correspondence' found between those two sets

of solutions. Therefore we could expect that there are as many Self-complementary solutions as this Complete Pan-magic type, however big solution counts those two types might have, although I have not yet counted them up to the last.

### #3. Type Conversion

The Self-complementary type could be transformed into the Complete Pan-magic type whose complementary pairs of 65 are placed only on its pan-triagonals.

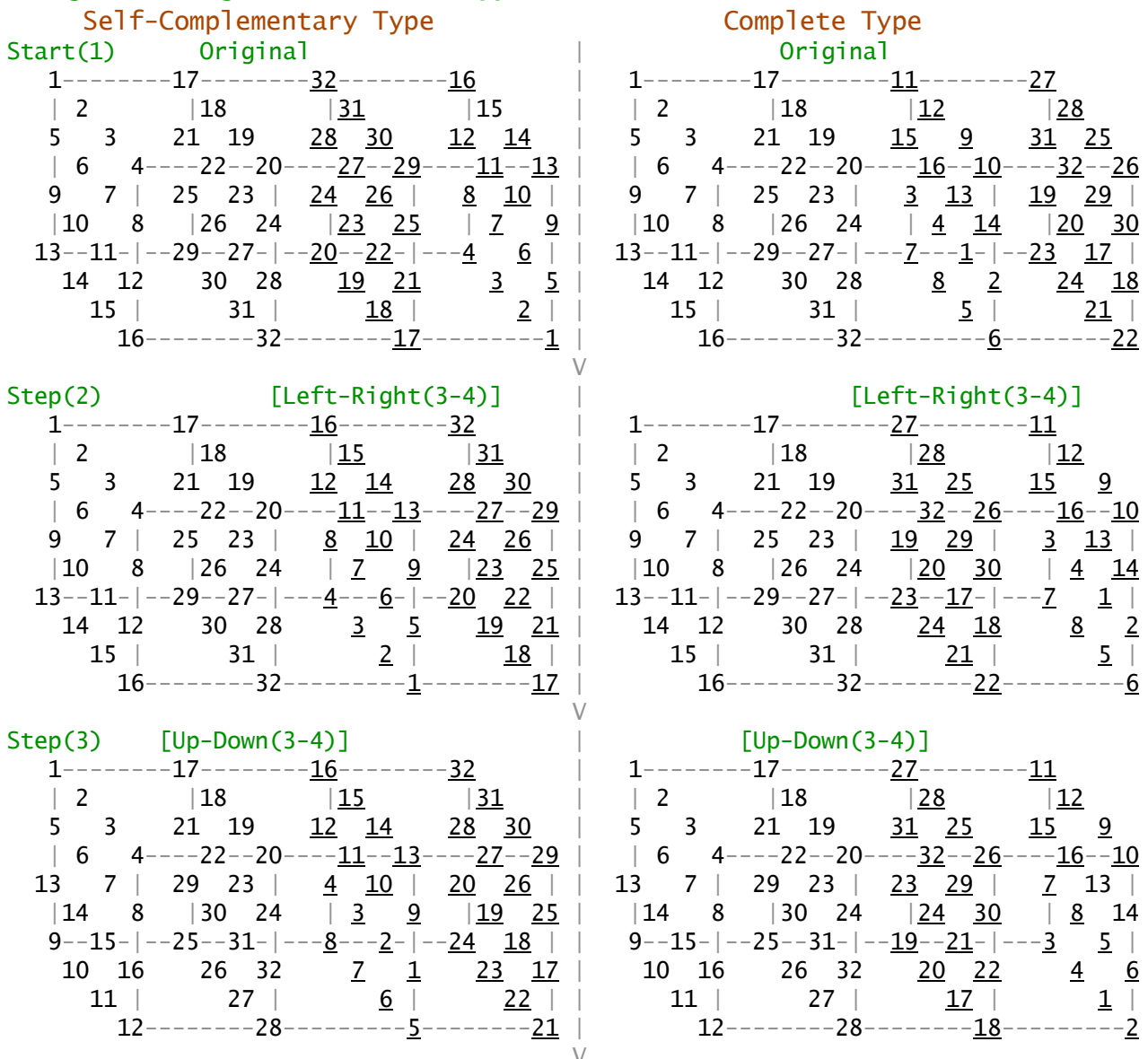
Let me show you the way how to transform the one type into the other by the next step-diagrams below, instead of showing the proof of 'one-to-one correspondence'.

It is the key point of this method that you should exchange the 3-rd plane with the 4-th one each other equally in all three directions at the same time.

You can also transform Complete Pan-magic type back into Self-complementary one, only when all complementary pairs of 65 are located on its pan-triagonals.

But, you cannot always make all types of Pan-magic cubes into Self-complementary ones. The only type you can transform is limited to the one with  $n_1=1$  and  $n_4=64$ , that is so-called "Complete Pan-magic" type.

[Figure 7: Magic Cube 4x4x4: Type Conversions]



Step(4) [Back-Front(3-4)]

```

1-----17-----16-----32
| 2         |18         |15         |31
5  4      21 20   12 13   28 29
| 6  3----22--19---11--14---27--30
13  8 | 29 24 | 4  9 | 20 25 |
|14  7 |30 23 | 3 10 |19 26 |
9--16-|--25--32-|--8---1---24 17 |
10 15  26 31   7  2   23 18
12 |         28 | 5 |         21 |
11-----27-----6-----22

```

[Goal: Complete Type]

Check all Pan-triagonals:  
 $n1+n22+n1+n22=K$ ;  $n1+n23+n1+n23=K$ ;  
 $n1+n26+n1+n26=K$ ;  $n1+n27+n1+n27=K$ ;  
 $n2+n24+n2+n24=K$ ;  $n2+n21+n2+n21=K$ ;  
 $n2+n28+n2+n28=K$ ;  $n2+n25+n2+n25=K$ ;  
 $n3+n24+n3+n24=K$ ;  $n3+n21+n3+n21=K$ ;  
 $n3+n28+n3+n28=K$ ;  $n3+n25+n3+n25=K$ ;  
 $n4+n23+n4+n23=K$ ;  $n4+n22+n4+n22=K$ ;  
.....  
 $n16+n27+n16+n27=K$ ;  $n16+n22+n16+n22=K$ ;

[Back-Front(3-4)]

```

1-----17-----27-----11
| 2         |18         |28         |12
5  4      21 20   31 26   15 10
| 6  3----22--19---32--25---16--9
13  8 | 29 24 | 23 30 | 7 14 |
|14  7 |30 23 |24 29 | 8 13 |
9--16-|--25--32-|--19--22-|--3  6 |
10 15  26 31   20 21   4  5
12 |         28 | 18 |         2 |
11-----27-----17-----1

```

[Goal: Self-Complementary Type]

Check all Complementary pairs:  
 $n1+n1=n2+n2=n3+n3=n4+n4=n5+n5=n6+n6$   
 $=n7+n7=n8+n8=n9+n9=n10+n10=n11+n11$   
 $=n12+n12=n13+n13=n14+n14=n15+n15$   
 $=n16+n16=n17+n17=.....$   
 $..=n30+n30=n31+n31=n32+n32=C$

All pairs are located symmetrically with respect to the geometric center.

I would like to revive the name 'Complete Pan-magic' back to us now for this special type with all complementary pairs of 65 placed only on its pan-triagonals and that can be transformed into the Self-complementary type by such simple rules. This request arises from our Japanese tradition. We have loved it best for a long time.

The next lists below show some examinations of two-way applicability of this type converter between the Self-complementary type and the Complete Pan-magic.

**\*\* Examine the Conversion from Complete Type into Self-complementary one \*\***

1/C =>

```

1-----61-----32-----36
|63         6         8         |53
62 42      7 14   35 43   26 31
| 5 24----19--49---54--47---52--10
22 25 | 34 48   23 20   51 37 |
|18 38   55 56   41 21   |16 15
45--33-|--28--29---40--64---17  4 |
44 57   50 12   27  2     9 59
30 |         39         3         58 |
11-----13-----60-----46

```

1/S

```

1-----61-----36-----32
|63         6         53        | 8
62 24      7 49   26 10   35 47
| 5 42----19--14---52--31---54--43
45 38 | 28 56   17 15   40 21 |
|44 25   50 48     9 37   |27 20
22--11-|--34--13---51--46---23 60 |
18 30   55 39   16 58   41  3
57 |         12         59         2 |
33-----29-----4-----64

```

2/C =>

```

1-----61-----21-----47
|63         6         35         |26
62 32      7 36   19 22   42 40
| 5 34----20--27---54--52---51--17
43  8 | 25 53   33 41   29 28 |
|13 55   48 50   31 16   |38  9
24--44-|--37--18---57--64---12  4 |
49 30   56 39   10  2    15 59
46 |         23         3         58 |
11-----14-----60-----45

```

2/S

```

1-----61-----47-----21
|63         6         26         |35
62 34      7 27   42 17   19 52
| 5 32----20--36---51--40---54--22
24 55 | 37 50   12  9   57 16 |
|49  8   56 53   15 28   |10 41
43--11-|--25--14---29--45---33 60 |
13 46   48 23   38 58   31  3
30 |         39         59         2 |
44-----18-----4-----64

```

3/C =>

```

1-----61-----48-----20
|63         6         18         |43
62 32      7  8   19 49   42 41
| 5 34----29--55---44--15---52--26

```

3/S

```

1-----61-----20-----48
|63         6         43         |18
62 34      7 55   42 26   19 15
| 5 32----29-- 8---52--41---44--49

```

```

16 35 | 24 54   33 14   57 27 | 51 28 | 38 40   11 9   30 53 |
|50 28 | 39 40   31 53 |10 9 | |12 35 | 56 54   25 27 |37 14 |
51--17-|--38--45---30--64---11 4 | 16--21-|--24--13---57--36---33 60 |
 12 47 | 56 22   37 2   25 59 | 50 46 | 39 23   10 58 | 31 3 |
   46 |   23       3       58 | 47 |   22       59       2 |
   21-----13-----60-----36 | 17-----45-----4-----64

```

4/C =>

```

1-----61-----52-----16
|63       6       8       |53
62 22   7 24   15 33   46 51
| 5 44---29-39---54-37---42--10
32 45 | 14 38   43 30   41 17 |
|28 18   55 56   21 31 |26 25
35--13-|--48--49---20--64---27 4 |
 34 57 | 40 12   47 2   9 59 |
   50 |   19       3       58 |
   11-----23-----60-----36

```

4/S

```

1-----61-----16-----52
|63       6       53      | 8
62 44   7 39   46 10   15 37
| 5 22---29-24---42--51---54--33
35 18 | 48 56   27 25   20 31 |
|34 45   40 38   9 17   |47 30
32--11-|--14--23---41--36---43 60 |
 28 50 | 55 19   26 58 | 21 3 |
   57 |   12       59       2 |
   13-----49-----4-----64

```

5/C =>

```

1-----61-----48-----20
|63       6       18      |43
62 16   7 24   19 33   42 57
| 5 50---29-39---44--31---52--10
32 51 | 8 38   49 30   41 11 |
|34 12   55 56   15 37 |26 25
35--17-|--54--45---14--64---27 4 |
 28 47 | 40 22   53 2   9 59 |
   46 |   23       3       58 |
   21-----13-----60-----36

```

5/S

```

1-----61-----20-----48
|63       6       43      |18
62 50   7 39   42 10   19 31
| 5 16---29-24---52--57---44--33
35 12 | 54 56   27 25   14 37 |
|28 51   40 38   9 11   |53 30
32--21-|-- 8--13---41--36---49 60 |
 34 46 | 55 23   26 58 | 15 3 |
   47 |   22       59       2 |
   17-----45-----4-----64

```

.....

**/\*\* The Converter Program developed by Kanji Setsuda \*\*/**

```

void conv(void){
d[1][1]=d[0][1];   d[1][2]=d[0][2];   d[1][3]=d[0][4];   d[1][4]=d[0][3];
d[1][5]=d[0][5];   d[1][6]=d[0][6];   d[1][7]=d[0][8];   d[1][8]=d[0][7];
d[1][9]=d[0][13];  d[1][10]=d[0][14];  d[1][11]=d[0][16];  d[1][12]=d[0][15];
d[1][13]=d[0][9];  d[1][14]=d[0][10];  d[1][15]=d[0][12];  d[1][16]=d[0][11];
d[1][17]=d[0][17]; d[1][18]=d[0][18];  d[1][19]=d[0][20];  d[1][20]=d[0][19];
d[1][21]=d[0][21]; d[1][22]=d[0][22];  d[1][23]=d[0][24];  d[1][24]=d[0][23];
d[1][25]=d[0][29]; d[1][26]=d[0][30];  d[1][27]=d[0][32];  d[1][28]=d[0][31];
d[1][29]=d[0][25]; d[1][30]=d[0][26];  d[1][31]=d[0][28];  d[1][32]=d[0][27];
d[1][33]=d[0][49]; d[1][34]=d[0][50];  d[1][35]=d[0][52];  d[1][36]=d[0][51];
d[1][37]=d[0][53]; d[1][38]=d[0][54];  d[1][39]=d[0][56];  d[1][40]=d[0][55];
d[1][41]=d[0][61]; d[1][42]=d[0][62];  d[1][43]=d[0][64];  d[1][44]=d[0][63];
d[1][45]=d[0][57]; d[1][46]=d[0][58];  d[1][47]=d[0][60];  d[1][48]=d[0][59];
d[1][49]=d[0][33]; d[1][50]=d[0][34];  d[1][51]=d[0][36];  d[1][52]=d[0][35];
d[1][53]=d[0][37]; d[1][54]=d[0][38];  d[1][55]=d[0][40];  d[1][56]=d[0][39];
d[1][57]=d[0][45]; d[1][58]=d[0][46];  d[1][59]=d[0][48];  d[1][60]=d[0][47];
d[1][61]=d[0][41]; d[1][62]=d[0][42];  d[1][63]=d[0][44];  d[1][64]=d[0][43];
};

```

**\*\* Examine the Conversion from Self-complementary Type into Complete one \*\***

```

1/S => 1/C
1-----61-----36-----32
|63       6       53      | 8
62 24   7 49   26 10   35 47
| 5 42---19-14---52--31---54--43
45 38 | 28 56   17 15   40 21 |
|44 25   50 48   9 37   |27 20
22--11-|--34--13---51--46---23 60 |
 18 30 | 55 39   16 58 | 41 3 |
   57 |   12       59       2 |
   33-----29-----4-----64

```

2/S =>

```

1-----61-----47-----21
|63      6      26      |35
62 34    7 27    42 17    19 52
| 5 32---20-36---51-40---+54--22
24 55 | 37 50    12 9     57 16 |
|49 8   56 53    15 28    |10 41
43--11-|--25--14---29--45---33 60 |
 13 46    48 23    38 58    31 3
    30 |      39      59      2 |
    44-----18-----4-----64

```

2/C

```

1-----61-----21-----47
|63      6      35      |26
62 32    7 36    19 22    42 40
| 5 34---20-27---54-52---+51--17
43 8 | 25 53    33 41    29 28 |
|13 55    48 50    31 16    |38 9
24--44-|--37--18---57--64---12 4 |
 49 30    56 39    10 2     15 59
    46 |      23      3      58 |
    11-----14-----60-----45

```

3/S =>

```

1-----61-----20-----48
|63      6      43      |18
62 34    7 55    42 26    19 15
| 5 32---29-8---52-41---+44--49
51 28 | 38 40    11 9     30 53 |
|12 35    56 54    25 27    |37 14
16--21-|--24--13---57--36---33 60 |
 50 46    39 23    10 58    31 3
    47 |      22      59      2 |
    17-----45-----4-----64

```

3/C

```

1-----61-----48-----20
|63      6      18      |43
62 32    7 8     19 49    42 41
| 5 34---29-55---44-15---+52--26
16 35 | 24 54    33 14    57 27 |
|50 28    39 40    31 53    |10 9
51--17-|--38--45---30--64---11 4 |
 12 47    56 22    37 2     25 59
    46 |      23      3      58 |
    21-----13-----60-----36

```

4/S =>

```

1-----61-----16-----52
|63      6      53      |8
62 44    7 39    46 10    15 37
| 5 22---29-24---42-51---+54--33
35 18 | 48 56    27 25    20 31 |
|34 45    40 38    9 17     |47 30
32--11-|--14--23---41--36---43 60 |
 28 50    55 19    26 58    21 3
    57 |      12      59      2 |
    13-----49-----4-----64

```

4/C

```

1-----61-----52-----16
|63      6      8      |53
62 22    7 24    15 33    46 51
| 5 44---29-39---54-37---+42--10
32 45 | 14 38    43 30    41 17 |
|28 18    55 56    21 31    |26 25
35--13-|--48--49---20--64---27 4 |
 34 57    40 12    47 2     9 59
    50 |      19      3      58 |
    11-----23-----60-----36

```

5/S =>

```

1-----61-----20-----48
|63      6      43      |18
62 50    7 39    42 10    19 31
| 5 16---29-24---52-57---+44--33
35 12 | 54 56    27 25    14 37 |
|28 51    40 38    9 11     |53 30
32--21-|--8--13---41--36---49 60 |
 34 46    55 23    26 58    15 3
    47 |      22      59      2 |
    17-----45-----4-----64

```

5/C

```

1-----61-----48-----20
|63      6      18      |43
62 16    7 24    19 33    42 57
| 5 50---29-39---44-31---+52--10
32 51 | 8 38     49 30    41 11 |
|34 12    55 56    15 37    |26 25
35--17-|--54--45---14--64---27 4 |
 28 47    40 22    53 2     9 59
    46 |      23      3      58 |
    21-----13-----60-----36

```

.....

[OK!]

I want you to compare those results with the original list in p.27 one after another, and check if this single method of conversion is applicable in both two ways.

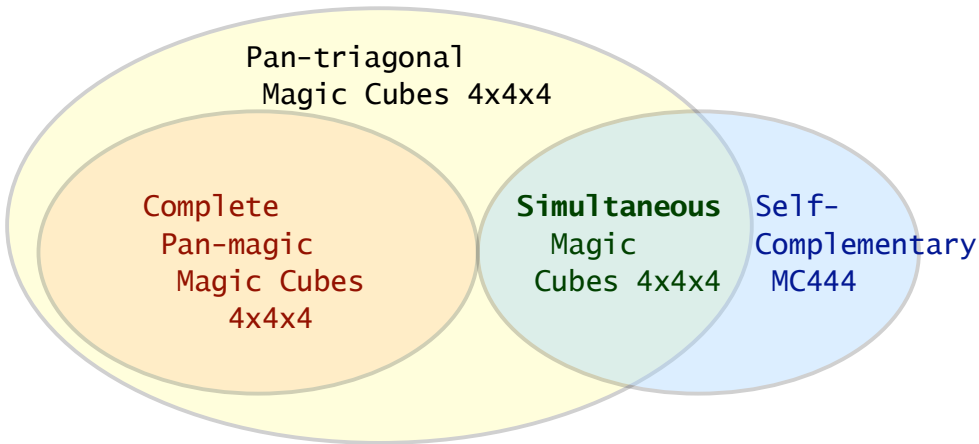
This beautiful reversibility and two-way applicability certainly indicates something. It is our type converter that should make the 'One-to-one Correspondence' between those two types possible to be realized, I think.

(Originally Written in English on Sep.10, 2001; Revised on Feb.27, 2007; Newest Revision on Jul.17, 2010 with MacOSX & Xcode3.1 by Kanji Setsuda)

# Section 3: 'Simultaneous' Magic Cubes of Order 4: Both Self-complementary and Pan-triagonal

## #1. Simultaneous Type of MC444: Both Self-complementary and Pan-triagonal

Set Diagrams for 'Simultaneous' Type

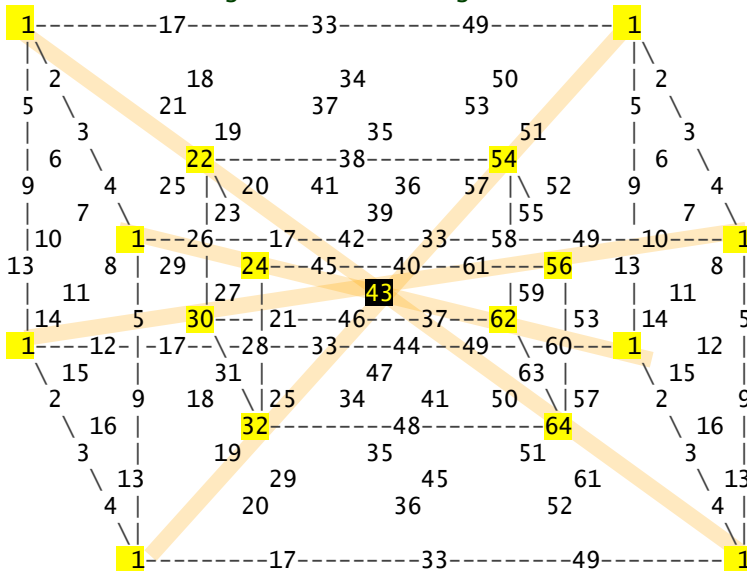


I put some Basic Conditions and Basic Form at the first definition stage as follows:

\* Basic Conditions for Rows, Columns and Pillars: S=130; \*

$n_1+n_2+n_3+n_4=S$	...bc01;	$n_1+n_5+n_9+n_{13}=S$	...bc02;	$n_1+n_{17}+n_{33}+n_{49}=S$	...bc03;
$n_5+n_6+n_7+n_8=S$	...bc04;	$n_2+n_6+n_{10}+n_{14}=S$	...bc05;	$n_2+n_{18}+n_{34}+n_{50}=S$	...bc06;
$n_9+n_{10}+n_{11}+n_{12}=S$	...bc07;	$n_3+n_7+n_{11}+n_{15}=S$	...bc08;	$n_3+n_{19}+n_{35}+n_{51}=S$	...bc09;
$n_{13}+n_{14}+n_{15}+n_{16}=S$	...bc10;	$n_4+n_8+n_{12}+n_{16}=S$	...bc11;	$n_4+n_{20}+n_{36}+n_{52}=S$	...bc12;
$n_{17}+n_{18}+n_{19}+n_{20}=S$	...bc13;	$n_{17}+n_{21}+n_{25}+n_{29}=S$	...bc14;	$n_5+n_{21}+n_{37}+n_{53}=S$	...bc15;
$n_{21}+n_{22}+n_{23}+n_{24}=S$	...bc16;	$n_{18}+n_{22}+n_{26}+n_{30}=S$	...bc17;	$n_6+n_{22}+n_{38}+n_{54}=S$	...bc18;
$n_{25}+n_{26}+n_{27}+n_{28}=S$	...bc19;	$n_{19}+n_{23}+n_{27}+n_{31}=S$	...bc20;	$n_7+n_{23}+n_{39}+n_{55}=S$	...bc21;
$n_{29}+n_{30}+n_{31}+n_{32}=S$	...bc22;	$n_{20}+n_{24}+n_{28}+n_{32}=S$	...bc23;	$n_8+n_{24}+n_{40}+n_{56}=S$	...bc24;
$n_{33}+n_{34}+n_{35}+n_{36}=S$	...bc25;	$n_{33}+n_{37}+n_{41}+n_{45}=S$	...bc26;	$n_9+n_{25}+n_{41}+n_{57}=S$	...bc27;
$n_{37}+n_{38}+n_{39}+n_{40}=S$	...bc28;	$n_{34}+n_{38}+n_{42}+n_{46}=S$	...bc29;	$n_{10}+n_{26}+n_{42}+n_{58}=S$	...bc30;
$n_{41}+n_{42}+n_{43}+n_{44}=S$	...bc31;	$n_{35}+n_{39}+n_{43}+n_{47}=S$	...bc32;	$n_{11}+n_{27}+n_{43}+n_{59}=S$	...bc33;
$n_{45}+n_{46}+n_{47}+n_{48}=S$	...bc34;	$n_{36}+n_{40}+n_{44}+n_{48}=S$	...bc35;	$n_{12}+n_{28}+n_{44}+n_{60}=S$	...bc36;
$n_{49}+n_{50}+n_{51}+n_{52}=S$	...bc37;	$n_{49}+n_{53}+n_{57}+n_{61}=S$	...bc38;	$n_{13}+n_{29}+n_{45}+n_{61}=S$	...bc39;
$n_{53}+n_{54}+n_{55}+n_{56}=S$	...bc40;	$n_{50}+n_{54}+n_{58}+n_{62}=S$	...bc41;	$n_{14}+n_{30}+n_{46}+n_{62}=S$	...bc42;
$n_{57}+n_{58}+n_{59}+n_{60}=S$	...bc43;	$n_{51}+n_{55}+n_{59}+n_{63}=S$	...bc44;	$n_{15}+n_{31}+n_{47}+n_{63}=S$	...bc45;
$n_{61}+n_{62}+n_{63}+n_{64}=S$	...bc46;	$n_{52}+n_{56}+n_{60}+n_{64}=S$	...bc47;	$n_{16}+n_{32}+n_{48}+n_{64}=S$	...bc48;

\*\* Extended Diagram for Pan-Magic Cubes of Order 4 \*\*



\* 'Self-complementary' Conditions:

SC=65; \*  
 $n_1+n_{64}=n_2+n_{63}=n_3+n_{62}=n_4+n_{61}=$   
 $n_5+n_{60}=n_6+n_{59}=n_7+n_{58}=n_8+n_{57}=$   
 $n_9+n_{56}=n_{10}+n_{55}=n_{11}+n_{54}=n_{12}+n_{53}=$   
 $n_{13}+n_{52}=n_{14}+n_{51}=n_{15}+n_{50}=n_{16}+n_{49}=$   
 $n_{17}+n_{48}=n_{18}+n_{47}=n_{19}+n_{46}=n_{20}+n_{45}=$   
 $n_{21}+n_{44}=n_{22}+n_{43}=n_{23}+n_{42}=n_{24}+n_{41}=$   
 $n_{25}+n_{40}=n_{26}+n_{39}=n_{27}+n_{38}=n_{28}+n_{37}=$   
 $n_{29}+n_{36}=n_{30}+n_{35}=n_{31}+n_{34}=n_{32}+n_{33}=$   
 $n_{33}+n_{32}=n_{34}+n_{31}= \dots =SC \dots scc;$

\* 'Pan-triagonal' Conditions: S=130; \*

$n_1+n_{22}+n_{43}+n_{64}=S$  ...pc01; |  $n_1+n_{24}+n_{43}+n_{62}=S$  ...pc02;



```

n1+n30+n43+n56=S ...pc03; | n1+n32+n43+n54=S ...pc04;
n2+n23+n44+n61=S ...pc05; | n2+n21+n44+n63=S ...pc06;
n2+n31+n44+n53=S ...pc07; | n2+n29+n44+n55=S ...pc08;
n3+n24+n41+n62=S ...pc09; | n3+n22+n41+n64=S ...pc10;
n3+n32+n41+n54=S ...pc11; | n3+n30+n41+n56=S ...pc12;
n4+n21+n42+n63=S ...pc13; | n4+n23+n42+n61=S ...pc14;
n4+n29+n42+n55=S ...pc15; | n4+n31+n42+n53=S ...pc16;
n5+n26+n47+n52=S ...pc17; | n5+n28+n47+n50=S ...pc18;
n5+n18+n47+n60=S ...pc19; | n5+n20+n47+n58=S ...pc20;
n6+n27+n48+n49=S ...pc21; | n6+n25+n48+n51=S ...pc22;
n6+n19+n48+n57=S ...pc23; | n6+n17+n48+n59=S ...pc24;
n7+n28+n45+n50=S ...pc25; | n7+n26+n45+n52=S ...pc26;
n7+n20+n45+n58=S ...pc27; | n7+n18+n45+n60=S ...pc28;
n8+n25+n46+n51=S ...pc29; | n8+n27+n46+n49=S ...pc30;
n8+n17+n46+n59=S ...pc31; | n8+n19+n46+n57=S ...pc32;
n9+n30+n35+n56=S ...pc33; | n9+n32+n35+n54=S ...pc34;
n9+n22+n35+n64=S ...pc35; | n9+n24+n35+n62=S ...pc36;
n10+n31+n36+n53=S ...pc37; | n10+n29+n36+n55=S ...pc38;
n10+n23+n36+n61=S ...pc39; | n10+n21+n36+n63=S ...pc40;
n11+n32+n33+n54=S ...pc41; | n11+n30+n33+n56=S ...pc42;
n11+n24+n33+n62=S ...pc43; | n11+n22+n33+n64=S ...pc44;
n12+n29+n34+n55=S ...pc45; | n12+n31+n34+n53=S ...pc46;
n12+n21+n34+n63=S ...pc47; | n12+n23+n34+n61=S ...pc48;
n13+n18+n39+n60=S ...pc49; | n13+n20+n39+n58=S ...pc50;
n13+n26+n39+n52=S ...pc51; | n13+n28+n39+n50=S ...pc52;
n14+n19+n40+n57=S ...pc53; | n14+n17+n40+n59=S ...pc54;
n14+n27+n40+n49=S ...pc55; | n14+n25+n40+n51=S ...pc56;
n15+n20+n37+n58=S ...pc57; | n15+n18+n37+n60=S ...pc58;
n15+n28+n37+n50=S ...pc59; | n15+n26+n37+n52=S ...pc60;
n16+n17+n38+n59=S ...pc61; | n16+n19+n38+n57=S ...pc62;
n16+n25+n38+n51=S ...pc63; | n16+n27+n38+n49=S ...pc64;

```

\* List-forming Inequality Conditions for Standard Solutions \*  
 $n1 < n49 < n13 < n4$ ;  $n1 < n16$ ;  $n1 < n52$ ;  $n1 < n61$ ; and  $n1 < n64$ ;

I was once surprised at the news that someone in Europe discovered and reported about the 'Simultaneous' MC444. But I wondered if it was really true. I knew that we could not make any Simultaneous type of MC444 for both Self-complementary and 'Complete' Pan-magic type. But, I soon began to think we might be able to make any Simultaneous type for both Self-complementary and 'Pan-triagonal' type in general.

Let's follow him and try to examine and count up all the possible solutions of them.

## 2. How many Simultaneous MC444 are there in all?

But, it is always a hard job for us to compose them in such an ordinary way as usual, without using any new method of composition. It really took too much time and energy to go on doing our job, and we often gave it up on the way.

I had to design and write the program carefully, using all our knowledge and skills. I tried as hard as possible. I have now got it! I am glad to report here of our result.

\*\* **Simultaneous** Magic Cubes of Order 4: Both **Self-Complementary & Pan-Triagonal** \*\*  
 \*\* Abstract List of **Standard Solutions** with  $n1=1$  &  $n43=32, 48, 56, 60, 62$  and  $63$ ; \*\*

[List 1: Solutions with  $n1=1$  &  $n64=64$  &  $n43=32$ ]

1/	67/
1-----52-----16-----61	1-----46-----24-----59
58            23            43              6	58            20            47              5
55   8        26 41        38 21        11 60	52   8        26 43        37 17        15 62
20 63-----33--14-----29--50-----48-- 3	14 63-----33--21-----27--42-----56-- 4
12 46   37 31        25 35        56 18	16 53   35 31        25 36        54 10
47   9        30 40        34 28         19 53	55 11        29 40        34 30         12 49
62--17- --15--36-----51--32----- 2 45	61-- 9- --23--38-----44--32----- 2 51
5 54        44 27        24 39        57 10	3 50        48 28        22 39        57 13
59              22            42            7	60              18            45            7
4-----49-----13-----64	6-----41-----19-----64

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1-----54-----16-----59  
 |58           23           45           | 4  
 55 8   26 41   36 19   13 62  
 |22 63---33--12---27--50---48-- 5  
 14 44 | 35 31   25 37   56 18 |  
 |47 9   28 40   34 30   |21 51  
 60--17-|--15--38---53--32--- 2 43 |  
   3 52   46 29   24 39   57 10  
   61 |       20           42           7 |  
   6-----49-----11-----64

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1-----58-----16-----55  
 |54           27           45           | 4  
 59 12   22 37   36 19   13 62  
 |26 63---33-- 8---23--50---48-- 9  
 14 40 | 35 31   21 41   60 18 |  
 |47 5   24 44   34 30   |25 51  
 56--17-|--15--42---57--32--- 2 39 |  
   3 52   46 29   28 43   53 6  
   61 |       20           38           11 |  
   10-----49----- 7-----64

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1-----58-----24-----47  
 |46           27           53           | 4  
 59 20   14 37   36 11   21 62  
 |26 63---33-- 8---15--42---56--17  
 22 40 | 35 31   13 49   60 10 |  
 |55 5   16 52   34 30   |25 43  
 48-- 9-|--23--50---57--32--- 2 39 |  
   3 44   54 29   28 51   45 6  
   61 |       12           38           19 |  
   18-----41----- 7-----64

475/

1-----47-----24-----58  
 |60           22           45           | 3  
 54 8   28 42   35 17   13 63  
 |15 61---33--19---26--44---56-- 6  
 16 51 | 34 29   25 38   55 12 |  
 |53 10   27 40   36 31   |14 49  
 59-- 9-|--21--39---46--32--- 4 50 |  
   2 52   48 30   23 37   57 11  
   62 |       20           43           5 |  
   7-----41-----18-----64

631/

1-----50-----25-----54  
 |58           28           41           | 3  
 60 12   26 36   35 19   9 63  
 |18 59---33--16---22--45---57--10  
 14 48 | 34 27   21 42   61 13 |  
 |52 4   23 44   38 31   |17 51  
 55-- 8-|--20--43---49--32--- 6 47 |  
   2 56   46 30   29 39   53 5  
   62 |       24           37           7 |  
   11-----40-----15-----64

661/

1-----58-----27-----44  
 |50           28           48           | 4  
 60 22   18 35   36 10   16 63  
 |26 57---33-- 9---12--45---59--19  
 23 41 | 34 25   11 51   62 13 |  
 |52 3   14 54   40 31   |24 42  
 46-- 6-|--20--53---56--32--- 8 39 |

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1-----44-----30-----55  
 |52           22           47           | 9  
 54 14   20 39   41 17   15 60  
 |12 63---33--25---23--36---62-- 6  
 16 57 | 37 31   19 38   58 4 |  
 |61 7   27 46   34 28   | 8 49  
 59-- 3-|--29--42---40--32--- 2 53 |  
   5 50   48 24   26 45   51 11  
   56 |       18           43           13 |  
   10-----35-----21-----64

217/

1-----52-----30-----47  
 |40           14           59           |17  
 46 26   8 43   49 5   27 56  
 |20 63---33--21---15--36---62--10  
 28 53 | 41 31   7 42   54 4 |  
 |61 11   23 58   34 24   |12 37  
 55-- 3-|--29--50---44--32--- 2 45 |  
   9 38   60 16   22 57   39 19  
   48 |       6           51           25 |  
   18-----35-----13-----64

271/

1-----47-----24-----58  
 |59           20           46           | 5  
 52 8   27 42   37 17   14 63  
 |15 62---33--21---26--43---56-- 4  
 16 53 | 34 30   25 36   55 11 |  
 |54 10   29 40   35 31   |12 49  
 61-- 9-|--22--39---44--32--- 3 50 |  
   2 51   48 28   23 38   57 13  
   60 |       19           45           6 |  
   7-----41-----18-----64

493/

1-----47-----24-----58  
 |61           22           44           | 3  
 54 8   29 42   35 17   12 63  
 |15 60---33--19---26--45---56-- 6  
 16 51 | 34 28   25 38   55 13 |  
 |52 10   27 40   37 31   |14 49  
 59-- 9-|--20--39---46--32--- 5 50 |  
   2 53   48 30   23 36   57 11  
   62 |       21           43           4 |  
   7-----41-----18-----64

649/

1-----46-----31-----52  
 |56           27           42           | 5  
 59 15   24 36   37 17   10 62  
 |14 58---33--21---20--40---63--11  
 16 53 | 35 26   18 43   61 8 |  
 |57 4   22 47   39 30   |12 49  
 54-- 2-|--25--45---44--32--- 7 51 |  
   3 55   48 28   29 41   50 6  
   60 |       23           38           9 |  
   13-----34-----19-----64

667/

1-----47-----28-----54  
 |61           26           40           | 3  
 58 12   29 38   35 17   8 63  
 |15 56---33--19---22--45---60--10  
 16 51 | 34 24   21 42   59 13 |  
 |52 6   23 44   41 31   |14 49  
 55-- 5-|--20--43---46--32--- 9 50 |

```

2 49 55 29 30 47 43 5
61 | 17 37 15 |
21-----38-----7-----64

```

```

2 57 48 30 27 36 53 7
62 | 25 39 4 |
11-----37-----18-----64

```

751/

```

1-----59-----30-----40
|48 22 51 | 9
54 26 16 36 41 5 19 63
|27 55----33--13----8--44----62--18
28 45 | 34 23 7 50 61 12 |
|53 4 15 58 42 31 |20 37
47-- 3-|--21--57----52--32----10 38 |
2 46 60 24 29 49 39 11
56 | 14 43 17 |
25-----35-----6-----64

```

757/

```

1-----58-----31-----40
|48 23 50 | 9
55 27 16 36 41 5 18 62
|26 54----33--13----8--44----63--19
28 45 | 35 22 6 51 61 12 |
|53 4 14 59 43 30 |20 37
46-- 2-|--21--57----52--32----11 39 |
3 47 60 24 29 49 38 10
56 | 15 42 17 |
25-----34-----7-----64

```

763/

```

1-----58-----31-----40
|48 23 50 | 9
55 29 16 38 41 3 18 60
|26 52----33--11----8--46----63--21
30 43 | 37 20 4 53 59 14 |
|51 6 12 61 45 28 |22 35
44-- 2-|--19--57----54--32----13 39 |
5 47 62 24 27 49 36 10
56 | 15 42 17 |
25-----34-----7-----64

```

769/

```

1-----55-----28-----46
|61 26 40 | 3
58 20 29 38 35 9 8 63
|23 48----33--11----14--53----60--18
24 43 | 34 16 13 50 59 21 |
|44 6 15 52 49 31 |22 41
47-- 5-|--12--51----54--32----17 42 |
2 57 56 30 27 36 45 7
62 | 25 39 4 |
19-----37-----10-----64

```

[Count = 804]

[List 2: Solutions with n1=1 & n64=64 & n43=48]

1/

```

1-----52-----16-----61
|58 39 27 | 6
55 8 42 25 22 37 11 60
|36 63----17--14----45--50----32-- 3
12 30 | 21 47 41 19 56 34 |
|31 9 46 24 18 44 |35 53
62--33-|--15--20----51--48---- 2 29 |
5 54 28 43 40 23 57 10
59 | 38 26 7 |
4-----49-----13-----64

```

67/

```

1-----30-----40-----59
|58 36 31 | 5
52 8 42 27 21 33 15 62
|14 63----17--37----43--26----56-- 4
16 53 | 19 47 41 20 54 10 |
|55 11 45 24 18 46 |12 49
61-- 9-|--39--22----28--48---- 2 51 |
3 50 32 44 38 23 57 13
60 | 34 29 7 |
6-----25-----35-----64

```

85/

```

1-----54-----16-----59
|58 39 29 | 4
55 8 42 25 20 35 13 62
|38 63----17--12----43--50----32-- 5
14 28 | 19 47 41 21 56 34 |
|31 9 44 24 18 46 |37 51
60--33-|--15--22----53--48---- 2 27 |
3 52 30 45 40 23 57 10
61 | 36 26 7 |
6-----49-----11-----64

```

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```

1-----28-----46-----55
|52 38 31 | 9
54 14 36 23 25 33 15 60
|12 63----17--41----39--20----62-- 6
16 57 | 21 47 35 22 58 4 |
|61 7 43 30 18 44 | 8 49
59-- 3-|--45--26----24--48---- 2 53 |
5 50 32 40 42 29 51 11
56 | 34 27 13 |
10-----19-----37-----64

```

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```

1-----58-----16-----55
|54 43 29 | 4
59 12 38 21 20 35 13 62
|42 63----17-- 8----39--50----32-- 9
14 24 | 19 47 37 25 60 34 |
|31 5 40 28 18 46 |41 51
56--33-|--15--26----57--48---- 2 23 |
3 52 30 45 44 27 53 6
61 | 36 22 11 |
10-----49-----7-----64

```

211/

```

1-----52-----46-----31
|24 14 59 |33
30 42 8 27 49 5 43 56
|36 63----17--37----15--20----62--10
44 53 | 25 47 7 26 54 4 |
|61 11 39 58 18 40 |12 21
55-- 3-|--45--50----28--48---- 2 29 |
9 22 60 16 38 57 23 35
32 | 6 51 41 |
34-----19-----13-----64

```

217/

265/

1-----58-----40-----31  
 |30 43 53 | 4  
 59 36 14 21 20 11 37 62  
 |42 63---17-- 8---15--26---56--33  
 38 24 | 19 47 13 49 60 10 |  
 |55 5 16 52 18 46 |41 27  
 32-- 9-|--39--50---57--48--- 2 23 |  
 3 28 54 45 44 51 29 6  
 61 | 12 22 35 |  
 34-----25----- 7-----64

463/

1-----31-----40-----58  
 |60 38 29 | 3  
 54 8 44 26 19 33 13 63  
 |15 61---17--35---42--28---56-- 6  
 16 51 | 18 45 41 22 55 12 |  
 |53 10 43 24 20 47 |14 49  
 59-- 9-|--37--23---30--48--- 4 50 |  
 2 52 32 46 39 21 57 11  
 62 | 36 27 5 |  
 7-----25-----34-----64

619/

1-----31-----46-----52  
 |56 42 27 | 5  
 58 14 40 20 21 33 11 63  
 |15 59---17--37---36--24---62--10  
 16 53 | 18 43 35 26 61 8 |  
 |57 4 39 30 22 47 |12 49  
 55-- 3-|--41--29---28--48--- 6 50 |  
 2 54 32 44 45 25 51 7  
 60 | 38 23 9 |  
 13-----19-----34-----64

643/

1-----31-----44-----54  
 |61 42 24 | 3  
 58 12 45 22 19 33 8 63  
 |15 56---17--35---38--29---60--10  
 16 51 | 18 40 37 26 59 13 |  
 |52 6 39 28 25 47 |14 49  
 55-- 5-|--36--27---30--48--- 9 50 |  
 2 57 32 46 43 20 53 7  
 62 | 41 23 4 |  
 11-----21-----34-----64

733/

1-----58-----47-----24  
 |32 39 50 | 9  
 55 43 16 20 25 5 34 62  
 |42 54---17--13--- 8--28---63--35  
 44 29 | 19 38 6 51 61 12 |  
 |53 4 14 59 27 46 |36 21  
 30-- 2-|--37--57---52--48---11 23 |  
 3 31 60 40 45 49 22 10  
 56 | 15 26 33 |  
 41-----18----- 7-----64

745/

1-----55-----44-----30  
 |61 42 24 | 3  
 58 36 45 22 19 9 8 63  
 |39 32---17--11---14--53---60--34  
 40 27 | 18 16 13 50 59 37 |  
 |28 6 15 52 49 47 |38 25  
 31-- 5-|--12--51---54--48---33 26 |  
 2 57 56 46 43 20 29 7

1-----31-----40-----58  
 |59 36 30 | 5  
 52 8 43 26 21 33 14 63  
 |15 62---17--37---42--27---56-- 4  
 16 53 | 18 46 41 20 55 11 |  
 |54 10 45 24 19 47 |12 49  
 61-- 9-|--38--23---28--48--- 3 50 |  
 2 51 32 44 39 22 57 13  
 60 | 35 29 6 |  
 7-----25-----34-----64

481/

1-----31-----40-----58  
 |61 38 28 | 3  
 54 8 45 26 19 33 12 63  
 |15 60---17--35---42--29---56-- 6  
 16 51 | 18 44 41 22 55 13 |  
 |52 10 43 24 21 47 |14 49  
 59-- 9-|--36--23---30--48--- 5 50 |  
 2 53 32 46 39 20 57 11  
 62 | 37 27 4 |  
 7-----25-----34-----64

631/

1-----30-----47-----52  
 |56 43 26 | 5  
 59 15 40 20 21 33 10 62  
 |14 58---17--37---36--24---63--11  
 16 53 | 19 42 34 27 61 8 |  
 |57 4 38 31 23 46 |12 49  
 54-- 2-|--41--29---28--48--- 7 51 |  
 3 55 32 44 45 25 50 6  
 60 | 39 22 9 |  
 13-----18-----35-----64

727/

1-----59-----46-----24  
 |32 38 51 | 9  
 54 42 16 20 25 5 35 63  
 |43 55---17--13--- 8--28---62--34  
 44 29 | 18 39 7 50 61 12 |  
 |53 4 15 58 26 47 |36 21  
 31-- 3-|--37--57---52--48---10 22 |  
 2 30 60 40 45 49 23 11  
 56 | 14 27 33 |  
 41-----19----- 6-----64

739/

1-----58-----47-----24  
 |32 39 50 | 9  
 55 45 16 22 25 3 34 60  
 |42 52---17--11--- 8--30---63--37  
 46 27 | 21 36 4 53 59 14 |  
 |51 6 12 61 29 44 |38 19  
 28-- 2-|--35--57---54--48---13 23 |  
 5 31 62 40 43 49 20 10  
 56 | 15 26 33 |  
 41-----18----- 7-----64

62 | 41 23 4 |  
35-----21-----10-----64

[Count = 780]

[List 3: Solutions with n1=1 & n64=64 & n43=56]

1/

1-----44-----24-----61  
|58 39 27 | 6  
47 8 50 25 14 37 19 60  
|36 63---- 9--22----53--42----32-- 3  
20 30 | 13 55 49 11 48 34 |  
|31 17 54 16 10 52 |35 45  
62--33-|--23--12----43--56---- 2 29 |  
5 46 28 51 40 15 57 18  
59 | 38 26 7 |  
4-----41-----21-----64

85/

1-----46-----24-----59  
|58 39 29 | 4  
47 8 50 25 12 35 21 62  
|38 63---- 9--20----51--42----32-- 5  
22 28 | 11 55 49 13 48 34 |  
|31 17 52 16 10 54 |37 43  
60--33-|--23--14----45--56---- 2 27 |  
3 44 30 53 40 15 57 18  
61 | 36 26 7 |  
6-----41-----19-----64

157/

1-----58-----24-----47  
|46 51 29 | 4  
59 20 38 13 12 35 21 62  
|50 63---- 9-- 8----39--42----32--17  
22 16 | 11 55 37 25 60 34 |  
|31 5 40 28 10 54 |49 43  
48--33-|--23--26----57--56---- 2 15 |  
3 44 30 53 52 27 45 6  
61 | 36 14 19 |  
18-----41----- 7-----64

217/

1-----58-----40-----31  
|30 51 45 | 4  
59 36 22 13 12 19 37 62  
|50 63---- 9-- 8----23--26----48--33  
38 16 | 11 55 21 41 60 18 |  
|47 5 24 44 10 54 |49 27  
32--17-|--39--42----57--56---- 2 15 |  
3 28 46 53 52 43 29 6  
61 | 20 14 35 |  
34-----25----- 7-----64

463/

1-----31-----40-----58  
|60 38 29 | 3  
46 8 52 26 11 33 21 63  
|23 61---- 9--35----50--28----48-- 6  
24 43 | 10 53 49 14 47 20 |  
|45 18 51 16 12 55 |22 41  
59--17-|--37--15----30--56---- 4 42 |  
2 44 32 54 39 13 57 19  
62 | 36 27 5 |  
7-----25-----34-----64

619/

1-----31-----54-----44  
|48 50 27 | 5

67/

1-----30-----40-----59  
|58 36 31 | 5  
44 8 50 27 13 33 23 62  
|22 63---- 9--37----51--26----48-- 4  
24 45 | 11 55 49 12 46 18 |  
|47 19 53 16 10 54 |20 41  
61--17-|--39--14----28--56---- 2 43 |  
3 42 32 52 38 15 57 21  
60 | 34 29 7 |  
6-----25-----35-----64

145/

1-----28-----54-----47  
|44 38 31 |17  
46 22 36 15 25 33 23 60  
|20 63---- 9--49----39--12----62-- 6  
24 57 | 13 55 35 14 58 4 |  
|61 7 51 30 10 52 | 8 41  
59-- 3-|--53--26----16--56---- 2 45 |  
5 42 32 40 50 29 43 19  
48 | 34 27 21 |  
18-----11-----37-----64

211/

1-----44-----54-----31  
|16 22 59 |33  
30 50 8 27 41 5 51 48  
|36 63---- 9--37----23--12----62--18  
52 45 | 25 55 7 26 46 4 |  
|61 19 39 58 10 40 |20 13  
47-- 3-|--53--42----28--56---- 2 29 |  
17 14 60 24 38 57 15 35  
32 | 6 43 49 |  
34-----11-----21-----64

265/

1-----31-----40-----58  
|59 36 30 | 5  
44 8 51 26 13 33 22 63  
|23 62---- 9--37----50--27----48-- 4  
24 45 | 10 54 49 12 47 19 |  
|46 18 53 16 11 55 |20 41  
61--17-|--38--15----28--56---- 3 42 |  
2 43 32 52 39 14 57 21  
60 | 35 29 6 |  
7-----25-----34-----64

481/

1-----31-----40-----58  
|61 38 28 | 3  
46 8 53 26 11 33 20 63  
|23 60---- 9--35----50--29----48-- 6  
24 43 | 10 52 49 14 47 21 |  
|44 18 51 16 13 55 |22 41  
59--17-|--36--15----30--56---- 5 42 |  
2 45 32 54 39 12 57 19  
62 | 37 27 4 |  
7-----25-----34-----64

631/

1-----30-----55-----44  
|48 51 26 | 5

```

58 22 40 12 13 33 19 63
|23 59---- 9--37----36--16----62--18
24 45 | 10 51 35 26 61 8 |
|57 4 39 30 14 55 |20 41
47-- 3-|--49--29----28--56---- 6 42 |
 2 46 32 52 53 25 43 7
 60 | 38 15 17 |
 21-----11-----34-----64

```

643/

```

1-----31-----52-----46
|61 50 16 | 3
58 20 53 14 11 33 8 63
|23 48---- 9--35----38--29----60--18
24 43 | 10 40 37 26 59 21 |
|44 6 39 28 25 55 |22 41
47-- 5-|--36--27----30--56----17 42 |
 2 57 32 54 51 12 45 7
 62 | 49 15 4 |
 19-----13-----34-----64

```

733/

```

1-----58-----55-----16
|32 39 42 |17
47 51 24 12 25 5 34 62
|50 46---- 9--21---- 8--28----63--35
52 29 | 11 38 6 43 61 20 |
|45 4 22 59 27 54 |36 13
30-- 2-|--37--57----44--56----19 15 |
 3 31 60 40 53 41 14 18
 48 | 23 26 33 |
 49-----10----- 7-----64

```

745/

```

1-----47-----52-----30
|61 50 16 | 3
58 36 53 14 11 17 8 63
|39 32---- 9--19----22--45----60--34
40 27 | 10 24 21 42 59 37 |
|28 6 23 44 41 55 |38 25
31-- 5-|--20--43----46--56----33 26 |
 2 57 48 54 51 12 29 7
 62 | 49 15 4 |
 35-----13-----18-----64

```

[Count = 780]

[List 4: Solutions with n1=1 & n64=64 & n43=60]

1/

```

1-----40-----28-----61
|54 43 23 |10
47 12 50 21 14 41 19 56
|36 63---- 5--26----57--38----32-- 3
20 30 | 13 59 49 7 48 34 |
|31 17 58 16 6 52 |35 45
62--33-|--27-- 8----39--60---- 2 29 |
 9 46 24 51 44 15 53 18
 55 | 42 22 11 |
 4-----37-----25-----64

```

85/

```

1-----46-----28-----55
|54 43 29 | 4
47 12 50 21 8 35 25 62
|42 63---- 5--20----51--38----32-- 9
26 24 | 7 59 49 13 48 34 |
|31 17 52 16 6 58 |41 39
56--33-|--27--14----45--60---- 2 23 |

```

```

59 23 40 12 13 33 18 62
|22 58---- 9--37----36--16----63--19
24 45 | 11 50 34 27 61 8 |
|57 4 38 31 15 54 |20 41
46-- 2-|--49--29----28--56---- 7 43 |
 3 47 32 52 53 25 42 6
 60 | 39 14 17 |
 21-----10-----35-----64

```

727/

```

1-----59-----54-----16
|32 38 43 |17
46 50 24 12 25 5 35 63
|51 47---- 9--21---- 8--28----62--34
52 29 | 10 39 7 42 61 20 |
|45 4 23 58 26 55 |36 13
31-- 3-|--37--57----44--56----18 14 |
 2 30 60 40 53 41 15 19
 48 | 22 27 33 |
 49-----11----- 6-----64

```

739/

```

1-----58-----55-----16
|32 39 42 |17
47 53 24 14 25 3 34 60
|50 44---- 9--19---- 8--30----63--37
54 27 | 13 36 4 45 59 22 |
|43 6 20 61 29 52 |38 11
28-- 2-|--35--57----46--56----21 15 |
 5 31 62 40 51 41 12 18
 48 | 23 26 33 |
 49-----10----- 7-----64

```

67/

```

1-----30-----44-----55
|54 36 31 | 9
40 12 50 23 13 33 27 62
|26 63---- 5--41----51--22----48-- 4
28 45 | 7 59 49 8 46 18 |
|47 19 57 16 6 58 |20 37
61--17-|--43--14----24--60---- 2 39 |
 3 38 32 52 42 15 53 25
 56 | 34 29 11 |
 10-----21-----35-----64

```

145/

```

1-----24-----58-----47
|40 42 31 |17
46 26 36 15 21 33 27 56
|20 63---- 5--49----43-- 8----62--10
28 53 | 13 59 35 14 54 4 |
|61 11 51 30 6 52 |12 37
55-- 3-|--57--22----16--60---- 2 45 |

```

3 40 30 57 44 15 53 18  
61 | 36 22 11 |  
10-----37-----19-----64

157/

1-----54-----28-----47  
|46 51 29 | 4  
55 20 42 13 8 35 25 62  
|50 63---- 5--12---43--38---32--17  
26 16 | 7 59 41 21 56 34 |  
|31 9 44 24 6 58 |49 39  
48--33-|--27--22---53--60--- 2 15 |  
3 40 30 57 52 23 45 10  
61 | 36 14 19 |  
18-----37-----11-----64

217/

1-----54-----44-----31  
|30 51 45 | 4  
55 36 26 13 8 19 41 62  
|50 63---- 5--12---27--22---48--33  
42 16 | 7 59 25 37 56 18 |  
|47 9 28 40 6 58 |49 23  
32--17-|--43--38---53--60--- 2 15 |  
3 24 46 57 52 39 29 10  
61 | 20 14 35 |  
34-----21-----11-----64

463/

1-----31-----44-----54  
|56 42 29 | 3  
46 12 52 22 7 33 25 63  
|27 61---- 5--35---50--24---48--10  
28 39 | 6 57 49 14 47 20 |  
|45 18 51 16 8 59 |26 37  
55--17-|--41--15---30--60--- 4 38 |  
2 40 32 58 43 13 53 19  
62 | 36 23 9 |  
11-----21-----34-----64

619/

1-----31-----58-----40  
|48 50 23 | 9  
54 26 44 8 13 33 19 63  
|27 55---- 5--41---36--16---62--18  
28 45 | 6 51 35 22 61 12 |  
|53 4 43 30 14 59 |20 37  
47-- 3-|--49--29---24--60---10 38 |  
2 46 32 52 57 21 39 11  
56 | 42 15 17 |  
25-----7-----34-----64

643/

1-----31-----52-----46  
|61 50 16 | 3  
54 20 57 14 7 33 12 63  
|27 48---- 5--35---42--29---56--18  
28 39 | 6 44 41 22 55 25 |  
|40 10 43 24 21 59 |26 37  
47-- 9-|--36--23---30--60---17 38 |  
2 53 32 58 51 8 45 11  
62 | 49 15 4 |  
19-----13-----34-----64

733/

1-----54-----59-----16  
|32 43 38 |17  
47 51 28 8 21 9 34 62  
|50 46---- 5--25---12--24---63--35

9 38 32 44 50 29 39 19  
48 | 34 23 25 |  
18-----7-----41-----64

211/

1-----40-----58-----31  
|16 26 55 |33  
30 50 12 23 37 9 51 48  
|36 63---- 5--41---27-- 8---62--18  
52 45 | 21 59 11 22 46 4 |  
|61 19 43 54 6 44 |20 13  
47-- 3-|--57--38---24--60--- 2 29 |  
17 14 56 28 42 53 15 35  
32 | 10 39 49 |  
34-----7-----25-----64

265/

1-----31-----44-----54  
|55 36 30 | 9  
40 12 51 22 13 33 26 63  
|27 62---- 5--41---50--23---48-- 4  
28 45 | 6 58 49 8 47 19 |  
|46 18 57 16 7 59 |20 37  
61--17-|--42--15---24--60--- 3 38 |  
2 39 32 52 43 14 53 25  
56 | 35 29 10 |  
11-----21-----34-----64

481/

1-----31-----44-----54  
|61 42 24 | 3  
46 12 57 22 7 33 20 63  
|27 56---- 5--35---50--29---48--10  
28 39 | 6 52 49 14 47 25 |  
|40 18 51 16 13 59 |26 37  
55--17-|--36--15---30--60--- 9 38 |  
2 45 32 58 43 8 53 19  
62 | 41 23 4 |  
11-----21-----34-----64

631/

1-----30-----59-----40  
|48 51 22 | 9  
55 27 44 8 13 33 18 62  
|26 54---- 5--41---36--16---63--19  
28 45 | 7 50 34 23 61 12 |  
|53 4 42 31 15 58 |20 37  
46-- 2-|--49--29---24--60---11 39 |  
3 47 32 52 57 21 38 10  
56 | 43 14 17 |  
25-----6-----35-----64

727/

1-----55-----58-----16  
|32 42 39 |17  
46 50 28 8 21 9 35 63  
|51 47---- 5--25---12--24---62--34  
52 29 | 6 43 11 38 61 20 |  
|45 4 27 54 22 59 |36 13  
31-- 3-|--41--53---40--60---18 14 |  
2 30 56 44 57 37 15 19  
48 | 26 23 33 |  
49-----7-----10-----64

739/

1-----54-----59-----16  
|32 43 38 |17  
47 57 28 14 21 3 34 56  
|50 40---- 5--19---12--30---63--41

```

52 29 | 7 42 10 39 61 20 |
|45 4 26 55 23 58 |36 13
30-- 2-|--41--53---40--60---19 15 |
 3 31 56 44 57 37 14 18
 48 | 27 22 33 |
 49----- 6-----11-----64

```

```

58 23 | 13 36 4 45 55 26 |
|39 10 20 61 29 52 |42 7
24-- 2-|--35--53---46--60---25 15 |
 9 31 62 44 51 37 8 18
 48 | 27 22 33 |
 49----- 6-----11-----64

```

745/

```

1-----47-----52-----30
|61 50 16 | 3
54 36 57 14 7 17 12 63
|43 32---- 5--19---26--45---56--34
44 23 | 6 28 25 38 55 41 |
|24 10 27 40 37 59 |42 21
31-- 9-|--20--39---46--60---33 22 |
 2 53 48 58 51 8 29 11
 62 | 49 15 4 |
 35-----13-----18-----64

```

[Count = 780]

[List 5: Solutions with n1=1 & n64=64 & n43=62]

1/

```

1-----40-----30-----59
|52 45 23 |10
47 14 50 19 12 41 21 56
|38 63---- 3--26---57--36---32-- 5
22 28 | 11 61 49 7 48 34 |
|31 17 58 16 4 54 |37 43
60--33-|--29-- 8---39--62---- 2 27 |
 9 44 24 53 46 15 51 18
 55 | 42 20 13 |
 6-----35-----25-----64

```

67/

```

1-----28-----46-----55
|52 38 31 | 9
40 14 50 23 11 33 29 60
|26 63---- 3--41---53--20---48-- 6
30 43 | 7 61 49 8 44 18 |
|47 21 57 16 4 58 |42 35
59--17-|--45--12---24--62---- 2 39 |
 5 36 32 54 42 15 51 25
 56 | 34 27 13 |
 10-----19-----37-----64

```

85/

```

1-----44-----30-----55
|52 45 27 | 6
47 14 50 19 8 37 25 60
|42 63---- 3--22---53--36---32-- 9
26 24 | 7 61 49 11 48 34 |
|31 17 54 16 4 58 |41 39
56--33-|--29--12---43--62---- 2 23 |
 5 40 28 57 46 15 51 18
 59 | 38 20 13 |
 10-----35-----21-----64

```

145/

```

1-----24-----58-----47
|40 42 31 |17
44 26 38 15 19 33 29 56
|22 63---- 3--49---45-- 8---60--10
30 51 | 11 61 37 12 52 6 |
|59 13 53 28 4 54 |14 35
55-- 5-|--57--20---16--62---- 2 43 |
 9 36 32 46 50 27 39 21
 48 | 34 23 25 |
 18----- 7-----41-----64

```

157/

```

1-----52-----30-----47
|44 53 27 | 6
55 22 42 11 8 37 25 60
|50 63---- 3--14---45--36---32--17
26 16 | 7 61 41 19 56 34 |
|31 9 46 24 4 58 |49 39
48--33-|--29--20---51--62---- 2 15 |
 5 40 28 57 54 23 43 10
 59 | 38 12 21 |
 18-----35-----13-----64

```

211/

```

1-----40-----58-----31
|16 26 55 |33
28 50 14 23 35 9 53 48
|38 63---- 3--41---29-- 8---60--18
54 43 | 19 61 13 20 44 6 |
|59 21 45 52 4 46 |22 11
47-- 5-|--57--36---24--62---- 2 27 |
 17 12 56 30 42 51 15 37
 32 | 10 39 49 |
 34----- 7-----25-----64

```

217/

```

1-----52-----46-----31
|28 53 43 | 6
55 38 26 11 8 21 41 60
|50 63---- 3--14---29--20---48--33
42 16 | 7 61 25 35 56 18 |
|47 9 30 40 4 58 |49 23
32--17-|--45--36---51--62---- 2 15 |
 5 24 44 57 54 39 27 10
 59 | 22 12 37 |

```

265/

```

1-----31-----46-----52
|55 38 28 | 9
40 14 53 20 11 33 26 63
|29 60---- 3--41---50--23---48-- 6
30 43 | 4 58 49 8 47 21 |
|44 18 57 16 7 61 |22 35
59--17-|--42--15---24--62---- 5 36 |
 2 39 32 54 45 12 51 25
 56 | 37 27 10 |

```



34-----19-----13-----64

463/

```

1-----31-----46-----52
|56      42      27      | 5
44 14   54 20   7 33   25 63
|29 59---- 3--37----50--24----48--10
30 39 | 4 57   49 12   47 22 |
|43 18   53 16   8 61   |26 35
55--17-|--41--15----28--62---- 6 36 |
  2 40   32 58   45 11   51 21
  60 |    38      23      9 |
13-----19-----34-----64

```

619/

```

1-----31-----58-----40
|48      50      23      | 9
52 26   46 8    11 33   21 63
|29 55---- 3--41----38--16----60--18
30 43 | 4 53   37 20   59 14 |
|51 6    45 28   12 61   |22 35
47-- 5-|--49--27----24--62----10 36 |
  2 44   32 54   57 19   39 13
  56 |    42      15      17 |
25----- 7-----34-----64

```

643/

```

1-----31-----54-----44
|59      50      16      | 5
52 22   57 12   7 33   14 63
|29 48---- 3--37----42--27----56--18
30 39 | 4 46   41 20   55 25 |
|40 10   45 24   19 61   |26 35
47-- 9-|--38--23----28--62----17 36 |
  2 51   32 58   53 8    43 13
  60 |    49      15      6 |
21-----11-----34-----64

```

733/

```

1-----52-----61-----16
|32      45      36      |17
47 53   30 8    19 9    34 60
|50 44---- 3--25----14--24----63--37
54 27 | 7 42   10 39   59 22 |
|43 6    26 55   23 58   |38 11
28-- 2-|--41--51----40--62----21 15 |
  5 31   56 46   57 35   12 18
  48 |    29      20      33 |
49----- 4-----13-----64

```

745/

```

1-----47-----54-----28
|59      50      16      | 5
52 38   57 12   7 17   14 63
|45 32---- 3--21----26--43----56--34
46 23 | 4 30   25 36   55 41 |
|24 10   29 40   35 61   |42 19
31-- 9-|--22--39----44--62----33 20 |
  2 51   48 58   53 8    27 13
  60 |    49      15      6 |
37-----11-----18-----64

```

[Count = 780]

[List 6: Solutions with n1=1 & n64=64 & n43=63]

1/

```

1-----40-----31-----58
|52      45      22      |11

```

13-----19-----34-----64

481/

```

1-----31-----46-----52
|59      42      24      | 5
44 14   57 20   7 33   22 63
|29 56---- 3--37----50--27----48--10
30 39 | 4 54   49 12   47 25 |
|40 18   53 16   11 61   |26 35
55--17-|--38--15----28--62---- 9 36 |
  2 43   32 58   45 8    51 21
  60 |    41      23      6 |
13-----19-----34-----64

```

631/

```

1-----28-----61-----40
|48      53      20      | 9
55 29   46 8    11 33   18 60
|26 52---- 3--41----38--16----63--21
30 43 | 7 50   34 23   59 14 |
|51 6    42 31   15 58   |22 35
44-- 2-|--49--27----24--62----13 39 |
  5 47   32 54   57 19   36 10
  56 |    45      12      17 |
25----- 4-----37-----64

```

727/

```

1-----55-----58-----16
|32      42      39      |17
44 50   30 8    19 9    37 63
|53 47---- 3--25----14--24----60--34
54 27 | 4 45   13 36   59 22 |
|43 6    29 52   20 61   |38 11
31-- 5-|--41--51----40--62----18 12 |
  2 28   56 46   57 35   15 21
  48 |    26      23      33 |
49----- 7-----10-----64

```

739/

```

1-----52-----61-----16
|32      45      36      |17
47 57   30 12   19 5    34 56
|50 40---- 3--21----14--28----63--41
58 23 | 11 38   6 43   55 26 |
|39 10   22 59   27 54   |42 7
24-- 2-|--37--51----44--62----25 15 |
  9 31   60 46   53 35   8 18
  48 |    29      20      33 |
49----- 4-----13-----64

```

46 15 51 18 12 41 21 56  
|39 62---- 2--27----57--36----32-- 5  
23 28 | 10 61 49 6 48 35 |  
|30 17 59 16 4 55 |37 42  
60--33-|--29-- 8----38--63---- 3 26 |  
9 44 24 53 47 14 50 19  
54 | 43 20 13 |  
7-----34-----25-----64

85/

1-----44-----31-----54  
|52 45 26 | 7  
46 15 51 18 8 37 25 60  
|43 62---- 2--23----53--36----32-- 9  
27 24 | 6 61 49 10 48 35 |  
|30 17 55 16 4 59 |41 38  
56--33-|--29--12----42--63---- 3 22 |  
5 40 28 57 47 14 50 19  
58 | 39 20 13 |  
11-----34-----21-----64

163/

1-----52-----31-----46  
|44 53 26 | 7  
54 23 43 10 8 37 25 60  
|51 62---- 2--15----45--36----32--17  
27 16 | 6 61 41 18 56 35 |  
|30 9 47 24 4 59 |49 38  
48--33-|--29--20----50--63---- 3 14 |  
5 40 28 57 55 22 42 11  
58 | 39 12 21 |  
19-----34-----13-----64

223/

1-----52-----47-----30  
|28 53 42 | 7  
54 39 27 10 8 21 41 60  
|51 62---- 2--15----29--20----48--33  
43 16 | 6 61 25 34 56 19 |  
|46 9 31 40 4 59 |49 22  
32--17-|--45--36----50--63---- 3 14 |  
5 24 44 57 55 38 26 11  
58 | 23 12 37 |  
35-----18-----13-----64

475/

1-----30-----47-----52  
|56 43 26 | 5  
44 15 55 20 6 33 25 62  
|29 58---- 2--37----51--24----48--11  
31 38 | 4 57 49 12 46 23 |  
|42 19 53 16 8 61 |27 34  
54--17-|--41--14----28--63---- 7 36 |  
3 40 32 59 45 10 50 21  
60 | 39 22 9 |  
13-----18-----35-----64

631/

1-----36-----49-----44  
|52 55 18 | 5  
56 23 51 8 6 37 17 62  
|35 54---- 2--31----43--26----50--19  
27 32 | 4 53 41 20 58 25 |  
|40 7 45 24 12 61 |33 38  
46--15-|--39--22----34--63----11 30 |  
3 48 28 59 57 14 42 9  
60 | 47 10 13 |  
21-----16-----29-----64

40 15 51 22 10 33 29 60  
|27 62---- 2--41----53--20----48-- 7  
31 42 | 6 61 49 8 44 19 |  
|46 21 57 16 4 59 |23 34  
58--17-|--45--12----24--63---- 3 38 |  
5 36 32 55 43 14 50 25  
56 | 35 26 13 |  
11-----18-----37-----64

151/

1-----24-----59-----46  
|40 43 30 |17  
44 27 39 14 18 33 29 56  
|23 62---- 2--49----45-- 8----60--11  
31 50 | 10 61 37 12 52 7 |  
|58 13 53 28 4 55 |15 34  
54-- 5-|--57--20----16--63---- 3 42 |  
9 36 32 47 51 26 38 21  
48 | 35 22 25 |  
19----- 6-----41-----64

217/

1-----40-----59-----30  
|16 27 54 |33  
28 51 15 22 34 9 53 48  
|39 62---- 2--41----29-- 8----60--19  
55 42 | 18 61 13 20 44 7 |  
|58 21 45 52 4 47 |23 10  
46-- 5-|--57--36----24--63---- 3 26 |  
17 12 56 31 43 50 14 37  
32 | 11 38 49 |  
35----- 6-----25-----64

271/

1-----30-----47-----52  
|54 39 28 | 9  
40 15 53 20 10 33 27 62  
|29 60---- 2--41----51--22----48-- 7  
31 42 | 4 59 49 8 46 21 |  
|44 19 57 16 6 61 |23 34  
58--17-|--43--14----24--63---- 5 36 |  
3 38 32 55 45 12 50 25  
56 | 37 26 11 |  
13-----18-----35-----64

493/

1-----30-----47-----52  
|58 43 24 | 5  
44 15 57 20 6 33 23 62  
|29 56---- 2--37----51--26----48--11  
31 38 | 4 55 49 12 46 25 |  
|40 19 53 16 10 61 |27 34  
54--17-|--39--14----28--63---- 9 36 |  
3 42 32 59 45 8 50 21  
60 | 41 22 7 |  
13-----18-----35-----64

649/

1-----28-----61-----40  
|48 53 20 | 9  
54 29 47 8 10 33 19 60  
|27 52---- 2--41----39--16----62--21  
31 42 | 6 51 35 22 58 15 |  
|50 7 43 30 14 59 |23 34  
44-- 3-|--49--26----24--63----13 38 |  
5 46 32 55 57 18 36 11  
56 | 45 12 17 |  
25----- 4-----37-----64

661/

```

1-----52-----53-----24
|36      55      32      | 7
56 43   35   6   8 19   31 62
|51 50---- 2--17---23--26---54--37
45 18 | 4 49   21 38   60 25 |
|40 5   27 44   16 61   |47 20
28--11-|--39--42---48--63---15 14 |
 3 34   46 57   59 30   22 9
 58 |      33      10      29 |
 41-----12-----13-----64

```

667/

```

1-----30-----55-----44
|58      51      16      | 5
52 23   57 12   6 33   15 62
|29 48---- 2--37---43--26---56--19
31 38 | 4 47   41 20   54 25 |
|40 11   45 24   18 61   |27 34
46-- 9-|--39--22---28--63---17 36 |
 3 50   32 59   53 8    42 13
 60 |      49      14      7 |
 21-----10-----35-----64

```

751/

```

1-----54-----59-----16
|32      43      38      |17
44 51   31   8   18 9    37 62
|53 46---- 2--25---15--24---60--35
55 26 | 4 45   13 36   58 23 |
|42 7   29 52   20 61   |39 10
30-- 5-|--41--50---40--63---19 12 |
 3 28   56 47   57 34   14 21
 48 |      27      22      33 |
 49----- 6-----11-----64

```

757/

```

1-----52-----61-----16
|32      45      36      |17
46 53   31   8   18 9    35 60
|51 44---- 2--25---15--24---62--37
55 26 | 6 43   11 38   58 23 |
|42 7   27 54   22 59   |39 10
28-- 3-|--41--50---40--63---21 14 |
 5 30   56 47   57 34   12 19
 48 |      29      20      33 |
 49----- 4-----13-----64

```

763/

```

1-----52-----61-----16
|32      45      36      |17
46 57   31 12   18 5    35 56
|51 40---- 2--21---15--28---62--41
59 22 | 10 39   7 42   54 27 |
|38 11   23 58   26 55   |43 6
24-- 3-|--37--50---44--63---25 14 |
 9 30   60 47   53 34   8 19
 48 |      29      20      33 |
 49----- 4-----13-----64

```

769/

```

1-----46-----55-----28
|58      51      16      | 5
52 39   57 12   6 17   15 62
|45 32---- 2--21---27--42---56--35
47 22 | 4 31   25 36   54 41 |
|24 11   29 40   34 61   |43 18
30-- 9-|--23--38---44--63---33 20 |
 3 50   48 59   53 8    26 13
 60 |      49      14      7 |
 37-----10-----19-----64

```

[Count = 804]

[Solution Counts in Total when n1==1: 4728]

[Counts according to the Value of n43]

```

1: ..., 2: ..., 3: ..., 4: ..., 5: ..., 6: ..., 7: ..., 8: ...,
9: ..., 10: ..., 11: ..., 12: ..., 13: ..., 14: ..., 15: ..., 16: ...,
17: ..., 18: ..., 19: ..., 20: ..., 21: ..., 22: ..., 23: ..., 24: ...,
25: ..., 26: ..., 27: ..., 28: ..., 29: ..., 30: ..., 31: ..., 32: 804,
33: ..., 34: ..., 35: ..., 36: ..., 37: ..., 38: ..., 39: ..., 40: ...,
41: ..., 42: ..., 43: ..., 44: ..., 45: ..., 46: ..., 47: ..., 48: 780,
49: ..., 50: ..., 51: ..., 52: ..., 53: ..., 54: ..., 55: ..., 56: 780,
57: ..., 58: ..., 59: ..., 60: 780, 61: ..., 62: 780, 63: 804, 64: ...,

```

[OK!]

\*\* Calculated and Listed by Kanji Setsuda on Mar. 3, 2011 with MacOSX & Xcode 3 \*\*

I am happy I have finally known the counts and can report about them here.

I must report about another new result of my challenge to count all the 'Complete Euler Type' of this Simultaneous MC444: Self-complementary and Pan-triagonal:

There are 4680 'Complete Euler Type' built by binary number system among all our 4728 Simultaneous MC444. You may guess there are 48 'Non-Euler type' of them.

I would like you to read [my newest article about them in Part 4, Chapter 4](#).

(Originally written in 2003; Revised in 2005 and 2007 by Kanji Setsuda; Newest Revision on Mar. 4, 2011 by Kanji Setsuda on MacOSX and Xcode 3.1)

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