

# Part 3: New Algebraic Studies of Magic Squares: Kanji Setsuda

## Chapter 4: Algebraic Study of Magic Squares of Order 6

### Section 1: Self-Complementary Magic Square of Order 6

#1. Let's start our discussion about a Self-complementary Magic type with the next diagram and simultaneous equations below. They are the basic form and conditions assumed at the first definition stage. Any one of serial Natural Numbers {1, 2, 3, 4, 5, ..., 35, 36} must be used strictly once and be placed in the 6 x 6 regular array.

The equation (15) below means all 18 Complementary Pairs of 37 must be located symmetrically with respect to the geometric center(C) of the square. They should surely make this square into a Self-complementary type.

[Figure 1: Basic Form and Conditions]

1   2   3   4   5   6	$n1+ n2+ n3+ n4+ n5+ n6=C \dots (1)$
7   8   9   10   11   12	$n7+ n8+ n9+n10+n11+n12=C \dots (2)$
13   14   15   16   17   18	$n13+n14+n15+n16+n17+n18=C \dots (3)$
19   20   21   22   23   24	$n19+n20+n21+n22+n23+n24=C \dots (4)$
25   26   27   28   29   30	$n25+n26+n27+n28+n29+n30=C \dots (5)$
31   32   33   34   35   36	$n31+n32+n33+n34+n35+n36=C \dots (6)$
19   20   21   22   23   24	$n1+ n7+n13+n19+n25+n31=C \dots (7)$
7   8   9   10   11   12	$n2+ n8+n14+n20+n26+n32=C \dots (8)$
13   14   15   16   17   18	$n3+ n9+n15+n21+n27+n33=C \dots (9)$
25   26   27   28   29   30	$n4+n10+n16+n22+n28+n34=C \dots (10)$
31   32   33   34   35   36	$n5+n11+n17+n23+n29+n35=C \dots (11)$
19   20   21   22   23   24	$n6+n12+n18+n24+n30+n36=C \dots (12)$
7   8   9   10   11   12	$n1+ n8+n15+n22+n29+n36=C \dots (13)$
13   14   15   16   17   18	$n6+n11+n16+n21+n26+n31=C \dots (14)$

$(C=111)$

$$n1+n36=n2+n35=n3+n34=n4+n33=n5+n32=n6+n31=n7+n30=n8+n29=n9+n28=n10+n27=n11+n26=n12+n25=n13+n24=n14+n23=n15+n22=n16+n21=n17+n20=n18+n19=CS=37 \dots (15)$$

The next figure is rewritten with the new notation of Complementary Pairs.

$$n36=CS-n1=\underline{n1}; n35=CS-n2=\underline{n2}; n34=CS-n3=\underline{n3}; \dots ; n20=CS-n17=\underline{n17}; n19=CS-n18=\underline{n18}$$

[Figure 2: Basic Form and Conditions again in the New Notation]

1   2   3   4   5   6	$n1+ n2+ n3+ n4+ n5+ n6=C \dots (1)$
7   8   9   10   11   12	$n7+ n8+ n9+n10+n11+n12=C \dots (2)$
13   14   15   16   17   18	$n13+n14+n15+n16+n17+n18=C \dots (3)$
18   17   16   15   14   13	$\underline{n18+n17+n16+n15+n14+n13}=C \dots (4)$
12   11   10   9   8   7	$\underline{n12+n11+n10+ n9+ n8+ n7}=C \dots (5)$
6   5   4   3   2   1	$\underline{n6+ n5+ n4+ n3+ n2+ n1}=C \dots (6)$
18   17   16   15   14   13	$n1+ n7+n13+\underline{n18+n12+ n6}=C \dots (7)$
12   11   10   9   8   7	$n2+ n8+n14+\underline{n17+n11+ n5}=C \dots (8)$
6   5   4   3   2   1	$n3+ n9+n15+\underline{n16+n10+ n4}=C \dots (9)$
12   11   10   9   8   7	$n4+n10+n16+\underline{n15+ n9+ n3}=C \dots (10)$
6   5   4   3   2   1	$n5+n11+n17+\underline{n14+ n8+ n2}=C \dots (11)$
18   17   16   15   14   13	$n6+n12+n18+\underline{n13+ n7+ n1}=C \dots (12)$
12   11   10   9   8   7	$n1+ n8+n15+\underline{n15+ n8+ n1}=C \dots (13)$
6   5   4   3   2   1	$n6+n11+n16+\underline{n16+n11+ n6}=C \dots (14)$

$(C=111)$

$$n1+\underline{n1}=n2+\underline{n2}=n3+\underline{n3}=n4+\underline{n4}=n5+\underline{n5}=n6+\underline{n6}=n7+\underline{n7}=n8+\underline{n8}=n9+\underline{n9}=n10+\underline{n10}=n11+\underline{n11}=n12+\underline{n12}=n13+\underline{n13}=n14+\underline{n14}=n15+\underline{n15}=n16+\underline{n16}=n17+\underline{n17}=n18+\underline{n18}=CS=37 \dots (15)$$

It is very useful for you to calculate such a sum of each primary diagonal as:

$$n_1+n_8+n_{15}+n_{22}+n_{29}+n_{36}=n_1+n_8+n_{15}+n_{15}+n_8+n_1$$

$$=(n_1+n_1)+(n_8+n_8)+(n_{15}+n_{15})=CS+CS+CS=3 \times 37=111=C$$

$$n_6+n_{11}+n_{16}+n_{21}+n_{26}+n_{31}=(n_6+n_6)+(n_{11}+n_{11})+(n_{16}+n_{16})=3 \times CS=C$$

#2. It is known that we can never make a Self-complementary type of Magic Square of Order 6. No answers could really be found.

Are there any ways of proving that could demonstrate its impossibility?

Let me introduce you such a proof example by 'reductio ad absurdum' as follows.

Let me put the name to each block of the figure above:

$$P=n_1+n_2+n_3+n_7+n_8+n_9+n_{13}+n_{14}+n_{15};$$

$$Q=n_4+n_5+n_6+n_{10}+n_{11}+n_{12}+n_{16}+n_{17}+n_{18};$$

$$R=n_{19}+n_{20}+n_{21}+n_{25}+n_{26}+n_{27}+n_{31}+n_{32}+n_{33}$$

$$=n_{18}+n_{17}+n_{16}+n_{12}+n_{11}+n_{10}+n_6+n_5+n_4;$$

$$S=n_{22}+n_{23}+n_{24}+n_{28}+n_{29}+n_{30}+n_{34}+n_{35}+n_{36}$$

$$=n_{15}+n_{14}+n_{13}+n_9+n_8+n_7+n_3+n_2+n_1;$$

$$P+Q=3C \text{ [Because } n_1+n_2+n_3+n_4+n_5+n_6=C; n_7+n_8+n_9+n_{10}+n_{11}+n_{12}=C; \text{ and } n_{13}+n_{14}+n_{15}+n_{16}+n_{17}+n_{18}=C]$$

$$P+S=3C \text{ [Because } n_1+n_2+n_3+n_3+n_2+n_1=3CS=C; n_7+n_8+n_9+n_9+n_8+n_7=3CS=C; \text{ and } n_{13}+n_{14}+n_{15}+n_{15}+n_{14}+n_{13}=3CS=C]$$

$$Q+S=3C \text{ [Because } n_4+n_{10}+n_{16}+n_{22}+n_{28}+n_{34}=C; n_5+n_{11}+n_{17}+n_{23}+n_{29}+n_{35}=C; \text{ and } n_6+n_{12}+n_{18}+n_{24}+n_{30}+n_{36}=C]$$

$$\text{Therefore } Q = S = 3C/2 = 333/2$$

This quotient is not an integer at all.

But the block-sums Q and S must be integers, because the sum of integers must be an integer.

These conclusions are contradictory to each other. They should demonstrate that any Self-complementary Magic Square of Order 6 is impossible for us to build.

[This proof was worked out by Prof. M. Suzuki and K. Setsuda.]

## Section 2: Pan-Diagonal Magic Square of Order 6

[Figure 3: Extended Space for PMS66]

34 35 36 31 32 33 34 35 36 31 32 33 34	$n_1+ n_2+ n_3+ n_4+ n_5+ n_6=C \dots (1)$
4 5 6   1   2   3   4   5   6   1 2 3 4	$n_7+ n_8+ n_9+n_{10}+n_{11}+n_{12}=C \dots (2)$
10 11 12   7   8   9   10   11   12   7 8 9 10	$n_{13}+n_{14}+n_{15}+n_{16}+n_{17}+n_{18}=C \dots (3)$
16 17 18   13   14   15   16   17   18   13 14 15 16	$n_{19}+n_{20}+n_{21}+n_{22}+n_{23}+n_{24}=C \dots (4)$
22 23 24   19   20   21   22   23   24   19 20 21 22	$n_{25}+n_{26}+n_{27}+n_{28}+n_{29}+n_{30}=C \dots (5)$
28 29 30   25   26   27   28   29   30   25 26 27 28	$n_{31}+n_{32}+n_{33}+n_{34}+n_{35}+n_{36}=C \dots (6)$
34 35 36   31   32   33   34   35   36   31 32 33 34	$n_1+ n_7+n_{13}+n_{19}+n_{25}+n_{31}=C \dots (7)$
4 5 6   1 2 3 4 5 6   1 2 3 4	$n_2+ n_8+n_{14}+n_{20}+n_{26}+n_{32}=C \dots (8)$
	$n_3+ n_9+n_{15}+n_{21}+n_{27}+n_{33}=C \dots (9)$
	$n_4+n_{10}+n_{16}+n_{22}+n_{28}+n_{34}=C \dots (10)$
	$n_5+n_{11}+n_{17}+n_{23}+n_{29}+n_{35}=C \dots (11)$
	$n_6+n_{12}+n_{18}+n_{24}+n_{30}+n_{36}=C \dots (12)$
	<b>Pan-Diagonals:</b>
	$n_1+ n_8+n_{15}+n_{22}+n_{29}+n_{36}=C \dots (13)$
	$n_1+n_{12}+n_{17}+n_{22}+n_{27}+n_{32}=C \dots (14)$
	$n_2+ n_9+n_{16}+n_{23}+n_{30}+n_{31}=C \dots (15)$
	$n_2+ n_7+n_{18}+n_{23}+n_{28}+n_{33}=C \dots (16)$

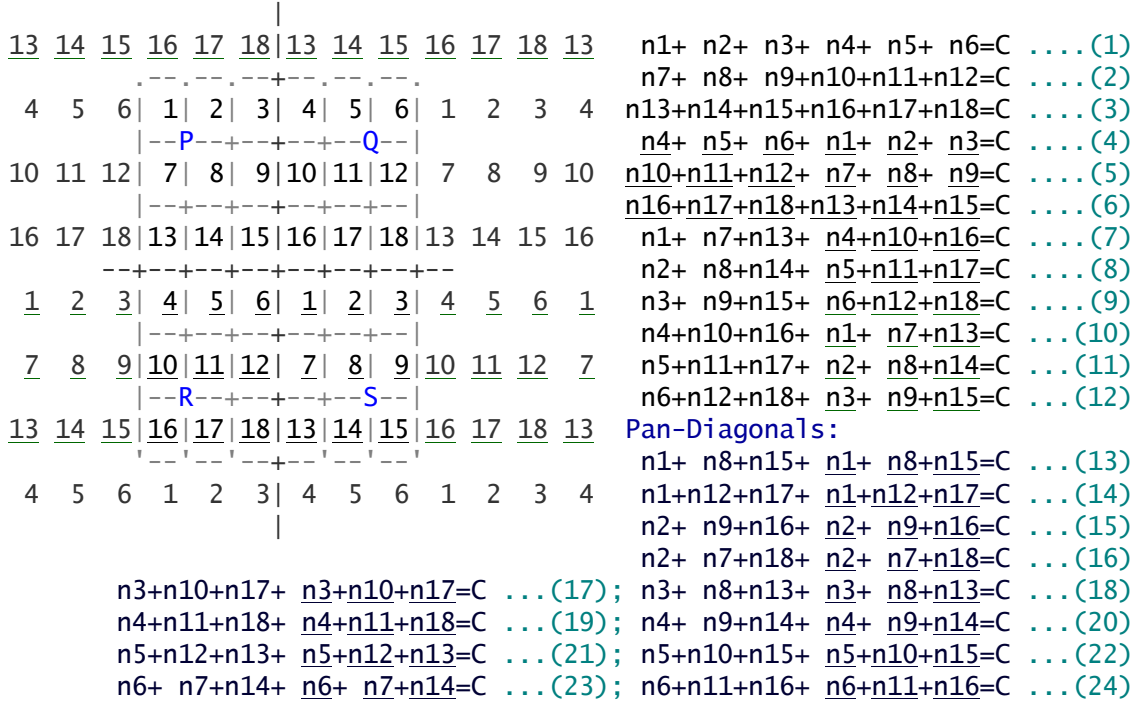
[Classical Style]

$$\begin{aligned}
n_3+n_{10}+n_{17}+n_{24}+n_{25}+n_{32} &= C \dots (17); & n_3+ n_8+n_{13}+n_{24}+n_{29}+n_{34} &= C \dots (18) \\
n_4+n_{11}+n_{18}+n_{19}+n_{26}+n_{33} &= C \dots (19); & n_4+ n_9+n_{14}+n_{19}+n_{30}+n_{35} &= C \dots (20) \\
n_5+n_{12}+n_{13}+n_{20}+n_{27}+n_{34} &= C \dots (21); & n_5+n_{10}+n_{15}+n_{20}+n_{25}+n_{36} &= C \dots (22) \\
n_6+ n_7+n_{14}+n_{21}+n_{28}+n_{35} &= C \dots (23); & n_6+n_{11}+n_{16}+n_{21}+n_{26}+n_{31} &= C \dots (24)
\end{aligned}$$

$$n_1+n_{22}=n_2+n_{23}=n_3+n_{24}=n_4+n_{19}=n_5+n_{20}=n_6+n_{21}=n_7+n_{28}=n_8+n_{29}=n_9+n_{30}=n_{10}+n_{25}=n_{11}+n_{26}=n_{12}+n_{27}=n_{13}+n_{34}=n_{14}+n_{35}=n_{15}+n_{36}=n_{16}+n_{31}=n_{17}+n_{32}=n_{18}+n_{33}=CS=37 \dots (25)$$

[Figure 4: Basic Concept of the 'Complete' Type in the new notation]

$$n_1+n_1=n_2+n_2=n_3+n_3=n_4+n_4=n_5+n_5=n_6+n_6=n_7+n_7=n_8+n_8=n_9+n_9=n_{10}+n_{10}=n_{11}+n_{11}=n_{12}+n_{12}=n_{13}+n_{13}=n_{14}+n_{14}=n_{15}+n_{15}=n_{16}+n_{16}=n_{17}+n_{17}=n_{18}+n_{18}=CS=37 \dots (25)$$



#1. It is also known that we cannot make any Pan-diagonal Magic Squares of Order 6. No answers could really be found. Are there any proof to show its impossibility? See the next diagram [Figure 3] and simultaneous equations above. Let's take them for the first definitions of basic form and the basic conditions.

At first I like to start our discussion about 'Complete' Magic Squares of Order 6. It is a special type of Pan-diagonal Magic Squares. All complementary pairs of 37 exist only on its pan-diagonals and no one elsewhere. See the equation (25) above.

Imagine what form the complete type could take if it did exist.

The next figure [Figure 4] above shows its concept of basic form and simultaneous equations assumed.

Let me introduce you such a proof example by 'reductio ad absurdum' as follows.

Let me put the name to each block of the Figure 4 above:

$$\begin{aligned}
P &= n_1+n_2+n_3+n_7+n_8+n_9+n_{13}+n_{14}+n_{15}; \\
Q &= n_4+n_5+n_6+n_{10}+n_{11}+n_{12}+n_{16}+n_{17}+n_{18}; \\
R &= n_{19}+n_{20}+n_{21}+n_{25}+n_{26}+n_{27}+n_{31}+n_{32}+n_{33} \\
&= n_4+n_5+n_6+n_{10}+n_{11}+n_{12}+n_{16}+n_{17}+n_{18}; \\
S &= n_{22}+n_{23}+n_{24}+n_{28}+n_{29}+n_{30}+n_{34}+n_{35}+n_{36} \\
&= n_1+n_2+n_3+n_7+n_8+n_9+n_{13}+n_{14}+n_{15};
\end{aligned}$$

$$P+Q=3C \text{ [Because } n_1+n_2+n_3+n_4+n_5+n_6=C; n_7+n_8+n_9+n_{10}+n_{11}+n_{12}=C; \text{ and}$$

$n_{13}+n_{14}+n_{15}+n_{16}+n_{17}+n_{18}=C$   
 $P+S=3C$  [Because  $n_1+n_2+n_3+n_{13}+n_{14}+n_{15}=3CS=C$ ;  $n_7+n_8+n_9+n_{17}+n_{18}+n_{19}=3CS=C$ ; and  $n_{13}+n_{14}+n_{15}+n_{13}+n_{14}+n_{15}=3CS=C$ ]  
 $Q+S=3C$  [Because  $n_4+n_{10}+n_{16}+n_{22}+n_{28}+n_{34}=C$ ;  $n_5+n_{11}+n_{17}+n_{23}+n_{29}+n_{35}=C$ ; and  $n_6+n_{12}+n_{18}+n_{24}+n_{30}+n_{36}=C$ ]

Therefore  $Q = S = 3C/2 = 333/2$

This quotient is not an integer at all.

But the block-sums  $Q$  and  $S$  must be integers, because the sum of integers must be an integer.

These conclusions are contradictory to each other. They should demonstrate that we could not make any 'Complete' type of Magic Squares of Order 6 at all.

[This proof was worked out by Prof. M. Suzuki and K. Setsuda.]

**#2.** How about the other 'Pan-diagonal' type? It doesn't always have to have all the Complementary Pairs of 37 on its pandiagonals, though the sum of every pandiagonal must be equal to the magic constant 111. Could we find any solutions of this Pan-diagonal type of Magic Squares of Order 6?

No answers could really be found. Then how do we prove that?

I don't know what to do with it yet. But I think the Euler's method seems to help us. We will probably be able to find it by the figures of binary notation, especially by binary decomposition, I hope.

**#3.** If you want to know more about the new notation, Positional Writing System of Numbers, Binary Numbers, and especially about "Euler Squares", would you please read my first articles in Chapter 4?

(1) Watch the next list below carefully and please find out the beautiful law in it.

**\*\* Pan-Magic Square of Order 4 Decomposed by the Base 2 \*\***

1/Classic		/Math(Ext)	
<pre> -----   1  8 13 12  12 11  0  7 12 11  0 7 -----+-----  14 11  2  7   1  6 13 10  1  6 13 10 -----+-----   4  5 16  9  15  8  3  4 15  8  3  4 -----+-----  15 10  3  6   2  5 14  9  2  5 14  9 ----- </pre>			
<b>/N2i</b>		<b>/D2i</b>	
0000 0111 1100 1011		0011 0110 0101 0101	
1101 1010 0001 0110		1100 1001 0101 1010	
0011 0100 1111 1000		0011 0110 1010 1010	
1110 1001 0010 0101		1100 1001 1010 0101	
		<b>*8/ *4/ *2/ *1/</b>	

\* Check Line-Sums and See if they are equal to 30:

$0+7+12+11=(0+0+1+1)*8+(0+1+1+0)*4+(0+1+0+1)*2+(0+1+0+1)=30$  ... rw1  
 $13+10+1+6=(1+1+0+0)*8+(1+0+0+1)*4+(0+1+0+1)*2+(1+0+1+0)=30$  ... rw2  
 $3+4+15+8=(0+0+1+1)*8+(0+1+1+0)*4+(1+0+1+0)*2+(1+0+1+0)=30$  ... rw3  
 $14+9+2+5=(1+1+0+0)*8+(1+0+0+1)*4+(1+0+1+0)*2+(0+1+0+1)=30$  ... rw4  
 $0+13+3+14=(0+1+0+1)*8+(0+1+0+1)*4+(0+0+1+1)*2+(0+1+1+0)=30$  ... c1  
 $7+10+4+9=(0+1+0+1)*8+(1+0+1+0)*4+(1+1+0+0)*2+(1+0+0+1)=30$  ... c2  
 $12+1+15+2=(1+0+1+0)*8+(1+0+1+0)*4+(0+0+1+1)*2+(0+1+1+0)=30$  ... c3  
 $11+6+8+5=(1+0+1+0)*8+(0+1+0+1)*4+(1+1+0+0)*2+(1+0+0+1)=30$  ... c4

... This is certainly a Basic type of MS44!

$0+10+15+5=(0+1+1+0)*8+(0+0+1+1)*4+(0+1+1+0)*2+(0+0+1+1)=30 \dots \text{pd1}$   
 $7+1+8+14=(0+0+1+1)*8+(1+0+0+1)*4+(1+0+0+1)*2+(1+1+0+0)=30 \dots \text{pd2}$   
 $12+6+3+9=(1+0+0+1)*8+(1+1+0+0)*4+(0+1+1+0)*2+(0+0+1+1)=30 \dots \text{pd3}$   
 $11+13+4+2=(1+1+0+0)*8+(0+1+1+0)*4+(1+0+0+1)*2+(1+1+0+0)=30 \dots \text{pd4}$   
 $0+6+15+9=(0+0+1+1)*8+(0+1+1+0)*4+(0+1+1+0)*2+(0+0+1+1)=30 \dots \text{pb1}$   
 $7+13+8+2=(0+1+1+0)*8+(1+1+0+0)*4+(1+0+0+1)*2+(1+1+0+0)=30 \dots \text{pb2}$   
 $12+10+3+5=(1+1+0+0)*8+(1+0+0+1)*4+(0+1+1+0)*2+(0+0+1+1)=30 \dots \text{pb3}$   
 $11+1+4+14=(1+0+0+1)*8+(0+0+1+1)*4+(1+0+0+1)*2+(1+1+0+0)=30 \dots \text{pb4}$

... This is certainly a Pan-diagonal type!

$0+5=5 \dots \text{cp1}; 7+2=9 \dots \text{cp2}; 12+9=21 \dots \text{cp3}; 11+14=25 \dots \text{cp4};$   
 $13+8=21 \dots \text{cp5}; 10+15=25 \dots \text{cp6}; 1+4=5 \dots \text{cp7}; 6+3=9 \dots \text{cp8};$

... This is not a Self-complementary type!

2/Classic	/Math	/N2i	/D2i
1 8 11 14	10 13   0   7   10   13   0 7	0000 0111 1010 1101	0011 0101 0110 0101
12 13 2 7	1 6   11   12   1   6   11 12	1011 1100 0001 0110	1100 0101 1001 1010
6 3 16 9	15 8   5   2   15   8   5 2	0101 0010 1111 1000	0011 1010 0110 1010
15 10 5 4	4 3   14   9   4   3   14 9	1110 1001 0100 0011	1100 1010 1001 0101
			*8/ *4/ *2/ *1/

3/Classic	/Math	/N2i	/D2i
1 12 7 14	6 13   0   11   6   13   0 11	0000 1011 0110 1101	0101 0011 0110 0101
8 13 2 11	1 10   7   12   1   10   7 12	0111 1100 0001 1010	0101 1100 1001 1010
10 3 16 5	15 4   9   2   15   4   9 2	1001 0010 1111 0100	1010 0011 0110 1010
15 6 9 4	8 3   14   5   8   3   14 5	1110 0101 1000 0011	1010 1100 1001 0101
			*8/ *4/ *2/ *1/

I would just like you to know that any Pandiagonal Magic Square has the constant combination of same number patterns in each line in each decomposed layer such as {0,0,1,1}, {0,1,0,1}, {0,1,1,0}, ..., {1,0,1,0}, {1,1,0,0}.

It makes each line add up to the same sum, so called 'Magic Constant' by the next form of equation:

$$(0+0+1+1)*8+(0+0+1+1)*4+(0+0+1+1)*2+(0+0+1+1)=30$$

It is supposed to be the basic condition for every Pandiagonal Magic Square 4x4.

(2) In the case of order 8, those which have the same property in each line in each decomposed layer as the ones of order 4 are all the Pandiagonal Magic Squares:

**\*\* Pan-Magic Square of Order 8 Decomposed by the Base 2 \*\***

1/Classic	/Math(Ext)
1   63   4   62   8   58   5   59	7 57 4 58   0   62   3   61   7   57   4   58   0 62 3 61
56   10   53   11   49   15   52   14	48 14 51 13   55   9   52   10   48   14   51   13   55 9 52 10
25   39   28   38   32   34   29   35	31 33 28 34   24   38   27   37   31   33   28   34   24 38 27 37
48   18   45   19   41   23   44   22	40 22 43 21   47   17   44   18   40   22   43   21   47 17 44 18
57   7   60   6   64   2   61   3	63 1 60 2   56   6   59   5   63   1   60   2   56 6 59 5
16   50   13   51   9   55   12   54	8 54 11 53   15   49   12   50   8   54   11   53   15 49 12 50
33   31   36   30   40   26   37   27	39 25 36 26   32   30   35   29   39   25   36   26   32 30 35 29
24   42   21   43   17   47   20   46	16 46 19 45   23   41   20   42   16   46   19   45   23 41 20 42

/N2i

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000000 111110 000011 111101 000111 111001 000100 111010
110111 001001 110100 001010 110000 001110 110011 001101
011000 100110 011011 100101 011111 100001 011100 100010
101111 010001 101100 010010 101000 010110 101011 010101
111000 000110 111011 000101 111111 000001 111100 000010
001111 110001 001100 110010 001000 110110 001011 110101
100000 011110 100011 011101 100111 011001 100100 011010
010111 101001 010100 101010 010000 101110 010011 101101

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/D2i*32/      *16/      *8/      *4/      *2/      *1/
01010101 01010101 01010101 01011010 01101001 00111100
10101010 10101010 01010101 10100101 10010110 11000011
01010101 10101010 10101010 01011010 01101001 00111100
10101010 01010101 10101010 10100101 10010110 11000011
10101010 10101010 10101010 01011010 01101001 00111100
01010101 01010101 10101010 10100101 10010110 11000011
10101010 01010101 01010101 01011010 01101001 00111100
01010101 10101010 01010101 10100101 10010110 11000011

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\* Check Line-Sums and See if they are equal to 252:

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0+62+3+61+7+57+4+58= (0+1+0+1+0+1+0+1)*32+(0+1+0+1+0+1+0+1)*16+(0+1+0+1+0+1+0+1)*8+
(0+1+0+1+1+0+1+0)*4+(0+1+1+0+1+0+0+1)*2+(0+0+1+1+1+1+0+0)= 252 ... rw1
55+9+52+10+48+14+51+13= (1+0+1+0+1+0+1+0)*32+(1+0+1+0+1+0+1+0)*16+(0+1+0+1+0+1+0+1)*8+
(1+0+1+0+0+1+0+1)*4+(1+0+0+1+0+1+1+0)*2+(1+1+0+0+0+0+1+1)= 252 ... rw2
24+38+27+37+31+33+28+34= (0+1+0+1+0+1+0+1)*32+(1+0+1+0+1+0+1+0)*16+(1+0+1+0+1+0+1+0)*8+
(0+1+0+1+1+0+1+0)*4+(0+1+1+0+1+0+0+1)*2+(0+0+1+1+1+1+0+0)= 252 ... rw3
47+17+44+18+40+22+43+21= (1+0+1+0+1+0+1+0)*32+(0+1+0+1+0+1+0+1)*16+(1+0+1+0+1+0+1+0)*8+
(1+0+1+0+0+1+0+1)*4+(1+0+0+1+0+1+1+0)*2+(1+1+0+0+0+0+1+1)= 252 ... rw4

0+55+24+47+56+15+32+23= (0+1+0+1+1+0+1+0)*32+(0+1+1+0+1+0+0+1)*16+(0+0+1+1+1+1+0+0)*8+
(0+1+0+1+0+1+0+1)*4+(0+1+0+1+0+1+0+1)*2+(0+1+0+1+0+1+0+1)= 252 ... c11
62+9+38+17+6+49+30+41= (1+0+1+0+0+1+0+1)*32+(1+0+0+1+0+1+1+0)*16+(1+1+0+0+0+0+1+1)*8+
(1+0+1+0+1+0+1+0)*4+(1+0+1+0+1+0+1+0)*2+(0+1+0+1+0+1+0+1)= 252 ... c12
3+52+27+44+59+12+35+20= (0+1+0+1+1+0+1+0)*32+(0+1+1+0+1+0+0+1)*16+(0+0+1+1+1+1+0+0)*8+
(0+1+0+1+0+1+0+1)*4+(1+0+1+0+1+0+1+0)*2+(1+0+1+0+1+0+1+0)= 252 ... c13
61+10+37+18+5+50+29+42= (1+0+1+0+0+1+0+1)*32+(1+0+0+1+0+1+1+0)*16+(1+1+0+0+0+0+1+1)*8+
(1+0+1+0+1+0+1+0)*4+(0+1+0+1+0+1+0+1)*2+(1+0+1+0+1+0+1+0)= 252 ... c14

```

... This is certainly a Basic type of MS88!

```

0+9+27+18+63+54+36+45= (0+0+0+0+1+1+1+1)*32+(0+0+1+1+1+1+0+0)*16+(0+1+1+0+1+0+0+1)*8+
(0+0+0+0+1+1+1+1)*4+(0+0+1+1+1+1+0+0)*2+(0+1+1+0+1+0+0+1)= 252 ... pd1
62+52+37+40+1+11+26+23= (1+1+1+1+0+0+0+0)*32+(1+1+0+0+0+0+1+1)*16+(1+0+0+1+0+1+1+0)*8+
(1+1+1+0+0+0+0+1)*4+(1+0+0+0+0+1+1+1)*2+(0+0+1+0+1+1+0+1)= 252 ... pd2
3+10+31+22+60+53+32+41= (0+0+0+0+1+1+1+1)*32+(0+0+1+1+1+1+0+0)*16+(0+1+1+0+1+0+0+1)*8+
(0+0+1+1+1+1+0+0)*4+(1+1+1+1+0+0+0+0)*2+(1+0+1+0+0+1+0+1)= 252 ... pd3
61+48+33+43+2+15+30+20= (1+1+1+1+0+0+0+0)*32+(1+1+0+0+0+0+1+1)*16+(1+0+0+1+0+1+1+0)*8+
(1+0+0+0+0+1+1+1)*4+(0+0+0+1+1+1+1+0)*2+(1+0+1+1+0+1+0+0)= 252 ... pd4

7+10+27+17+56+53+36+46= (0+0+0+0+1+1+1+1)*32+(0+0+1+1+1+1+0+0)*16+(0+1+1+0+1+0+0+1)*8+
(1+0+0+0+0+1+1+1)*4+(1+1+1+0+0+0+0+1)*2+(1+0+1+1+0+1+0+0)= 252 ... pb5
57+48+37+44+6+15+26+19= (1+1+1+1+0+0+0+0)*32+(1+1+0+0+0+0+1+1)*16+(1+0+0+1+0+1+1+0)*8+
(0+0+1+1+1+1+0+0)*4+(0+0+0+0+1+1+1+1)*2+(1+0+1+0+0+1+0+1)= 252 ... pb6
4+14+31+18+59+49+32+45= (0+0+0+0+1+1+1+1)*32+(0+0+1+1+1+1+0+0)*16+(0+1+1+0+1+0+0+1)*8+
(1+1+1+0+0+0+0+1)*4+(0+1+1+1+1+0+0+0)*2+(0+0+1+0+1+1+0+1)= 252 ... pb7
58+51+33+40+5+12+30+23= (1+1+1+1+0+0+0+0)*32+(1+1+0+0+0+0+1+1)*16+(1+0+0+1+0+1+1+0)*8+
(0+0+0+0+1+1+1+1)*4+(1+1+0+0+0+0+1+1)*2+(0+1+1+0+1+0+0+1)= 252 ... pb8

```

... This is certainly a Pan-diagonal type!

```

0+45=45 ... cp1; 62+19=81 ... cp2; 3+46=49 ... cp3; 61+16=77 ... cp4;
7+42=49 ... cp5; 57+20=77 ... cp6; 4+41=45 ... cp7; 58+23=81 ... cp8;
55+26=81 ... cp9; 9+36=45 ... cp10; 52+25=77 ... cp11; 10+39=49 ... cp12;
48+29=77 ... cp13; 14+35=49 ... cp14; 51+30=81 ... cp15; 13+32=45 ... cp16;
24+53=77 ... cp17; 38+11=49 ... cp18; 27+54=81 ... cp19; 37+8=45 ... cp20;
31+50=81 ... cp21; 33+12=45 ... cp22; 28+49=77 ... cp23; 34+15=49 ... cp24;
47+2=49 ... cp25; 17+60=77 ... cp26; 44+1=45 ... cp27; 18+63=81 ... cp28;

```

40+5=45 ... cp29; 22+59=81 ... cp30; 43+6=49 ... cp31; 21+56=77 ... cp32;  
 ... This is not a Self-complementary type!

**\* New Euler's Condition for order 8:**

$$(0+0+0+0+1+1+1+1)*2^5+(0+0+0+0+1+1+1+1)*2^4+(0+0+0+0+1+1+1+1)*2^3+ \\
 (0+0+0+0+1+1+1+1)*2^2+(0+0+0+0+1+1+1+1)*2^1+(0+0+0+0+1+1+1+1)*2^0 = 252$$

This beautiful structure is well known as 'Greco-Latin Square' or 'Euler Square'.

I would like to call "**Complete Euler Squares**" for those ones whose pandiagonals also obey the same condition at the same time along with all rows and columns.

**#4.** How about the case of Order 6?

Could we make such a kind of 'Euler Squares' of Order 6 just as order 4 and 8?

If we could, then we would know there certainly exist any Pan-diagonal Magic Squares of Order 6. If we could not, then we would finally know there are no such things as Pandiagonal MS66.

At first let me decompose each of our Basic Form by the base 6, 3 and 2 as follows:

See the lists below. It doesn't seem that decompositions by the base 3 and 2 could possibly make any "Complete Euler Squares", but it seems that the decomposition by the base 6 might make any 'CES'.

**\*\* Basic Forms of Order 6 Decomposed by the Base 6 \*\***

0/Classic

/Math(Ext)

1	2	3	4	5	6	3	4	5	0	1	2	3	4	5	0	1	2
7	8	9	10	11	12	9	10	11	6	7	8	9	10	11	6	7	8
13	14	15	16	17	18	15	16	17	12	13	14	15	16	17	12	13	14
19	20	21	22	23	24	21	22	23	18	19	20	21	22	23	18	19	20
25	26	27	28	29	30	27	28	29	24	25	26	27	28	29	24	25	26
31	32	33	34	35	36	33	34	35	30	31	32	33	34	35	30	31	32

/N6i

/D6i

\*6/

\*1/

00	01	02	03	04	05	0	0	0	0	0	0	0	1	2	3	4	5
10	11	12	13	14	15	1	1	1	1	1	1	0	1	2	3	4	5
20	21	22	23	24	25	2	2	2	2	2	2	0	1	2	3	4	5
30	31	32	33	34	35	3	3	3	3	3	3	0	1	2	3	4	5
40	41	42	43	44	45	4	4	4	4	4	4	0	1	2	3	4	5
50	51	52	53	54	55	5	5	5	5	5	5	0	1	2	3	4	5

**\*\* Decomposed by the Base 3 \*\***

/N3i

/D3i

0000	0001	0002	0010	0011	0012	000000	000000	000111	012012
0020	0021	0022	0100	0101	0102	000000	000111	222000	012012
0110	0111	0112	0120	0121	0122	000000	111111	111222	012012
0200	0201	0202	0210	0211	0212	000000	222222	000111	012012
0220	0221	0222	1000	1001	1002	000111	222000	222000	012012
1010	1011	1012	1020	1021	1022	111111	000000	111222	012012
						*27/	*9/	*3/	*1/

**\*\* Decomposed by the Base 2 \*\***

/N2i

```

000000 000001 000010 000011 000100 000101
000110 000111 001000 001001 001010 001011
001100 001101 001110 001111 010000 010001
010010 010011 010100 010101 010110 010111
011000 011001 011010 011011 011100 011101
011110 011111 100000 100001 100010 100011

```

/D2i

```

000000 000000 000000 000011 001100 010101
000000 000000 001111 110000 110011 010101
000000 000011 111100 111100 001100 010101
000000 111111 000000 001111 110011 010101
000000 111111 111111 000011 001100 010101
001111 110000 110000 110000 110011 010101
    *32/    *16/    *8/    *4/    *2/    *1/

```

Do you notice the left-most layer of the highest position  $*3^3/$  is not yet filled full with the same counts of {0, 1 and 2}. There are only 9 counts of 1, which stand for the values exceeding 26. It is because the positional layer  $*3^3/$  is made for the values from 27 to 80. But our square of Order 6 could only give the largest value 35 ( $=6^2 - 1$ ).

The left-most layer of the highest position  $*2^5/$  is not yet filled full with 18 counts of {0 and 1}. There are only 4 counts of 1, which stand for the numbers exceeding 31. It is because the positional layer  $*2^5/$  is intended to express the values from 32 to 63. But our 6x6 square could only give the largest value 35.

Because you cannot fill the layer of the highest position full with equal counts of every value, you cannot make the beautiful structure 'Euler Square' of Order 6 at all.

The square could fill the highest layer full with its binary values, only when it is of order 4, 8, 16, . . . . It is because  $4 \times 4 = 2^4$ ;  $8 \times 8 = 2^6$ ;  $16 \times 16 = 2^8$ ; . . .

But  $2^5 < 6 \times 6 < 2^6$ ;  $2^6 < 10 \times 10 < 2^7$ ;  $2^7 < 14 \times 14 < 2^8$ ; . . .

As a consequence the Pan-diagonal Magic Squares of Order 6, 10, 14, . . . . prove to be impossible for us to construct by any ordinary method.

I believe Legendary Leonhard Euler(1707-1783) must probably have known this fact. They say he already knew the impossibility of those squares about 250 years ago.

I actually tried to compose "Complete Euler Squares" of Order 6 by our "New Euler's Method" the other day, but I could not even compose any 'Latin Squares' decomposed by the base 6. It proves to be impossible for us to compose any CES of Order 6.

That seems to mean there are no such things as the Pan-diagonal MS66 at all.

## Section 3: Standard Magic Squares of Order 6

What type of magic squares can we make for order 6, then? Can we really make the most basic type of Standard Magic Squares of Order 6? Let's try to make that here.

**#1.** The following diagram and list of equations demonstrate our assumption for the first definitions. We want to make the most basic type of Standard Magic Squares of Order 6, whose rows and columns add up to the magic constant, and two primary diagonals also add up to the same sum. We add no other conditions than those.

We use each of the serial Natural Numbers {1, 2, 3, 4, 5, . . . , 34, 35, 36} strictly once and never use any number twice or more often.



**\*\* Basic Diagram for Magic Square 6x6 and Basic Equations: \*\***

5	6	1	2	3	4	5	6	1	2	$n1+n2+n3+n4+n5+n6=C$	....(rw1)
11	12	7	8	9	10	11	12	7	8	$n7+n8+n9+n10+n11+n12=C$	....(rw2)
17	18	13	14	15	16	17	18	13	14	$n13+n14+n15+n16+n17+n18=C$	....(rw3)
23	24	19	20	21	22	23	24	19	20	$n19+n20+n21+n22+n23+n24=C$	....(rw4)
29	30	25	26	27	28	29	30	25	26	$n25+n26+n27+n28+n29+n30=C$	....(rw5)
35	36	31	32	33	34	35	36	31	32	$n31+n32+n33+n34+n35+n36=C$	....(rw6)
										$n1+n7+n13+n19+n25+n31=C$	....(c11)
										$n2+n8+n14+n20+n26+n32=C$	....(c12)
										$n3+n9+n15+n21+n27+n33=C$	....(c13)
										$n4+n10+n16+n22+n28+n34=C$	....(c14)
										$n5+n11+n17+n23+n29+n35=C$	....(c15)
										$n6+n12+n18+n24+n30+n36=C$	....(c16)
										$n1+n8+n15+n22+n29+n36=C$	....(pd1)
										$n6+n11+n16+n21+n26+n31=C$	....(pd2)

#2. The next list of solutions shows the result of my recent computation, only a part of innumerable solutions. I don't yet know how many solutions exist in all. I am afraid it will take too long a time for you to count them through.

**\*\* Standard Magic Squares of Order 6: Compact List of Standard Solutions \*\***

1/	39/	75/	131/
1 34 9 33 32 2	1 34 9 33 32 2	1 34 9 33 32 2	1 34 9 33 32 2
31 14 25 7 13 21	31 16 25 7 20 12	31 16 26 7 18 13	31 16 25 10 17 12
6 20 26 16 24 19	6 18 27 14 22 24	6 20 24 15 25 21	6 23 27 15 21 19
30 11 18 12 17 23	30 15 17 10 13 26	30 14 19 11 10 27	30 11 20 7 14 29
8 27 29 15 22 10	8 23 29 19 21 11	8 22 29 17 23 12	8 22 26 18 24 13
35 5 4 28 3 36	35 5 4 28 3 36	35 5 4 28 3 36	35 5 4 28 3 36
284/	374/	545/	745/
1 34 9 33 32 2	1 34 9 33 32 2	1 34 9 33 32 2	1 34 9 33 32 2
31 16 26 10 17 11	31 18 24 12 16 10	31 20 23 10 14 13	31 14 29 7 20 10
6 21 27 15 23 19	6 20 27 14 25 19	6 19 25 15 29 17	6 24 25 15 22 19
30 13 20 7 12 29	30 11 21 7 13 29	30 12 24 7 11 27	30 13 18 12 11 27
8 22 25 18 24 14	8 23 26 17 22 15	8 21 26 18 22 16	8 21 26 16 23 17
35 5 4 28 3 36	35 5 4 28 3 36	35 5 4 28 3 36	35 5 4 28 3 36
941/	1167/	1459/	1654/
1 34 9 33 32 2	1 34 9 33 32 2	1 34 9 33 32 2	1 34 9 33 32 2
31 12 24 14 19 11	31 17 22 10 15 16	31 18 21 10 16 15	31 16 25 7 20 12
6 25 29 13 21 17	6 25 26 20 23 11	6 25 26 19 24 11	6 24 26 19 23 13
30 15 22 7 10 27	30 12 21 7 14 27	30 12 22 7 13 27	30 15 18 10 11 27
8 20 23 16 26 18	8 18 29 13 24 19	8 17 29 14 23 20	8 17 29 14 22 21
35 5 4 28 3 36	35 5 4 28 3 36	35 5 4 28 3 36	35 5 4 28 3 36
2036/	2275/	2573/	2784/
1 34 9 33 32 2	1 34 9 33 32 2	1 34 9 33 32 2	1 34 9 33 32 2
31 14 26 10 19 11	31 14 27 10 17 12	31 15 26 12 16 11	31 15 22 13 16 14
6 25 29 17 21 13	6 21 29 22 20 13	6 27 29 17 19 13	6 27 29 19 20 10
30 15 20 7 12 27	30 18 16 7 15 25	30 10 21 7 18 25	30 12 21 7 17 24
8 18 23 16 24 22	8 19 26 11 24 23	8 20 22 14 23 24	8 18 26 11 23 25
35 5 4 28 3 36	35 5 4 28 3 36	35 5 4 28 3 36	35 5 4 28 3 36
3103/	3303/	3599/	3850/
1 34 9 33 32 2	1 34 9 33 32 2	1 34 9 33 32 2	1 34 9 33 32 2
31 19 22 11 21 7	31 15 25 7 21 12	31 18 27 7 16 12	31 12 22 10 15 21
6 24 27 16 23 15	6 24 29 19 22 11	6 24 26 19 25 11	6 23 29 18 16 19
30 12 20 10 14 25	30 17 18 10 13 23	30 13 22 10 15 21	27 13 17 8 20 26
8 17 29 13 18 26	8 16 26 14 20 27	8 17 23 14 20 29	11 24 30 14 25 7
35 5 4 28 3 36	35 5 4 28 3 36	35 5 4 28 3 36	35 5 4 28 3 36

3875/ 1 34 9 33 32 2 31 12 22 10 16 20 6 24 30 18 14 19 27 13 17 7 21 26 11 23 29 15 25 8 35 5 4 28 3 36	3928/ 1 34 9 33 32 2 31 16 29 8 13 14 6 21 25 18 22 19 27 12 20 7 15 30 11 23 24 17 26 10 35 5 4 28 3 36	4098/ 1 34 9 33 32 2 31 17 20 10 15 18 6 25 29 14 24 13 27 8 23 7 16 30 11 22 26 19 21 12 35 5 4 28 3 36	4247/ 1 34 9 33 32 2 31 8 23 16 14 19 6 24 29 17 20 15 27 18 21 7 12 26 11 22 25 10 30 13 35 5 4 28 3 36
4335/ 1 34 9 33 32 2 31 15 26 13 16 10 6 25 29 12 20 19 27 8 22 7 17 30 11 24 21 18 23 14 35 5 4 28 3 36	4605/ 1 34 9 33 32 2 31 16 25 8 24 7 6 23 26 13 22 21 27 14 18 12 10 30 11 19 29 17 20 15 35 5 4 28 3 36	4722/ 1 34 9 33 32 2 31 18 29 7 12 14 6 21 26 20 25 13 27 10 19 8 17 30 11 23 24 15 22 16 35 5 4 28 3 36	4963/ 1 34 9 33 32 2 31 18 20 14 21 7 6 26 25 13 22 19 27 12 24 8 10 30 11 16 29 15 23 17 35 5 4 28 3 36

.....

#3. I tried to pick up an example of those solutions above and examined it carefully by decomposing of the base 6, 3 and 2. The following lists show the exam results.

**\*\* Standard Magic Square of Order 6 Examined \*\***

1/Classic	/Math(Ext)																								
<table border="0" style="width: 100%; border-collapse: collapse;"> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">1 34 9 33 32 2 </td><td style="padding: 2px 5px;">32 31 1 </td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">31 16 20 12 14 18 </td><td style="padding: 2px 5px;">11 13 17 </td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">7 21 27 13 24 19 </td><td style="padding: 2px 5px;">12 23 18 </td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">26 10 22 8 15 30 </td><td style="padding: 2px 5px;">7 14 29 </td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">11 25 29 17 23 6 </td><td style="padding: 2px 5px;">16 22 5 </td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">35 5 4 28 3 36 </td><td style="padding: 2px 5px;">27 2 35 </td></tr> </table>	1 34 9 33 32 2	32 31 1	31 16 20 12 14 18	11 13 17	7 21 27 13 24 19	12 23 18	26 10 22 8 15 30	7 14 29	11 25 29 17 23 6	16 22 5	35 5 4 28 3 36	27 2 35	<table border="0" style="width: 100%; border-collapse: collapse;"> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">0 33 8 32 31 1 </td><td style="padding: 2px 5px;">0 33 8</td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">30 15 19 11 13 17 </td><td style="padding: 2px 5px;">30 15 19</td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">6 20 26 12 23 18 </td><td style="padding: 2px 5px;">6 20 26</td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">25 9 21 7 14 29 </td><td style="padding: 2px 5px;">25 9 21</td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">10 24 28 16 22 5 </td><td style="padding: 2px 5px;">10 24 28</td></tr> <tr><td style="border-right: 1px dashed black; padding: 2px 5px;">34 4 3 27 2 35 </td><td style="padding: 2px 5px;">34 4 3</td></tr> </table>	0 33 8 32 31 1	0 33 8	30 15 19 11 13 17	30 15 19	6 20 26 12 23 18	6 20 26	25 9 21 7 14 29	25 9 21	10 24 28 16 22 5	10 24 28	34 4 3 27 2 35	34 4 3
1 34 9 33 32 2	32 31 1																								
31 16 20 12 14 18	11 13 17																								
7 21 27 13 24 19	12 23 18																								
26 10 22 8 15 30	7 14 29																								
11 25 29 17 23 6	16 22 5																								
35 5 4 28 3 36	27 2 35																								
0 33 8 32 31 1	0 33 8																								
30 15 19 11 13 17	30 15 19																								
6 20 26 12 23 18	6 20 26																								
25 9 21 7 14 29	25 9 21																								
10 24 28 16 22 5	10 24 28																								
34 4 3 27 2 35	34 4 3																								

/N6i	/D6i	*6/	*1/
00 53 12 52 51 01	0 5 1 5 5 0	0 3 2 2 1 1	
50 23 31 15 21 25	5 2 3 1 2 2	0 3 1 5 1 5	
10 32 42 20 35 30	1 3 4 2 3 3	0 2 2 0 5 0	
41 13 33 11 22 45	4 1 3 1 2 4	1 3 3 1 2 5	
14 40 44 24 34 05	1 4 4 2 3 0	4 0 4 4 4 5	
54 04 03 43 02 55	5 0 0 4 0 5	4 4 3 3 2 5	

\* Check Line-Sums and See if they are equal to 105:

0+33+8+32+31+1= (0+5+1+5+5+0)\*6+(0+3+2+2+1+1)= 105 ... rw1  
30+15+19+11+13+17= (5+2+3+1+2+2)\*6+(0+3+1+5+1+5)= 105 ... rw2  
6+20+26+12+23+18= (1+3+4+2+3+3)\*6+(0+2+2+0+5+0)= 105 ... rw3  
25+9+21+7+14+29= (4+1+3+1+2+4)\*6+(1+3+3+1+2+5)= 105 ... rw4  
10+24+28+16+22+5= (1+4+4+2+3+0)\*6+(4+0+4+4+4+5)= 105 ... rw5  
34+4+3+27+2+35= (5+0+0+4+0+5)\*6+(4+4+3+3+2+5)= 105 ... rw6  
0+30+6+25+10+34= (0+5+1+4+1+5)\*6+(0+0+0+1+4+4)= 105 ... c11  
33+15+20+9+24+4= (5+2+3+1+4+0)\*6+(3+3+2+3+0+4)= 105 ... c12  
8+19+26+21+28+3= (1+3+4+3+4+0)\*6+(2+1+2+3+4+3)= 105 ... c13  
32+11+12+7+16+27= (5+1+2+1+2+4)\*6+(2+5+0+1+4+3)= 105 ... c14  
31+13+23+14+22+2= (5+2+3+2+3+0)\*6+(1+1+5+2+4+2)= 105 ... c15  
1+17+18+29+5+35= (0+2+3+4+0+5)\*6+(1+5+0+5+5+5)= 105 ... c16  
0+15+26+7+22+35= (0+2+4+1+3+5)\*6+(0+3+2+1+4+5)= 105 ... pd1  
1+13+12+21+24+34= (0+2+2+3+4+5)\*6+(1+1+0+3+0+4)= 105 ... pb6

... This is certainly a basic type of Standard MS66!  
... But, this is not a Pan-diagonal one!

0+35=35 ... cp1; 33+2=35 ... cp2; 8+27=35 ... cp3; 32+3=35 ... cp4;  
31+4=35 ... cp5; 1+34=35 ... cp6; 30+5=35 ... cp7; 15+22=37 ... cp8;

19+16=35 ... cp9; 11+28=39 ... cp10; 13+24=37 ... cp11; 17+10=27 ... cp12;  
 6+29=35 ... cp13; 20+14=34 ... cp14; 26+7=33 ... cp15; 12+21=33 ... cp16;  
 23+9=32 ... cp17; 18+25=43 ... cp18;  
 ... This is not a Self-complementary type!

**\*\* The Same Solution Decomposed by the Base 3 \*\***

/N3i						/D3i					
0000	1020	0022	1012	1011	0001	010110	000000	022110	002211		
1010	0120	0201	0102	0111	0122	100000	012111	120012	001212		
0020	0202	0222	0110	0212	0200	000000	022122	202110	022020		
0221	0100	0210	0021	0112	1002	000001	212010	201210	100122		
0101	0220	1001	0121	0211	0012	001000	120120	020211	101112		
1021	0011	0010	1000	0002	1022	100101	000000	211002	110022		

**\*\* The Same Solution Decomposed by the Base 2 \*\***

/N2i					
000000	100001	001000	100000	011111	000001
011110	001111	010011	001011	001101	010001
000110	010100	011010	001100	010111	010010
011001	001001	010101	000111	001110	011101
001010	011000	011100	010000	010110	000101
100010	000100	000011	011011	000010	100011

/D2i					
010100	000010	001010	000010	000010	010011
000000	101001	110110	110010	111100	011111
000000	011011	001100	110110	101011	000010
000000	101001	110011	001111	000110	111101
000000	011110	111000	001011	100010	000001
100001	000100	000100	010000	101111	001101

As you see, this is certainly not an example of Euler Squares by any means. Any line in any layer of the decompositions does not always obey the Euler's Condition. It is not always expressed in the same forms to the next equations:

$$(0+1+2+3+4+5) \times 6 + (0+1+2+3+4+5) = 105 \quad \dots /D6i$$

$$(0+0+1+1+2+2) \times 27 + (0+0+1+1+2+2) \times 9 + (0+0+1+1+2+2) \times 3 + (0+0+1+1+2+2) = 105 \quad \dots /D3i$$

$$(0+0+0+1+1+1) \times 32 + (0+0+0+1+1+1) \times 16 + (0+0+0+1+1+1) \times 8 + (0+0+0+1+1+1) \times 4 + (0+0+0+1+1+1) \times 2 + (0+0+0+1+1+1) = 105 \quad \dots /D2i$$

**#4.** The next list shows the same solutions with /D6i diagrams.

It seems that all the innumerable solutions of Standard Magic Squares of Order 6 may be "Non-Euler" type of Magic Squares, doesn't it? How surprising it is!

**\*\* Standard Magic Squares of Order 6: Standard Solutions with /D6i \*\***

1/ /D6i						3/ /D6i									
1	34	9	33	32	2	051550	032211	1	34	9	33	32	2	051550	032211
31	14	25	7	13	21	524123	010002	31	11	25	7	18	19	514123	040050
6	20	26	16	24	19	034233	511350	6	20	27	13	24	21	034233	512052
30	11	18	12	17	23	412123	545544	30	15	17	14	12	23	422213	524154
8	27	29	15	22	10	144231	124233	8	26	29	16	22	10	144231	114333
35	5	4	28	3	36	500405	443325	35	5	4	28	3	36	500405	443325

8/ /D6i						9/ /D6i									
1	34	9	33	32	2	051550	032211	1	34	9	33	32	2	051550	032211
31	12	27	7	19	15	514132	052002	31	11	25	7	18	19	514123	040050
6	20	24	14	21	26	033234	515121	6	27	24	14	17	23	043223	525144
30	17	18	13	11	22	422213	545043	30	12	20	13	15	21	413223	551022
8	23	29	16	25	10	134241	144303	8	22	29	16	26	10	134241	134313
35	5	4	28	3	36	500405	443325	35	5	4	28	3	36	500405	443325

13/ /D6i						16/ /D6i									
1	34	9	33	32	2	051550	032211	1	34	9	33	32	2	051550	032211
31	12	24	13	16	15	513222	055032	31	14	25	11	17	13	524122	010440
6	18	29	11	20	27	024134	554412	6	19	27	12	18	29	034124	502554
30	17	22	7	14	21	423123	543012	30	16	22	7	15	21	423123	533022
8	25	23	19	26	10	143341	104013	8	23	24	20	26	10	133341	145113
35	5	4	28	3	36	500405	443325	35	5	4	28	3	36	500405	443325
17/ /D6i						19/ /D6i									
1	34	9	33	32	2	051550	032211	1	34	9	33	32	2	051550	032211
31	11	26	7	17	19	514123	041040	31	14	18	12	13	23	522123	015504
6	21	25	12	23	24	034133	520545	6	25	27	15	17	21	044223	502242
30	13	18	16	14	20	422223	505311	30	11	24	7	20	19	413133	545010
8	27	29	15	22	10	144231	124233	8	22	29	16	26	10	134241	134313
35	5	4	28	3	36	500405	443325	35	5	4	28	3	36	500405	443325
20/ /D6i						21/ /D6i									
1	34	9	33	32	2	051550	032211	1	34	9	33	32	2	051550	032211
31	16	22	13	14	15	523222	033012	31	11	24	12	14	19	513123	045510
6	20	27	12	17	29	034124	512544	6	17	27	15	20	26	024234	542211
30	11	23	7	21	19	413133	544020	30	21	22	7	13	18	433122	523005
8	25	26	18	24	10	144231	101553	8	23	25	16	29	10	134241	140343
35	5	4	28	3	36	500405	443325	35	5	4	28	3	36	500405	443325
25/ /D6i						27/ /D6i									
1	34	9	33	32	2	051550	032211	1	34	9	33	32	2	051550	032211
31	14	23	7	11	25	523114	014040	31	11	22	14	12	21	513213	043152
6	18	26	16	24	21	024233	551352	6	20	29	13	18	25	034224	514050
30	13	20	12	19	17	423132	501504	30	15	23	7	19	17	423132	524004
8	27	29	15	22	10	144231	124233	8	26	24	16	27	10	143241	115323
35	5	4	28	3	36	500405	443325	35	5	4	28	3	36	500405	443325
32/ /D6i						33/ /D6i									
1	34	9	33	32	2	051550	032211	1	34	9	33	32	2	051550	032211
31	16	23	7	15	19	523123	034020	31	12	19	7	15	27	513124	050022
6	24	20	12	22	27	033134	551532	6	22	26	14	23	20	034233	531141
30	11	26	13	14	17	414222	541014	30	17	24	11	13	16	423122	545403
8	21	29	18	25	10	134241	124503	8	21	29	18	25	10	134241	124503
35	5	4	28	3	36	500405	443325	35	5	4	28	3	36	500405	443325
34/ /D6i						38/ /D6i									
1	34	9	33	32	2	051550	032211	1	34	9	33	32	2	051550	032211
31	11	22	7	19	21	513133	043002	31	12	19	7	15	27	513124	050022
6	25	24	12	17	27	043124	505542	6	22	26	14	20	23	034233	531114
30	16	23	13	14	15	423222	534012	30	17	24	11	16	13	423122	545430
8	20	29	18	26	10	134241	114513	8	21	29	18	25	10	134241	124503
35	5	4	28	3	36	500405	443325	35	5	4	28	3	36	500405	443325

.....

I cannot help wondering whether there might be any Euler Squares among them or not. The solution count would surely be so big, almost as uncountable, that there might probably exist any 'Euler Type' among those solutions, don't you think?

I made up my mind to examine it at once. Let's make a new experiment of trying to compose Euler Squares of Order 6 by our "New Euler's Method".

I gave up my idea of composing any "Complete Euler Squares", for I know it is really impossible. I just want to compose any basic type of Euler Squares, only the two primary diagonals of which obey the Euler's Condition beside their rows and columns. And I use the decompositions by the base 6 this time.

**#5.** The next list demonstrates the core part of my recent program.

```
/** Composing 'Euler Squares' of Order 6 **
```

```

/** by the "New Euler's Method" with /D6i **
/** 'EulerMS6.c' Dictated by Kanji Setsuda **
/** on Nov. 5, 2005 and on Apr.11, 2006; **
/** Recompiled on Feb. 11, 2015 **
/** with MacOSX 10.10.2 & Xcode 6.1.1 **
//
#include <stdio.h>
//
long int cnt, bcnt;
long cntr[4], cfc[37];
short hcnt, lcnt, cnt2, cnt3;
short u1,u2;
short nm[37], uflg[37];
short anm[3][39];
char thlu[7681][37], tllu[92161][37];
char mtc[7681][92161];
//
short rw[7][7], cl[7][7];
short pd[7], pb[7];
//
void stp01(void), stp02(void), stp03(void), stp04(void), stp05(void);
void stp06(void), stp07(void), stp08(void), stp09(void), stp10(void);
void stp11(void), stp12(void), stp13(void), stp14(void), stp15(void);
void stp16(void), stp17(void), stp18(void), stp19(void), stp20(void);
void stp21(void), stp22(void), stp23(void), stp24(void), stp25(void);
void stp26(void), stp27(void), stp28(void), stp29(void), stp30(void);
void stp31(void), stp32(void), stp33(void), stp34(void), stp35(void);
void stp36(void), ansrecord(void);
void prlunit(void);
void cmbcmp(void);
void prans(void), pr3ans(void), pr2ans(void), pr1ans(void);
void cmissn(void);
//
int main(){
short m,n;
printf("\n** Composing 'Euler Squares' of Order 6 **\n");
printf("** by the 'New Euler's Method' with /D6i *\n");
for(m=0;m<7;m++){
for(n=0;n<7;n++){
rw[m][n]=0; cl[m][n]=0;
}}
for(n=0;n<7;n++){pd[n]=0; pb[n]=0;}
for(n=0;n<37;n++){nm[n]=0;}
hcnt=0; cnt=0; bcnt=0; cnt3=0;
stp01(); /* Make the Latin Squares *
printf("\n [List of Latin Squares]\n");
prlunit(); /* Print the Latin Squares *
lcnt=cnt; cnt=0; cnt3=0;
printf("\n [List of Composed Solutions of 'Euler' MS66]\n");
//printf("\n [Examples of Wrong Compositions of 'Euler' MS66]\n");
cmbcmp(); /* Combine, Compose and Print *
if(cnt3==1){pr1ans();} /* Print the Rest One *
printf(" [Count = %ld]\n",cnt);
cmissn();
printf(" [OK!]\n");
return 0;
}
/** Make the Latin Squares *
/** Line #1 & #6 *
/** Set n1 *
void stp01(){
short i;
for(i=0;i<6;i++){
if((rw[1][i]==0)&&(cl[1][i]==0)&&(pd[i]==0)){nm[1]=i;

```

```

        rw[1][i]=1; cl[1][i]=1; pd[i]=1;
        stp02();
        rw[1][i]=0; cl[1][i]=0; pd[i]=0;}
    }
}
//
/** Set n36 *
void stp02(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[6][i]==0)&&(cl[6][i]==0)&&(pd[i]==0)){nm[36]=i;
            rw[6][i]=1; cl[6][i]=1; pd[i]=1;
            stp03();
            rw[6][i]=0; cl[6][i]=0; pd[i]=0;}
    }
}
//
/** Set n2 *
void stp03(){
    short i;
    for(i=0;i<6;i++){
        if((rw[1][i]==0)&&(cl[2][i]==0)){nm[2]=i;
            rw[1][i]=1; cl[2][i]=1;
            stp04();
            rw[1][i]=0; cl[2][i]=0;}
    }
}
//
/** Set n35 *
void stp04(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[6][i]==0)&&(cl[5][i]==0)){nm[35]=i;
            rw[6][i]=1; cl[5][i]=1;
            stp05();
            rw[6][i]=0; cl[5][i]=0;}
    }
}
//
/** Set n3 *
void stp05(){
    short i;
    for(i=0;i<6;i++){
        if((rw[1][i]==0)&&(cl[3][i]==0)){nm[3]=i;
            rw[1][i]=1; cl[3][i]=1;
            stp06();
            rw[1][i]=0; cl[3][i]=0;}
    }
}
//
/** Set n34 *
void stp06(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[6][i]==0)&&(cl[4][i]==0)){nm[34]=i;
            rw[6][i]=1; cl[4][i]=1;
            stp07();
            rw[6][i]=0; cl[4][i]=0;}
    }
}
//
/** Set n4 *
void stp07(){
    short i;
    for(i=0;i<6;i++){

```

```

        if((rw[1][i]==0)&&(cl[4][i]==0)){nm[4]=i;
            rw[1][i]=1; cl[4][i]=1;
            stp08();
            rw[1][i]=0; cl[4][i]=0;}
    }
}
//
/** Set n33 *
void stp08(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[6][i]==0)&&(cl[3][i]==0)){nm[33]=i;
            rw[6][i]=1; cl[3][i]=1;
            stp09();
            rw[6][i]=0; cl[3][i]=0;}
    }
}
//
/** Set n5 *
void stp09(){
    short i;
    for(i=0;i<6;i++){
        if((rw[1][i]==0)&&(cl[5][i]==0)){nm[5]=i;
            rw[1][i]=1; cl[5][i]=1;
            stp10();
            rw[1][i]=0; cl[5][i]=0;}
    }
}
//
/** Set n32 *
void stp10(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[6][i]==0)&&(cl[2][i]==0)){nm[32]=i;
            rw[6][i]=1; cl[2][i]=1;
            stp11();
            rw[6][i]=0; cl[2][i]=0;}
    }
}
//
/** Set n6 *
void stp11(){
    short i;
    for(i=0;i<6;i++){
        if((rw[1][i]==0)&&(cl[6][i]==0)&&(pb[i]==0)){nm[6]=i;
            rw[1][i]=1; cl[6][i]=1; pb[i]=1;
            stp12();
            rw[1][i]=0; cl[6][i]=0; pb[i]=0;}
    }
}
//
/** Set n31 *
void stp12(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[6][i]==0)&&(cl[1][i]==0)&&(pb[i]==0)){nm[31]=i;
            rw[6][i]=1; cl[1][i]=1; pb[i]=1;
            stp13();
            rw[6][i]=0; cl[1][i]=0; pb[i]=0;}
    }
}
//
/** Line #2 & #5 *
/** Set n7 *
void stp13(){

```

```

short i;
for(i=0;i<6;i++){
    if((rw[2][i]==0)&&(cl[1][i]==0)){nm[7]=i;
        rw[2][i]=1; cl[1][i]=1;
        stp14();
        rw[2][i]=0; cl[1][i]=0;}
}
}
/* Set n30 *
void stp14(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[5][i]==0)&&(cl[6][i]==0)){nm[30]=i;
            rw[5][i]=1; cl[6][i]=1;
            stp15();
            rw[5][i]=0; cl[6][i]=0;}
    }
}
//
/* Set n8 *
void stp15(){
    short i;
    for(i=0;i<6;i++){
        if((rw[2][i]==0)&&(cl[2][i]==0)&&(pd[i]==0)){nm[8]=i;
            rw[2][i]=1; cl[2][i]=1; pd[i]=1;
            stp16();
            rw[2][i]=0; cl[2][i]=0; pd[i]=0;}
    }
}
//
/* Set n29 *
void stp16(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[5][i]==0)&&(cl[5][i]==0)&&(pd[i]==0)){nm[29]=i;
            rw[5][i]=1; cl[5][i]=1; pd[i]=1;
            stp17();
            rw[5][i]=0; cl[5][i]=0; pd[i]=0;}
    }
}
//
/* Set n9 *
void stp17(){
    short i;
    for(i=0;i<6;i++){
        if((rw[2][i]==0)&&(cl[3][i]==0)){nm[9]=i;
            rw[2][i]=1; cl[3][i]=1;
            stp18();
            rw[2][i]=0; cl[3][i]=0;}
    }
}
//
/* Set n28 *
void stp18(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[5][i]==0)&&(cl[4][i]==0)){nm[28]=i;
            rw[5][i]=1; cl[4][i]=1;
            stp19();
            rw[5][i]=0; cl[4][i]=0;}
    }
}
//
/* Set n10 *
void stp19(){

```



```

short i;
for(i=0;i<6;i++){
    if((rw[2][i]==0)&&(cl[4][i]==0)){nm[10]=i;
        rw[2][i]=1; cl[4][i]=1;
        stp20();
        rw[2][i]=0; cl[4][i]=0;}
}
}
//
/** Set n27 *
void stp20(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[5][i]==0)&&(cl[3][i]==0)){nm[27]=i;
            rw[5][i]=1; cl[3][i]=1;
            stp21();
            rw[5][i]=0; cl[3][i]=0;}
    }
}
//
/** Set n11 *
void stp21(){
    short i;
    for(i=0;i<6;i++){
        if((rw[2][i]==0)&&(cl[5][i]==0)&&(pb[i]==0)){nm[11]=i;
            rw[2][i]=1; cl[5][i]=1; pb[i]=1;
            stp22();
            rw[2][i]=0; cl[5][i]=0; pb[i]=0;}
    }
}
//
/** Set n26 *
void stp22(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[5][i]==0)&&(cl[2][i]==0)&&(pb[i]==0)){nm[26]=i;
            rw[5][i]=1; cl[2][i]=1; pb[i]=1;
            stp23();
            rw[5][i]=0; cl[2][i]=0; pb[i]=0;}
    }
}
//
/** Set n12 *
void stp23(){
    short i;
    for(i=0;i<6;i++){
        if((rw[2][i]==0)&&(cl[6][i]==0)){nm[12]=i;
            rw[2][i]=1; cl[6][i]=1;
            stp24();
            rw[2][i]=0; cl[6][i]=0;}
    }
}
//
/** Set n25 *
void stp24(){
    short i;
    for(i=5;i>=0;i--){
        if((rw[5][i]==0)&&(cl[1][i]==0)){nm[25]=i;
            rw[5][i]=1; cl[1][i]=1;
            stp25();
            rw[5][i]=0; cl[1][i]=0;}
    }
}
//
/** Line #3 & #4 *

```

```

/** Set n13 *
void stp25(){
  short i;
  for(i=0;i<6;i++){
    if((rw[3][i]==0)&&(cl[1][i]==0)){nm[13]=i;
      rw[3][i]=1; cl[1][i]=1;
      stp26();
      rw[3][i]=0; cl[1][i]=0;}
  }
}
//
/** Set n24 *
void stp26(){
  short i;
  for(i=5;i>=0;i--){
    if((rw[4][i]==0)&&(cl[6][i]==0)){nm[24]=i;
      rw[4][i]=1; cl[6][i]=1;
      stp27();
      rw[4][i]=0; cl[6][i]=0;}
  }
}
//
/** Set n14 *
void stp27(){
  short i;
  for(i=0;i<6;i++){
    if((rw[3][i]==0)&&(cl[2][i]==0)){nm[14]=i;
      rw[3][i]=1; cl[2][i]=1;
      stp28();
      rw[3][i]=0; cl[2][i]=0;}
  }
}
//
/** Set n23 *
void stp28(){
  short i;
  for(i=5;i>=0;i--){
    if((rw[4][i]==0)&&(cl[5][i]==0)){nm[23]=i;
      rw[4][i]=1; cl[5][i]=1;
      stp29();
      rw[4][i]=0; cl[5][i]=0;}
  }
}
//
/** Set n15 *
void stp29(){
  short i;
  for(i=0;i<6;i++){
    if((rw[3][i]==0)&&(cl[3][i]==0)&&(pd[i]==0)){nm[15]=i;
      rw[3][i]=1; cl[3][i]=1; pd[i]=1;
      stp30();
      rw[3][i]=0; cl[3][i]=0; pd[i]=0;}
  }
}
//
/** Set n22 *
void stp30(){
  short i;
  for(i=5;i>=0;i--){
    if((rw[4][i]==0)&&(cl[4][i]==0)&&(pd[i]==0)){nm[22]=i;
      rw[4][i]=1; cl[4][i]=1; pd[i]=1;
      stp31();
      rw[4][i]=0; cl[4][i]=0; pd[i]=0;}
  }
}
}

```

```

//
/** Set n16 *
void stp31(){
short i;
for(i=0;i<6;i++){
if((rw[3][i]==0)&&(cl[4][i]==0)&&(pb[i]==0)){nm[16]=i;
rw[3][i]=1; cl[4][i]=1; pb[i]=1;
stp32();
rw[3][i]=0; cl[4][i]=0; pb[i]=0;}
}
}
//
/** Set n21 *
void stp32(){
short i;
for(i=5;i>=0;i--){
if((rw[4][i]==0)&&(cl[3][i]==0)&&(pb[i]==0)){nm[21]=i;
rw[4][i]=1; cl[3][i]=1; pb[i]=1;
stp33();
rw[4][i]=0; cl[3][i]=0; pb[i]=0;}
}
}
//
/** Set n17 *
void stp33(){
short i;
for(i=0;i<6;i++){
if((rw[3][i]==0)&&(cl[5][i]==0)){nm[17]=i;
rw[3][i]=1; cl[5][i]=1;
stp34();
rw[3][i]=0; cl[5][i]=0;}
}
}
//
/** Set n20 *
void stp34(){
short i;
for(i=5;i>=0;i--){
if((rw[4][i]==0)&&(cl[2][i]==0)){nm[20]=i;
rw[4][i]=1; cl[2][i]=1;
stp35();
rw[4][i]=0; cl[2][i]=0;}
}
}
//
/** Set n18 *
void stp35(){
short i;
for(i=0;i<6;i++){
if((rw[3][i]==0)&&(cl[6][i]==0)){nm[18]=i;
rw[3][i]=1; cl[6][i]=1;
stp36();
rw[3][i]=0; cl[6][i]=0;}
}
}
//
/** Set n19 *
void stp36(){
short i;
for(i=5;i>=0;i--){
if((rw[4][i]==0)&&(cl[1][i]==0)){nm[19]=i;
rw[4][i]=1; cl[1][i]=1;
ansrecord();
rw[4][i]=0; cl[1][i]=0;}
}
}

```

```

}
//
/** Record the Answers */
void ansrecord(){
    short n;
    tllu[cnt][0]=cnt+1;
    for(n=1;n<37;n++){tllu[cnt][n]=nm[n];}
    if((nm[1]==0)&&(nm[6]<nm[31])){
        thlu[hcnt][0]=hcnt+1;
        for(n=1;n<37;n++){thlu[hcnt][n]=nm[n];}
        hcnt++;
    }
    cnt++;
}
//
/** Classify and Print the Latin Squares */
void prlunit(){
    long int t,m,n;
    short l,l6;
    printf(" [for High]");
    for(t=0;t<hcnt;t=t+768){
        printf("\n%7ld/%7ld/%7ld/%7ld/%7ld/%7ld/%7ld/%7ld/\n",
            t+1,t+2,t+3,t+4,t+5,t+6,t+7,t+8);
        for(l=0;l<6;l++){l6=l*6;
            for(m=t;m<(t+8);m++){
                printf(" ");
                for(n=1;n<7;n++){printf("%d",thlu[m][l6+n]);}
            }
            printf("\n");
        }
    }
    printf("\n [for Low]");
    for(t=0;t<cnt;t=t+9216){
        printf("\n%7ld/%7ld/%7ld/%7ld/%7ld/%7ld/%7ld/%7ld/\n",
            t+1,t+2,t+3,t+4,t+5,t+6,t+7,t+8);
        for(l=0;l<6;l++){l6=l*6;
            for(m=t;m<(t+8);m++){
                printf(" ");
                for(n=1;n<7;n++){printf("%d",tllu[m][l6+n]);}
            }
            printf("\n");
        }
    }
    printf("\n [Counts of Latin Squares = %dH/%dL]\n",hcnt,cnt);
    for(m=0;m<hcnt;m++){
        for(n=0;n<cnt;n++){
            t=0;
            for(l=1;l<37;l++){if(thlu[m][l]==tllu[n][l]){t++;}}
            mtc[m][n]=t;
            if(mtc[m][n]==6){bcnt++;}
        }
    }
    printf("\n [Reference Table for the Best Combinations = %ld]\n",bcnt);
    printf("\n T1");
    for(n=0;n<20;n++){printf("%3ld",n+1);}
    printf("\n-----");
    for(m=0;m<20;m++){
        printf("\n%3ld",m+1);
        for(n=0;n<20;n++){printf("%3d",mtc[m][n]);}
    }
    printf("\n\n T2");
    for(n=20;n<40;n++){printf("%3ld",n+1);}
    printf("\n-----");
    for(m=20;m<40;m++){
        printf("\n%3ld",m+1);
    }
}

```

```

    for(n=20;n<40;n++){printf("%3d",mtc[m][n]);}
}
printf("\n");
}
//
/** Combine, Compose and Print *
void cmbcmp(){
short n,d,fc;
for(u1=0;u1<hcnt;u1++){cnt2=0;
for(u2=0;u2<lcnt;u2++){
if(mtc[u1][u2]==6){
for(n=1;n<37;n++){uflg[n]=0;}
for(n=1;n<37;n++){
d=thlu[u1][n]*6+tllu[u2][n]+1;
nm[n]=d; uflg[d]++;
}
fc=0;
for(n=1;n<37;n++){if(uflg[n]==0){fc++;}}
cfc[fc]++;
if(fc==0){ //if(fc<=6){
if((nm[1]==1)&&(nm[6]<nm[31])){prans();}
}
}
}
}
}
//
/** Print the Solution List *
void prans(){
short n;
cnt++; cnt2++;
if(cnt2==1){
cntr[cnt3]=cnt; cntr[cnt3+1]=0;
for(n=1;n<37;n++){anm[cnt3][n]=nm[n]; anm[cnt3+1][n]=0;}
anm[cnt3][37]=u1+1; anm[cnt3][38]=u2+1;
cnt3++;
if(cnt3==2){pr2ans(); cnt3=0;}
}
}
//
/** Print 2 Answers *
void pr2ans(){
short lu1,lu2,l,l6,m,n;
printf("%6d/%6d/%18ld#%8d/%6d/%18ld#\n",
anm[0][37],anm[0][38],cntr[0],anm[1][37],anm[1][38],cntr[1]);
for(l=0;l<6;l++){l6=l*6;
for(m=0;m<2;m++){
lu1=anm[m][37]-1; lu2=anm[m][38]-1;
printf(" ");
for(n=1;n<7;n++){printf("%d",thlu[lu1][l6+n]);}
printf(" ");
for(n=1;n<7;n++){printf("%d",tllu[lu2][l6+n]);}
printf(" ");
for(n=1;n<7;n++){printf("%3d",anm[m][l6+n]);}
if(m<1){printf(" ");}
}
printf("\n");
}
}
//
/** Print the Rest One *
void pr1ans(){
short lu1,lu2,l,l6,n;
lu1=anm[0][37]; lu2=anm[0][38];
printf("%6d/%6d/ /N6i%31ld#\n",lu1,lu2,cntr[0]);

```

```

for(l=0;l<6;l++){l6=l*6;
printf(" ");
for(n=1;n<7;n++){printf("%d",thlu[l6-1][l6+n]);}
printf(" ");
for(n=1;n<7;n++){printf("%d",tllu[l6-1][l6+n]);}
printf(" ");
for(n=1;n<7;n++){printf(" %d%d",thlu[l6-1][l6+n],tllu[l6-1][l6+n]);}
printf(" ");
for(n=1;n<7;n++){printf("%3d",anm[0][l6+n]);}
printf("\n");
}
}
//
void cmissn(){
short n;
printf("\n [Count of Missing Numbers]");
for(n=0;n<36;n++){
if(n%6==0){printf("\n");}
printf(" %2d:%7ld,",36-n,cfc[36-n]);
}
printf("\n [No Missing: %ld]\n",cfc[0]);
}
//

```

I was extremely surprised when I knew I had to compose a lot of Latin Squares. Let me show you the next report of my newest result.

**\*\* Composing 'Euler Squares' of Order 6 \*\***  
\* by the 'New Euler's Method' with /D6i \*

[List of Latin Squares]

[for High]

1/	2/	3/	4/	5/	6/	7/	8/
012453	012453	012453	012453	012453	012453	012453	012453
340521	340521	340512	340512	540321	540321	540312	540312
253140	253104	153240	153204	253140	253104	153240	153204
135204	135240	235104	235140	135204	135240	235104	235140
504312	504312	504321	504321	304512	304512	304521	304521
421035	421035	421035	421035	421035	421035	421035	421035
769/	770/	771/	772/	773/	774/	775/	776/
041352	041352	041352	041352	041352	041352	041352	041352
124503	324501	532014	532014	124503	324501	530214	530214
253140	253140	324501	124503	253140	253140	324501	124503
305421	105423	253140	253140	305421	105423	253140	253140
532014	532014	105423	305421	530214	530214	105423	305421
410235	410235	410235	410235	412035	412035	412035	412035
1537/	1538/	1539/	1540/	1541/	1542/	1543/	1544/
012543	012543	012543	012543	012543	012543	012543	012543
350421	350421	350412	350412	450321	450321	450312	450312
243150	243105	143250	143205	243150	243105	143250	143205
134205	134250	234105	234150	134205	134250	234105	234150
405312	405312	405321	405321	305412	305412	305421	305421
521034	521034	521034	521034	521034	521034	521034	521034
2305/	2306/	2307/	2308/	2309/	2310/	2311/	2312/
041532	041532	041532	041532	041532	041532	041532	041532
123405	452013	452013	523401	123405	450213	450213	523401
235140	523401	123405	235140	235140	523401	123405	235140
504321	235140	235140	104325	504321	235140	235140	104325
452013	104325	504321	452013	450213	104325	504321	450213
310254	310254	310254	310254	312054	312054	312054	312054

3073/	3074/	3075/	3076/	3077/	3078/	3079/	3080/
012534	012534	012534	012534	012534	012534	012534	012534
350421	350421	350412	350412	450321	450321	450312	450312
234150	234105	134250	134205	234150	234105	134250	134205
143205	143250	243105	243150	143205	143250	243105	243150
405312	405312	405321	405321	305412	305412	305421	305421
521043	521043	521043	521043	521043	521043	521043	521043

3841/	3842/	3843/	3844/	3845/	3846/	3847/	3848/
031542	031542	031542	031542	031524	031524	031524	031524
345201	345201	315204	315204	325401	325401	315402	315402
152430	102435	452130	402135	154230	104235	254130	204135
403125	453120	103425	153420	203145	253140	103245	153240
210354	210354	240351	240351	410352	410352	420351	420351
524013	524013	524013	524013	542013	542013	542013	542013

4609/	4610/	4611/	4612/	4613/	4614/	4615/	4616/
012534	012534	012534	012534	012534	012534	012534	012534
154023	354021	435210	435210	150423	350421	435210	435210
243105	243105	354021	154023	324051	124053	241305	241305
320451	120453	243105	243105	241305	241305	350421	150423
435210	435210	120453	320451	435210	435210	124053	324051
501342	501342	501342	501342	503142	503142	503142	503142

5377/	5378/	5379/	5380/	5381/	5382/	5383/	5384/
032451	032451	032451	032451	032541	032541	032541	032541
415230	415230	410235	410235	145230	145230	350124	450123
153024	253014	153024	253014	203415	203415	421053	321054
204513	104523	204513	104523	450123	350124	203415	203415
320145	320145	325140	325140	321054	421053	145230	145230
541302	541302	541302	541302	514302	514302	514302	514302

6145/	6146/	6147/	6148/	6149/	6150/	6151/	6152/
012543	012543	012543	012543	012543	012543	012543	012543
153024	153024	234105	534102	150324	150324	234105	534102
534102	234105	345210	345210	534102	234105	345210	345210
345210	345210	501432	201435	345210	345210	501432	201435
201435	501432	153024	153024	201435	501432	150324	150324
420351	420351	420351	420351	423051	423051	423051	423051

6913/	6914/	6915/	6916/	6917/	6918/	6919/	6920/
031524	031524	031524	031524	031542	031542	031542	031542
250413	350412	425130	425130	324150	324150	320154	320154
314052	214053	103245	103245	145023	245013	145023	245013
103245	103245	350412	250413	203415	103425	203415	103425
425130	425130	214053	314052	510234	510234	514230	514230
542301	542301	542301	542301	452301	452301	452301	452301

[for Low]

1/	2/	3/	4/	5/	6/	7/	8/
012453	012453	012453	012453	012534	012534	012534	012534
231504	431502	543021	543021	124053	324051	435210	435210
354210	354210	431502	231504	543102	543102	324051	124053
405132	205134	354210	354210	350421	150423	543102	543102
543021	543021	205134	405132	435210	435210	150423	350421
120345	120345	120345	120345	201345	201345	201345	201345

9217/	9218/	9219/	9220/	9221/	9222/	9223/	9224/
012345	012345	012345	012345	012543	012543	012543	012543
135024	435021	543210	543210	451320	451320	450321	450321
254103	254103	435021	135024	234015	324015	234015	324015
420531	120534	254103	254103	325104	235104	325104	235104
543210	543210	120534	420531	540231	540231	541230	541230
301452	301452	301452	301452	103452	103452	103452	103452

18433/	18434/	18435/	18436/	18437/	18438/	18439/	18440/
102435	102435	102435	102435	102543	102543	102543	102543
025143	325140	534201	534201	230415	453120	453120	530412
453012	453012	325140	025143	345201	530412	230415	345201
341520	041523	453012	453012	514032	345201	345201	214035
534201	534201	041523	341520	453120	214035	514032	453120
210354	210354	210354	210354	021354	021354	021354	021354
27649/	27650/	27651/	27652/	27653/	27654/	27655/	27656/
102534	102534	102534	102534	102543	102543	102543	102543
054123	054123	243015	543012	053214	053214	043215	043215
543012	243015	435201	435201	524031	514032	425031	415032
435201	435201	510342	210345	415302	425301	514302	524301
210345	510342	054123	054123	340125	340125	350124	350124
321450	321450	321450	321450	231450	231450	231450	231450
36865/	36866/	36867/	36868/	36869/	36870/	36871/	36872/
201345	201345	201345	201345	201534	201534	201534	201534
015234	415230	543102	543102	140325	354210	354210	540321
354021	354021	415230	015234	435102	540321	140325	435102
432510	032514	354021	354021	523041	435102	435102	123045
543102	543102	032514	432510	354210	123045	523041	354210
120453	120453	120453	120453	012453	012453	012453	012453
46081/	46082/	46083/	46084/	46085/	46086/	46087/	46088/
301254	301254	301254	301254	301254	301254	301254	301254
423510	423510	423501	423501	523410	523410	523401	523401
154032	154023	054132	054123	154032	154023	054132	054123
045123	045132	145023	145032	045123	045132	145023	145032
532401	532401	532410	532410	432501	432501	432510	432510
210345	210345	210345	210345	210345	210345	210345	210345
55297/	55298/	55299/	55300/	55301/	55302/	55303/	55304/
301245	301245	301245	301245	301542	301542	301542	301542
025314	425310	542103	542103	453210	453210	450213	450213
154032	154032	425310	025314	124305	214305	124305	214305
413520	013524	154032	154032	215034	125034	215034	125034
542103	542103	013524	413520	540123	540123	543120	543120
230451	230451	230451	230451	032451	032451	032451	032451
64513/	64514/	64515/	64516/	64517/	64518/	64519/	64520/
401235	401235	401235	401235	401235	401235	401235	401235
324510	324510	324501	324501	524310	524310	524301	524301
135042	135024	035142	035124	135042	135024	035142	035124
053124	053142	153024	153042	053124	053142	153024	153042
542301	542301	542310	542310	342501	342501	342510	342510
210453	210453	210453	210453	210453	210453	210453	210453
73729/	73730/	73731/	73732/	73733/	73734/	73735/	73736/
401523	401523	401523	401523	401532	401532	401532	401532
054312	054312	132045	532041	054213	054213	123045	523041
532041	132045	325104	325104	523041	123045	235104	235104
325104	325104	540231	140235	235104	235104	540321	140325
140235	540231	054312	054312	140325	540321	054213	054213
213450	213450	213450	213450	312450	312450	312450	312450
82945/	82946/	82947/	82948/	82949/	82950/	82951/	82952/
501234	501234	501234	501234	501234	501234	501234	501234
042513	042513	042315	042315	142503	142503	142305	142305
253041	453021	253041	453021	253140	453120	253140	453120
435120	235140	435120	235140	435021	235041	435021	235041
124305	124305	124503	124503	024315	024315	024513	024513
310452	310452	310452	310452	310452	310452	310452	310452

[Counts of Latin Squares = 7680H/92160L]



[Reference Table for the Best Combinations = 112008960]

T1\	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	13	14	13	15	8	11	6	6	13	11	11	9	11	9	9	7	8	10	5	6
2	12	12	15	18	9	12	5	5	11	13	9	11	9	11	7	9	7	9	6	7
3	17	20	10	10	6	7	9	11	12	10	12	10	10	8	10	8	7	9	7	8
4	16	18	12	13	7	8	8	10	10	12	10	12	8	10	8	10	6	8	8	9
5	11	12	14	16	8	10	6	7	12	10	10	8	12	10	10	8	8	9	5	7
6	10	10	16	19	9	11	5	6	10	12	8	10	10	12	8	10	7	8	6	8
7	15	18	11	11	6	6	9	12	11	9	11	9	11	9	11	9	7	8	7	9
8	14	16	13	14	7	7	8	11	9	11	9	11	9	11	9	11	6	7	8	10
9	4	6	11	12	18	16	7	8	6	7	5	6	7	8	6	7	11	10	9	9
10	6	4	12	13	16	18	8	9	8	8	7	7	9	9	8	8	11	12	8	10
11	5	6	4	6	9	10	18	16	8	7	8	7	7	6	7	6	11	11	11	10
12	6	7	6	4	10	11	16	18	9	9	9	9	8	8	8	8	10	12	11	12
13	10	12	15	16	6	8	6	7	18	16	14	12	16	14	12	10	8	11	5	5
14	9	10	17	19	7	9	5	6	16	18	12	14	14	16	10	12	7	10	6	6
15	12	14	14	15	5	7	8	9	14	12	18	16	12	10	16	14	6	7	8	10
16	11	12	16	18	6	8	7	8	12	14	16	18	10	12	14	16	5	6	9	11
17	8	10	18	19	7	8	5	7	16	14	12	10	18	16	14	12	9	11	4	5
18	7	8	20	22	8	9	4	6	14	16	10	12	16	18	12	14	8	10	5	6
19	10	12	17	18	6	7	7	9	12	10	16	14	14	12	18	16	7	7	7	10
20	9	10	19	21	7	8	6	8	10	12	14	16	12	14	16	18	6	6	8	11
T2\	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
21	10	9	8	7	9	8	7	6	5	7	9	10	5	6	6	5	13	11	13	11
22	10	9	8	7	9	8	7	6	6	6	11	10	4	5	7	6	11	13	11	13
23	6	7	8	9	7	8	9	10	5	6	7	9	6	5	7	8	11	9	15	13
24	6	7	8	9	7	8	9	10	7	6	8	8	7	6	6	7	9	11	13	15
25	6	7	6	7	7	8	7	8	5	7	9	10	5	5	12	12	22	18	8	8
26	8	9	4	5	9	10	5	6	7	5	10	11	5	5	12	12	18	22	8	8
27	8	7	8	7	7	6	7	6	9	10	5	7	10	10	5	5	10	10	22	18
28	6	5	10	9	5	4	9	8	10	11	7	5	10	10	5	5	10	10	18	22
29	9	8	7	6	10	9	8	7	6	8	9	10	6	7	6	5	11	9	11	9
30	9	8	7	6	10	9	8	7	7	7	11	10	5	6	7	6	9	11	9	11
31	7	8	9	10	6	7	8	9	7	8	8	10	8	7	8	9	11	9	13	11
32	7	8	9	10	6	7	8	9	9	8	9	9	9	8	7	8	9	11	11	13
33	10	13	7	10	10	13	7	10	10	8	8	7	9	7	12	11	24	22	9	10
34	7	5	9	7	7	5	9	7	6	5	10	8	8	7	9	7	9	10	24	22
35	7	5	11	9	8	6	12	10	7	6	8	10	9	8	7	9	10	11	22	24
36	13	16	8	11	12	15	7	10	8	10	9	8	7	9	13	12	22	24	10	11
37	16	12	14	10	15	11	13	9	12	10	11	10	14	12	15	14	10	12	9	10
38	12	16	10	14	11	15	9	13	12	10	13	12	12	11	18	16	13	15	7	8
39	14	10	14	10	13	9	13	9	12	10	12	11	14	12	16	15	9	11	9	10
40	10	14	10	14	9	13	9	13	12	10	14	13	12	11	19	17	12	14	7	8

\* List of Composed Solutions of 'Euler' MS66 \*  
 [Count = 0]

I was really surprised at the big counts of Latin Squares. All of them are of Euler type, as you see. I must confess I was delighted by these wonderful results at first.

But I was shocked at the next moment when I read the report from my computer that it could not really make any compositions at all. It checked all the cases of 7680 x 92160 combinations, but it could find no correct answers at all.

I wondered what was wrong with it. I tried first to find any bugs in my program, tested and checked everything many times. But finally I came to know everything tells

me that it is impossible for us to compose any 'Greco-Latin' Squares of Order 6, even when we make only the basic type of Standard Magic Squares 6x6.

#6. The next list is printed out so that we could know what the problem is.

[Count of Missing Numbers]

36: 0, 35: 0, 34: 0, 33: 0, 32: 0, 31: 0,  
 30: 585568, 29: 0, 28: 628176, 27: 0, 26: 725728, 25: 0,  
 24: 232656, 23: 0, 22: 0, 21: 120954, 20: 112112, 19: 0,  
 18:1062664, 17:1028688, 16:1424014, 15:1912124, 14:5295216, 13:1384918,  
 12:5973378, 11:1640468, 10:7582804, 9: 988896, 8: 0, 7: 0,  
 6:1236860, 5: 0, 4: 0, 3: 0, 2: 0, 1: 0;

[No Missing: 0]

[Examples of Wrong Compositions of 'Euler' MS66]

1/ 012453 340521 253140 135204 504312 421035	51/ 013524 425310 541203 230451 154032 302145	1# 1 8 16 30 33 23 23 27 6 34 14 7 18 35 20 9 25 4 9 22 31 17 6 26 32 6 29 19 10 15 28 13 9 2 23 36	2/ 012453 340521 253104 135240 504312 421035	82/ 014532 342150 521403 250314 435021 103245	199# 1 8 17 30 34 21 22 29 3 32 18 7 18 33 20 11 1 28 9 24 31 16 26 5 35 4 30 19 9 14 26 13 10 3 23 36
3/ 012453 340512 153240 235104 504321 421035	2314/ 051432 542103 403521 314250 235014 120345	259# 1 12 14 29 34 21 24 29 3 32 7 16 11 31 22 18 27 2 16 20 35 9 6 25 33 4 30 19 14 11 26 15 7 4 23 36	4/ 012453 340512 153204 235140 504321 421035	506/ 015234 143520 532041 401352 250413 324105	315# 1 8 18 27 34 23 20 29 4 36 9 13 12 34 21 13 5 26 17 19 32 10 30 3 33 6 25 23 14 10 28 15 11 2 19 36
5/ 012453 540321 253140 135204 304512 421035	511/ 015234 240513 431052 502341 153420 324105	377# 1 8 18 27 34 23 33 29 1 24 14 10 17 34 20 7 30 3 12 19 33 16 5 26 20 6 28 35 9 13 28 15 11 2 19 36	6/ 012453 540321 253104 135240 304512 421035	2311/ 051432 235104 314520 403251 542013 120345	439# 1 12 14 29 34 21 33 28 6 20 13 11 16 32 23 12 3 25 11 19 34 15 30 2 24 5 27 31 8 16 26 15 7 4 23 36
7/ 012453 540312 153240 235104 304521 421035	87/ 014532 432051 251304 520413 345120 103245	493# 1 8 17 30 34 21 35 28 3 19 12 14 9 36 20 16 25 5 18 21 31 11 2 28 22 5 30 32 15 7 26 13 10 3 23 36	8/ 012453 540312 153204 235140 304521 421035	172/ 013524 324051 541203 450312 235140 102435	557# 1 8 16 30 33 23 34 27 5 19 12 14 12 35 20 15 1 28 17 24 31 10 26 3 21 4 30 32 17 7 26 13 9 5 22 36
9/ 014523 143052 532104 250341 325410 401235	95/ 014352 540231 253410 425103 301524 132045	753# 1 8 29 34 18 21 12 29 19 3 34 14 33 24 16 11 2 25 17 33 6 20 25 10 22 13 32 30 9 5 26 4 9 13 23 36	10/ 014523 243051 532104 150342 325410 401235	77/ 014352 325104 401523 253410 542031 130245	815# 1 8 29 34 18 21 16 27 24 2 31 11 35 19 14 12 3 28 9 36 4 23 26 13 24 17 33 25 10 2 26 4 7 15 23 36
11/ 014523 325410 243051 532104 150342 401235	161/ 013254 245103 301542 452310 534021 120435	879# 1 8 28 33 18 23 21 17 36 26 7 4 16 25 20 6 35 9 35 24 15 10 2 25 12 34 5 19 27 14 26 3 7 17 22 36	12/ 014523 325410 143052 532104 250341 401235	655/ 024351 541032 153420 415203 302514 230145	941# 1 9 29 34 18 20 24 17 32 25 10 3 8 30 22 5 33 13 35 20 18 9 1 28 16 31 3 24 26 11 27 4 7 14 23 36

13/	101/		1001#	14/	61/		1061#
014253	015432	1	8 30 17 34 21	014253	013452	1	8 28 17 36 21
345021	341250	22	29 32 3 18 7	345021	321504	22	27 32 6 13 11
132540	253014	9	24 16 31 26 5	132504	254310	9	24 17 34 2 25
251304	524103	18	33 11 20 1 28	251340	405123	17	31 12 20 27 4
503412	430521	35	4 19 30 9 14	503412	540231	36	5 19 27 10 14
420135	102345	26	13 3 10 23 36	420135	132045	26	16 3 7 23 36
15/	385/		1259#	16/	2433/		1321#
014253	012534	1	8 27 18 34 23	014253	054132	1	12 29 14 34 21
345012	145320	20	29 36 4 9 13	345012	541203	24	29 32 3 7 16
231540	403152	17	19 10 32 30 3	231504	312450	16	20 9 35 6 25
152304	530241	12	34 13 21 5 26	152340	405321	11	31 18 22 27 2
503421	254013	33	6 23 25 14 10	503421	230514	33	4 19 30 14 11
420135	321405	28	15 2 11 19 36	420135	123045	26	15 4 7 23 36
17/	2432/		1377#	18/	392/		1431#
014253	054132	1	12 29 14 34 21	014253	012534	1	8 27 18 34 23
543021	231504	33	28 20 6 13 11	543021	245013	33	29 24 1 14 10
132540	402351	11	19 15 34 30 2	132504	503241	12	19 16 33 5 26
251304	315420	16	32 12 23 3 25	251340	430152	17	34 7 20 30 3
305412	540213	24	5 31 27 8 16	305412	154320	20	6 35 28 9 13
420135	123045	26	15 4 7 23 36	420135	321405	28	15 2 11 19 36
19/	243/		1493#	20/	107/		1689#
014253	015324	1	8 30 16 33 23	014253	015432	1	8 30 17 34 21
543012	320451	34	27 19 5 12 14	543012	430251	35	28 19 3 12 14
231540	453012	17	24 10 31 26 3	231504	254013	15	24 11 31 2 28
152304	542103	12	35 15 20 1 28	152340	523104	12	33 16 20 25 5
305421	231540	21	4 32 30 17 7	305421	341520	22	5 32 30 15 7
420135	104235	26	13 5 9 22 36	420135	102345	26	13 3 10 23 36
1001/	513/		94001#	1002/	1/		94055#
043521	015243	1	26 24 33 17 10	043521	012453	1	26 21 35 18 10
125340	120354	8	15 31 22 30 5	125340	231504	9	16 32 24 25 5
514203	453021	35	12 28 13 3 20	514203	354210	34	12 29 15 2 19
430152	501432	30	19 2 11 34 15	230154	405132	17	19 6 8 34 27
251034	342510	16	35 9 6 20 25	451032	543021	30	35 10 1 21 14
302415	234105	21	4 17 26 7 36	302415	120345	20	3 13 28 11 36
1003/	514/		94251#	1004/	2/		94307#
043521	015243	1	26 24 33 17 10	043521	012453	1	26 21 35 18 10
230154	342510	16	23 3 12 32 25	430152	431502	29	22 2 12 31 15
451032	501432	30	31 8 5 22 15	251034	354210	16	36 11 3 20 25
514203	420351	35	9 25 16 6 20	514203	205134	33	7 30 14 4 23
125340	153024	8	18 34 19 27 5	125340	543021	12	17 34 19 27 2
302415	234105	21	4 17 26 7 36	302415	120345	20	3 13 28 11 36
1005/	17/		94505#	1006/	19/		94567#
043152	012534	1	26 21 12 34 17	043152	012534	1	26 21 12 34 17
124503	120453	8	15 25 35 6 22	324501	435210	23	16 30 33 2 7
251340	354021	16	36 11 19 27 2	251340	541302	18	35 8 22 25 3
305421	541302	24	5 32 28 13 9	105423	320451	10	3 31 29 18 20
512034	435210	35	10 18 3 20 25	512034	154023	32	12 17 1 21 28
430215	203145	27	19 4 14 11 36	430215	203145	27	19 4 14 11 36
1007/	19/		94627#	1008/	17/		94689#
043152	012534	1	26 21 12 34 17	043152	012534	1	26 21 12 34 17
512034	435210	35	10 18 3 20 25	512034	120453	32	9 13 5 24 28
324501	541302	24	17 26 34 1 9	124503	354021	10	18 29 31 3 20
251340	320451	16	33 7 23 30 2	251340	541302	18	35 8 22 25 3
105423	154023	8	6 35 25 15 22	305421	435210	23	4 36 27 14 7
430215	203145	27	19 4 14 11 36	430215	203145	27	19 4 14 11 36

1009/	67/		94753#	1010/	65/		94813#
043521	014253	1	26 23 33 18 10	043521	014253	1	26 23 33 18 10
125340	543021	12	17 34 19 27 2	125340	235104	9	16 36 20 25 5
431052	352410	28	24 9 5 32 13	231054	401532	17	19 8 6 34 27
512403	435102	35	10 18 26 1 21	512403	352410	34	12 15 29 2 19
250134	201534	15	31 2 12 22 29	450132	543021	30	35 4 7 21 14
304215	120345	20	3 25 16 11 36	304215	120345	20	3 25 16 11 36
.	.	.	.	.	.	.	.

Although all Latin Squares certainly belong to the Euler type, any combination of the two units picked up out of them could not make any correct answer at all. It is because we could not avoid repeating usage of certain numbers and missing of some others in any composed solution, while we must obey another Euler's Condition every time: We must use each one of the serial integers {00, 01, 02, 03, 04, 05, 10, 11, 12, 13, 14, 15, 20, . . . , 50, 51, 52, 53, 54, 55<sub>(N6i)</sub>} strictly once, and must not use any number twice or more often.

In fact we could not compose any Greco-Latin Square of Order 6, even when we want to make them for the basic type of Standard Magic Squares 6x6.

That means all the solutions in page 9-10 are really the "Non-Euler type" of them and there are no Euler Squares among the solution set of Standard Magic Squares of Order 6. What a conclusion it is! I have had no idea about that.

But, we cannot help accepting it, however embarrassing this conclusion might be.

It is in the Order 6 that we have got any "Non-Euler type" of Magic Squares for the first time. We now know there exist Complete Euler Squares and nothing else among the solution set of Pan-diagonal Magic Squares of Order 4 and 5. We also know there exist both the Complete Euler Squares and many Non-Euler ones among the solution sets of Pan-diagonal Magic Squares of Orders 7, 8, 9, . . .

It is in the case of Order 6 when anything new like 'Non-Euler type' was born. It is also in the case of Order 6 when 'nothingness' begins to exist. Why!

(Written on Sep. 1, 2001; Revised on Nov. 5, 2005 and Apr.12, 2006;  
 Recompiled on Feb. 11, 2015 with MacOSX 10.10.2 and Xcode 6.1.1)

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 by Kanji Setsuda: E-Mail Address:<jag12001@nifty.com>