

Part 3 : New Algebraic Studies of Magic Squares: Kanji Setsuda
Chapter 2 : New Algebraic Study of Magic Squares of Order 4
Section 1 : Self-Complementary Magic Squares of Order 4

#0. First Definition

What I am studying intensively now is concentrated to some specific types of magic squares and cubes: One is **Self-Complementary** type, and another is **Pan-Diagonal** one.

The former is sometimes called in a different way such as 'Self-Similar' type or 'Symmetrical type' (with respect to the geometric center), etc.

The concept of **Self-Complementary** type is described like this:
 Let's take an example of order 4 and use those diagrams below.

1. When you rotate the object in 180 degrees, add the result to the original, and you will find each sum of the two numbers in corresponding positions is always equal to the magic constant $C(=17)$.
 When you move every number to the symmetric position, you will get the same result. Add this to the original, and you will find each sum of the two numbers in corresponding positions is always equal to 17.
2. When you transform the square by some simple rules of exchanging every Complementary Pairs, rotate the result in 180 degrees, and you will find the similar square to the original, say, the same with itself.

**** Concept of Self-Complementary Magic Square 4*4 ****

[1]

1	15	14	4	16	2	3	13	17	17	17	17
12	6	7	9	5	11	10	8	17	17	17	17
8	10	11	5	9	7	6	12	17	17	17	17
13	3	2	16	4	14	15	1	17	17	17	17
1	15	14	4	16	2	3	13	1	15	14	4
12	6	7	9	5	11	10	8	12	6	7	9
8	10	11	5	9	7	6	12	8	10	11	5
13	3	2	16	4	14	15	1	13	3	2	16

[2]

1	15	12	6	16	2	5	11	17	17	17	17
14	4	7	9	3	13	10	8	17	17	17	17
8	10	13	3	9	7	4	14	17	17	17	17
11	5	2	16	6	12	15	1	17	17	17	17
1	15	12	6	16	2	5	11	1	15	12	6
14	4	7	9	3	13	10	8	14	4	7	9
8	10	13	3	9	7	4	14	8	10	13	3
11	5	2	16	6	12	15	1	11	5	2	16

[3]

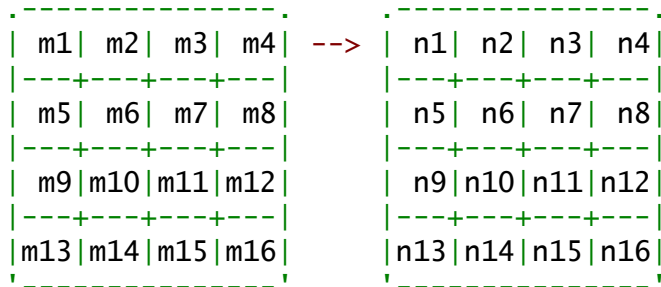
1	14	12	7	16	3	5	10	17	17	17	17
15	4	6	9	2	13	11	8	17	17	17	17
8	11	13	2	9	6	4	15	17	17	17	17
10	5	3	16	7	12	14	1	17	17	17	17

```

1 14 12 7   16 3 5 10   1 14 12 7
15 4 6 9    2 13 11 8   15 4 6 9
8 11 13 2   9 6 4 15    8 11 13 2
10 5 3 16   7 12 14 1   10 5 3 16

```

[Complementary Transformation]



```

[ n1=17-m1;   n2=17-m2;   n3=17-m3;   n4=17-m4;
  n5=17-m5;   n6=17-m6;   n7=17-m7;   n8=17-m8;
  n9=17-m9;   n10=17-m10; n11=17-m11;  n12=17-m12;
  n13=17-m13; n14=17-m14; n15=17-m15;  n16=17-m16;]

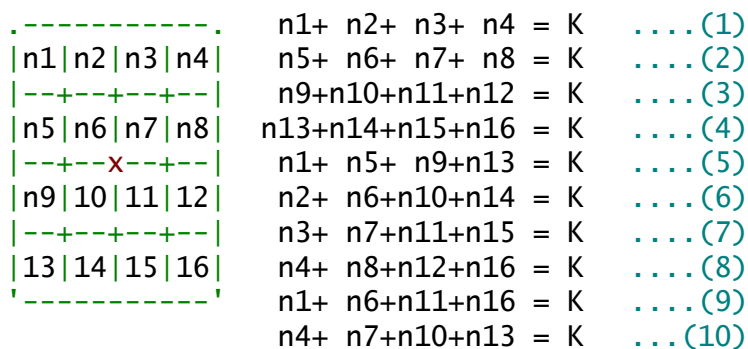
```

Because both tell us the same concept, I call this type of magic square 'Self-Complementary type' (with all Complementary Pairs located symmetrically with respect to the geometric center), and at the first stage of mathematical argument I define it as shown below:

#1. Basic Form and Basic Conditions

We prepare the basic diagram and basic equations for our object.

[Figure 1: Basic Form and Basic Conditions]



```

and n1+n16=n2+n15=n3+n14=n4+n13
    =n5+n12+n6+n11=n7+n10=n8+n9=C   ... (11)

```

The 8 pairs (n1, n16), (n2, n15), (n3, n14), (n4, n13), (n5, n12), (n6, n11), (n7, n10), and (n8, n9) of (11) are usually called "Complementary Pairs of C."

When m+n=C(onstant), we say "(m, n) is one of the Complementary Pairs of C." They must be placed symmetrically with respect to geometric center 'x' of the square.

The constants C and K could be determined in advance.

(1)+(2)+(3)+(4)

$$n1+n2+n3+\dots+n15+n16 = 4K$$

$$(16+1) \times 16 / 2 = 4 \times K$$

$$136 = 4 \times K$$

Therefore $K = 136/4 = 34$

(9) $n1+n6+n11+n16=(n1+n16)+(n6+n11)= CC + CC = K = 34$
 Therefore $CC = K / 2 = 34/2 = 17$

#2. Complementary Pairs and New Notations

Let me introduce you a new notation of Complementary Pairs:

$$n1+n1=n2+n2=n3+n3=\dots=n8+n8=\dots=n15+n15=n16+n16=C$$

These new notations mean $n1=C-n1=n16$, $n2=C-n2=n15$,

$$n3=C-n3=n14, \dots, n8=C-n8=n9 (\dots, n16=C-n16=n1)$$

Of course $C=17$

We can rewrite the Basic Figure 1 by these notations as follows:

[Figure 2: Complementary Pairs in their New Notation]

.-----.	$n1+ n2+ n3+ n4 = K$(1)'
n1 n2 n3 n4	$n5+ n6+ n7+ n8 = K$(2)'
---+---+---+	$n8+ n7+ n6+ n5 = K$(3)'
n5 n6 n7 n8	$n4+ n3+ n2+ n1 = K$(4)'
---+---+---+	$n1+ n5+ n8+ n4 = K$(5)'
n8 n7 n6 n5	$n2+ n6+ n7+ n3 = K$(6)'
---+---+---+	$n3+ n7+ n6+ n2 = K$(7)'
n4 n3 n2 n1	$n4+ n8+ n5+ n1 = K$(8)'
.-----.	$n1+ n6+ n6+ n1 = K$(9)'
	$n4+ n7+ n7+ n4 = K$... (10)'

$$n1+n2+n2+n1=(n1+n1)+(n2+n2)=34(=K)$$

17 17

$$n1+n2+n3+n4 = K \dots(1)'$$

Therefore $n2+n1 = n3+n4 = K-(n1+n2)$

That is $n3+n4 = n15+n16$

In the same way $n7+n8 = n6+n5 = n11+n12$ is worked out.

$$n1+n2 = n4+n3 = n13+n14; n5+n6 = n8+n7 = n9+n10;$$

$$n1+n5 = n4+n8; n2+n6 = n3+n7; n9+n13 = n12+n16; n10+n14 = n11+n15$$

#3. Block-Sums and their Relations

Calculate four block-sums below and compare the results.

Block A Block B [Figure 2(again)]

.-----.	$n1+ n2+ n3+ n4 = K$(1)'
n1 n2 n3 n4	$n5+ n6+ n7+ n8 = K$(2)'
---+--- ---+---	$n8+ n7+ n6+ n5 = K$(3)'
n5 n6 n7 n8	$n4+ n3+ n2+ n1 = K$(4)'
---+--- ---+---	$n1+ n5+ n8+ n4 = K$(5)'
n8 n7 n6 n5	$n2+ n6+ n7+ n3 = K$(6)'
---+--- ---+---	$n3+ n7+ n6+ n2 = K$(7)'
n4 n3 n2 n1	$n4+ n8+ n5+ n1 = K$(8)'
.-----.	$n1+ n6+ n6+ n1 = K$(9)'
	$n4+ n7+ n7+ n4 = K$... (10)'

Block C Block D

$$\{Block A\}+\{Block B\}=(n1+n2+n3+n4)+(n5+n6+n7+n8)=34+34=68$$

$$\{Block A\}+\{Block D\}=(n1+n1)+(n2+n2)+(n5+n5)+(n6+n6)=4 \times 17=68$$

17 17 17 17

$$\{Block B\}+\{Block D\}=(n3+n7+n6+n2)+(n4+n8+n5+n1)=34+34=68$$

34 34

$$\{\text{Block B}\} + \{\text{Block C}\} = \underbrace{(n3+n3)}_{17} + \underbrace{(n4+n4)}_{17} + \underbrace{(n7+n7)}_{17} + \underbrace{(n8+n8)}_{17} = 4 \times 17 = 68$$

Therefore $\{\text{Block A}\} = \{\text{Block B}\} = \{\text{Block C}\} = \{\text{Block D}\} = 34$

#4. Which Lines Can We Exchange with Each Other?

New notation of Complementary Pairs is very useful for you to find which rows and columns you can exchange with each other.

[Figure 2(again):

Which rows and columns can we exchange?]

	(ii-iii)Exchange	(II-III)Exchange	(ii-iii)&(II-III)
I n1 n2 n3 n4 --+--+--+--+ II n5 n6 n7 n8 --+--+--+--+ III n8 n7 n6 n5 --+--+--+--+ IV n4 n3 n2 n1 --+--+--+--+ i ii iii iv	n1 n3 n2 n4 I --+--+--+--+ II n5 n7 n6 n8 II --+--+--+--+ III n8 n6 n7 n5 III --+--+--+--+ IV n4 n2 n3 n1 IV --+--+--+--+ i iii ii iv	n1 n2 n3 n4 I --+--+--+--+ III n8 n7 n6 n5 III --+--+--+--+ II n5 n6 n7 n8 II --+--+--+--+ IV n4 n3 n2 n1 IV --+--+--+--+ i ii iii iv	n1 n3 n2 n4 I --+--+--+--+ III n8 n6 n7 n5 III --+--+--+--+ II n5 n7 n6 n8 II --+--+--+--+ IV n4 n2 n3 n1 IV --+--+--+--+ i iii ii iv

Every row and every column adds up to the same constant, because the members of each row and column are not changed but just for their permutations.

The primary diagonals are partially changed, but the sums of new diagonals are all equal to the magic constant.

$$n1+n6+n6+n1 = K; \quad n1+n7+n7+n1 = K; \quad n1+n7+n7+n1 = K; \quad n1+n6+n6+n1 = K;$$

$$n4+n7+n7+n4 = K; \quad n4+n6+n6+n4 = K; \quad n4+n6+n6+n4 = K; \quad n4+n7+n7+n4 = K;$$

We can exchange (2nd ↔ 3rd) rows/columns with each other.

In the same way (1st ↔ 4th) rows/columns are always exchangeable.

Next diagrams will tell you another way how to exchange lines.

[Figure 3:

Which rows and columns can we exchange?]

	(i-ii)&(iii-iv)	(I-II)&(III-IV)	
I n1 n2 n3 n4 --+--+--+--+ II n5 n6 n7 n8 --+--+--+--+ III n8 n7 n6 n5 --+--+--+--+ IV n4 n3 n2 n1 --+--+--+--+ i ii iii iv	n2 n1 n4 n3 I --+--+--+--+ II n6 n5 n8 n7 II --+--+--+--+ III n7 n8 n5 n6 III --+--+--+--+ IV n3 n4 n1 n2 IV --+--+--+--+ ii i iv iii	n5 n6 n7 n8 II --+--+--+--+ I n1 n2 n3 n4 I --+--+--+--+ IV n4 n3 n2 n1 IV --+--+--+--+ III n8 n7 n6 n5 III --+--+--+--+ i ii iii iv	n6 n5 n8 n7 II --+--+--+--+ I n2 n1 n4 n3 I --+--+--+--+ IV n3 n4 n1 n2 IV --+--+--+--+ III n7 n8 n5 n6 III --+--+--+--+ ii i iv iii
	$\{(i-ii)\&(iii-iv)\}\&\{(I-II)\&(III-IV)\}$		

If you want to exchange 1st ↔ 2nd rows/columns, then you must 3rd ↔ 4th rows/columns each other at the same time.

#5. How many Solutions can we find?

Let's make all the self-complementary magic squares of order 4 actually with these properties. How many solutions in all can we make for them?

There are three levels of solution sets for any object of any order:

- (1) 'Primitive Solutions' mean all object forms pre-normalized without any selection.
- (2) 'Standard Solutions' mean the selected ones removing the other same forms when transformed by some simple reflection and repeating rotations.
- (3) 'Fundamental Solutions' mean the fewest ones, selected strictly by some reasonable transformation system. They usually have their $n1=1$ on the left top.

Let me show you the list of 'Primitive Solutions' of our object at first.

**** Self-Complementary Magic Squares of Order 4: ****

**** Compact List of the 384 Primitive Solutions ****

1/		2/		3/		4/		5/		6/													
1	8	15	10	1	8	14	11	1	8	15	10	1	8	12	13	1	8	14	11	1	8	12	13
12	13	6	3	12	13	7	2	14	11	4	5	14	11	7	2	15	10	4	5	15	10	6	3
14	11	4	5	15	10	4	5	12	13	6	3	15	10	6	3	12	13	7	2	14	11	7	2
7	2	9	16	6	3	9	16	7	2	9	16	4	5	9	16	6	3	9	16	4	5	9	16
7/		8/		9/		10/		11/		12/													
1	12	15	6	1	12	14	7	1	12	15	6	1	12	8	13	1	12	14	7	1	12	8	13
8	13	10	3	8	13	11	2	14	7	4	9	14	7	11	2	15	6	4	9	15	6	10	3
14	7	4	9	15	6	4	9	8	13	10	3	15	6	10	3	8	13	11	2	14	7	11	2
11	2	5	16	10	3	5	16	11	2	5	16	4	9	5	16	10	3	5	16	4	9	5	16
13/		14/		15/		16/		17/		18/													
1	14	15	4	1	14	12	7	1	14	15	4	1	14	8	11	1	14	12	7	1	14	8	11
8	11	10	5	8	11	13	2	12	7	6	9	12	7	13	2	15	4	6	9	15	4	10	5
12	7	6	9	15	4	6	9	8	11	10	5	15	4	10	5	8	11	13	2	12	7	13	2
13	2	3	16	10	5	3	16	13	2	3	16	6	9	3	16	10	5	3	16	6	9	3	16
19/		20/		21/		22/		23/		24/													
1	15	14	4	1	15	12	6	1	15	14	4	1	15	8	10	1	15	12	6	1	15	8	10
8	10	11	5	8	10	13	3	12	6	7	9	12	6	13	3	14	4	7	9	14	4	11	5
12	6	7	9	14	4	7	9	8	10	11	5	14	4	11	5	8	10	13	3	12	6	13	3
13	3	2	16	11	5	2	16	13	3	2	16	7	9	2	16	11	5	2	16	7	9	2	16
25/		26/		27/		28/		29/		30/													
2	7	16	9	2	7	13	12	2	7	16	9	2	7	11	14	2	7	13	12	2	7	11	14
11	14	5	4	11	14	8	1	13	12	3	6	13	12	8	1	16	9	3	6	16	9	5	4
13	12	3	6	16	9	3	6	11	14	5	4	16	9	5	4	11	14	8	1	13	12	8	1
8	1	10	15	5	4	10	15	8	1	10	15	3	6	10	15	5	4	10	15	3	6	10	15
31/		32/		33/		34/		35/		36/													
2	11	16	5	2	11	13	8	2	11	16	5	2	11	7	14	2	11	13	8	2	11	7	14
7	14	9	4	7	14	12	1	13	8	3	10	13	8	12	1	16	5	3	10	16	5	9	4
13	8	3	10	16	5	3	10	7	14	9	4	16	5	9	4	7	14	12	1	13	8	12	1
12	1	6	15	9	4	6	15	12	1	6	15	3	10	6	15	9	4	6	15	3	10	6	15
37/		38/		39/		40/		41/		42/													
2	13	16	3	2	13	11	8	2	13	16	3	2	13	7	12	2	13	11	8	2	13	7	12
7	12	9	6	7	12	14	1	11	8	5	10	11	8	14	1	16	3	5	10	16	3	9	6
11	8	5	10	16	3	5	10	7	12	9	6	16	3	9	6	7	12	14	1	11	8	14	1
14	1	4	15	9	6	4	15	14	1	4	15	5	10	4	15	9	6	4	15	5	10	4	15
43/		44/		45/		46/		47/		48/													
2	16	13	3	2	16	11	5	2	16	13	3	2	16	7	9	2	16	11	5	2	16	7	9
7	9	12	6	7	9	14	4	11	5	8	10	11	5	14	4	13	3	8	10	13	3	12	6
11	5	8	10	13	3	8	10	7	9	12	6	13	3	12	6	7	9	14	4	11	5	14	4
14	4	1	15	12	6	1	15	14	4	1	15	8	10	1	15	12	6	1	15	8	10	1	15
49/		50/		51/		52/		53/		54/													
3	6	16	9	3	6	13	12	3	6	16	9	3	6	10	15	3	6	13	12	3	6	10	15
10	15	5	4	10	15	8	1	13	12	2	7	13	12	8	1	16	9	2	7	16	9	5	4
13	12	2	7	16	9	2	7	10	15	5	4	16	9	5	4	10	15	8	1	13	12	8	1
8	1	11	14	5	4	11	14	8	1	11	14	2	7	11	14	5	4	11	14	2	7	11	14

55/ 3 10 16 5 6 15 9 4 13 8 2 11 12 1 7 14	56/ 3 10 13 8 6 15 12 1 16 5 2 11 9 4 7 14	57/ 3 10 16 5 13 8 2 11 6 15 9 4 12 1 7 14	58/ 3 10 6 15 13 8 12 1 16 5 9 4 2 11 7 14	59/ 3 10 13 8 16 5 2 11 6 15 12 1 9 4 7 14	60/ 3 10 6 15 16 5 9 4 13 8 12 1 2 11 7 14
61/ 3 13 16 2 6 12 9 7 10 8 5 11 15 1 4 14	62/ 3 13 10 8 6 12 15 1 16 2 5 11 9 7 4 14	63/ 3 13 16 2 10 8 5 11 6 12 9 7 15 1 4 14	64/ 3 13 6 12 10 8 15 1 16 2 9 7 5 11 4 14	65/ 3 13 10 8 16 2 5 11 6 12 15 1 9 7 4 14	66/ 3 13 6 12 16 2 9 7 10 8 15 1 5 11 4 14
67/ 3 16 13 2 6 9 12 7 10 5 8 11 15 4 1 14	68/ 3 16 10 5 6 9 15 4 13 2 8 11 12 7 1 14	69/ 3 16 13 2 10 5 8 11 6 9 12 7 15 4 1 14	70/ 3 16 6 9 10 5 15 4 13 2 12 7 8 11 1 14	71/ 3 16 10 5 13 2 8 11 6 9 15 4 12 7 1 14	72/ 3 16 6 9 13 2 12 7 10 5 15 4 8 11 1 14
73/ 4 5 15 10 9 16 6 3 14 11 1 8 7 2 12 13	74/ 4 5 14 11 9 16 7 2 15 10 1 8 6 3 12 13	75/ 4 5 15 10 14 11 1 8 9 16 6 3 7 2 12 13	76/ 4 5 9 16 14 11 7 2 15 10 6 3 1 8 12 13	77/ 4 5 14 11 15 10 1 8 9 16 7 2 6 3 12 13	78/ 4 5 9 16 15 10 6 3 14 11 7 2 1 8 12 13

.....

361/ 16 2 9 7 3 13 6 12 5 11 4 14 10 8 15 1	362/ 16 2 5 11 3 13 10 8 9 7 4 14 6 12 15 1	363/ 16 2 9 7 5 11 4 14 3 13 6 12 10 8 15 1	364/ 16 2 3 13 5 11 10 8 9 7 6 12 4 14 15 1	365/ 16 2 5 11 9 7 4 14 3 13 10 8 6 12 15 1	366/ 16 2 3 13 9 7 6 12 5 11 10 8 4 14 15 1
367/ 16 3 9 6 2 13 7 12 5 10 4 15 11 8 14 1	368/ 16 3 5 10 2 13 11 8 9 6 4 15 7 12 14 1	369/ 16 3 9 6 5 10 4 15 2 13 7 12 11 8 14 1	370/ 16 3 2 13 5 10 11 8 9 6 7 12 4 15 14 1	371/ 16 3 5 10 9 6 4 15 2 13 11 8 7 12 14 1	372/ 16 3 2 13 9 6 7 12 5 10 11 8 4 15 14 1
373/ 16 5 9 4 2 11 7 14 3 10 6 15 13 8 12 1	374/ 16 5 3 10 2 11 13 8 9 4 6 15 7 14 12 1	375/ 16 5 9 4 3 10 6 15 2 11 7 14 13 8 12 1	376/ 16 5 2 11 3 10 13 8 9 4 7 14 6 15 12 1	377/ 16 5 3 10 9 4 6 15 2 11 13 8 7 14 12 1	378/ 16 5 2 11 9 4 7 14 3 10 13 8 6 15 12 1
379/ 16 9 5 4 2 7 11 14 3 6 10 15 13 12 8 1	380/ 16 9 3 6 2 7 13 12 5 4 10 15 11 14 8 1	381/ 16 9 5 4 3 6 10 15 2 7 11 14 13 12 8 1	382/ 16 9 2 7 3 6 13 12 5 4 11 14 10 15 8 1	383/ 16 9 3 6 5 4 10 15 2 7 13 12 11 14 8 1	384/ 16 9 2 7 5 4 11 14 3 6 13 12 10 15 8 1

* Counts according to the value of n1 *
 [1: 24] [2: 24] [3: 24] [4: 24] [5: 24] [6: 24] [7: 24] [8: 24]
 [9: 24] [10: 24] [11: 24] [12: 24] [13: 24] [14: 24] [15: 24] [16: 24]
 * Total Count of Solutions = 384 *

If you want to have the 'Standard Solutions' of our object, you have to put such the 'Normalizing Inequality Conditions' as {n1<n4; n1<n13; n1<n16; and n2<n5}, for instance, to prevent you from counting double any reflection or rotation.

Let me show you (1) a sample list of my recent computer program, and (2) the list of Standard Solutions calculated and printed out by that program.

```
/** Self-Complementary Magic Squares of Order 4: **
```

```

/** File: 'MS44SC48.c' Dictated by Kanji Setsuda **
/** on Aug.10, 2002; Dec.13, 2005; **
/** Recompiled on February 11, 2015; **
/** with MacOSX 10.10.2 and Xcode 6.1.1 **
//
// .----- [Basic Conditions: C=34;]
// | 1| 2| 3| 4| n1+n2+n3+n4=C ...rw1;
// |---+---+---| n5+n6+n7+n8=C ...rw2;
// | 5| 6| 7| 8| n9+n10+n11+n12=C ...rw3;
// |---+---+---| n13+n14+n15+n16=C ...rw4;
// | 9|10|11|12| n1+n5+n9+n13=C ...c1;
// |---+---+---| n2+n6+n10+n14=C ...c2;
// |13|14|15|16| n3+n7+n11+n15=C ...c3;
// |'-----' n4+n8+n12+n16=C ...c4;
//[Conditions for Symmetric Complementary Pairs: CC=17;]
// n1+n16=CC ...Cmp1; n2+n15=CC ...Cmp2;
// n3+n14=CC ...Cmp3; n4+n13=CC ...Cmp4;
// n5+n12=CC ...Cmp5; n6+n11=CC ...Cmp6;
// n7+n10=CC ...Cmp7; n8+n9=CC ...Cmp8;
//
/** [Normalizing Inequality Condtions: n1<n16, n1<n4, n1<n13 and n2<n5;]
//
#include <stdio.h>
//
short cnt, cnt3;
short LSM, PSM;
short nm[17], uflg[17];
short anm[9][17];
//
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void ansprint(void);
void printans(short x);
//
/** Main Program *
int main(){
short n;
printf("\n");
printf("*** Self-Complementary Magic Squares of Order 4: **\n");
printf(" ** List of the 48 Standard Solutions **\n");
printf("[Normalizing Inequality Condtions: n1<n16, n1<n4, n1<n13 and n2<n5;]\n");
for(n=0;n<17;n++){nm[n]=0; uflg[n]=0;}
LSM=34; PSM=17; cnt=0; cnt3=0;
stp01(); /* Begin the Calculations *
if(cnt3>0){printans(cnt3);}
printf(" [Count = %d] OK!\n",cnt);
printf("\n");
printf("*** Calculated and Listed by Kanji Setsuda on\n");
printf(" February 11, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 **\n");
printf("\n");
return 0;
}
//
/** Begin the Calculations *
/** Set n1 & n16=PSM-n1 & n1<n16 *
void stp01(){
short a,b;
for(a=1;a<9;a++){b=PSM-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[1]=a; nm[16]=b;
uflg[a]=1; uflg[b]=1;
stp02();
uflg[b]=0; uflg[a]=0;}
}
}
}

```

```

/** Set n2 & n15=PSM-n2 *
void stp02(){
short a,b;
for(a=1;a<17;a++){b=PSM-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[2]=a; nm[15]=b;
uflg[a]=1; uflg[b]=1;
stp03();
uflg[b]=0; uflg[a]=0;}}
}
}
/** Set n5 & n12=PSM-n5 & n5>n2 *
void stp03(){
short a,b;
for(a=nm[2]+1;a<17;a++){b=PSM-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[5]=a; nm[12]=b;
uflg[a]=1; uflg[b]=1;
stp04();
uflg[b]=0; uflg[a]=0;}}
}
}
/** Set n6 & n11=PSM-n6 *
void stp04(){
short a,b;
for(a=1;a<17;a++){b=PSM-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[6]=a; nm[11]=b;
uflg[a]=1; uflg[b]=1;
stp05();
uflg[b]=0; uflg[a]=0;}}
}
}
/** Set n4 & n13=PSM-n4 & n1<n4 & n1<n13 *
void stp05(){
short a,b;
for(a=nm[1]+1;a<17;a++){b=PSM-a;
if((b>nm[1])&&(uflg[a]==0)&&(uflg[b]==0)){
nm[4]=a; nm[13]=b;
uflg[a]=1; uflg[b]=1;
stp06();
uflg[b]=0; uflg[a]=0;}}
}
}
/** Set n3=LSM-n1-n2-n4 & n14=PSM-n3 *
void stp06(){
short a,b;
a=LSM-nm[1]-nm[2]-nm[4]; b=PSM-a;
if((0<a)&&(a<17)){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[3]=a; nm[14]=b;
uflg[a]=1; uflg[b]=1;
stp07();
uflg[b]=0; uflg[a]=0;}}}
}
}
/** Set n9=LSM-n1-n5-n13 & n8=LSM-n4-n12-n16 *
/** & Check n8+n9==PSM? *
void stp07(){
short a,b;
a=LSM-nm[1]-nm[5]-nm[13];
b=LSM-nm[4]-nm[12]-nm[16];
if((0<a)&&(a<17)&&(a+b==PSM)){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[9]=a; nm[8]=b;
uflg[a]=1; uflg[b]=1;
}
}
}
}

```



```

        stp08();
        uflg[b]=0; uflg[a]=0;}}
}
/** Set n7=LSM-n5-n6-n8 & n10=LSM-n2-n6-n14 *
** & Check n7+n10==PSM? *
void stp08(){
    short a,b;
    a=LSM-nm[5]-nm[6]-nm[8];
    b=LSM-nm[2]-nm[6]-nm[14];
    if((0<a)&&(a<17)&&(a+b==PSM)){
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[7]=a; nm[10]=b;
            uflg[a]=1; uflg[b]=1;
            ansprint();
            uflg[b]=0; uflg[a]=0;}}
}
//
/** Print the Answers *
void ansprint(){
    short n;
    cnt++;
    anm[cnt3][0]=cnt;
    for(n=1;n<17;n++){anm[cnt3][n]=nm[n];}
    cnt3++; if(cnt3==6){printans(6); cnt3=0;}
}
/** Print 6 Answers *
void printans(short x){
    short m;
    for(m=0;m<x;m++){printf("%13d/",anm[m][0]);}
    printf("\n");
    for(m=0;m<x;m++){
        printf("%5d%3d%3d%3d",anm[m][1],anm[m][2],anm[m][3],anm[m][4]);}
    printf("\n");
    for(m=0;m<x;m++){
        printf("%5d%3d%3d%3d",anm[m][5],anm[m][6],anm[m][7],anm[m][8]);}
    printf("\n");
    for(m=0;m<x;m++){
        printf("%5d%3d%3d%3d",anm[m][9],anm[m][10],anm[m][11],anm[m][12]);}
    printf("\n");
    for(m=0;m<x;m++){
        printf("%5d%3d%3d%3d",anm[m][13],anm[m][14],anm[m][15],anm[m][16]);}
    printf("\n");
}
//

```

**** Self-Complementary Magic Squares of Order 4: ****

**** List of the 48 Standard Solutions ****

[Normalizing Inequality Conditions: n1<n16, n1<n4, n1<n13 and n2<n5;]

1/		2/		3/		4/		5/		6/	
1 8 15 10	1 8 14 11	1 8 15 10	1 8 12 13	1 8 14 11	1 8 12 13	12 13 6 3	12 13 7 2	14 11 4 5	14 11 7 2	15 10 4 5	15 10 6 3
14 11 4 5	15 10 4 5	12 13 6 3	15 10 6 3	12 13 7 2	14 11 7 2	7 2 9 16	6 3 9 16	7 2 9 16	4 5 9 16	6 3 9 16	4 5 9 16
7/		8/		9/		10/		11/		12/	
1 12 15 6	1 12 8 13	1 12 14 7	1 12 8 13	1 14 12 7	1 14 8 11	14 7 4 9	14 7 11 2	15 6 4 9	15 6 10 3	15 4 6 9	15 4 10 5
8 13 10 3	15 6 10 3	8 13 11 2	14 7 11 2	8 11 13 2	12 7 13 2	11 2 5 16	4 9 5 16	10 3 5 16	4 9 5 16	10 5 3 16	6 9 3 16
13/		14/		15/		16/		17/		18/	
2 7 16 9	2 7 13 12	2 7 16 9	2 7 11 14	2 7 13 12	2 7 11 14	11 14 5 4	11 14 8 1	13 12 3 6	13 12 8 1	16 9 3 6	16 9 5 4
13 12 3 6	16 9 3 6	11 14 5 4	16 9 5 4	11 14 8 1	13 12 8 1	8 1 10 15	5 4 10 15	8 1 10 15	3 6 10 15	5 4 10 15	3 6 10 15

19/ 2 11 16 5 13 8 3 10 7 14 9 4 12 1 6 15	20/ 2 11 7 14 13 8 12 1 16 5 9 4 3 10 6 15	21/ 2 11 13 8 16 5 3 10 7 14 12 1 9 4 6 15	22/ 2 11 7 14 16 5 9 4 13 8 12 1 3 10 6 15	23/ 2 13 11 8 16 3 5 10 7 12 14 1 9 6 4 15	24/ 2 13 7 12 16 3 9 6 11 8 14 1 5 10 4 15
25/ 3 6 16 9 10 15 5 4 13 12 2 7 8 1 11 14	26/ 3 6 13 12 10 15 8 1 16 9 2 7 5 4 11 14	27/ 3 6 16 9 13 12 2 7 10 15 5 4 8 1 11 14	28/ 3 6 13 12 16 9 2 7 10 15 8 1 5 4 11 14	29/ 3 10 16 5 13 8 2 11 6 15 9 4 12 1 7 14	30/ 3 10 13 8 16 5 2 11 6 15 12 1 9 4 7 14
31/ 3 13 10 8 16 2 5 11 6 12 15 1 9 7 4 14	32/ 3 13 6 12 16 2 9 7 10 8 15 1 5 11 4 14	33/ 4 5 15 10 9 16 6 3 14 11 1 8 7 2 12 13	34/ 4 5 14 11 9 16 7 2 15 10 1 8 6 3 12 13	35/ 4 5 15 10 14 11 1 8 9 16 6 3 7 2 12 13	36/ 4 5 14 11 15 10 1 8 9 16 7 2 6 3 12 13
37/ 4 9 15 6 14 7 1 12 5 16 10 3 11 2 8 13	38/ 4 9 14 7 15 6 1 12 5 16 11 2 10 3 8 13	39/ 4 14 9 7 15 1 6 12 5 11 16 2 10 8 3 13	40/ 4 14 5 11 15 1 10 8 9 7 16 2 6 12 3 13	41/ 5 4 16 9 10 15 3 6 11 14 2 7 8 1 13 12	42/ 5 4 16 9 11 14 2 7 10 15 3 6 8 1 13 12
43/ 5 10 11 8 16 3 2 13 4 15 14 1 9 6 7 12	44/ 5 11 10 8 16 2 3 13 4 14 15 1 9 7 6 12	45/ 6 3 15 10 9 16 4 5 12 13 1 8 7 2 14 11	46/ 6 3 15 10 12 13 1 8 9 16 4 5 7 2 14 11	47/ 6 9 12 7 15 4 1 14 3 16 13 2 10 5 8 11	48/ 6 12 9 7 15 1 4 14 3 13 16 2 10 8 5 11

[Count = 48] OK!

** Calculated and Listed by Kanji Setsuda on
February 11, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 **

If you want to have the 'Fundamental Solutions' of our object, you have to put such the conditions as {n1=1; n16=16; n2<n3; n5<n9; and n2<n5;}. You will finally get the three Fundamental Solutions as follows.

** Self-Complementary Magic Squares of Order 4: **

** List of the Three 'Fundamental Solutions' **

1/ 1 8 15 10 12 13 6 3 14 11 4 5 7 2 9 16	2/ 1 8 14 11 12 13 7 2 15 10 4 5 6 3 9 16	3/ 1 8 12 13 14 11 7 2 15 10 6 3 4 5 9 16
---	---	---

What kinds of relations are there among those three levels of solutions? What is the meaning of the next factors of divisions? $384 = 48 \times 8 = 2 \times 4 \times 16 \times 3$

Can we assume any transformation systems to make them meaningful?

#6. What Transformation Systems can we assume?

Take your careful look at the next conceptual diagrams below I prepared.

** Transformation System for Self-Complementary Magic Squares 4x4 **

[2x4 Transformation making 1 into 8: Rotate and Reflect]

00/				01/				02/				03/			
1	2	3	4	13	9	5	1	16	15	14	13	4	8	12	16
5	6	7	8	14	10	6	2	12	11	10	9	3	7	11	15
9	10	11	12	15	11	7	3	8	7	6	5	2	6	10	14
13	14	15	16	16	12	8	4	4	3	2	1	1	5	9	13

10/				11/				12/				13/			
1	5	9	13	4	3	2	1	16	12	8	4	13	14	15	16
2	6	10	14	8	7	6	5	15	11	7	3	9	10	11	12
3	7	11	15	12	11	10	9	14	10	6	2	5	6	7	8
4	8	12	16	16	15	14	13	13	9	5	1	1	2	3	4

[4x4 Transformation making 1 into 16: Exchange some Lines]

00/				01/				02/				03/			
1	2	3	4	1	3	2	4	2	1	4	3	2	4	1	3
5	6	7	8	5	7	6	8	6	5	8	7	6	8	5	7
9	10	11	12	9	11	10	12	10	9	12	11	10	12	9	11
13	14	15	16	13	15	14	16	14	13	16	15	14	16	13	15
10/				11/				12/				13/			
1	2	3	4	1	3	2	4	2	1	4	3	2	4	1	3
9	10	11	12	9	11	10	12	10	9	12	11	10	12	9	11
5	6	7	8	5	7	6	8	6	5	8	7	6	8	5	7
13	14	15	16	13	15	14	16	14	13	16	15	14	16	13	15
20/				21/				22/				23/			
5	6	7	8	5	7	6	8	6	5	8	7	6	8	5	7
1	2	3	4	1	3	2	4	2	1	4	3	2	4	1	3
13	14	15	16	13	15	14	16	14	13	16	15	14	16	13	15
9	10	11	12	9	11	10	12	10	9	12	11	10	12	9	11
30/				31/				32/				33/			
5	6	7	8	5	7	6	8	6	5	8	7	6	8	5	7
13	14	15	16	13	15	14	16	14	13	16	15	14	16	13	15
1	2	3	4	1	3	2	4	2	1	4	3	2	4	1	3
9	10	11	12	9	11	10	12	10	9	12	11	10	12	9	11

The upper set illustrates the combination of rotation by 90 degrees and the mirror reflection. These 8 patterns are always possible in every type and in every order.

The lower set of 16 patterns illustrates the combination of row exchangeability and column exchangeability.

You can really make different forms by those two sets of transformation and can even combine them to make one fundamental into 8x16 different patterns.

The next long list shows the result of those transformations.

** Reconstruction of All 'Self-Complementary' Magic Squares 4x4: **

** Start: The 'Fundamental' Three Solutions **

1/				2/				3/			
1	8	15	10	1	8	14	11	1	8	12	13
12	13	6	3	12	13	7	2	14	11	7	2
14	11	4	5	15	10	4	5	15	10	6	3
7	2	9	16	6	3	9	16	4	5	9	16

** Transform them into the Primitive Solutions
and Compare the Result with the Old List **

[Fundamental #1]

	1/	163/	384/	222/	8/	235/	377/	150/
1	8 15 10	7 14 12 1	16 9 2 7	10 3 5 16	1 12 14 7	10 15 8 1	16 5 3 10	7 2 9 16
12	13 6 3	2 11 13 8	5 4 11 14	15 6 4 9	8 13 11 2	3 6 13 12	9 4 6 15	14 11 4 5
14	11 4 5	9 4 6 15	3 6 13 12	8 13 11 2	15 6 4 9	5 4 11 14	2 11 13 8	12 13 6 3
7	2 9 16	16 5 3 10	10 15 8 1	1 12 14 7	10 3 5 16	16 9 2 7	7 14 12 1	1 8 15 10
	22/	165/	363/	220/	11/	229/	374/	156/
1	15 8 10	7 14 12 1	16 2 9 7	10 3 5 16	1 12 14 7	10 8 15 1	16 5 3 10	7 9 2 16
12	6 13 3	9 4 6 15	5 11 4 14	8 13 11 2	15 6 4 9	3 13 6 12	2 11 13 8	14 4 11 5
14	4 11 5	2 11 13 8	3 13 6 12	15 6 4 9	8 13 11 2	5 11 4 14	9 4 6 15	12 6 13 3
7	9 2 16	16 5 3 10	10 8 15 1	1 12 14 7	10 3 5 16	16 2 9 7	7 14 12 1	1 15 8 10
	174/	32/	211/	353/	187/	360/	198/	25/
8	1 10 15	2 11 13 8	9 16 7 2	15 6 4 9	8 13 11 2	15 10 1 8	9 4 6 15	2 7 16 9
13	12 3 6	7 14 12 1	4 5 14 11	10 3 5 16	1 12 14 7	6 3 12 13	16 5 3 10	11 14 5 4
11	14 5 4	16 5 3 10	6 3 12 13	1 12 14 7	10 3 5 16	4 5 14 11	7 14 12 1	13 12 3 6
2	7 16 9	9 4 6 15	15 10 1 8	8 13 11 2	15 6 4 9	9 16 7 2	2 11 13 8	8 1 10 15
	180/	35/	205/	350/	189/	339/	196/	46/
8	10 1 15	2 11 13 8	9 7 16 2	15 6 4 9	8 13 11 2	15 1 10 8	9 4 6 15	2 16 7 9
13	3 12 6	16 5 3 10	4 14 5 11	1 12 14 7	10 3 5 16	6 12 3 13	7 14 12 1	11 5 14 4
11	5 14 4	7 14 12 1	6 12 3 13	10 3 5 16	1 12 14 7	4 14 5 11	16 5 3 10	13 3 12 6
2	16 7 9	9 4 6 15	15 1 10 8	8 13 11 2	15 6 4 9	9 7 16 2	2 11 13 8	8 10 1 15
	3/	157/	382/	228/	14/	237/	371/	148/
1	8 15 10	7 12 14 1	16 9 2 7	10 5 3 16	1 14 12 7	10 15 8 1	16 3 5 10	7 2 9 16
14	11 4 5	2 13 11 8	3 6 13 12	15 4 6 9	8 11 13 2	5 4 11 14	9 6 4 15	12 13 6 3
12	13 6 3	9 6 4 15	5 4 11 14	8 11 13 2	15 4 6 9	3 6 13 12	2 13 11 8	14 11 4 5
7	2 9 16	16 3 5 10	10 15 8 1	1 14 12 7	10 5 3 16	16 9 2 7	7 12 14 1	1 8 15 10
	24/	159/	361/	226/	17/	231/	368/	154/
1	15 8 10	7 12 14 1	16 2 9 7	10 5 3 16	1 14 12 7	10 8 15 1	16 3 5 10	7 9 2 16
14	4 11 5	9 6 4 15	3 13 6 12	8 11 13 2	15 4 6 9	5 11 4 14	2 13 11 8	12 6 13 3
12	6 13 3	2 13 11 8	5 11 4 14	15 4 6 9	8 11 13 2	3 13 6 12	9 6 4 15	14 4 11 5
7	9 2 16	16 3 5 10	10 8 15 1	1 14 12 7	10 5 3 16	16 2 9 7	7 12 14 1	1 15 8 10
	172/	38/	213/	347/	181/	358/	204/	27/
8	1 10 15	2 13 11 8	9 16 7 2	15 4 6 9	8 11 13 2	15 10 1 8	9 6 4 15	2 7 16 9
11	14 5 4	7 12 14 1	6 3 12 13	10 5 3 16	1 14 12 7	4 5 14 11	16 3 5 10	13 12 3 6
13	12 3 6	16 3 5 10	4 5 14 11	1 14 12 7	10 5 3 16	6 3 12 13	7 12 14 1	11 14 5 4
2	7 16 9	9 6 4 15	15 10 1 8	8 11 13 2	15 4 6 9	9 16 7 2	2 13 11 8	8 1 10 15
	178/	41/	207/	344/	183/	337/	202/	48/
8	10 1 15	2 13 11 8	9 7 16 2	15 4 6 9	8 11 13 2	15 1 10 8	9 6 4 15	2 16 7 9
11	5 14 4	16 3 5 10	6 12 3 13	1 14 12 7	10 5 3 16	4 14 5 11	7 12 14 1	13 3 12 6
13	3 12 6	7 12 14 1	4 14 5 11	10 5 3 16	1 14 12 7	6 12 3 13	16 3 5 10	11 5 14 4
2	16 7 9	9 6 4 15	15 1 10 8	8 11 13 2	15 4 6 9	9 7 16 2	2 13 11 8	8 10 1 15
	284/	330/	101/	55/	269/	50/	116/	335/
12	13 6 3	14 7 1 12	5 4 11 14	3 10 16 5	12 1 7 14	3 6 13 12	5 16 10 3	14 11 4 5
1	8 15 10	11 2 8 13	16 9 2 7	6 15 9 4	13 8 2 11	10 15 8 1	4 9 15 6	7 2 9 16
7	2 9 16	4 9 15 6	10 15 8 1	13 8 2 11	6 15 9 4	16 9 2 7	11 2 8 13	1 8 15 10
14	11 4 5	5 16 10 3	3 6 13 12	12 1 7 14	3 10 16 5	5 4 11 14	14 7 1 12	12 13 6 3
	271/	328/	114/	57/	266/	64/	119/	321/
12	6 13 3	14 7 1 12	5 11 4 14	3 10 16 5	12 1 7 14	3 13 6 12	5 16 10 3	14 4 11 5
1	15 8 10	4 9 15 6	16 2 9 7	13 8 2 11	6 15 9 4	10 8 15 1	11 2 8 13	7 9 2 16
7	9 2 16	11 2 8 13	10 8 15 1	6 15 9 4	13 8 2 11	16 2 9 7	4 9 15 6	1 15 8 10
14	4 11 5	5 16 10 3	3 13 6 12	12 1 7 14	3 10 16 5	5 11 4 14	14 7 1 12	12 6 13 3
	311/	245/	74/	140/	306/	125/	79/	260/
13	12 3 6	11 2 8 13	4 5 14 11	6 15 9 4	13 8 2 11	6 3 12 13	4 9 15 6	11 14 5 4
8	1 10 15	14 7 1 12	9 16 7 2	3 10 16 5	12 1 7 14	15 10 1 8	5 16 10 3	2 7 16 9
2	7 16 9	5 16 10 3	15 10 1 8	12 1 7 14	3 10 16 5	9 16 7 2	14 7 1 12	8 1 10 15
11	14 5 4	4 9 15 6	6 3 12 13	13 8 2 11	6 15 9 4	4 5 14 11	11 2 8 13	13 12 3 6
	297/	242/	88/	143/	304/	138/	81/	247/
13	3 12 6	11 2 8 13	4 14 5 11	6 15 9 4	13 8 2 11	6 12 3 13	4 9 15 6	11 5 14 4
8	10 1 15	5 16 10 3	9 7 16 2	12 1 7 14	3 10 16 5	15 1 10 8	14 7 1 12	2 16 7 9
2	16 7 9	14 7 1 12	15 1 10 8	3 10 16 5	12 1 7 14	9 7 16 2	5 16 10 3	8 10 1 15
11	5 14 4	4 9 15 6	6 12 3 13	13 8 2 11	6 15 9 4	4 14 5 11	11 2 8 13	13 3 12 6

287/	317/	98/	68/	282/	53/	103/	332/
12 13 6 3 14 1 7 12	5 4 11 14	3 16 10 5	12 7 1 14	3 6 13 12	5 10 16 3	14 11 4 5	
7 2 9 16	11 8 2 13	10 15 8 1	6 9 15 4	13 2 8 11	16 9 2 7	4 15 9 6	1 8 15 10
1 8 15 10	4 15 9 6	16 9 2 7	13 2 8 11	6 9 15 4	10 15 8 1	11 8 2 13	7 2 9 16
14 11 4 5	5 10 16 3	3 6 13 12	12 7 1 14	3 16 10 5	5 4 11 14	14 1 7 12	12 13 6 3
273/	314/	112/	71/	280/	66/	105/	319/
12 6 13 3	14 1 7 12	5 11 4 14	3 16 10 5	12 7 1 14	3 13 6 12	5 10 16 3	14 4 11 5
7 9 2 16	4 15 9 6	10 8 15 1	13 2 8 11	6 9 15 4	16 2 9 7	11 8 2 13	1 15 8 10
1 15 8 10	11 8 2 13	16 2 9 7	6 9 15 4	13 2 8 11	10 8 15 1	4 15 9 6	7 9 2 16
14 4 11 5	5 10 16 3	3 13 6 12	12 7 1 14	3 16 10 5	5 11 4 14	14 1 7 12	12 6 13 3
308/	258/	77/	127/	293/	122/	92/	263/
13 12 3 6	11 8 2 13	4 5 14 11	6 9 15 4	13 2 8 11	6 3 12 13	4 15 9 6	11 14 5 4
2 7 16 9	14 1 7 12	15 10 1 8	3 16 10 5	12 7 1 14	9 16 7 2	5 10 16 3	8 1 10 15
8 1 10 15	5 10 16 3	9 16 7 2	12 7 1 14	3 16 10 5	15 10 1 8	14 1 7 12	2 7 16 9
11 14 5 4	4 15 9 6	6 3 12 13	13 2 8 11	6 9 15 4	4 5 14 11	11 8 2 13	13 12 3 6
295/	256/	90/	129/	290/	136/	95/	249/
13 3 12 6	11 8 2 13	4 14 5 11	6 9 15 4	13 2 8 11	6 12 3 13	4 15 9 6	11 5 14 4
2 16 7 9	5 10 16 3	15 1 10 8	12 7 1 14	3 16 10 5	9 7 16 2	14 1 7 12	8 10 1 15
8 10 1 15	14 1 7 12	9 7 16 2	3 16 10 5	12 7 1 14	15 1 10 8	5 10 16 3	2 16 7 9
11 5 14 4	4 15 9 6	6 12 3 13	13 2 8 11	6 9 15 4	4 14 5 11	11 8 2 13	13 3 12 6

[Fundamental #2]

2/	139/	383/	246/	7/	259/	378/	126/
1 8 14 11	6 15 12 1	16 9 3 6	11 2 5 16	1 12 15 6	11 14 8 1	16 5 2 11	6 3 9 16
12 13 7 2	3 10 13 8	5 4 10 15	14 7 4 9	8 13 10 3	2 7 13 12	9 4 7 14	15 10 4 5
15 10 4 5	9 4 7 14	2 7 13 12	8 13 10 3	14 7 4 9	5 4 10 15	3 10 13 8	12 13 7 2
6 3 9 16	16 5 2 11	11 14 8 1	1 12 15 6	11 2 5 16	16 9 3 6	6 15 12 1	1 8 14 11
16/	141/	369/	244/	9/	253/	376/	132/
1 14 8 11	6 15 12 1	16 3 9 6	11 2 5 16	1 12 15 6	11 8 14 1	16 5 2 11	6 9 3 16
12 7 13 2	9 4 7 14	5 10 4 15	8 13 10 3	14 7 4 9	2 13 7 12	3 10 13 8	15 4 10 5
15 4 10 5	3 10 13 8	2 13 7 12	14 7 4 9	8 13 10 3	5 10 4 15	9 4 7 14	12 7 13 2
6 9 3 16	16 5 2 11	11 8 14 1	1 12 15 6	11 2 5 16	16 3 9 6	6 15 12 1	1 14 8 11
173/	56/	212/	329/	188/	336/	197/	49/
8 1 11 14	3 10 13 8	9 16 6 3	14 7 4 9	8 13 10 3	14 11 1 8	9 4 7 14	3 6 16 9
13 12 2 7	6 15 12 1	4 5 15 10	11 2 5 16	1 12 15 6	7 2 12 13	16 5 2 11	10 15 5 4
10 15 5 4	16 5 2 11	7 2 12 13	1 12 15 6	11 2 5 16	4 5 15 10	6 15 12 1	13 12 2 7
3 6 16 9	9 4 7 14	14 11 1 8	8 13 10 3	14 7 4 9	9 16 6 3	3 10 13 8	8 1 11 14
186/	59/	199/	326/	191/	315/	194/	70/
8 11 1 14	3 10 13 8	9 6 16 3	14 7 4 9	8 13 10 3	14 1 11 8	9 4 7 14	3 16 6 9
13 2 12 7	16 5 2 11	4 15 5 10	1 12 15 6	11 2 5 16	7 12 2 13	6 15 12 1	10 5 15 4
10 5 15 4	6 15 12 1	7 12 2 13	11 2 5 16	1 12 15 6	4 15 5 10	16 5 2 11	13 2 12 7
3 16 6 9	9 4 7 14	14 1 11 8	8 13 10 3	14 7 4 9	9 6 16 3	3 10 13 8	8 11 1 14
5/	133/	380/	252/	20/	261/	365/	124/
1 8 14 11	6 12 15 1	16 9 3 6	11 5 2 16	1 15 12 6	11 14 8 1	16 2 5 11	6 3 9 16
15 10 4 5	3 13 10 8	2 7 13 12	14 4 7 9	8 10 13 3	5 4 10 15	9 7 4 14	12 13 7 2
12 13 7 2	9 7 4 14	5 4 10 15	8 10 13 3	14 4 7 9	2 7 13 12	3 13 10 8	15 10 4 5
6 3 9 16	16 2 5 11	11 14 8 1	1 15 12 6	11 5 2 16	16 9 3 6	6 12 15 1	1 8 14 11
18/	135/	367/	250/	23/	255/	362/	130/
1 14 8 11	6 12 15 1	16 3 9 6	11 5 2 16	1 15 12 6	11 8 14 1	16 2 5 11	6 9 3 16
15 4 10 5	9 7 4 14	2 13 7 12	8 10 13 3	14 4 7 9	5 10 4 15	3 13 10 8	12 7 13 2
12 7 13 2	3 13 10 8	5 10 4 15	14 4 7 9	8 10 13 3	2 13 7 12	9 7 4 14	15 4 10 5
6 9 3 16	16 2 5 11	11 8 14 1	1 15 12 6	11 5 2 16	16 3 9 6	6 12 15 1	1 14 8 11
170/	62/	215/	323/	175/	334/	210/	51/
8 1 11 14	3 13 10 8	9 16 6 3	14 4 7 9	8 10 13 3	14 11 1 8	9 7 4 14	3 6 16 9
10 15 5 4	6 12 15 1	7 2 12 13	11 5 2 16	1 15 12 6	4 5 15 10	16 2 5 11	13 12 2 7
13 12 2 7	16 2 5 11	4 5 15 10	1 15 12 6	11 5 2 16	7 2 12 13	6 12 15 1	10 15 5 4
3 6 16 9	9 7 4 14	14 11 1 8	8 10 13 3	14 4 7 9	9 16 6 3	3 13 10 8	8 1 11 14
184/	65/	201/	320/	177/	313/	208/	72/
8 11 1 14	3 13 10 8	9 6 16 3	14 4 7 9	8 10 13 3	14 1 11 8	9 7 4 14	3 16 6 9
10 5 15 4	16 2 5 11	7 12 2 13	1 15 12 6	11 5 2 16	4 15 5 10	6 12 15 1	13 2 12 7
13 2 12 7	6 12 15 1	4 15 5 10	1 15 12 6	1 15 12 6	7 12 2 13	16 2 5 11	10 5 15 4
3 16 6 9	9 7 4 14	14 1 11 8	8 10 13 3	14 4 7 9	9 6 16 3	3 13 10 8	8 11 1 14

283/	354/	102/	31/	270/	26/	115/	359/
12 13 7 2	15 6 1 12	5 4 10 15	2 11 16 5	12 1 6 15	2 7 13 12	5 16 11 2	15 10 4 5
1 8 14 11	10 3 8 13	16 9 3 6	7 14 9 4	13 8 3 10	11 14 8 1	4 9 14 7	6 3 9 16
6 3 9 16	4 9 14 7	11 14 8 1	13 8 3 10	7 14 9 4	16 9 3 6	10 3 8 13	1 8 14 11
15 10 4 5	5 16 11 2	2 7 13 12	12 1 6 15	2 11 16 5	5 4 10 15	15 6 1 12	12 13 7 2
277/	352/	108/	33/	268/	40/	117/	345/
12 7 13 2	15 6 1 12	5 10 4 15	2 11 16 5	12 1 6 15	2 13 7 12	5 16 11 2	15 4 10 5
1 14 8 11	4 9 14 7	16 3 9 6	13 8 3 10	7 14 9 4	11 8 14 1	10 3 8 13	6 9 3 16
6 9 3 16	10 3 8 13	11 8 14 1	7 14 9 4	13 8 3 10	16 3 9 6	4 9 14 7	1 14 8 11
15 4 10 5	5 16 11 2	2 13 7 12	12 1 6 15	2 11 16 5	5 10 4 15	15 6 1 12	12 7 13 2
312/	221/	73/	164/	305/	149/	80/	236/
13 12 2 7	10 3 8 13	4 5 15 10	7 14 9 4	13 8 3 10	7 2 12 13	4 9 14 7	10 15 5 4
8 1 11 14	15 6 1 12	9 16 6 3	2 11 16 5	12 1 6 15	14 11 1 8	5 16 11 2	3 6 16 9
3 6 16 9	5 16 11 2	14 11 1 8	12 1 6 15	2 11 16 5	9 16 6 3	15 6 1 12	8 1 11 14
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291/	218/	94/	167/	302/	162/	83/	223/
13 2 12 7	10 3 8 13	4 15 5 10	7 14 9 4	13 8 3 10	7 12 2 13	4 9 14 7	10 5 15 4
8 11 1 14	5 16 11 2	9 6 16 3	12 1 6 15	2 11 16 5	14 1 11 8	15 6 1 12	3 16 6 9
3 16 6 9	15 6 1 12	14 1 11 8	2 11 16 5	12 1 6 15	9 6 16 3	5 16 11 2	8 11 1 14
10 5 15 4	4 9 14 7	7 12 2 13	13 8 3 10	7 14 9 4	4 15 5 10	10 3 8 13	13 2 12 7
285/	341/	100/	44/	276/	29/	109/	356/
12 13 7 2	15 1 6 12	5 4 10 15	2 16 11 5	12 6 1 15	2 7 13 12	5 11 16 2	15 10 4 5
6 3 9 16	10 8 3 13	11 14 8 1	7 9 14 4	13 3 8 10	16 9 3 6	4 14 9 7	1 8 14 11
1 8 14 11	4 14 9 7	16 9 3 6	13 3 8 10	7 9 14 4	11 14 8 1	10 8 3 13	6 3 9 16
15 10 4 5	5 11 16 2	2 7 13 12	12 6 1 15	2 16 11 5	5 4 10 15	15 1 6 12	12 13 7 2
279/	338/	106/	47/	274/	42/	111/	343/
12 7 13 2	15 1 6 12	5 10 4 15	2 16 11 5	12 6 1 15	2 13 7 12	5 11 16 2	15 4 10 5
6 9 3 16	4 14 9 7	11 8 14 1	13 3 8 10	7 9 14 4	16 3 9 6	10 8 3 13	1 14 8 11
1 14 8 11	10 8 3 13	16 3 9 6	7 9 14 4	13 3 8 10	11 8 14 1	4 14 9 7	6 9 3 16
15 4 10 5	5 11 16 2	2 13 7 12	12 6 1 15	2 16 11 5	5 10 4 15	15 1 6 12	12 7 13 2
310/	234/	75/	151/	299/	146/	86/	239/
13 12 2 7	10 8 3 13	4 5 15 10	7 9 14 4	13 3 8 10	7 2 12 13	4 14 9 7	10 15 5 4
3 6 16 9	15 1 6 12	14 11 1 8	2 16 11 5	12 6 1 15	9 16 6 3	5 11 16 2	8 1 11 14
8 1 11 14	5 11 16 2	9 16 6 3	12 6 1 15	2 16 11 5	14 11 1 8	15 1 6 12	3 6 16 9
10 15 5 4	4 14 9 7	7 2 12 13	13 3 8 10	7 9 14 4	4 5 15 10	10 8 3 13	13 12 2 7
289/	232/	96/	153/	296/	160/	89/	225/
13 2 12 7	10 8 3 13	4 15 5 10	7 9 14 4	13 3 8 10	7 12 2 13	4 14 9 7	10 5 15 4
3 16 6 9	5 11 16 2	14 1 11 8	12 6 1 15	2 16 11 5	9 6 16 3	15 1 6 12	8 11 1 14
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10 5 15 4	4 14 9 7	7 12 2 13	13 3 8 10	7 9 14 4	4 15 5 10	10 8 3 13	13 2 12 7

[Fundamental #3]

4/	91/	381/	294/	13/	307/	372/	78/
1 8 12 13	4 15 14 1	16 9 5 4	13 2 3 16	1 14 15 4	13 12 8 1	16 3 2 13	4 5 9 16
14 11 7 2	5 10 11 8	3 6 10 15	12 7 6 9	8 11 10 5	2 7 11 14	9 6 7 12	15 10 6 3
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4 5 9 16	16 3 2 13	13 12 8 1	1 14 15 4	13 2 3 16	16 9 5 4	4 15 14 1	1 8 12 13
10/	93/	375/	292/	15/	301/	370/	84/
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14 7 11 2	9 6 7 12	3 10 6 15	8 11 10 5	12 7 6 9	2 11 7 14	5 10 11 8	15 6 10 3
15 6 10 3	5 10 11 8	2 11 7 14	12 7 6 9	8 11 10 5	3 10 6 15	9 6 7 12	14 7 11 2
4 9 5 16	16 3 2 13	13 8 12 1	1 14 15 4	13 2 3 16	16 5 9 4	4 15 14 1	1 12 8 13
171/	104/	214/	281/	182/	288/	203/	97/
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11 14 2 7	4 15 14 1	6 3 15 10	13 2 3 16	1 14 15 4	7 2 14 11	16 3 2 13	10 15 3 6
10 15 3 6	16 3 2 13	7 2 14 11	1 14 15 4	13 2 3 16	6 3 15 10	4 15 14 1	11 14 2 7
5 4 16 9	9 6 7 12	12 13 1 8	8 11 10 5	12 7 6 9	9 16 4 5	5 10 11 8	8 1 13 12
192/	107/	193/	278/	185/	267/	200/	118/
8 13 1 12	5 10 11 8	9 4 16 5	12 7 6 9	8 11 10 5	12 1 13 8	9 6 7 12	5 16 4 9
11 2 14 7	16 3 2 13	6 15 3 10	1 14 15 4	13 2 3 16	7 14 2 11	4 15 14 1	10 3 15 6
10 3 15 6	4 15 14 1	7 14 2 11	13 2 3 16	1 14 15 4	6 15 3 10	16 3 2 13	11 2 14 7
5 16 4 9	9 6 7 12	12 1 13 8	8 11 10 5	12 7 6 9	9 4 16 5	5 10 11 8	8 13 1 12

	6/	85/	379/	300/	19/	309/	366/	76/
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15	10 6 3	5 11 10 8	2 7 11 14	12 6 7 9	8 10 11 5	3 6 10 15	9 7 6 12	14 11 7 2
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	12/	87/	373/	298/	21/	303/	364/	82/
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	169/	110/	216/	275/	176/	286/	209/	99/
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	190/	113/	195/	272/	179/	265/	206/	120/
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	331/	348/	54/	37/	318/	28/	67/	357/
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4	5 9 16	6 9 12 7	13 12 8 1	11 8 5 10	7 12 9 6	16 9 5 4	10 5 8 11	1 8 12 13
15	10 6 3	3 16 13 2	2 7 11 14	14 1 4 15	2 13 16 3	3 6 10 15	15 4 1 14	14 11 7 2
	325/	346/	60/	39/	316/	34/	69/	351/
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4	9 5 16	10 5 8 11	13 8 12 1	7 12 9 6	11 8 5 10	16 5 9 4	6 9 12 7	1 12 8 13
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	264/	227/	121/	158/	257/	147/	128/	238/
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8	1 13 12	15 4 1 14	9 16 4 5	2 13 16 3	14 1 4 15	12 13 1 8	3 16 13 2	5 4 16 9
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	243/	224/	142/	161/	254/	168/	131/	217/
11	2 14 7	10 5 8 11	6 15 3 10	7 12 9 6	11 8 5 10	7 14 2 11	6 9 12 7	10 3 15 6
8	13 1 12	3 16 13 2	9 4 16 5	14 1 4 15	2 13 16 3	12 1 13 8	15 4 1 14	5 16 4 9
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	333/	342/	52/	43/	324/	30/	61/	355/
14	11 7 2	15 1 4 14	3 6 10 15	2 16 13 3	14 4 1 15	2 7 11 14	3 13 16 2	15 10 6 3
4	5 9 16	10 8 5 11	13 12 8 1	7 9 12 6	11 5 8 10	16 9 5 4	6 12 9 7	1 8 12 13
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15	10 6 3	3 13 16 2	2 7 11 14	14 4 1 15	2 16 13 3	3 6 10 15	15 1 4 14	14 11 7 2
	327/	340/	58/	45/	322/	36/	63/	349/
14	7 11 2	15 1 4 14	3 10 6 15	2 16 13 3	14 4 1 15	2 11 7 14	3 13 16 2	15 6 10 3
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15	6 10 3	3 13 16 2	2 11 7 14	14 4 1 15	2 16 13 3	3 10 6 15	15 1 4 14	14 7 11 2
	262/	233/	123/	152/	251/	145/	134/	240/
11	14 2 7	10 8 5 11	6 3 15 10	7 9 12 6	11 5 8 10	7 2 14 11	6 12 9 7	10 15 3 6
5	4 16 9	15 1 4 14	12 13 1 8	2 16 13 3	14 4 1 15	9 16 4 5	3 13 16 2	8 1 13 12
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10	15 3 6	6 12 9 7	7 2 14 11	11 5 8 10	7 9 12 6	6 3 15 10	10 8 5 11	11 14 2 7
	241/	230/	144/	155/	248/	166/	137/	219/
11	2 14 7	10 8 5 11	6 15 3 10	7 9 12 6	11 5 8 10	7 14 2 11	6 12 9 7	10 3 15 6
5	16 4 9	3 13 16 2	12 1 13 8	14 4 1 15	2 16 13 3	9 4 16 5	15 1 4 14	8 13 1 12
8	13 1 12	15 1 4 14	9 4 16 5	2 16 13 3	14 4 1 15	12 1 13 8	3 13 16 2	5 16 4 9
10	3 15 6	6 12 9 7	7 14 2 11	11 5 8 10	7 9 12 6	6 15 3 10	10 8 5 11	11 2 14 7

