

**Part 3: "New Algebraic Studies of Magic Squares"**  
**Chapter 3: Study of Magic Squares of Order 5: Kanji Setsuda**  
**Section 1: Self-Complementary Magic Squares of Order 5**

**#1. Definitions of Self-Complementary Magic Squares of Order 5**

Let's study the Self-Complementary type of Magic Squares of Order 5, first of all. It is defined by such the basic form and the simultaneous equations as follows.

The last multi-equation (13) means the existence of 'Complementary Pairs of C' placed symmetrically with respect to  $n_{13}$  on the geometric center of this square.

[Figure 1: Definitions of Self-Complementary Type]

$n_1--n_2--n_3--n_4--n_5$		$n_1+ n_2+ n_3+ n_4+ n_5=K$ .... (1)	
		$n_6+ n_7+ n_8+ n_9+n_{10}=K$ .... (2)	
$n_6--n_7--n_8--n_9--n_{10}$		$n_{11}+n_{12}+n_{13}+n_{14}+n_{15}=K$ .... (3)	
		$n_{16}+n_{17}+n_{18}+n_{19}+n_{20}=K$ .... (4)	
$11--12--13--14--15$		$n_{21}+n_{22}+n_{23}+n_{24}+n_{25}=K$ .... (5)	
		$n_1+ n_6+n_{11}+n_{16}+n_{21}=K$ .... (6)	
$16--17--18--19--20$		$n_2+ n_7+n_{12}+n_{17}+n_{22}=K$ .... (7)	
		$n_3+ n_8+n_{13}+n_{18}+n_{23}=K$ .... (8)	
$21--22--23--24--25$		$n_4+ n_9+n_{14}+n_{19}+n_{24}=K$ .... (9)	
		$n_5+ n_{10}+n_{15}+n_{20}+n_{25}=K$ ... (10)	
$n_1+n_{25}=n_2+n_{24}=n_3+n_{23}$		$n_1+ n_7+n_{13}+n_{19}+n_{25}=K$ ... (11)	
$=n_4+n_{22}=n_5+n_{21}=n_6+n_{20}$		$n_5+ n_9+n_{13}+n_{17}+n_{21}=K$ ... (12)	
$=n_7+n_{19}+n_8+n_{18}+n_9+n_{17}$			
$=n_{10}+n_{16}=n_{11}+n_{15}=n_{12}+n_{14}=n_{13}+n_{13}=C$		... (13)	

The two Magic Constants K and C should be determined by our simple calculations.

(1)+(2)+(3)+(4)+(5)

$$n_1+n_2+n_3+\dots+n_{13}+\dots+n_{24}+n_{25}=5*K$$

$$1+2+3+4+\dots+13+14+\dots+24+25=5*K$$

$$(25+1) \times 25 / 2 = 5*K$$

Therefore  $K=65$  .... (14)

(11)  $n_1+n_7+n_{13}+n_{19}+n_{25}=(n_1+n_{25})+(n_7+n_{19})+n_{13}=K$

$C + C + C/2 = K$  ( $n_{13}=C/2$ ; because  $n_{13}+n_{13}=C$ )

Multiply both sides by 2

$$5*C=2*K$$

As a consequence  $C=2*65/5=26$  .... (15)

Therefore, the equations (13) actually take such the value pairs as:

$(1, 25), (2, 24), (3, 23), (4, 22), (5, 21), (6, 20), \dots, (11, 15), (12, 14), (13, 13);$

You may rewrite these equations (13) into:

$n_{25}=26-n_1; n_{24}=26-n_2; n_{23}=26-n_3; n_{22}=26-n_4; \dots; n_{15}=26-n_{11}; n_{14}=26-n_{12};$   
 and  $n_{13}=26-n_{13}=13;$

Let me introduce you here a new notation for those Complementary Pairs.

$\underline{n_1}=26-n_1; \underline{n_2}=26-n_2; \underline{n_3}=26-n_3; \underline{n_4}=26-n_4; \dots; \underline{n_{11}}=26-n_{11}; \underline{n_{12}}=26-n_{12};$  and  
 $\underline{n_{13}}=26-n_{13};$

You could rewrite them back to the original style anytime you like:

$\underline{n_1}=n_{25}; \underline{n_2}=n_{24}; \underline{n_3}=n_{23}; \underline{n_4}=n_{22}; \dots; \underline{n_{12}}=n_{14};$  and  $\underline{n_{13}}=n_{13};$

Or you could write them as:

$n_1+\underline{n_1}=26; n_2+\underline{n_2}=26; n_3+\underline{n_3}=26; n_4+\underline{n_4}=26; \dots; n_{11}+\underline{n_{11}}=26; n_{12}+\underline{n_{12}}=26;$  and  
 $n_{13}+\underline{n_{13}}=26;$

You could also write the basic diagram and conditions as follows.

[Figure 2: Basic Conditions for Self-Complementary Type]

n1--n2--n3--n4--n5		n1+ n2+ n3+ n4+ n5=K ....(1)
		n6+ n7+ n8+ n9+n10=K ....(2)
n6--n7--n8--n9--10		n11+n12+n13+n12+n11=K ....(3)
		n10+ n9+ n8+ n7+ n6=K ....(4)
11--12--13--12--11		n5+ n4+ n3+ n2+ n1=K ....(5)
		n1+ n6+n11+n10+ n5=K ....(6)
10--n9--n8--n7--n6		n2+ n7+n12+ n9+ n4=K ....(7)
		n3+ n8+n13+ n8+ n3=K ....(8)
n5--n4--n3--n2--n1		n4+ n9+n12+ n7+ n2=K ....(9)
		n5+n10+n11+ n6+ n1=K ... (10)
n1+n1=n2+n2=n3+n3		n1+ n7+n13+ n7+ n1=K ... (11)
=n4+n4=n5+n5=n6+n6		n5+ n9+n13+ n9+ n5=K ... (12)
=n7+n7=n8+n8=n9+n9		(Of course K=65)
=n10+n10=n11+n11=n12+n12=n13+n13=C=26		... (13)

You might easily find where any complementary pair is placed in the diagram.

Take the 2 primary diagonals, for instance.

$$n1+n7+n13+n7+n1=K \dots (11); \quad n5+n9+n13+n9+n5=K \dots (12)$$

$$K=(n1+n1)+(n7+n7)+n13=(n5+n5)+(n9+n9)+n13=C+C+13=26+26+13=65$$

It is the locations of those complementary pairs that must really make the sums of 2 diagonals equal to the magic constant.

Find this kind of characteristic structure in any line, and you can just know its sum. You don't have to calculate the constant sums any more. You may well only see it.

## #2. How Many Solutions could we find for that type?

Under the basic conditions of Figure 1, I dictated some new program codes for my personal computer and tried to count all the solutions of our object.

```

/** Self-Complementary Magic Squares of Order 5: **
/** Compact List of 48544 Standard Solutions **
/** File: 'SCMS55S.c' Dictated by Kanji Setsuda **
/** on Feb. 5, 2005; Sep.30, 2006 **
/** Revised on Jan.28, 2015 **
/** with MacOSX 10.10.2 & Xcode 6.1.1 **
//
#include <stdio.h>
//
long cnt;
long cntr[5];
short cnt11, cnt13;
short cnt2, cnt3;
short LSM, CC;
short nm[26], uflg[26], area[26];
short nt[5][26];
//
void stp01(void), stp02(void), stp03(void), stp04(void);
void stp05(void), stp06(void), stp07(void), stp08(void);
void stp09(void), stp10(void), stp11(void), stp12(void);
//
void stp21(void), stp22(void), stp23(void), stp24(void);
void stp25(void), stp26(void), stp27(void), stp28(void);
void stp29(void), stp30(void), stp31(void), stp32(void);
//
void ansprint(void);

```

```

void prans(short x);
//
/** Main Program *
int main(){
    short n;
    printf("\n");
    printf(" *** Self-Complementary Magic Squares of Order 5: ***\n");
    printf(" ** Compact List 1 of 48544 Standard Solutions starting with n1=1 **\n");
    for(n=0;n<26;n++){nm[n]=0; uflg[n]=0;}
    LSM=65; CC=26; cnt=0; cnt11=0; cnt13=0; cnt3=0;
    nm[13]=13; uflg[13]=1;
    stp01();
    uflg[13]=0; nm[13]=0;
    if(cnt3>0){prans(cnt3);}
    printf(" [Solution Counts(n1=1/All) = %d/%ld] OK!\n",cnt11,cnt);
    printf("\n");
    printf(" ** Compact List 2 of 48544 Standard Solutions starting with n3=1 **\n");
    for(n=0;n<26;n++){nm[n]=0; uflg[n]=0;}
    LSM=65; CC=26; cnt=0; cnt11=0; cnt13=0; cnt3=0;
    nm[13]=13; uflg[13]=1;
    stp21();
    uflg[13]=0; nm[13]=0;
    if(cnt3>0){prans(cnt3);}
    printf(" [Solution Counts(n3=1/All) = %d/%ld] OK!\n",cnt13,cnt);
    printf("\n");
    printf("*** Calculated and Listed by Kanji Setsuda on\n");
    printf(" Jan.28, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 **\n");
    printf("\n");
    return 0;
}
/** Begin The Search 1 *
/** Normalizing Inequality Conditions: n1<n5<n21 and n1<n25;
/** Set N1 & n25 & N1<n25 *
void stp01(){
    short a,b;
    for(a=1;a<13;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){cnt2=0;
            nm[1]=a; nm[25]=b;
            uflg[a]=1; uflg[b]=1;
            stp02();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set N3 & n23 *
void stp02(){
    short a,b;
    for(a=1;a<26;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){if(nm[1]==1){cnt2=0;}
            nm[3]=a; nm[23]=b;
            uflg[a]=1; uflg[b]=1;
            stp03();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set N2 & n24 *
void stp03(){
    short a,b;
    for(a=1;a<26;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[2]=a; nm[24]=b;
            uflg[a]=1; uflg[b]=1;
            stp04();
            uflg[b]=0; uflg[a]=0;}
    }
}
}

```

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/** Set N4 & n22 *
void stp04(){
short a,b;
for(a=1;a<26;a++){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[4]=a; nm[22]=b;
uflg[a]=1; uflg[b]=1;
stp05();
uflg[b]=0; uflg[a]=0;}}
}
}
/** Set n5=LSM-n1-n2-n3-n4 & n21 & n1<n5<n21 *
void stp05(){
short a,b;
a=LSM-nm[1]-nm[2]-nm[3]-nm[4];
if((nm[1]<a)&&(a<26)){
b=LSM-nm[25]-nm[24]-nm[23]-nm[22];
if((a<b)&&(a+b==CC)){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[5]=a; nm[21]=b;
uflg[a]=1; uflg[b]=1;
stp06();
uflg[b]=0; uflg[a]=0;}}}}
}
}
/** Set N6 & n20 *
void stp06(){
short a,b;
for(a=1;a<26;a++){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[6]=a; nm[20]=b;
uflg[a]=1; uflg[b]=1;
stp07();
uflg[b]=0; uflg[a]=0;}}
}
}
}
/** Set N16 & n10 *
void stp07(){
short a,b;
for(a=25;a>0;a--){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[16]=a; nm[10]=b;
uflg[a]=1; uflg[b]=1;
stp08();
uflg[b]=0; uflg[a]=0;}}
}
}
}
/** Set n11=LSM-n1-n6-n16-n21 & n15 *
void stp08(){
short a,b;
a=LSM-nm[1]-nm[6]-nm[16]-nm[21];
if((0<a)&&(a<26)){
b=LSM-nm[25]-nm[20]-nm[10]-nm[5];
if(a+b==CC){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[11]=a; nm[15]=b;
uflg[a]=1; uflg[b]=1;
stp09();
uflg[b]=0; uflg[a]=0;}}}}
}
}
}
/** Set N7 & n19 *
void stp09(){
short a,b;
for(a=1;a<26;a++){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[7]=a; nm[19]=b;

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        uflg[a]=1; uflg[b]=1;
        if(nm[1]+nm[7]+nm[13]+nm[19]+nm[25]==LSM){stp10();}
        uflg[b]=0; uflg[a]=0;}
    }
}
/** Set N9 & n17 *
void stp10(){
    short a,b;
    for(a=25;a>0;a--){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[9]=a; nm[17]=b;
            uflg[a]=1; uflg[b]=1;
            if(nm[5]+nm[9]+nm[13]+nm[17]+nm[21]==LSM){stp11();}
            uflg[b]=0; uflg[a]=0;}
        }
}
/** Set n8=LSM-n6-n7-n9-n10 & n18 *
void stp11(){
    short a,b;
    a=LSM-nm[6]-nm[7]-nm[9]-nm[10];
    if((0<a)&&(a<26)){
        b=LSM-nm[20]-nm[19]-nm[17]-nm[16];
        if(a+b==CC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[8]=a; nm[18]=b;
                uflg[a]=1; uflg[b]=1;
                if(nm[3]+nm[8]+nm[13]+nm[18]+nm[23]==LSM){stp12();}
                uflg[b]=0; uflg[a]=0;}}}
}
/** Set n12=LSM-n2-n7-n17-n22 & n14 *
void stp12(){
    short a,b;
    a=LSM-nm[2]-nm[7]-nm[17]-nm[22];
    if((0<a)&&(a<26)){
        b=LSM-nm[24]-nm[19]-nm[9]-nm[4];
        if(a+b==CC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[12]=a; nm[14]=b;
                uflg[a]=1; uflg[b]=1;
                if(nm[11]+nm[12]+nm[13]+nm[14]+nm[15]==LSM){ansprint();}
                uflg[b]=0; uflg[a]=0;}}}
}
//
/** Begin The Search 2 *
/** Normalizing Inequality Conditions:  n3<n11<n15 and n3<n23;
/** Set N3 & n23 & n3<n23 *
void stp21(){
    short a,b;
    for(a=1;a<13;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){cnt2=0;
            nm[3]=a; nm[23]=b;
            uflg[a]=1; uflg[b]=1;
            stp22();
            uflg[b]=0; uflg[a]=0;}
        }
}
/** Set N1 & n25 *
void stp22(){
    short a,b;
    for(a=1;a<26;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){if(nm[3]==1){cnt2=0;}
            nm[1]=a; nm[25]=b;
            uflg[a]=1; uflg[b]=1;
            stp23();
            uflg[b]=0; uflg[a]=0;}
        }
}

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}
}
/** Set N2 & n24 *
void stp23(){
short a,b;
for(a=1;a<26;a++){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[2]=a; nm[24]=b;
uflg[a]=1; uflg[b]=1;
stp24();
uflg[b]=0; uflg[a]=0;}}
}
}
/** Set N4 & n22 *
void stp24(){
short a,b;
for(a=1;a<26;a++){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[4]=a; nm[22]=b;
uflg[a]=1; uflg[b]=1;
stp25();
uflg[b]=0; uflg[a]=0;}}
}
}
/** Set n5=LSM-n1-n2-n3-n4 & n21 *
void stp25(){
short a,b;
a=LSM-nm[1]-nm[2]-nm[3]-nm[4];
if((0<a)&&(a<26)){
b=LSM-nm[25]-nm[24]-nm[23]-nm[22];
if(a+b==CC){
if((uflg[a]==0)&&(uflg[b]==0)){
nm[5]=a; nm[21]=b;
uflg[a]=1; uflg[b]=1;
stp26();
uflg[b]=0; uflg[a]=0;}}}
}
}
/** Set N6 & n20 *
void stp26(){
short a,b;
for(a=1;a<26;a++){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[6]=a; nm[20]=b;
uflg[a]=1; uflg[b]=1;
stp27();
uflg[b]=0; uflg[a]=0;}}
}
}
/** Set N16 & n10 *
void stp27(){
short a,b;
for(a=25;a>0;a--){b=CC-a;
if((uflg[a]==0)&&(uflg[b]==0)){
nm[16]=a; nm[10]=b;
uflg[a]=1; uflg[b]=1;
stp28();
uflg[b]=0; uflg[a]=0;}}
}
}
/** Set n11=LSM-n1-n6-n16-n21 & n15 & n3<n11<n15 *
void stp28(){
short a,b;
a=LSM-nm[1]-nm[6]-nm[16]-nm[21];
if((nm[3]<a)&&(a<26)){
b=LSM-nm[25]-nm[20]-nm[10]-nm[5];

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    if((a<b)&&(a+b==CC)){
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[11]=a; nm[15]=b;
            uflg[a]=1; uflg[b]=1;
            stp29();
            uflg[b]=0; uflg[a]=0;}}
}
/* Set N7 & n19 *
void stp29(){
    short a,b;
    for(a=1;a<26;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[7]=a; nm[19]=b;
            uflg[a]=1; uflg[b]=1;
            if(nm[1]+nm[7]+nm[13]+nm[19]+nm[25]==LSM){stp30();}
            uflg[b]=0; uflg[a]=0;}}
}
}
/* Set N9 & n17 *
void stp30(){
    short a,b;
    for(a=25;a>0;a--){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[9]=a; nm[17]=b;
            uflg[a]=1; uflg[b]=1;
            if(nm[5]+nm[9]+nm[13]+nm[17]+nm[21]==LSM){stp31();}
            uflg[b]=0; uflg[a]=0;}}
}
}
/* Set n8=LSM-n6-n7-n9-n10 & n18 *
void stp31(){
    short a,b;
    a=LSM-nm[6]-nm[7]-nm[9]-nm[10];
    if((0<a)&&(a<26)){
        b=LSM-nm[20]-nm[19]-nm[17]-nm[16];
        if(a+b==CC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[8]=a; nm[18]=b;
                uflg[a]=1; uflg[b]=1;
                if(nm[3]+nm[8]+nm[13]+nm[18]+nm[23]==LSM){stp32();}
                uflg[b]=0; uflg[a]=0;}}}}
}
}
/* Set n12=LSM-n2-n7-n17-n22 & n14 *
void stp32(){
    short a,b;
    a=LSM-nm[2]-nm[7]-nm[17]-nm[22];
    if((0<a)&&(a<26)){
        b=LSM-nm[24]-nm[19]-nm[9]-nm[4];
        if(a+b==CC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[12]=a; nm[14]=b;
                uflg[a]=1; uflg[b]=1;
                if(nm[11]+nm[12]+nm[13]+nm[14]+nm[15]==LSM){ansprint();}
                uflg[b]=0; uflg[a]=0;}}}}
}
}
//
/* Print the Answers *
void ansprint(){
    short n;
    cnt++;
    if(nm[1]==1){cnt11++;}
    if(nm[3]==1){cnt13++;}
    cnt2++;
    if(cnt2==1){
        for(n=1;n<26;n++){nt[cnt3][n]=nm[n];}
}

```

```

    cnt3++; if(cnt3==5){prans(cnt3); cnt3=0;}}
}
//
/** Print the Answers *
void prans(short x){
short l,l5,m,n;
for(n=0;n<x;n++){
printf("%15ld#",cnt3[n]);
if(n+1<x){printf(" ");}
}
printf("\n");
for(l=0;l<5;l++){l5=l*5;
for(m=0;m<x;m++){
printf(" %3d%3d%3d%3d%3d",
nt[m][l5+1],nt[m][l5+2],nt[m][l5+3],nt[m][l5+4],nt[m][l5+5]);
if(m+1<x){printf(" ");}
}
printf("\n");
}
}
}

```

Let me show you here the following 2 types of lists for the 48544 Standard Solutions that I have got by the recent execution of my program above.

(1) For the set of Standard Solutions starting with  $n_1=1$ , I assumed the normalizing inequality condition as  $n_1 < n_5 < n_{21}$  and  $n_1 < n_{25}$ .

(2) For another set of 48544 Standard Solutions starting with  $n_3=1$ , I assumed the normalizing inequality condition as  $n_3 < n_{11} < n_{15}$  and  $n_3 < n_{23}$ .

Watch the next two lists and know about their unique contrast of demonstrations.

**\*\* Self-Complementary Magic Squares of Order 5: \*\***

**\*\* Compact List 1 of the 48544 Standard Solutions starting with  $n_1=1$  \*\***

1#	41#	77#	113#	245#
1 23 5 24 12	1 22 6 24 12	1 21 7 24 12	1 20 8 24 12	1 20 9 23 12
8 15 19 17 6	8 21 16 17 3	8 17 23 11 6	7 23 16 15 4	7 24 11 18 5
22 16 13 10 4	19 11 13 15 7	22 10 13 16 4	21 9 13 17 5	22 10 13 16 4
20 9 7 11 18	23 9 10 5 18	20 15 3 9 18	22 11 10 3 19	21 8 15 2 19
14 2 21 3 25	14 2 20 4 25	14 2 19 5 25	14 2 18 6 25	14 3 17 6 25
337#	509#	677#	1073#	1553#
1 18 10 24 12	1 17 11 24 12	1 17 12 24 11	1 16 14 23 11	1 16 15 21 12
11 23 4 21 6	7 16 21 18 3	7 23 10 21 4	4 19 20 17 5	9 22 7 24 3
19 17 13 9 7	20 22 13 4 6	20 18 13 8 6	24 18 13 8 2	18 20 13 6 8
20 5 22 3 15	23 8 5 10 19	22 5 16 3 19	21 9 6 7 22	23 2 19 4 17
14 2 16 8 25	14 2 15 9 25	15 2 14 9 25	15 3 12 10 25	14 5 11 10 25
1881#	2417#	2801#	3345#	3861#
1 14 16 23 11	1 11 17 24 12	1 10 18 24 12	1 9 19 24 12	1 8 20 24 12
6 18 22 17 2	8 23 21 7 6	7 21 22 9 6	18 23 5 15 4	9 21 16 15 4
19 21 13 5 7	22 10 13 16 4	23 15 13 11 3	10 20 13 6 16	19 23 13 3 7
24 9 4 8 20	20 19 5 3 18	20 17 4 5 19	22 11 21 3 8	22 11 10 5 17
15 3 10 12 25	14 2 9 15 25	14 2 8 16 25	14 2 7 17 25	14 2 6 18 25
4525#	4969#	5829#	6601#	7793#
1 8 21 24 11	1 6 22 24 12	1 5 23 24 12	1 5 24 23 12	2 23 4 25 11
12 22 19 3 9	16 19 18 9 3	15 16 19 6 9	11 22 16 9 7	8 14 20 16 7
20 10 13 16 6	11 21 13 5 15	18 22 13 4 8	20 18 13 8 6	21 17 13 9 5
17 23 7 4 14	23 17 8 7 10	17 20 7 10 11	19 17 10 4 15	19 10 6 12 18
15 2 5 18 25	14 2 4 20 25	14 2 3 21 25	14 3 2 21 25	15 1 22 3 24
14601#	21805#	27597#	33193#	37313#
3 21 4 25 12	4 23 2 25 11	5 23 1 24 12	6 22 1 24 12	7 21 1 24 12
7 15 16 18 9	7 17 20 16 5	6 15 19 17 8	7 16 21 18 3	6 15 23 17 4
24 20 13 6 2	18 14 13 12 8	22 16 13 10 4	15 17 13 9 11	16 18 13 8 10
17 8 10 11 19	21 10 6 9 19	18 9 7 11 20	23 8 5 10 19	22 9 3 11 20
14 1 22 5 23	15 1 24 3 22	14 2 25 3 21	14 2 25 4 20	14 2 25 5 19



41081#	44129#	46533#	48065#
8 20 1 24 12	9 19 1 24 12	10 18 1 24 12	11 17 1 24 12
3 22 17 16 7	3 22 15 20 5	4 15 21 19 6	3 18 19 20 5
21 11 13 15 5	18 16 13 10 8	17 23 13 3 9	16 22 13 4 10
19 10 9 4 23	21 6 11 4 23	20 7 5 11 22	21 6 7 8 23
14 2 25 6 18	14 2 25 7 17	14 2 25 8 16	14 2 25 9 15

[Solution Counts(when n1=1/A11) = 7792/48544] OK!

\*\* Compact List 2 of the 48544 Standard Solutions starting with n3=1 \*\*

1#	53#	117#	165#	269#
3 18 1 24 19	4 17 1 23 20	5 16 1 24 19	6 15 1 19 24	7 10 1 24 23
21 15 9 16 4	21 12 19 11 2	22 12 17 11 3	22 9 21 10 3	21 15 17 8 4
12 20 13 6 14	10 18 13 8 16	8 20 13 6 18	12 18 13 8 14	12 20 13 6 14
22 10 17 11 5	24 15 7 14 5	23 15 9 14 4	23 16 5 17 4	22 18 9 11 5
7 2 25 8 23	6 3 25 9 22	7 2 25 10 21	2 7 25 11 20	3 2 25 16 19
469#	761#	981#	1281#	1633#
8 10 1 24 22	9 10 1 21 24	10 7 1 23 24	11 6 1 23 24	12 6 1 22 24
19 20 9 14 3	19 22 15 6 3	20 15 17 8 5	19 16 17 8 5	19 18 15 10 3
11 21 13 5 15	12 8 13 18 14	12 22 13 4 14	12 22 13 4 14	9 21 13 5 17
23 12 17 6 7	23 20 11 4 7	21 18 9 11 6	21 18 9 10 7	23 16 11 8 7
4 2 25 16 18	2 5 25 16 17	2 3 25 19 16	2 3 25 20 15	2 4 25 20 14
2077#	2405#	2845#	3205#	3617#
14 5 1 23 22	15 3 1 24 22	16 3 1 21 24	17 3 1 20 24	18 2 1 21 23
17 15 19 8 6	16 19 17 5 8	15 17 19 8 6	15 18 21 4 7	15 16 19 6 9
10 24 13 2 16	12 20 13 6 14	12 22 13 4 14	12 16 13 10 14	12 22 13 4 14
20 18 7 11 9	18 21 9 7 10	20 18 7 9 11	19 22 5 8 11	17 20 7 10 11
4 3 25 21 12	4 2 25 23 11	2 5 25 23 10	2 6 25 23 9	3 5 25 24 8
4025#	4849#	5433#	6081#	6737#
19 2 1 20 23	20 2 1 23 19	21 2 1 18 23	22 2 1 19 21	23 2 1 18 21
15 21 9 12 8	14 21 11 9 10	11 19 17 12 6	14 23 11 9 8	12 22 15 9 7
10 22 13 4 16	8 22 13 4 18	10 22 13 4 16	6 16 13 10 20	6 16 13 10 20
18 14 17 5 11	16 17 15 5 12	20 14 9 7 15	18 17 15 3 12	19 17 11 4 14
3 6 25 24 7	7 3 25 24 6	3 8 25 24 5	5 7 25 24 4	5 8 25 24 3
7681#	8689#	17309#	23157#	29177#
24 3 1 15 22	3 16 2 25 19	1 20 3 22 19	1 18 4 23 19	1 20 5 22 17
7 21 17 14 6	21 17 8 15 4	21 17 15 10 2	21 15 10 17 2	23 11 19 10 2
10 18 13 8 16	12 20 13 6 14	12 8 13 18 14	12 20 13 6 14	8 14 13 12 18
20 12 9 5 19	22 11 18 9 5	24 16 11 9 5	24 9 16 11 5	24 16 7 15 3
4 11 25 23 2	7 1 24 10 23	7 4 23 6 25	7 3 22 8 25	9 4 21 6 25
33689#	38109#	41085#	43829#	46033#
1 15 6 24 19	1 17 7 22 18	1 15 8 20 21	1 15 9 22 18	1 14 10 22 18
22 17 18 5 3	20 23 5 15 2	23 19 4 17 2	21 24 7 10 3	21 17 6 19 2
12 10 13 16 14	12 10 13 16 14	12 16 13 10 14	12 6 13 20 14	11 23 13 3 15
23 21 8 9 4	24 11 21 3 6	24 9 22 7 3	23 16 19 2 5	24 7 20 9 5
7 2 20 11 25	8 4 19 9 25	5 6 18 11 25	8 4 17 11 25	8 4 16 12 25
47585#				
1 16 11 18 19				
21 17 3 22 2				
12 20 13 6 14				
24 4 23 9 5				
7 8 15 10 25				

[Solution Counts(when n3=1/A11) = 8688/48544] OK!

\*\* Calculated and Listed by Kanji Setsuda on  
Jan.25, 2015 with MacOSX 10.10.1 & Xcode 6.1.1 \*\*

These two sets look quite different, but each solution of the former set should have the one-to-one correspondence between the other one of the latter.

Difference is derived from their own unique styles of making lists smart, because I had to assume the different 'normalizing inequality conditions' for them.

If you want to find the correspondence between the two, or if you want to find out your answer there, you should have to know how the list order was determined and

how to adjust to those conditions when you have to reflect or rotate your answer.

### #3. What Parts of this Square can we Exchange?

Let's consider about the 'fundamental solutions' of our object now.

Can't we have any smaller set of solutions, the essence of the Self-complementary Magic Squares of Order 5?

Before trying to get such the essential solutions, we have to know which kinds of solutions should be classified into the same group, I would say, the same 'family'.

Give your careful look at the next 4 squares below.

[Figure 3-1: Members of the same family]

Exchange Lines: (2<->4)

No.1

1	23	17	22	2
21	8	11	6	19
12	10	13	16	14
7	20	15	18	5
24	4	9	3	25

Original

No.2

1	23	17	22	2
7	20	15	18	5
12	10	13	16	14
21	8	11	6	19
24	4	9	3	25

Row(2<->4)  
Exchange

No.3

1	22	17	23	2
21	6	11	8	19
12	16	13	10	14
7	18	15	20	5
24	3	9	4	25

Column(2<->4)  
Exchange

Double Exchange:

No.4

1	22	17	23	2
7	18	15	20	5
12	16	13	10	14
21	6	11	8	19
24	3	9	4	25

Row & C1m(2<->4)  
Exchange

They are very similar, but you can find a little difference among them, can't you?

No.2 has got its 2nd and 4th rows exchanged with each other from No.1.

No.3 has got its 2nd and 4th columns exchanged with each other from No.1.

No.4 has got its 2nd and 4th rows and columns both exchanged at the same time.

Could you find that the contents of any column or any row are not changed at all and they always add up to the same constant sum? Only the precise positions or orders of the contents are partially changed.

Watch the 2 primary diagonals now. Both of the contents and positions are changed, but the sums of them are just the same with the magic constant 65, for they only exchange the positions of the same 'Complementary Pairs of 26': (8,18) and (6,20).

Should we still recognize these 4 squares as different and independent?

Or should we now classify them into the same group of No.1 and suppose them as the 'family' members of a single 'fundamental' solution?

How do you like to do with it? It is the problem of your personal decision.

I decide to take them for the family members of a single 'fundamental' solution.

Watch the next basic forms. They tell us another possibilities how to exchange lines. They are designed for us to know clearly about the transformations we have wanted.

[Figure 3-2: Which lines can we exchange 2 by 2?]

Exchange Lines(1<->2)&(4<->5):

No.1

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
10	9	8	7	6
5	4	3	2	1

Original

No.5

6	7	8	9	10
1	2	3	4	5
11	12	13	14	15
5	4	3	2	1
10	9	8	7	6

Row(1<->2)&(4<->5)  
Exchange

No.6

2	1	3	5	4
7	6	8	10	9
12	11	13	14	15
9	10	8	6	7
4	5	3	1	2

Column(1<->2)&(4<->5)  
Exchange

Double Exchange:

No.7

7	6	8	10	9
2	1	3	5	4
12	11	13	14	15
4	5	3	1	2
9	10	8	6	7

R&C(1<->2)&(4<->5)  
Exchange

You can now exchange the 2x2 lines at the same time just as in No.5 and No.6.

In No.7, you can exchange both rows and columns 2 by 2 at the same time. Check every primary diagonal whether the sum of each is equal to 65 or not. For instance,

$$n6+n2+n13+n2+n6 = (n6+n6)+(n2+n2)+n13 = C+C+13=65; \text{ Very good!}$$

$$n10+n4+n13+n4+n10 = (n10+n10)+(n4+n4)+n13 = C+C+13=65; \text{ Very good!}$$

$$n7+n1+n13+n1+n7 = (n7+n7)+(n1+n1)+n13 = C+C+13=65; \text{ Very good!}$$

$$n9+n5+n13+n5+n9 = (n9+n9)+(n5+n5)+n13 = C+C+13=65; \text{ Very good!}$$

You can accept these three types of line-exchange and classify the results into the same family, can't you?

Watch the following diagrams for the last transformation. They also show another possibilities how to make a family by exchanging lines 4 by 4 at the same time.

[Figure 3-3: Which lines can we exchange 4 by 4?]

Change Lines: (1->2)&(2->5)&(4->1)&(5->4);

Double Change:

No.1	No.8	No.9	No.10																																																																																																				
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Now let's come to my conclusion for the summary of these new possibilities.

I want to combine all of them into our transformation system so that we could have got every set of 'family' solutions from the fewer 'fundamental' ones.

Let me show you here the conceptual diagram of our new transformation system.

\*\* Concepts of 15 Transformations for our Object: \*\*

[Original] 00/T	01/T	02/T	03/T																																																																																																				
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I put those three types of row exchanges on the X-axis(01/T, 02/T and 03/T) and those three of the column exchanges on the Y-axis(10/T, 20/T and 30/T).

For the various double exchanges by the combinations of row and column, I put the results on the other positions. Watch the line(11/T, 22/T and 33/T) especially.

I have finally taken the 16 transformations in all, including 'no operation' to the original (00/T), as you see above.

It means we could classify all these 16 transformed results into the single group led by the 'fundamental solution' placed on 00/T.

As a consequence, it is implied that we should have got the 3034 Fundamental Solutions in all from the 48544 Standard Solutions. It is because  $48544 = 16 \times 3034$ .

#### #4. But why are these 16 Patterns only available for our SC Magic Squares 5x5?

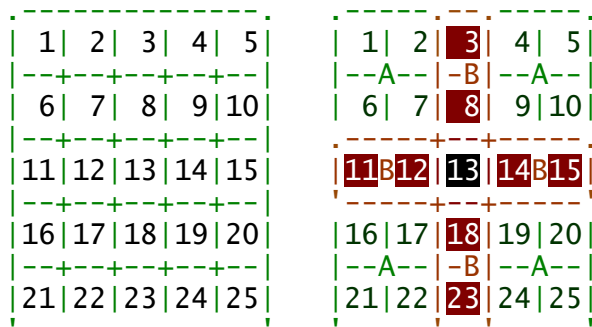
Watch the above diagrams again for the Concept of Transformations.

It seems you can move any number to anywhere you like, when you apply these 16 transformations and the additional reflection or rotation to the results if necessary.

But, you may soon notice that you can never move any number from the 3rd row or 3rd column into the other area by any combination of those line-exchanges.

The contents of the 3rd row and the 3rd column stay still as  $\{n_3, n_8, n_{13}, n_8, n_3\}$  and  $\{n_{11}, n_{12}, n_{13}, n_{12}, n_{11}\}$ , while their positions or orders may be changed among them.

[Figure 5: Exclusive Areas]



There are certainly three 'Exclusive Areas' in the square shown as above:

- (1)  $n_{13}$ : takes only the fixed value 13 and stays still.
- (2) **A** :  $\{n_1, n_2, n_6, n_7, n_4, n_5, n_9, n_{10}, n_{16}, n_{17}, n_{21}, n_{22}, n_{19}, n_{20}, n_{24}, n_{25}\}$
- (3) **B** :  $\{n_3, n_8, n_{13}, n_{18}, n_{23}\}, \{n_{11}, n_{12}, n_{13}, n_{14}, n_{15}\}$

Any number in any area can never move to the other area.

$n_1$  can surely move to any position in Area **A**  $\{n_1, n_2, n_6, n_7, n_4, n_5, \dots, n_{25}\}$ , but it can never move into Area **B**  $\{n_3, n_8, n_{11}, n_{12}, \dots, n_{23}\}$ .

$n_3$  can move to any position in Area **B**  $\{n_3, n_8, n_{11}, n_{12}, \dots, n_{23}\}$ , but it can never move into Area **A**  $\{n_1, n_2, n_6, n_7, n_4, n_5, n_9, \dots, n_{25}\}$ .

The Area **B** is always critical, where the symmetrical centers always exist.

As a result, we are forced to assume the 2 types of different 'Normalizing Inequality Conditions' for the two sets of 'Standard Solutions' of our Self-Complementary Magic Squares: The one starting with  $n_1=1$  and the other starting with  $n_3=1$ .

And our 16 Transformations Method only concerns about the Area **A** and nothing about Area **B** at all, while the positions or orders of 4 contents in **B** change a little.

Therefore, you can use only the 16 Patterns in total, because  $16 = 25 - 9$ ;

#### #5. Find the Fundamental Solutions for each type

Let's try to find the 3034 Fundamental Solutions actually for those two types now.

For the present I tried to presume such the 2 different sets of 'Selecting Inequality Conditions' for the two types as follows.

(1) For the type starting with  $n1=1$ :  $n1 < n2 < n4 < n5 < n21$ ,  $n1 < n6 < n16$  and  $n1 < n25$ ;

(2) For the type starting with  $n3=1$ :  $n3 < n8 < n18 < n23$ ,  $n3 < n11 < n12 < n14 < n15$ ;

Let me show you the recent results of my computation in compact style.

**\*\* Self-Complementary Magic Squares of Order 5: \*\***

**\*\* Compact List 1 of Fundamental Solutions starting with  $n1=1$  \*\***

(Selecting Inequality Conditions:  $n1 < n2 < n4 < n5 < n21$ ,  $n1 < n6 < n16$ ; and  $n1 < n25$ ;) )

1#	11#	20#	29#	62#
1 23 5 24 12	1 22 6 24 12	1 21 7 24 12	1 20 8 24 12	1 20 9 23 12
8 15 19 17 6	8 21 16 17 3	8 17 23 11 6	7 23 16 15 4	7 24 11 18 5
22 16 13 10 4	19 11 13 15 7	22 10 13 16 4	21 9 13 17 5	22 10 13 16 4
20 9 7 11 18	23 9 10 5 18	20 15 3 9 18	22 11 10 3 19	21 8 15 2 19
14 2 21 3 25	14 2 20 4 25	14 2 19 5 25	14 2 18 6 25	14 3 17 6 25
85#	128#	170#	269#	389#
1 18 10 24 12	1 17 11 24 12	1 17 12 24 11	1 16 14 23 11	1 16 15 21 12
11 23 4 21 6	7 16 21 18 3	7 23 10 21 4	4 19 20 17 5	9 22 7 24 3
19 17 13 9 7	20 22 13 4 6	20 18 13 8 6	24 18 13 8 2	18 20 13 6 8
20 5 22 3 15	23 8 5 10 19	22 5 16 3 19	21 9 6 7 22	23 2 19 4 17
14 2 16 8 25	14 2 15 9 25	15 2 14 9 25	15 3 12 10 25	14 5 11 10 25
471#	605#	701#	837#	966#
1 14 16 23 11	1 11 17 24 12	1 10 18 24 12	1 9 19 24 12	1 8 20 24 12
6 18 22 17 2	8 23 21 7 6	7 21 22 9 6	18 23 5 15 4	9 21 16 15 4
19 21 13 5 7	22 10 13 16 4	23 15 13 11 3	10 20 13 6 16	19 23 13 3 7
24 9 4 8 20	20 19 5 3 18	20 17 4 5 19	22 11 21 3 8	22 11 10 5 17
15 3 10 12 25	14 2 9 15 25	14 2 8 16 25	14 2 7 17 25	14 2 6 18 25
1132#	1243#	1458#	1651#	1949#
1 8 21 24 11	1 6 22 24 12	1 5 23 24 12	1 5 24 23 12	2 23 4 25 11
12 22 19 3 9	16 19 18 9 3	15 16 19 6 9	11 22 16 9 7	8 14 20 16 7
20 10 13 16 6	11 21 13 5 15	18 22 13 4 8	20 18 13 8 6	21 17 13 9 5
17 23 7 4 14	23 17 8 7 10	17 20 7 10 11	19 17 10 4 15	19 10 6 12 18
15 2 5 18 25	14 2 4 20 25	14 2 3 21 25	14 3 2 21 25	15 1 22 3 24
3651#	5358#	6569#	7560#	8132#
3 21 4 25 12	4 23 2 25 11	5 23 1 24 12	6 22 1 24 12	7 21 1 24 12
7 15 16 18 9	7 17 20 16 5	6 15 19 17 8	7 16 21 18 3	8 11 23 17 6
24 20 13 6 2	18 14 13 12 8	22 16 13 10 4	15 17 13 9 11	16 22 13 4 10
17 8 10 11 19	21 10 6 9 19	18 9 7 11 20	23 8 5 10 19	20 9 3 15 18
14 1 22 5 23	15 1 24 3 22	14 2 25 3 21	14 2 25 4 20	14 2 25 5 19
8498#	8691#	8781#	8836#	
8 20 1 24 12	9 20 1 23 12	10 18 1 24 12	11 17 1 24 12	
10 19 9 23 4	11 18 7 24 5	11 20 9 22 3	16 21 3 19 6	
11 21 13 5 15	10 22 13 4 16	7 21 13 5 19	4 18 13 8 22	
22 3 17 7 16	21 2 19 8 15	23 4 17 6 15	20 7 23 5 10	
14 2 25 6 18	14 3 25 6 17	14 2 25 8 16	14 2 25 9 15	

[Solution Counts( $n1=1/A11$ ) = 1948/8846] OK!

**\*\* Compact List 2 of Fundamental Solutions starting with  $n3=1$  \*\***

(Selecting Inequality Conditions:  $n3 < n8 < n18 < n23$ ,  $n3 < n11 < n12 < n14 < n15$ ;) )

1#	3#	11#	15#	24#
3 20 1 22 19	4 17 1 23 20	5 16 1 24 19	6 17 1 19 22	7 18 1 24 15
21 18 9 15 2	24 18 7 11 5	22 20 9 11 3	24 18 3 15 5	20 21 9 12 3
10 12 13 14 16	10 12 13 14 16	8 12 13 14 18	10 12 13 14 16	4 10 13 16 22
24 11 17 8 5	21 15 19 8 2	23 15 17 6 4	21 11 23 8 2	23 14 17 5 6
7 4 25 6 23	6 3 25 9 22	7 2 25 10 21	4 7 25 9 20	11 2 25 8 19
39#	61#	83#	125#	168#
8 15 1 24 17	9 10 1 22 23	10 9 1 21 24	11 8 1 22 23	12 9 1 19 24
23 20 7 10 5	24 19 11 6 5	23 20 11 7 4	20 24 7 9 5	22 21 11 8 3
4 12 13 14 22	8 12 13 14 18	8 12 13 14 18	10 12 13 14 16	6 10 13 16 20
21 16 19 6 3	21 20 15 7 2	22 19 15 6 3	21 17 19 2 6	23 18 15 5 4
9 2 25 11 18	3 4 25 16 17	2 5 25 17 16	3 4 25 18 15	2 7 25 17 14

224#	270#	328#	367#	417#
14 7 1 22 21	15 8 1 17 24	16 7 1 21 20	17 5 1 20 22	18 3 1 19 24
23 20 9 2 11	23 20 7 10 5	24 18 9 3 11	19 24 3 8 11	20 21 9 4 11
8 10 13 16 18	4 12 13 14 22	4 12 13 14 22	10 12 13 14 16	10 12 13 14 16
15 24 17 6 3	21 16 19 6 3	15 23 17 8 2	15 18 23 2 7	15 22 17 5 6
5 4 25 19 12	2 9 25 18 11	6 5 25 19 10	4 6 25 21 9	2 7 25 23 8
455#	567#	632#	720#	815#
19 4 1 18 23	20 5 1 15 24	21 3 1 16 24	22 3 1 15 24	23 4 1 16 21
16 24 11 9 5	23 17 7 4 14	17 22 11 8 7	19 20 9 5 12	14 24 11 7 9
6 12 13 14 20	8 10 13 16 18	6 12 13 14 20	8 10 13 16 18	6 8 13 18 20
21 17 15 2 10	12 22 19 9 3	19 18 15 4 9	14 21 17 6 7	17 19 15 2 12
3 8 25 22 7	2 11 25 21 6	2 10 25 23 5	2 11 25 23 4	5 10 25 22 3
935#	1087#	1092#	1106#	1116#
24 3 1 15 22	3 19 2 21 20	4 15 2 25 19	5 17 2 25 16	6 14 2 21 22
17 21 7 6 14	22 18 10 14 1	23 20 8 9 5	20 22 8 12 3	25 18 10 9 3
8 10 13 16 18	9 11 13 15 17	10 12 13 14 16	7 11 13 15 19	7 11 13 15 19
12 20 19 5 9	25 12 16 8 4	21 17 18 6 3	23 14 18 4 6	23 17 16 8 1
4 11 25 23 2	6 5 24 7 23	7 1 24 11 22	10 1 24 9 21	4 5 24 12 20
1140#	1160#	1192#	1226#	1285#
7 15 2 21 20	8 11 2 21 23	9 12 2 23 19	10 8 2 20 25	11 7 2 20 25
25 17 12 8 3	25 20 4 9 7	20 25 4 11 5	22 21 12 7 3	21 23 8 9 4
4 10 13 16 22	10 12 13 14 16	8 10 13 16 18	9 11 13 15 17	10 12 13 14 16
23 18 14 9 1	19 17 22 6 1	21 15 22 1 6	23 19 14 5 4	22 17 18 3 5
6 5 24 11 19	3 5 24 15 18	7 3 24 14 17	1 6 24 18 16	1 6 24 19 15
1328#	1341#	1375#	1418#	1462#
12 9 2 22 20	14 5 2 19 25	15 5 2 20 23	16 11 2 14 22	17 4 2 23 19
19 25 8 10 3	23 20 10 4 8	17 25 12 7 4	25 18 6 7 9	20 21 8 1 15
5 11 13 15 21	9 11 13 15 17	8 10 13 16 18	3 5 13 21 23	10 12 13 14 16
23 16 18 1 7	18 22 16 6 3	22 19 14 1 9	17 19 20 8 1	11 25 18 5 6
6 4 24 17 14	1 7 24 21 12	3 6 24 21 11	4 12 24 15 10	7 3 24 22 9
1505#	1547#	1611#	1651#	1718#
18 4 2 16 25	19 3 2 21 20	20 7 2 11 25	21 3 2 17 22	22 6 2 10 25
23 19 6 5 12	15 25 12 4 9	17 21 8 16 3	15 25 8 10 7	14 23 8 15 5
9 11 13 15 17	8 10 13 16 18	4 12 13 14 22	6 12 13 14 20	7 9 13 17 19
14 21 20 7 3	17 22 14 1 11	23 10 18 5 9	19 16 18 1 11	21 11 18 3 12
1 10 24 22 8	6 5 24 23 7	1 15 24 19 6	4 9 24 23 5	1 16 24 20 4
1771#	1842#	2009#	2012#	2022#
23 4 2 11 25	25 3 2 17 18	2 19 3 21 20	4 15 3 25 18	5 18 3 20 19
16 21 12 9 7	15 21 4 6 19	22 18 9 15 1	24 20 5 9 7	24 17 11 12 1
6 8 13 18 20	10 12 13 14 16	10 12 13 14 16	10 12 13 14 16	4 10 13 16 22
19 17 14 5 10	7 20 22 5 11	25 11 17 8 4	19 17 21 6 2	25 14 15 9 2
1 15 24 22 3	8 9 24 23 1	6 5 23 7 24	8 1 23 11 22	7 6 23 8 21
2030#	2047#	2063#	2072#	2082#
6 11 3 21 24	7 12 3 22 21	8 15 3 25 14	9 14 3 21 18	10 9 3 24 19
25 19 9 8 4	20 24 9 11 1	24 20 5 7 9	19 24 11 10 1	25 20 5 4 11
10 12 13 14 16	8 10 13 16 18	4 10 13 16 22	4 6 13 20 22	8 12 13 14 18
22 18 17 7 1	25 15 17 2 6	17 19 21 6 2	25 16 15 2 7	15 22 21 6 1
2 5 23 15 20	5 4 23 14 19	12 1 23 11 18	8 5 23 12 17	7 2 23 17 16
2087#	2091#	2106#	2122#	2140#
11 8 3 22 21	12 11 3 21 18	14 7 3 17 24	15 4 3 18 25	16 9 3 12 25
19 25 9 10 2	25 19 9 2 10	25 18 11 5 6	20 24 5 9 7	24 19 5 11 6
6 12 13 14 20	4 6 13 20 22	4 10 13 16 22	10 12 13 14 16	4 8 13 18 22
24 16 17 1 7	16 24 17 7 1	20 21 15 8 1	19 17 21 2 6	20 15 21 7 2
5 4 23 18 15	8 5 23 15 14	2 9 23 19 12	1 8 23 22 11	1 14 23 17 10
2156#	2183#	2214#	2235#	2249#
17 4 3 16 25	18 5 3 15 24	19 4 3 18 21	20 1 3 17 24	21 1 3 18 22
19 24 5 11 6	17 25 7 12 4	24 16 9 1 15	19 21 11 4 10	19 20 9 2 15
8 12 13 14 18	6 10 13 16 20	6 12 13 14 20	8 12 13 14 18	10 12 13 14 16
20 15 21 2 7	22 14 19 1 9	11 25 17 10 2	16 22 15 5 7	11 24 17 6 7
1 10 23 22 9	2 11 23 21 8	5 8 23 22 7	2 9 23 25 6	4 8 23 25 5

2292#	2327#	2376#	2412#	2414#
22 2 3 17 21	24 1 3 17 20	25 2 3 14 21	1 20 4 21 19	2 17 4 23 19
14 25 11 7 8	16 22 7 5 15	18 19 9 4 15	23 17 8 15 2	25 18 6 11 5
6 10 13 16 20	8 12 13 14 18	6 10 13 16 20	10 12 13 14 16	10 12 13 14 16
18 19 15 1 12	11 21 19 4 10	11 22 17 7 8	24 11 18 9 3	21 15 20 8 1
5 9 23 24 4	6 9 23 25 2	5 12 23 24 1	7 5 22 6 25	7 3 22 9 24
2426#	2436#	2442#	2449#	2458#
3 17 4 21 20	5 16 4 25 15	6 16 4 24 15	7 16 4 23 15	8 20 4 24 9
24 18 10 12 1	19 23 12 9 2	25 17 12 3 8	18 25 6 14 2	12 25 10 15 3
7 11 13 15 19	6 8 13 18 20	5 7 13 19 21	5 9 13 17 21	5 7 13 19 21
25 14 16 8 2	24 17 14 3 7	18 23 14 9 1	24 12 20 1 8	23 11 16 1 14
6 5 22 9 23	11 1 22 10 21	11 2 22 10 20	11 3 22 10 19	17 2 22 6 18
2459#	2470#	2473#	2489#	2516#
9 8 4 25 19	10 18 4 9 24	11 6 4 19 25	12 5 4 19 25	14 5 4 17 25
24 21 6 3 11	25 12 6 19 3	21 23 10 9 2	20 24 10 8 3	20 24 8 10 3
10 12 13 14 16	5 11 13 15 21	8 12 13 14 18	9 11 13 15 17	7 11 13 15 19
15 23 20 5 2	23 7 20 14 1	24 17 16 3 5	23 18 16 2 6	23 16 18 2 6
7 1 22 18 17	2 17 22 8 16	1 7 22 20 15	1 7 22 21 14	1 9 22 21 12
2526#	2539#	2560#	2573#	2584#
15 5 4 24 17	16 2 4 18 25	17 3 4 16 25	18 15 4 16 12	19 1 4 17 24
19 25 6 3 12	19 23 12 5 6	24 19 6 5 11	25 9 6 2 23	21 20 10 3 11
8 10 13 16 18	9 11 13 15 17	8 12 13 14 18	5 7 13 19 21	8 12 13 14 18
14 23 20 1 7	20 21 14 3 7	15 21 20 7 2	3 24 20 17 1	15 23 16 6 5
9 2 22 21 11	1 8 22 24 10	1 10 22 23 9	14 10 22 11 8	2 9 22 25 7
2605#	2627#	2656#	2693#	2709#
20 1 4 17 23	21 1 4 15 24	23 1 4 19 18	24 3 4 9 25	25 2 4 11 23
19 21 12 2 11	17 23 10 8 7	17 21 10 2 15	15 21 12 10 7	18 19 10 6 12
8 10 13 16 18	6 12 13 14 20	6 12 13 14 20	6 8 13 18 20	5 9 13 17 21
15 24 14 5 7	19 18 16 3 9	11 24 16 5 9	19 16 14 5 11	14 20 16 7 8
3 9 22 25 6	2 11 22 25 5	8 7 22 25 3	1 17 22 23 2	3 15 22 24 1
2747#	2750#	2762#	2767#	2770#
1 20 5 24 15	2 17 5 23 18	3 16 5 22 19	4 17 5 25 14	7 8 5 22 23
22 19 9 12 3	20 22 7 15 1	25 18 9 11 2	19 23 11 10 2	20 24 11 9 1
8 10 13 16 18	10 12 13 14 16	6 12 13 14 20	6 8 13 18 20	10 12 13 14 16
23 14 17 7 4	25 11 19 4 6	24 15 17 8 1	24 16 15 3 7	25 17 15 2 6
11 2 21 6 25	8 3 21 9 24	7 4 21 10 23	12 1 21 9 22	3 4 21 18 19
2773#	2777#	2779#	2787#	2792#
9 6 5 22 23	11 22 5 18 9	12 19 5 4 25	14 19 5 3 24	15 16 5 22 7
19 25 11 8 2	25 3 7 6 24	20 11 9 23 2	25 4 11 17 8	23 8 9 1 24
10 12 13 14 16	10 12 13 14 16	8 10 13 16 18	6 10 13 16 20	6 12 13 14 20
24 18 15 1 7	2 20 19 23 1	24 3 17 15 6	18 9 15 22 1	2 25 17 18 3
3 4 21 20 17	17 8 21 4 15	1 22 21 7 14	2 23 21 7 12	19 4 21 10 11
2795#	2799#	2810#	2816#	2826#
16 19 5 1 24	17 2 5 16 25	18 15 5 4 23	19 3 5 14 24	20 8 5 9 23
22 3 11 20 9	19 23 11 8 4	14 17 7 25 2	16 25 11 9 4	7 24 11 22 1
8 12 13 14 18	6 12 13 14 20	6 10 13 16 20	6 8 13 18 20	10 12 13 14 16
17 6 15 23 4	22 18 15 3 7	24 1 19 9 12	22 17 15 1 10	25 4 15 2 19
2 25 21 7 10	1 10 21 24 9	3 22 21 11 8	2 12 21 23 7	3 17 21 18 6
2830#	2843#	2859#	2869#	2881#
22 2 5 19 17	23 8 5 4 25	24 1 5 12 23	25 2 5 10 23	3 15 6 25 16
16 23 11 1 14	24 7 9 10 15	17 22 7 8 11	19 17 11 4 14	21 22 12 8 2
6 8 13 18 20	6 12 13 14 20	6 10 13 16 20	6 8 13 18 20	7 9 13 17 19
12 25 15 3 10	11 16 17 19 2	15 18 19 4 9	12 22 15 9 7	24 18 14 4 5
9 7 21 24 4	1 22 21 18 3	3 14 21 25 2	3 16 21 24 1	10 1 20 11 23
2883#	2886#	2890#	2896#	2900#
4 11 6 25 19	10 21 6 4 24	11 19 6 4 25	12 18 6 24 5	14 2 6 22 21
21 24 8 9 3	25 3 12 18 7	21 9 10 23 2	22 9 10 1 23	19 25 10 3 8
10 12 13 14 16	9 11 13 15 17	8 12 13 14 18	7 11 13 15 19	9 11 13 15 17
23 17 18 2 5	19 8 14 23 1	24 3 16 17 5	3 25 16 17 4	18 23 16 1 7
7 1 20 15 22	2 22 20 5 16	1 22 20 7 15	21 2 20 8 14	5 4 20 24 12



2902#	2910#	2926#	2936#	2942#
15 1 6 19 24	21 2 6 22 14	22 8 6 10 19	23 12 6 2 22	24 1 6 12 22
21 23 8 4 9	16 23 8 1 17	24 5 12 1 23	10 17 8 25 5	21 16 8 3 17
10 12 13 14 16	7 11 13 15 19	9 11 13 15 17	7 11 13 15 19	7 11 13 15 19
17 22 18 3 5	9 25 18 3 10	3 25 14 21 2	21 1 18 9 16	9 23 18 10 5
2 7 20 25 11	12 4 20 24 5	7 16 20 18 4	4 24 20 14 3	4 14 20 25 2
2950#	2966#	2968#	2970#	2974#
25 3 6 7 24	3 24 7 22 9	4 25 7 9 20	9 20 7 4 25	11 4 7 23 20
21 11 12 4 17	12 21 11 20 1	21 8 11 23 2	21 8 11 23 2	24 21 9 1 10
8 10 13 16 18	8 10 13 16 18	10 12 13 14 16	10 12 13 14 16	8 12 13 14 18
9 22 14 15 5	25 6 15 5 14	24 3 15 18 5	24 3 15 18 5	16 25 17 5 2
2 19 20 23 1	17 4 19 2 23	6 17 19 1 22	1 22 19 6 17	6 3 19 22 15
2975#	2977#	2981#	2983#	2991#
14 22 7 1 21	21 8 7 4 25	22 5 7 11 20	24 3 7 6 25	25 2 7 8 23
23 2 9 20 11	9 20 11 23 2	24 8 9 1 23	9 22 11 18 5	21 11 9 4 20
8 10 13 16 18	10 12 13 14 16	10 12 13 14 16	10 12 13 14 16	10 12 13 14 16
15 6 17 24 3	24 3 15 6 17	3 25 17 18 2	21 8 15 4 17	6 22 17 15 5
5 25 19 4 12	1 22 19 18 5	6 15 19 21 4	1 20 19 23 2	3 18 19 24 1
2997#	2999#	3003#	3005#	3011#
3 25 8 22 7	12 2 8 22 21	3 24 9 22 7	4 8 9 25 19	5 7 9 20 24
10 20 12 21 2	20 25 10 3 7	25 5 11 6 18	21 24 11 6 3	23 22 11 8 1
9 11 13 15 17	9 11 13 15 17	10 12 13 14 16	10 12 13 14 16	10 12 13 14 16
24 5 14 6 16	19 23 16 1 6	8 20 15 21 1	23 20 15 2 5	25 18 15 4 3
19 4 18 1 23	5 4 18 24 14	19 4 17 2 23	7 1 17 18 22	2 6 17 19 21
3013#	3019#	3021#	3027#	3029#
8 4 9 21 23	20 7 9 5 24	21 4 9 8 23	22 6 9 3 25	25 3 9 4 24
20 25 11 7 2	8 22 11 23 1	7 25 11 20 2	24 5 11 7 18	8 21 11 19 6
10 12 13 14 16	10 12 13 14 16	10 12 13 14 16	10 12 13 14 16	10 12 13 14 16
24 19 15 1 6	25 3 15 4 18	24 6 15 1 19	8 19 15 21 2	20 7 15 5 18
3 5 17 22 18	2 21 17 19 6	3 18 17 22 5	1 23 17 20 4	2 22 17 23 1

[Solution Counts(n3=1/A11) = 1086/3034] OK!

\*\* Calculated and Listed by Kanji Setsuda on  
Jan.29, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 \*\*

Just as you see above, I have failed to have got the 3034 fundamental solutions for the **type 1**, on the other hand I have gladly succeeded in getting them for the **type 2**.

For the **type 1**, the results when n1=1 seems correct, since the 1948 solutions must be the answers we have really wanted to have. But, the total count is too many.

I have tried many times in vain to test various selecting conditions to have got the correct set of 3034 solutions for that **type 1**, but I have always failed.

The only thing I can proudly report about now is the fundamental solutions of the **type 2** starting with n3=1 just listed as above. I am really satisfied with the result.

But, this list is too weak for us to imagine easily what should be the 'standard' style of each solution reformed for the case when n1=1, n5<n21;

Look at those last 5 solutions above. They are all the 'rotated' or 'reflected' patterns.

But they are not false at all but correct, since the content orders of the 3rd row and the 3rd column are regular enough and obedient to their own selecting conditions.

Let me show you here how to reform the sample solution into the different type.

Suppose you take the 3013# in the above list, and try to transform it just as below.

00/T	3013#	11/T	22/T	Ref/T	01/T	362F1
8 4 9 21 23	8 21 9 4 23	1 24 15 6 19	1 21 14 22 7	1 21 14 22 7	1 21 14 22 7	
20 25 11 7 2	24 1 15 19 6	21 8 9 23 4	24 8 10 3 20	24 8 10 3 20	6 23 16 18 2	
10 12 13 14 16	10 14 13 12 16	14 10 13 16 12	15 9 13 17 11	15 9 13 17 11	15 9 13 17 11	
24 19 15 1 6	20 7 11 25 2	22 3 17 18 5	6 23 16 18 2	6 23 16 18 2	24 8 10 3 20	
3 5 17 22 18	3 22 17 5 18	7 20 11 2 25	19 4 12 5 25	19 4 12 5 25	19 4 12 5 25	



You can use our 16 Transformation Methods to do that: 11/T to remake the original into the second, 22/T to remake it into the third, Reflect it with respect to the primary diagonal into the fourth, and then use 01/T to get to the goal object.

You can finally obtain the fundamental solution 362# of the type 1, that has the one-to-one correspondence between the original sample of the type 2.

It means we should have the possibility to reform the 1948 fundamental solutions of the type 2 into the same count of type 1, because  $1948 = 3034 - 1086$ ;

## #6. How to prove that these 3034 are really the 'Fundamental Solutions' of all

Why do we always call these 3034 solutions as 'Fundamental'?

Is it only a custom that we have called them so for a long time?

Or is it because they are fewer than those 48544 Standard Solutions and they seem to be the 'essence' of the latter? Or is it because each member of the group of 16 Standard Solutions is very similar to the leading solution, and because we want to think it must be the 'representative' of those 16 similar Standard Solutions?

This is, however, the problem of 'classes', in terms of mathematical Set Theory.

The smaller set of 3034 Fundamental Solutions is the lower class contained among the larger set of 48544 Standard Solutions.

How was the former set made from the latter? It was picked up out of the standard solutions under the 'selecting inequality conditions', by which we always wanted our transformation methods didn't work. The hidden key was our 16 transformations.

Then, we can make these 3034 back and recompose those 48544 by our powerful bi-directional use of our transformation system revived, does it mean?

Let's try to reconstruct all those 48544 Standard Solutions from these fewer 3034 Solutions by our transformation system, shall we? We now want to prove directly by our powerful demonstration that these 3034 are really 'fundamental' to those 48544.

### \*\* Basic Forms for the Concepts our Transformation Method \*\*

0/T					1/T					2/T					3/T				
1	2	3	4	5	1	2	3	4	5	6	7	8	9	10	16	17	18	19	20
6	7	8	9	10	16	17	18	19	20	1	2	3	4	5	1	2	3	4	5
11	12	13	14	15	11	12	13	14	15	11	12	13	14	15	11	12	13	14	15
16	17	18	19	20	6	7	8	9	10	21	22	23	24	25	21	22	23	24	25
21	22	23	24	25	21	22	23	24	25	16	17	18	19	20	6	7	8	9	10
4/T					5/T					6/T					7/T				
1	4	3	2	5	1	4	3	2	5	6	9	8	7	10	16	19	18	17	20
6	9	8	7	10	16	19	18	17	20	1	4	3	2	5	1	4	3	2	5
11	14	13	12	15	11	14	13	12	15	11	14	13	12	15	11	14	13	12	15
16	19	18	17	20	6	9	8	7	10	21	24	23	22	25	21	24	23	22	25
21	24	23	22	25	21	24	23	22	25	16	19	18	17	20	6	9	8	7	10
8/T					9/T					10/T					11/T				
2	1	3	5	4	2	1	3	5	4	7	6	8	10	9	17	16	18	20	19
7	6	8	10	9	17	16	18	20	19	2	1	3	5	4	2	1	3	5	4
12	11	13	15	14	12	11	13	15	14	12	11	13	15	14	12	11	13	15	14
17	16	18	20	19	7	6	8	10	9	22	21	23	25	24	22	21	23	25	24
22	21	23	25	24	22	21	23	25	24	17	16	18	20	19	7	6	8	10	9
12/T					13/T					14/T					15/T				
4	1	3	5	2	4	1	3	5	2	9	6	8	10	7	19	16	18	20	17
9	6	8	10	7	19	16	18	20	17	4	1	3	5	2	4	1	3	5	2
14	11	13	15	12	14	11	13	15	12	14	11	13	15	12	14	11	13	15	12
19	16	18	20	17	9	6	8	10	7	24	21	23	25	22	24	21	23	25	22
24	21	23	25	22	24	21	23	25	22	19	16	18	20	17	9	6	8	10	7

Let me carry the conceptual diagrams for our Transformation System here again.  
 Put each of the 3034 fundamental solutions on 0/T, and transform it into the 15 variations according to the rules above, and you will get all the 16 Standard Solutions.

### #7. Result List of the first Transformation Test

Let me show you a sample list of raw results of the first experiment as follows.

[First Result of Reconstruction by the 16 Transformations]

[1/F]

	9/S	11/S	45611/S	0/S
3 20 1 22 19	3 20 1 22 19	21 18 9 15 2	24 11 17 8 5	
21 18 9 15 2	24 11 17 8 5	3 20 1 22 19	3 20 1 22 19	
10 12 13 14 16	10 12 13 14 16	10 12 13 14 16	10 12 13 14 16	
24 11 17 8 5	21 18 9 15 2	7 4 25 6 23	7 4 25 6 23	
7 4 25 6 23	7 4 25 6 23	24 11 17 8 5	21 18 9 15 2	
	22/S	24/S	45585/S	0/S
3 22 1 20 19	3 22 1 20 19	21 15 9 18 2	24 8 17 11 5	
21 15 9 18 2	24 8 17 11 5	3 22 1 20 19	3 22 1 20 19	
10 14 13 12 16	10 14 13 12 16	10 14 13 12 16	10 14 13 12 16	
24 8 17 11 5	21 15 9 18 2	7 6 25 4 23	7 6 25 4 23	
7 6 25 4 23	7 6 25 4 23	24 8 17 11 5	21 15 9 18 2	
	4857/S	4855/S	45172/S	0/S
20 3 1 19 22	20 3 1 19 22	18 21 9 2 15	11 24 17 5 8	
18 21 9 2 15	11 24 17 5 8	20 3 1 19 22	20 3 1 19 22	
12 10 13 16 14	12 10 13 16 14	12 10 13 16 14	12 10 13 16 14	
11 24 17 5 8	18 21 9 2 15	4 7 25 23 6	4 7 25 23 6	
4 7 25 23 6	4 7 25 23 6	11 24 17 5 8	18 21 9 2 15	
	0/S	0/S	0/S	0/S
22 3 1 19 20	22 3 1 19 20	15 21 9 2 18	8 24 17 5 11	
15 21 9 2 18	8 24 17 5 11	22 3 1 19 20	22 3 1 19 20	
14 10 13 16 12	14 10 13 16 12	14 10 13 16 12	14 10 13 16 12	
8 24 17 5 11	15 21 9 2 18	6 7 25 23 4	6 7 25 23 4	
6 7 25 23 4	6 7 25 23 4	8 24 17 5 11	15 21 9 2 18	

I put the number of Standard Solutions on each right shoulder by referring to the old list in page 13~16. But, 0/S means it was not really found in that old list.

Why couldn't I find those solutions 0/S in the old list as many as 7 among 16?

Because those transformed results are out of the Normalizing Inequality Conditions for the Standard Solutions:  $n3 < n23$  and  $n3 < n11 < n15$ ; they were not found there.

They were made as either 'rotated' or 'reflected' patterns of the standard solutions.

We need some modifications on those results of transformation.

Let's add the following additional rules to our Transformation Method here.

You have to rewrite all those results by the new rules if necessary, and make them smart to fit the strict normalizing inequality condition.

\*\* Modification of the results by Reflection and Rotation \*\*

	0/T	1/T	2/T	3/T
1 2 3 4 5	21 16 11 6 1	25 24 23 22 21	5 10 15 20 25	
6 7 8 9 10	22 17 12 7 2	20 19 18 17 16	4 9 14 19 24	
11 12 13 14 15	23 18 13 8 3	15 14 13 12 11	3 8 13 18 23	
16 17 18 19 20	24 19 14 9 4	10 9 8 7 6	2 7 12 17 22	
21 22 23 24 25	25 20 15 10 5	5 4 3 2 1	1 6 11 16 21	
	4/T	5/T	6/T	7/T
5 4 3 2 1	25 20 15 10 5	21 22 23 24 25	1 6 11 16 21	
10 9 8 7 6	24 19 14 9 4	16 17 18 19 20	2 7 12 17 22	
15 14 13 12 11	23 18 13 8 3	11 12 13 14 15	3 8 13 18 23	
20 19 18 17 16	22 17 12 7 2	6 7 8 9 10	4 9 14 19 24	
25 24 23 22 21	21 16 11 6 1	1 2 3 4 5	5 10 15 20 25	

## #8. Result List of the final Transformation by our Methods

The next list contains the final result of my recent elaborate works in extract style.

\*\*\* Self-Complementary Magic Squares of Order 5: \*\*\*

\*\* Reconstruct 48544 Solutions by Transforming the 3034 Fundamentals \*\*

[1/F]

9/S	11/S	45611/S	45612/S
3 20 1 22 19	3 20 1 22 19	21 18 9 15 2	21 18 9 15 2
21 18 9 15 2	24 11 17 8 5	3 20 1 22 19	7 4 25 6 23
10 12 13 14 16	10 12 13 14 16	10 12 13 14 16	10 12 13 14 16
24 11 17 8 5	21 18 9 15 2	7 4 25 6 23	3 20 1 22 19
7 4 25 6 23	7 4 25 6 23	24 11 17 8 5	24 11 17 8 5
22/S	24/S	45585/S	45586/S
3 22 1 20 19	3 22 1 20 19	21 15 9 18 2	21 15 9 18 2
21 15 9 18 2	24 8 17 11 5	3 22 1 20 19	7 6 25 4 23
10 14 13 12 16	10 14 13 12 16	10 14 13 12 16	10 14 13 12 16
24 8 17 11 5	21 15 9 18 2	7 6 25 4 23	3 22 1 20 19
7 6 25 4 23	7 6 25 4 23	24 8 17 11 5	24 8 17 11 5
4857/S	4855/S	45172/S	45171/S
20 3 1 19 22	20 3 1 19 22	18 21 9 2 15	18 21 9 2 15
18 21 9 2 15	11 24 17 5 8	20 3 1 19 22	4 7 25 23 6
12 10 13 16 14	12 10 13 16 14	12 10 13 16 14	12 10 13 16 14
11 24 17 5 8	18 21 9 2 15	4 7 25 23 6	20 3 1 19 22
4 7 25 23 6	4 7 25 23 6	11 24 17 5 8	11 24 17 5 8
5252/S	5250/S	45094/S	45093/S
20 19 1 3 22	20 19 1 3 22	18 2 9 21 15	18 2 9 21 15
18 2 9 21 15	11 5 17 24 8	20 19 1 3 22	4 23 25 7 6
12 16 13 10 14	12 16 13 10 14	12 16 13 10 14	12 16 13 10 14
11 5 17 24 8	18 2 9 21 15	4 23 25 7 6	20 19 1 3 22
4 23 25 7 6	4 23 25 7 6	11 5 17 24 8	11 5 17 24 8

[301/F]

2633/S	2635/S	10710/S	10748/S
15 20 1 22 7	15 20 1 22 7	12 15 2 19 17	12 19 2 15 17
12 23 5 16 9	17 10 21 3 14	23 20 8 4 10	23 4 8 20 10
2 8 13 18 24	2 8 13 18 24	5 1 13 25 21	5 25 13 1 21
17 10 21 3 14	12 23 5 16 9	16 22 18 6 3	16 6 18 22 3
19 4 25 6 11	19 4 25 6 11	9 7 24 11 14	9 11 24 7 14
2709/S	2711/S	10709/S	10747/S
15 22 1 20 7	15 22 1 20 7	12 15 2 19 17	12 19 2 15 17
12 16 5 23 9	17 3 21 10 14	16 22 18 6 3	16 6 18 22 3
2 18 13 8 24	2 18 13 8 24	5 1 13 25 21	5 25 13 1 21
17 3 21 10 14	12 16 5 23 9	23 20 8 4 10	23 4 8 20 10
19 6 25 4 11	19 6 25 4 11	9 7 24 11 14	9 11 24 7 14
5144/S	5138/S	32852/S	32850/S
20 15 1 7 22	20 15 1 7 22	23 12 5 9 16	23 12 5 9 16
23 12 5 9 16	10 17 21 14 3	20 15 1 7 22	4 19 25 11 6
8 2 13 24 18	8 2 13 24 18	8 2 13 24 18	8 2 13 24 18
10 17 21 14 3	23 12 5 9 16	4 19 25 11 6	20 15 1 7 22
4 19 25 11 6	4 19 25 11 6	10 17 21 14 3	10 17 21 14 3
4918/S	4912/S	32809/S	32807/S
20 7 1 15 22	20 7 1 15 22	23 9 5 12 16	23 9 5 12 16
23 9 5 12 16	10 14 21 17 3	20 7 1 15 22	4 11 25 19 6
8 24 13 2 18	8 24 13 2 18	8 24 13 2 18	8 24 13 2 18
10 14 21 17 3	23 9 5 12 16	4 11 25 19 6	20 7 1 15 22
4 11 25 19 6	4 11 25 19 6	10 14 21 17 3	10 14 21 17 3

[601/F]

5256/S	5255/S	19770/S	19768/S
20 19 1 4 21	20 19 1 4 21	17 10 3 24 11	17 10 3 24 11
17 10 3 24 11	15 2 23 16 9	20 19 1 4 21	5 22 25 7 6
8 12 13 14 18	8 12 13 14 18	8 12 13 14 18	8 12 13 14 18
15 2 23 16 9	17 10 3 24 11	5 22 25 7 6	20 19 1 4 21
5 22 25 7 6	5 22 25 7 6	15 2 23 16 9	15 2 23 16 9

4874/S	4873/S	20006/S	20004/S
20 4 1 19 21	20 4 1 19 21	17 24 3 10 11	17 24 3 10 11
17 24 3 10 11	15 16 23 2 9	20 4 1 19 21	5 7 25 22 6
8 14 13 12 18	8 14 13 12 18	8 14 13 12 18	8 14 13 12 18
15 16 23 2 9	17 24 3 10 11	5 7 25 22 6	20 4 1 19 21
5 7 25 22 6	5 7 25 22 6	15 16 23 2 9	15 16 23 2 9

4636/S	4635/S	18309/S	18310/S
19 20 1 21 4	19 20 1 21 4	10 17 3 11 24	10 17 3 11 24
10 17 3 11 24	2 15 23 9 16	19 20 1 21 4	22 5 25 6 7
12 8 13 18 14	12 8 13 18 14	12 8 13 18 14	12 8 13 18 14
2 15 23 9 16	10 17 3 11 24	22 5 25 6 7	19 20 1 21 4
22 5 25 6 7	22 5 25 6 7	2 15 23 9 16	2 15 23 9 16

4684/S	4683/S	18289/S	18290/S
19 21 1 20 4	19 21 1 20 4	10 11 3 17 24	10 11 3 17 24
10 11 3 17 24	2 9 23 15 16	19 21 1 20 4	22 6 25 5 7
12 18 13 8 14	12 18 13 8 14	12 18 13 8 14	12 18 13 8 14
2 9 23 15 16	10 11 3 17 24	22 6 25 5 7	19 21 1 20 4
22 6 25 5 7	22 6 25 5 7	2 9 23 15 16	2 9 23 15 16

[901/F]

7492/S	7494/S	23477/S	23491/S
23 19 1 20 2	23 19 1 20 2	5 23 4 24 9	5 24 4 23 9
5 18 11 14 17	9 12 15 8 21	18 19 10 6 12	18 6 10 19 12
4 10 13 16 22	4 10 13 16 22	11 1 13 25 15	11 25 13 1 15
9 12 15 8 21	5 18 11 14 17	14 20 16 7 8	14 7 16 20 8
24 6 25 7 3	24 6 25 7 3	17 2 22 3 21	17 3 22 2 21

7555/S	7557/S	23476/S	23490/S
23 20 1 19 2	23 20 1 19 2	5 23 4 24 9	5 24 4 23 9
5 14 11 18 17	9 8 15 12 21	14 20 16 7 8	14 7 16 20 8
4 16 13 10 22	4 16 13 10 22	11 1 13 25 15	11 25 13 1 15
9 8 15 12 21	5 14 11 18 17	18 19 10 6 12	18 6 10 19 12
24 7 25 6 3	24 7 25 6 3	17 2 22 3 21	17 3 22 2 21

4746/S	4745/S	46927/S	46873/S
19 23 1 2 20	19 23 1 2 20	18 19 10 6 12	18 6 10 19 12
18 5 11 17 14	12 9 15 21 8	5 23 4 24 9	5 24 4 23 9
10 4 13 22 16	10 4 13 22 16	11 1 13 25 15	11 25 13 1 15
12 9 15 21 8	18 5 11 17 14	17 2 22 3 21	17 3 22 2 21
6 24 25 3 7	6 24 25 3 7	14 20 16 7 8	14 7 16 20 8

4036/S	4035/S	46928/S	46874/S
19 2 1 23 20	19 2 1 23 20	18 19 10 6 12	18 6 10 19 12
18 17 11 5 14	12 21 15 9 8	17 2 22 3 21	17 3 22 2 21
10 22 13 4 16	10 22 13 4 16	11 1 13 25 15	11 25 13 1 15
12 21 15 9 8	18 17 11 5 14	5 23 4 24 9	5 24 4 23 9
6 3 25 24 7	6 3 25 24 7	14 20 16 7 8	14 7 16 20 8

[1201/F]

9692/S	9690/S	32810/S	32851/S
9 15 2 25 14	9 15 2 25 14	23 9 5 12 16	23 12 5 9 16
23 20 8 4 10	16 22 18 6 3	20 15 7 1 22	20 1 7 15 22
5 7 13 19 21	5 7 13 19 21	8 2 13 24 18	8 24 13 2 18
16 22 18 6 3	23 20 8 4 10	4 25 19 11 6	4 11 19 25 6
12 1 24 11 17	12 1 24 11 17	10 14 21 17 3	10 17 21 14 3

9869/S	9867/S	32808/S	32849/S
9 25 2 15 14	9 25 2 15 14	23 9 5 12 16	23 12 5 9 16
23 4 8 20 10	16 6 18 22 3	4 25 19 11 6	4 11 19 25 6
5 19 13 7 21	5 19 13 7 21	8 2 13 24 18	8 24 13 2 18
16 6 18 22 3	23 4 8 20 10	20 15 7 1 22	20 1 7 15 22
12 11 24 1 17	12 11 24 1 17	10 14 21 17 3	10 17 21 14 3

11213/S	11214/S	40058/S	39884/S
15 9 2 14 25	15 9 2 14 25	20 15 7 1 22	20 1 7 15 22
20 23 8 10 4	22 16 18 3 6	23 9 5 12 16	23 12 5 9 16
7 5 13 21 19	7 5 13 21 19	8 2 13 24 18	8 24 13 2 18
22 16 18 3 6	20 23 8 10 4	10 14 21 17 3	10 17 21 14 3
1 12 24 17 11	1 12 24 17 11	4 25 19 11 6	4 11 19 25 6

11263/S	11264/S	40055/S	39881/S
15 14 2 9 25	15 14 2 9 25	20 15 7 1 22	20 1 7 15 22
20 10 8 23 4	22 3 18 16 6	10 14 21 17 3	10 17 21 14 3
7 21 13 5 19	7 21 13 5 19	8 2 13 24 18	8 24 13 2 18
22 3 18 16 6	20 10 8 23 4	23 9 5 12 16	23 12 5 9 16
1 17 24 12 11	1 17 24 12 11	4 25 19 11 6	4 11 19 25 6

[1501/F]

12593/S	12591/S	26326/S	26325/S
17 25 2 6 15	17 25 2 6 15	19 3 4 21 18	19 3 4 21 18
19 3 4 21 18	8 5 22 23 7	17 25 2 6 15	11 20 24 1 9
10 12 13 14 16	10 12 13 14 16	10 12 13 14 16	10 12 13 14 16
8 5 22 23 7	19 3 4 21 18	11 20 24 1 9	17 25 2 6 15
11 20 24 1 9	11 20 24 1 9	8 5 22 23 7	8 5 22 23 7
12163/S	12161/S	26542/S	26541/S
17 6 2 25 15	17 6 2 25 15	19 21 4 3 18	19 21 4 3 18
19 21 4 3 18	8 23 22 5 7	17 6 2 25 15	11 1 24 20 9
10 14 13 12 16	10 14 13 12 16	10 14 13 12 16	10 14 13 12 16
8 23 22 5 7	19 21 4 3 18	11 1 24 20 9	17 6 2 25 15
11 1 24 20 9	11 1 24 20 9	8 23 22 5 7	8 23 22 5 7
16986/S	16989/S	23334/S	23333/S
25 17 2 15 6	25 17 2 15 6	3 19 4 18 21	3 19 4 18 21
3 19 4 18 21	5 8 22 7 23	25 17 2 15 6	20 11 24 9 1
12 10 13 16 14	12 10 13 16 14	12 10 13 16 14	12 10 13 16 14
5 8 22 7 23	3 19 4 18 21	20 11 24 9 1	25 17 2 15 6
20 11 24 9 1	20 11 24 9 1	5 8 22 7 23	5 8 22 7 23
16843/S	16846/S	23318/S	23317/S
25 15 2 17 6	25 15 2 17 6	3 18 4 19 21	3 18 4 19 21
3 18 4 19 21	5 7 22 8 23	25 15 2 17 6	20 9 24 11 1
12 16 13 10 14	12 16 13 10 14	12 16 13 10 14	12 16 13 10 14
5 7 22 8 23	3 18 4 19 21	20 9 24 11 1	25 15 2 17 6
20 9 24 11 1	20 9 24 11 1	5 7 22 8 23	5 7 22 8 23

[1801/F]

15710/S	15711/S	24367/S	24245/S
23 15 2 6 19	23 15 2 6 19	10 23 4 7 21	10 7 4 23 21
10 17 8 25 5	21 1 18 9 16	17 15 12 20 1	17 20 12 15 1
4 12 13 14 22	4 12 13 14 22	8 2 13 24 18	8 24 13 2 18
21 1 18 9 16	10 17 8 25 5	25 6 14 11 9	25 11 14 6 9
7 20 24 11 3	7 20 24 11 3	5 19 22 3 16	5 3 22 19 16
15428/S	15429/S	24368/S	24246/S
23 6 2 15 19	23 6 2 15 19	10 23 4 7 21	10 7 4 23 21
10 25 8 17 5	21 9 18 1 16	25 6 14 11 9	25 11 14 6 9
4 14 13 12 22	4 14 13 12 22	8 2 13 24 18	8 24 13 2 18
21 9 18 1 16	10 25 8 17 5	17 15 12 20 1	17 20 12 15 1
7 11 24 20 3	7 11 24 20 3	5 19 22 3 16	5 3 22 19 16
11538/S	11537/S	42697/S	42699/S
15 23 2 19 6	15 23 2 19 6	17 10 8 5 25	17 10 8 5 25
17 10 8 5 25	1 21 18 16 9	15 23 2 19 6	20 7 24 3 11
12 4 13 22 14	12 4 13 22 14	12 4 13 22 14	12 4 13 22 14
1 21 18 16 9	17 10 8 5 25	20 7 24 3 11	15 23 2 19 6
20 7 24 3 11	20 7 24 3 11	1 21 18 16 9	1 21 18 16 9
11378/S	11377/S	42662/S	42664/S
15 19 2 23 6	15 19 2 23 6	17 5 8 10 25	17 5 8 10 25
17 5 8 10 25	1 16 18 21 9	15 19 2 23 6	20 3 24 7 11
12 22 13 4 14	12 22 13 4 14	12 22 13 4 14	12 22 13 4 14
1 16 18 21 9	17 5 8 10 25	20 3 24 7 11	15 19 2 23 6
20 3 24 7 11	20 3 24 7 11	1 16 18 21 9	1 16 18 21 9

[2101/F]

18722/S	18719/S	28298/S	28213/S
12 25 3 7 18	12 25 3 7 18	24 12 4 8 17	24 8 4 12 17
24 5 11 16 9	17 10 15 21 2	5 25 6 19 10	5 19 6 25 10
4 6 13 20 22	4 6 13 20 22	11 3 13 23 15	11 23 13 3 15
17 10 15 21 2	24 5 11 16 9	16 7 20 1 21	16 1 20 7 21
8 19 23 1 14	8 19 23 1 14	9 18 22 14 2	9 14 22 18 2

18566/S	18563/S	28301/S	28216/S
12 7 3 25 18	12 7 3 25 18	24 12 4 8 17	24 8 4 12 17
24 16 11 5 9	17 21 15 10 2	16 7 20 1 21	16 1 20 7 21
4 20 13 6 22	4 20 13 6 22	11 3 13 23 15	11 23 13 3 15
17 21 15 10 2	24 16 11 5 9	5 25 6 19 10	5 19 6 25 10
8 1 23 19 14	8 1 23 19 14	9 18 22 14 2	9 14 22 18 2

22974/S	22975/S	34260/S	34212/S
25 12 3 18 7	25 12 3 18 7	5 25 6 19 10	5 19 6 25 10
5 24 11 9 16	10 17 15 2 21	24 12 4 8 17	24 8 4 12 17
6 4 13 22 20	6 4 13 22 20	11 3 13 23 15	11 23 13 3 15
10 17 15 2 21	5 24 11 9 16	9 18 22 14 2	9 14 22 18 2
19 8 23 14 1	19 8 23 14 1	16 7 20 1 21	16 1 20 7 21

23094/S	23095/S	34255/S	34207/S
25 18 3 12 7	25 18 3 12 7	5 25 6 19 10	5 19 6 25 10
5 9 11 24 16	10 2 15 17 21	9 18 22 14 2	9 14 22 18 2
6 22 13 4 20	6 22 13 4 20	11 3 13 23 15	11 23 13 3 15
10 2 15 17 21	5 9 11 24 16	24 12 4 8 17	24 8 4 12 17
19 14 23 8 1	19 14 23 8 1	16 7 20 1 21	16 1 20 7 21

[2401/F]

23063/S	23062/S	25687/S	25486/S
25 17 3 2 18	25 17 3 2 18	16 25 4 8 12	16 8 4 25 12
16 11 5 19 14	12 7 21 15 10	11 17 6 24 7	11 24 6 17 7
4 6 13 20 22	4 6 13 20 22	5 3 13 23 21	5 23 13 3 21
12 7 21 15 10	16 11 5 19 14	19 2 20 9 15	19 9 20 2 15
8 24 23 9 1	8 24 23 9 1	14 18 22 1 10	14 1 22 18 10

22679/S	22678/S	25689/S	25488/S
25 2 3 17 18	25 2 3 17 18	16 25 4 8 12	16 8 4 25 12
16 19 5 11 14	12 15 21 7 10	19 2 20 9 15	19 9 20 2 15
4 20 13 6 22	4 20 13 6 22	5 3 13 23 21	5 23 13 3 21
12 15 21 7 10	16 19 5 11 14	11 17 6 24 7	11 24 6 17 7
8 9 23 24 1	8 9 23 24 1	14 18 22 1 10	14 1 22 18 10

20064/S	20061/S	30225/S	30226/S
17 25 3 18 2	17 25 3 18 2	11 16 5 14 19	11 16 5 14 19
11 16 5 14 19	7 12 21 10 15	17 25 3 18 2	24 8 23 1 9
6 4 13 22 20	6 4 13 22 20	6 4 13 22 20	6 4 13 22 20
7 12 21 10 15	11 16 5 14 19	24 8 23 1 9	17 25 3 18 2
24 8 23 1 9	24 8 23 1 9	7 12 21 10 15	7 12 21 10 15

19924/S	19921/S	30217/S	30218/S
17 18 3 25 2	17 18 3 25 2	11 14 5 16 19	11 14 5 16 19
11 14 5 16 19	7 10 21 12 15	17 18 3 25 2	24 1 23 8 9
6 22 13 4 20	6 22 13 4 20	6 22 13 4 20	6 22 13 4 20
7 10 21 12 15	11 14 5 16 19	24 1 23 8 9	17 18 3 25 2
24 1 23 8 9	24 1 23 8 9	7 10 21 12 15	7 10 21 12 15

[2701/F]

28382/S	28380/S	29895/S	29922/S
24 16 4 20 1	24 16 4 20 1	8 24 5 25 3	8 25 5 24 3
8 15 12 7 23	3 19 14 11 18	15 16 9 6 19	15 6 9 16 19
5 9 13 17 21	5 9 13 17 21	12 4 13 22 14	12 22 13 4 14
3 19 14 11 18	8 15 12 7 23	7 20 17 10 11	7 10 17 20 11
25 6 22 10 2	25 6 22 10 2	23 1 21 2 18	23 2 21 1 18

28453/S	28451/S	29892/S	29919/S
24 20 4 16 1	24 20 4 16 1	8 24 5 25 3	8 25 5 24 3
8 7 12 15 23	3 11 14 19 18	7 20 17 10 11	7 10 17 20 11
5 17 13 9 21	5 17 13 9 21	12 4 13 22 14	12 22 13 4 14
3 11 14 19 18	8 7 12 15 23	15 16 9 6 19	15 6 9 16 19
25 10 22 6 2	25 10 22 6 2	23 1 21 2 18	23 2 21 1 18

25657/S	25659/S	44984/S	44966/S
16 24 4 1 20	16 24 4 1 20	15 16 9 6 19	15 6 9 16 19
15 8 12 23 7	19 3 14 18 11	8 24 5 25 3	8 25 5 24 3
9 5 13 21 17	9 5 13 21 17	12 4 13 22 14	12 22 13 4 14
19 3 14 18 11	15 8 12 23 7	23 1 21 2 18	23 2 21 1 18
6 25 22 2 10	6 25 22 2 10	7 20 17 10 11	7 10 17 20 11







\*\* Calculated and Listed by Kanji Setsuda on  
Jan.29, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 \*\*

I am glad to report that I have found each of the 3034 Fundamental Solutions could make its 15 brothers and sisters. All the family members really exist in the old list of 48544 Standard Solutions. I could find neither duplication nor lack of any solution.

I am sure we could really prove the meaning of equation  $3034 \times 16 = 48544$  directly by our transformation system.

No doubt, these 3034 solutions in page 13~16 are really fundamental to those 48544 Standard Solutions in page 9, that we have proved directly, I would say.

(Written by Kanji Setsuda in 2002; Revised in 2005;  
Newest Revision written on Feb. 1, 2015  
with MacOSX 10.10.2 and Xcode 6.1.1;)

-----  
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<< Appendix: The Newest Version of Program 2015 >>

```
/** Self-Complementary Magic Squares of Order 5: **  
/** Transform 3040 Fundamental into 48544 Standard **  
/** File: 'SCMS55Trnsf.c' Dictated by Kanji Setsuda **  
/** on Feb.20, 2005; Revised on Jan.29, 2015 **  
/** with MacOSX 10.10.2 & Xcode 6.1.1 **  
/**  
/** Basic From **          ** Basic Conditions: **  
/** .----- .          n1+n2+n3+n4+n5=C          ...rw1  
/** |n1|n2|n3|n4|n5|      n6+n7+n8+n9+n10=C         ...rw2  
/** |---+---+---+---|    n11+n12+n13+n14+n15=C        ...rw3  
/** |n6|n7|n8|n9|10|     n16+n17+n18+n19+n20=C       ...rw4  
/** |---+---+---+---|    n21+n22+n23+n24+n25=C       ...rw5  
/** |11|12|13|14|15|     n1+n6+n11+n16+n21=C         ...cl1  
/** |---+---+---+---|    n2+n7+n12+n17+n22=C         ...cl2  
/** |16|17|18|19|20|     n3+n8+n13+n18+n23=C         ...cl3  
/** |---+---+---+---|    n4+n9+n14+n19+n24=C         ...cl4  
/** |21|22|23|24|25|     n5+n10+n15+n20+n25=C        ...cl5  
/** '-----'          n1+n7+n13+n19+n25=C         ...pd1;  
/**                    n5+n9+n13+n17+n21=C         ...pd2;  
/** Conditions for Symmetric Complementary Pairs **  
/** n1+n25=n2+n24=n3+n23=n4+n22=n5+n21  
/** =n6+n20=n7+n19=n8+n18=n9+n17=n10+n16  
/** =n11+n15=n12+n14=n13+n13=CC          ...scc  
/**  
#include <stdio.h>  
/**  
long int cnt;  
long int tcnt[17];  
short cnt2, cnt3;  
short LSM, CC;  
short nm[26], uflg[26];  
short cnm[26], dnm[26];  
short ft[3035][26];  
short tsol[48545][26];  
short nt[17][26];  
short refnmb[48545];  
/**  
void stp01(void), stp02(void), stp03(void), stp04(void);  
void stp05(void), stp06(void), stp07(void), stp08(void);  
void stp09(void), stp10(void), stp11(void), stp12(void);  
void stp13(void);
```

```

void rcrdsol(void);
//
void prtrns(void), pr4ans(void);
void trnsf1(void), trnsf2(void), trnsf3(void);
void trnsf4(void), trnsf5(void), trnsf6(void);
void refrot(void), reflect3(void), reflect1(void), rotate(void);
void fndon(void), prmon(void);
//
int main(){
    long m;
    short n;
    printf("\n");
    printf("*** Self-Complementary Magic Squares of Order 5: ***\n");
    printf("** Find the 3040 Fundamental among 48544 Standard **\n");
    for(n=0;n<26;n++){nm[n]=0; uflg[n]=0;}
    for(m=0;m<48545;m++){refnmb[m]=0;}
    LSM=65; CC=26; cnt=0; cnt3=0;
    nm[13]=13; uflg[13]=1;
    stp01();    /* Calculations for Type: n3=1 *
    uflg[13]=0; nm[13]=0;
    printf(" [Count = %d/F, %ld/S]\n",cnt3,cnt);
    printf("\n");
    printf("*** Reconstruct 48544 Solutions by Transforming the 3034 Fundamentals **\n");
    prtrns();
    printf("\n");
    printf(" . . . . .\n");
    printf("\n");
    printf("** Monitor List of Solution Correspondence between Old and New *\n");
    prmon();
    printf(" [Count = %ld] OK!\n",cnt);
    printf("\n");
    printf("*** Calculated and Listed by Kanji Setsuda on\n");
    printf("    Jan.29, 2015 with MacOSX 10.10.2 & Xcode 6.1.1 **\n");
    printf("\n");
    return 0;
}
//
/* Begin the Calculations *
//(Normalizing Inequality Conditions: n3<n11<n15 and n3<n23;)
/* Set N3 & n23 & n3<n23 *
void stp01(){
    short a,b;
    for(a=1;a<13;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[3]=a; nm[23]=b;
            uflg[a]=1; uflg[b]=1;
            stp02();
            uflg[b]=0; uflg[a]=0;}
        }
}
/* Set N1 & n25 *
void stp02(){
    short a,b;
    for(a=1;a<26;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[1]=a; nm[25]=b;
            uflg[a]=1; uflg[b]=1;
            stp03();
            uflg[b]=0; uflg[a]=0;}
        }
}
/* Set N2 & n24 *
void stp03(){
    short a,b;
    for(a=1;a<26;a++){b=CC-a;

```

```

        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[2]=a; nm[24]=b;
            uflg[a]=1; uflg[b]=1;
            stp04();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set N4 & n22 *
void stp04(){
    short a,b;
    for(a=1;a<26;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[4]=a; nm[22]=b;
            uflg[a]=1; uflg[b]=1;
            stp05();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set n5=LSM-n1-n2-n3-n4 & n21 *
void stp05(){
    short a,b;
    a=LSM-nm[1]-nm[2]-nm[3]-nm[4];
    if((0<a)&&(a<26)){
        b=LSM-nm[25]-nm[24]-nm[23]-nm[22];
        if(a+b==CC){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[5]=a; nm[21]=b;
                uflg[a]=1; uflg[b]=1;
                stp06();
                uflg[b]=0; uflg[a]=0;}}
    }
}
/** Set N6 & n20 *
void stp06(){
    short a,b;
    for(a=1;a<26;a++){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[6]=a; nm[20]=b;
            uflg[a]=1; uflg[b]=1;
            stp07();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set N16 & n10 *
void stp07(){
    short a,b;
    for(a=25;a>0;a--){b=CC-a;
        if((uflg[a]==0)&&(uflg[b]==0)){
            nm[16]=a; nm[10]=b;
            uflg[a]=1; uflg[b]=1;
            stp08();
            uflg[b]=0; uflg[a]=0;}
    }
}
/** Set n11=LSM-n1-n6-n16-n21 & n15 & n3<n11<n15 *
void stp08(){
    short a,b;
    a=LSM-nm[1]-nm[6]-nm[16]-nm[21];
    if((nm[3]<a)&&(a<26)){
        b=LSM-nm[25]-nm[20]-nm[10]-nm[5];
        if((a<b)&&(a+b==CC)){
            if((uflg[a]==0)&&(uflg[b]==0)){
                nm[11]=a; nm[15]=b;
                uflg[a]=1; uflg[b]=1;
                stp09();
                uflg[b]=0; uflg[a]=0;}}
    }
}

```

```

}
/** Set N7 & n19 *
void stp09(){
  short a,b;
  for(a=1;a<26;a++){b=CC-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[7]=a; nm[19]=b;
      uflg[a]=1; uflg[b]=1;
      if(nm[1]+nm[7]+nm[13]+nm[19]+nm[25]==LSM){stp10();}
      uflg[b]=0; uflg[a]=0;}
    }
}
/** Set N9 & n17 *
void stp10(){
  short a,b;
  for(a=25;a>0;a--){b=CC-a;
    if((uflg[a]==0)&&(uflg[b]==0)){
      nm[9]=a; nm[17]=b;
      uflg[a]=1; uflg[b]=1;
      if(nm[5]+nm[9]+nm[13]+nm[17]+nm[21]==LSM){stp11();}
      uflg[b]=0; uflg[a]=0;}
    }
}
/** Set n8=LSM-n6-n7-n9-n10 & n18 *
void stp11(){
  short a,b;
  a=LSM-nm[6]-nm[7]-nm[9]-nm[10];
  if((0<a)&&(a<26)){
    b=LSM-nm[20]-nm[19]-nm[17]-nm[16];
    if(a+b==CC){
      if((uflg[a]==0)&&(uflg[b]==0)){
        nm[8]=a; nm[18]=b;
        uflg[a]=1; uflg[b]=1;
        if(nm[3]+nm[8]+nm[13]+nm[18]+nm[23]==LSM){stp12();}
        uflg[b]=0; uflg[a]=0;}}}
}
/** Set n12=LSM-n2-n7-n17-n22 & n14 *
void stp12(){
  short a,b;
  a=LSM-nm[2]-nm[7]-nm[17]-nm[22];
  if((0<a)&&(a<26)){
    b=LSM-nm[24]-nm[19]-nm[9]-nm[4];
    if(a+b==CC){
      if((uflg[a]==0)&&(uflg[b]==0)){
        nm[12]=a; nm[14]=b;
        uflg[a]=1; uflg[b]=1;
        if(nm[11]+nm[12]+nm[13]+nm[14]+nm[15]==LSM){rcrdsol();}
        uflg[b]=0; uflg[a]=0;}}}
}
//
/** Record the full Data to the Table
// and Select the Fundamental Solutions **
void rcrdsol(void){
  short n;
  for(n=1;n<26;n++){tsol[cnt][n]=nm[n];}
  cnt++;
  if((nm[3]<nm[8])&&(nm[8]<nm[18])&&(nm[18]<nm[23])){
    if((nm[3]<nm[11])&&(nm[11]<nm[12])&&(nm[12]<nm[14])&&(nm[14]<nm[15])){
      for(n=1;n<26;n++){ft[cnt3][n]=nm[n];}
      ft[cnt3][0]=cnt3+1; cnt3++;
    }
  }
}
//
/** Print the Transformations *
void ptrns(){

```

```

short m,n,p;
for(m=0;m<cnt3;m++){
    for(n=1;n<26;n++){cnm[n]=ft[m][n]; nt[0][n]=cnm[n]; }
    trnsf4(); for(n=1;n<26;n++){nt[1][n]=dnm[n];}
    trnsf5(); for(n=1;n<26;n++){nt[2][n]=dnm[n];}
    trnsf6(); for(n=1;n<26;n++){nt[3][n]=dnm[n];}
    //
    for(p=0;p<4;p++){
        for(n=1;n<26;n++){cnm[n]=nt[p][n];}
        trnsf1(); for(n=1;n<26;n++){nt[p+4][n]=dnm[n];}
        trnsf2(); for(n=1;n<26;n++){nt[p+8][n]=dnm[n];}
        trnsf3(); for(n=1;n<26;n++){nt[p+12][n]=dnm[n];}
    }
    refrot();
    fndon();
    //
    if(m%300==0){
        printf("%d/F\n",m+1);
        pr4ans();}
    //
}
}
//
/** Find the Old Number *
void fndon(){
    long l;
    short mtc=0;
    short m,n;
    for(m=0;m<16;m++){tcnt[m]=0;}
    for(m=0;m<16;m++){
        for(l=0;l<48544;l++){mtc=0;
            for(n=1;n<26;n++){
                if(nt[m][n]==tsol[l][n]){mtc++;}
                else{break;}
            }
            if(mtc==25){tcnt[m]=l+1; refnmb[l+1]++; break;}
        }
        if(mtc<25){refnmb[0]++;}
    }
}
//
/** Print the 4 Answers *
void pr4ans(){
    short k,k4,l,l5,m,n;
    for(k=0;k<4;k++){k4=k*4;
        printf("%14ld/S%15ld/S%15ld/S%15ld/S\n",
            tcnt[k4],tcnt[k4+1],tcnt[k4+2],tcnt[k4+3]);
        for(l=0;l<5;l++){l5=l*5;
            for(m=0;m<4;m++){
                printf(" ");
                for(n=1;n<6;n++){printf("%3d",nt[k4+m][l5+n]);}
                if(m+1<4){printf(" ");}
            }
            printf("\n");
        }
    }
}
//
/** Print the Reference Monitor *
void prmon(){
    int m;
    printf("   ???: %d",refnmb[0]);
    for(m=1;m<48545;m++){
        if(m%32==1){printf("\n%6d:",m);}
        printf("%2d",refnmb[m]);
    }
}

```

```

}
printf("\n");
}
//
/** Transform: Type 1 *
void trnsf1(){
    dnm[1]=cnm[1];    dnm[2]=cnm[4];    dnm[3]=cnm[3];    dnm[4]=cnm[2];    dnm[5]=cnm[5];
    dnm[6]=cnm[6];    dnm[7]=cnm[9];    dnm[8]=cnm[8];    dnm[9]=cnm[7];    dnm[10]=cnm[10];
    dnm[11]=cnm[11];  dnm[12]=cnm[14];  dnm[13]=cnm[13];  dnm[14]=cnm[12];  dnm[15]=cnm[15];
    dnm[16]=cnm[16];  dnm[17]=cnm[19];  dnm[18]=cnm[18];  dnm[19]=cnm[17];  dnm[20]=cnm[20];
    dnm[21]=cnm[21];  dnm[22]=cnm[24];  dnm[23]=cnm[23];  dnm[24]=cnm[22];  dnm[25]=cnm[25];
}
//
/** Transform: Type 2 *
void trnsf2(){
    dnm[1]=cnm[2];    dnm[2]=cnm[1];    dnm[3]=cnm[3];    dnm[4]=cnm[5];    dnm[5]=cnm[4];
    dnm[6]=cnm[7];    dnm[7]=cnm[6];    dnm[8]=cnm[8];    dnm[9]=cnm[10];   dnm[10]=cnm[9];
    dnm[11]=cnm[12];  dnm[12]=cnm[11];  dnm[13]=cnm[13];  dnm[14]=cnm[15];  dnm[15]=cnm[14];
    dnm[16]=cnm[17];  dnm[17]=cnm[16];  dnm[18]=cnm[18];  dnm[19]=cnm[20];  dnm[20]=cnm[19];
    dnm[21]=cnm[22];  dnm[22]=cnm[21];  dnm[23]=cnm[23];  dnm[24]=cnm[25];  dnm[25]=cnm[24];
}
//
/** Transform: Type 3 *
void trnsf3(){
    dnm[1]=cnm[4];    dnm[2]=cnm[1];    dnm[3]=cnm[3];    dnm[4]=cnm[5];    dnm[5]=cnm[2];
    dnm[6]=cnm[9];    dnm[7]=cnm[6];    dnm[8]=cnm[8];    dnm[9]=cnm[10];   dnm[10]=cnm[7];
    dnm[11]=cnm[14];  dnm[12]=cnm[11];  dnm[13]=cnm[13];  dnm[14]=cnm[15];  dnm[15]=cnm[12];
    dnm[16]=cnm[19];  dnm[17]=cnm[16];  dnm[18]=cnm[18];  dnm[19]=cnm[20];  dnm[20]=cnm[17];
    dnm[21]=cnm[24];  dnm[22]=cnm[21];  dnm[23]=cnm[23];  dnm[24]=cnm[25];  dnm[25]=cnm[22];
}
//
/** Transform: Type 4 *
void trnsf4(){
    dnm[1]=cnm[1];    dnm[2]=cnm[2];    dnm[3]=cnm[3];    dnm[4]=cnm[4];    dnm[5]=cnm[5];
    dnm[6]=cnm[16];   dnm[7]=cnm[17];   dnm[8]=cnm[18];   dnm[9]=cnm[19];   dnm[10]=cnm[20];
    dnm[11]=cnm[11];  dnm[12]=cnm[12];  dnm[13]=cnm[13];  dnm[14]=cnm[14];  dnm[15]=cnm[15];
    dnm[16]=cnm[6];   dnm[17]=cnm[7];   dnm[18]=cnm[8];   dnm[19]=cnm[9];   dnm[20]=cnm[10];
    dnm[21]=cnm[21];  dnm[22]=cnm[22];  dnm[23]=cnm[23];  dnm[24]=cnm[24];  dnm[25]=cnm[25];
}
//
/** Transform: Type 5 *
void trnsf5(){
    dnm[1]=cnm[6];    dnm[2]=cnm[7];    dnm[3]=cnm[8];    dnm[4]=cnm[9];    dnm[5]=cnm[10];
    dnm[6]=cnm[1];    dnm[7]=cnm[2];    dnm[8]=cnm[3];    dnm[9]=cnm[4];    dnm[10]=cnm[5];
    dnm[11]=cnm[11];  dnm[12]=cnm[12];  dnm[13]=cnm[13];  dnm[14]=cnm[14];  dnm[15]=cnm[15];
    dnm[16]=cnm[21];  dnm[17]=cnm[22];  dnm[18]=cnm[23];  dnm[19]=cnm[24];  dnm[20]=cnm[25];
    dnm[21]=cnm[16];  dnm[22]=cnm[17];  dnm[23]=cnm[18];  dnm[24]=cnm[19];  dnm[25]=cnm[20];
}
//
/** Transform: Type 6 *
void trnsf6(){
    dnm[1]=cnm[16];   dnm[2]=cnm[17];   dnm[3]=cnm[18];   dnm[4]=cnm[19];   dnm[5]=cnm[20];
    dnm[6]=cnm[1];    dnm[7]=cnm[2];    dnm[8]=cnm[3];    dnm[9]=cnm[4];    dnm[10]=cnm[5];
    dnm[11]=cnm[11];  dnm[12]=cnm[12];  dnm[13]=cnm[13];  dnm[14]=cnm[14];  dnm[15]=cnm[15];
    dnm[16]=cnm[21];  dnm[17]=cnm[22];  dnm[18]=cnm[23];  dnm[19]=cnm[24];  dnm[20]=cnm[25];
    dnm[21]=cnm[6];   dnm[22]=cnm[7];   dnm[23]=cnm[8];   dnm[24]=cnm[9];   dnm[25]=cnm[10];
}
//
/** Reflect & Rotate *
void refrot(){
    short m,n;
    for(m=0;m<16;m++){
        for(n=1;n<26;n++){cnm[n]=nt[m][n]; dnm[n]=nt[m][n];}
        while((cnm[3]>cnm[11])||(cnm[3]>cnm[15])||(cnm[3]>cnm[23])){rotate();}
        if(cnm[11]>cnm[15]){reflect3();}
    }
}

```

```

    for(n=1;n<26;n++){nt[m][n]=dnm[n];}
}
//
/* Mirror Reflection with Respect to n3+n8+n13+n18+n23 *
void reflect3C(){
    dnm[1]=cnm[5];    dnm[2]=cnm[4];    dnm[3]=cnm[3];    dnm[4]=cnm[2];    dnm[5]=cnm[1];
    dnm[6]=cnm[10];   dnm[7]=cnm[9];    dnm[8]=cnm[8];    dnm[9]=cnm[7];    dnm[10]=cnm[6];
    dnm[11]=cnm[15];  dnm[12]=cnm[14];   dnm[13]=cnm[13];  dnm[14]=cnm[12];  dnm[15]=cnm[11];
    dnm[16]=cnm[20];  dnm[17]=cnm[19];   dnm[18]=cnm[18];  dnm[19]=cnm[17];  dnm[20]=cnm[16];
    dnm[21]=cnm[25];  dnm[22]=cnm[24];   dnm[23]=cnm[23];  dnm[24]=cnm[22];  dnm[25]=cnm[21];
}
//
/* Mirror Reflection with Respect to n1+n7+n13+n19+n25 *
void reflect1C(){
    dnm[1]=cnm[1];    dnm[2]=cnm[6];    dnm[3]=cnm[11];   dnm[4]=cnm[16];   dnm[5]=cnm[21];
    dnm[6]=cnm[2];    dnm[7]=cnm[7];    dnm[8]=cnm[12];   dnm[9]=cnm[17];   dnm[10]=cnm[22];
    dnm[11]=cnm[3];   dnm[12]=cnm[8];   dnm[13]=cnm[13];  dnm[14]=cnm[18];  dnm[15]=cnm[23];
    dnm[16]=cnm[4];   dnm[17]=cnm[9];   dnm[18]=cnm[14];  dnm[19]=cnm[19];  dnm[20]=cnm[24];
    dnm[21]=cnm[5];   dnm[22]=cnm[10];  dnm[23]=cnm[15];  dnm[24]=cnm[20];  dnm[25]=cnm[25];
}
//
/* Rotation by 90 degrees *
void rotate(){
    short n;
    dnm[1]=cnm[21];   dnm[2]=cnm[16];   dnm[3]=cnm[11];   dnm[4]=cnm[6];    dnm[5]=cnm[1];
    dnm[6]=cnm[22];   dnm[7]=cnm[17];   dnm[8]=cnm[12];   dnm[9]=cnm[7];    dnm[10]=cnm[2];
    dnm[11]=cnm[23];  dnm[12]=cnm[18];  dnm[13]=cnm[13];  dnm[14]=cnm[8];   dnm[15]=cnm[3];
    dnm[16]=cnm[24];  dnm[17]=cnm[19];  dnm[18]=cnm[14];  dnm[19]=cnm[9];   dnm[20]=cnm[4];
    dnm[21]=cnm[25];  dnm[22]=cnm[20];  dnm[23]=cnm[15];  dnm[24]=cnm[10];  dnm[25]=cnm[5];
    for(n=1;n<26;n++){cnm[n]=dnm[n];}
}
//

```