

Part 3 : "New Algebraic Study of Magic Squares"

Chapter 9 : Fundamental Study of "Composite" Squares: Kanji Setsuda

Section1 : Study of "Composite" Squares of Order 6

#0. What is a 'Composite' Square?

A 'Composite' square has such a strict condition as "every 4 entries of any 2×2 block within must add up to the magic constant". The simultaneous equations below may be easier for you to understand this property. In the case of order 6 there are 25 simultaneous equations required at least.

The name 'Composite' was once given by Prof. Mutsumi Suzuki with his beautiful picture of Japanese royal flower in his page. I know there are some other names such as "Most-perfect" or "Compact", but I like this name 'Composite' best of all.

Let's study about this type of square of order 6. For we once failed in constructing any pan-diagonal magic squares or any self-complementary magic ones. This would be the last hope for us to make something like magic square of order 6.

See the basic diagram and the smallest set of Composite Conditions below.

[Figure 1: Basic Form and the Smallest Set of "Composite" Conditions]

Composite Conditions:

$n_6 n_1 n_2 n_3 n_4 n_5 n_6 n_1$	$n_1 + n_2 + n_7 + n_8 = S \dots (1)$
$ - + - + - + - + - $	$n_2 + n_3 + n_8 + n_9 = S \dots (2)$
$12 n_7 n_8 n_9 10 11 12 n_7$	$n_3 + n_4 + n_9 + n_{10} = S \dots (3)$
$ - + - + - + - + - $	$n_4 + n_5 + n_{10} + n_{11} = S \dots (4)$
$18 13 14 15 16 17 18 13$	$n_5 + n_6 + n_{11} + n_{12} = S \dots (5)$
$ - + - + - + - + - $	$n_7 + n_8 + n_{13} + n_{14} = S \dots (6)$
$24 19 20 21 22 23 24 19$	$n_8 + n_9 + n_{14} + n_{15} = S \dots (7)$
$ - + - + - + - + - $	$n_9 + n_{10} + n_{15} + n_{16} = S \dots (8)$
$30 25 26 27 28 29 30 25$	$n_{10} + n_{11} + n_{16} + n_{17} = S \dots (9)$
$ - + - + - + - + - $	$n_{11} + n_{12} + n_{17} + n_{18} = S \dots (10)$
$36 31 32 33 34 35 36 31$	$n_{13} + n_{14} + n_{19} + n_{20} = S \dots (11)$
$ - + - + - + - + - $	$n_{14} + n_{15} + n_{20} + n_{21} = S \dots (12)$
$'-----'$	$n_{15} + n_{16} + n_{21} + n_{22} = S \dots (13)$
$n_{16} + n_{17} + n_{22} + n_{23} = S \dots (14)$	$n_{23} + n_{24} + n_{29} + n_{30} = S \dots (20)$
$n_{17} + n_{18} + n_{23} + n_{24} = S \dots (15)$	$n_{25} + n_{26} + n_{31} + n_{32} = S \dots (21)$
$n_{19} + n_{20} + n_{25} + n_{26} = S \dots (16)$	$n_{26} + n_{27} + n_{32} + n_{33} = S \dots (22)$
$n_{20} + n_{21} + n_{26} + n_{27} = S \dots (17)$	$n_{27} + n_{28} + n_{33} + n_{34} = S \dots (23)$
$n_{21} + n_{22} + n_{27} + n_{28} = S \dots (18)$	$n_{28} + n_{29} + n_{34} + n_{35} = S \dots (24)$
$n_{22} + n_{23} + n_{28} + n_{29} = S \dots (19)$	$n_{29} + n_{30} + n_{35} + n_{36} = S \dots (25)$

#1. What do these 'Composite Conditions' mean?

Let's make some simple Algebraic calculations under these definitions.

First of all let's decide what value the above constant 'S(um)' should take.

$$(1) + (3) + (5) + (11) + (13 + (15) + (21) + (23) + (25)$$

$$n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 + n_8 + \dots + n_{33} + n_{34} + n_{35} + n_{36} = 9 * S;$$

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + \dots + 32 + 33 + 34 + 35 + 36 = 9 * S;$$

$$(1+36) + (2+35) + (3+34) + (4+33) + (5+32) + \dots + (17+20) + (18+19) = 9 * S;$$

$$37 * 18 = 9 * S;$$

Therefore $S=74$;

Next let's examine whether we can make the so-called 'Pan-magic Shift' move rows

and columns far to the opposite sides or not.

$$\begin{array}{llll}
 n1+n7+n2+n8=S & \dots(1)' ; & n2+n8+n3+n9=S & \dots(2)' ; \\
 n2+n8+n3+n9=S & \dots(2)' ; & n3+n9+n4+n10=S & \dots(3)' ; \\
 n3+n9+n4+n10=S & \dots(3)' ; & n4+n10+n5+n11=S & \dots(4)' ; \\
 n4+n10+n5+n11=S & \dots(4)' ; & n5+n11+n6+n12=S & \dots(5)' ;
 \end{array}
 \rightarrow \begin{array}{l}
 n1+n7=n3+n9; \\
 n2+n8=n4+n10; \\
 n3+n9=n5+n11; \\
 n4+n10=n6+n12;
 \end{array}$$

Therefore $n1+n7=n3+n9=n5+n11=P1$; $n2+n8=n4+n10=n6+n12=P2$; and $P1+P2=S$;

Consequently $n6+n1+n12+n7=(n1+n7)+(n6+n12)=P1+P2=S \dots(C1)$;

In the same way

$$\begin{array}{llll}
 n7+n13+n8+n14=S & \dots(6)' ; & n8+n14+n9+n15=S & \dots(7)' ; \\
 n8+n14+n9+n15=S & \dots(7)' ; & n9+n15+n10+n16=S & \dots(8)' ; \\
 n9+n15+n10+n16=S & \dots(8)' ; & n10+n16+n11+n17=S & \dots(9)' ; \\
 n10+n16+n11+n17=S & \dots(9)' ; & n11+n17+n12+n18=S & \dots(10)' ;
 \end{array}
 \rightarrow \begin{array}{l}
 n7+n13=n9+n15; \\
 n8+n14=n10+n16; \\
 n9+n15=n11+n17; \\
 n10+n16=n12+n18;
 \end{array}$$

Therefore $n7+n13=n9+n15=n11+n17=P3$; $n8+n14=n10+n16=n12+n18=P4$; and $P3+P4=S$;

Consequently $n12+n7+n18+n13=(n7+n13)+(n12+n18)=P3+P4=S \dots(C2)$;

In the same way

$$n13+n19=n15+n21=n17+n23=P5; \quad n14+n20=n16+n22=n18+n24=P6; \text{ and } P5+P6=S;$$

Consequently $n18+n13+n24+n19=(n13+n19)+(n18+n24)=P5+P6=S \dots(C3)$;

$$n19+n25=n21+n27=n23+n29=P7; \quad n20+n26=n22+n28=n24+n30=P8; \text{ and } P7+P8=S;$$

Consequently $n24+n19+n30+n25=(n19+n25)+(n24+n30)=P7+P8=S \dots(C4)$;

$$n25+n31=n27+n33=n29+n35=P9; \quad n26+n32=n28+n34=n30+n36=P10; \text{ and } P9+P10=S;$$

Consequently $n30+n25+n36+n31=(n25+n31)+(n30+n36)=P9+P10=S \dots(C5)$;

In the same way

$$\begin{array}{llll}
 n1+n2+n7+n8=S & \dots(1)' ; & n7+n8+n13+n14=S & \dots(6)' ; \\
 n7+n8+n13+n14=S & \dots(7)' ; & n13+n14+n19+n20=S & \dots(11)' ; \\
 n13+n14+n19+n20=S & \dots(11)' ; & n19+n20+n25+n26=S & \dots(16)' ; \\
 n19+n20+n25+n26=S & \dots(16)' ; & n25+n26+n31+n32=S & \dots(21)' ;
 \end{array}
 \rightarrow \begin{array}{l}
 n1+n2=n13+n14; \\
 n7+n8=n19+n20; \\
 n13+n14=n25+n26; \\
 n19+n20=n31+n32;
 \end{array}$$

Therefore $n1+n2=n13+n14=n25+n26=P11$; $n7+n8=n19+n20=n31+n32=P12$; and $P11+P12=S$;

Consequently $n31+n32+n1+n2=P12+P11=S \dots(C6)$;

In the same way

$$n2+n3=n14+n15=n26+n27=P13; \quad n8+n9=n20+n21=n32+n33=P14; \text{ and } P13+P14=S;$$

Consequently $n32+n33+n2+n3=P14+P13=S \dots(C7)$;

$$n3+n4=n15+n16=n27+n28=P15; \quad n9+n10=n21+n22=n33+n34=P16; \text{ and } P15+P16=S;$$

Consequently $n33+n34+n3+n4=P16+P15=S \dots(C8)$;

$$n4+n5=n16+n17=n28+n29=P17; \quad n10+n11=n22+n23=n34+n35=P18; \text{ and } P17+P18=S;$$

Consequently $n34+n35+n4+n5=P18+P17=S \dots(C9)$;

$$n5+n6=n17+n18=n29+n30=P19; \quad n11+n12=n23+n24=n35+n36=P20; \text{ and } P19+P20=S;$$

Consequently $n35+n36+n5+n6=P20+P19=S \dots(C10)$;

In the same way

$$\begin{array}{llll}
 n6+n1+n12+n7=S & \dots(C1)' ; & n12+n7+n18+n13=S & \dots(C2)' ; \\
 n18+n13+n24+n19=S & \dots(C3)' ; & n24+n19+n30+n25=S & \dots(C4)' ; \\
 n12+n7+n18+n13=S & \dots(C2)' ; & n18+n13+n24+n19=S & \dots(C3)' ; \\
 n24+n19+n30+n25=S & \dots(C4)' ; & n30+n25+n36+n31=S & \dots(C5)' ;
 \end{array}
 \rightarrow \begin{array}{l}
 n6+n1=n18+n13; \\
 n18+n13=n30+n25; \\
 n12+n7=n24+n19; \\
 n24+n19=n36+n31;
 \end{array}$$

Therefore $n6+n1=n18+n13=n30+n25=P13$; $n12+n7=n24+n19=n36+n31=P14$; and $P13+P14=S$;

Consequently $n36+n31+n6+n1=P14+P13=S \dots(C11)$;

What do these conclusions $(C1) \sim (C11)$ mean?

They mean the cylindrical structure of our object in two dimensions and the 'Pan-magic Shift' of rows and columns are surely possible to move each of them far to the opposite side. Any shift could never break those Composite Conditions, although it means no guarantee of any constant sum of rows, columns and pan-diagonals.

We can now draw the new Basic Form in the extended space as shown below.

We can also remake the definition of our 'Composite' Squares of Order 6 for the new extended one, adding our new knowledge of these properties.

#2. The New Definition of Basic Form and Composite Conditions

[Figure 2: New Basic Form and the Largest Set of Composite Conditions]

36 31 32 33 34 35 36 31		Composite Conditions:
.	.	$n_1 + n_2 + n_7 + n_8 = S \dots (1)$
n6 n1 n2 n3 n4 n5 n6 n1		$n_2 + n_3 + n_8 + n_9 = S \dots (2)$
-+---+-+---+-+--		$n_3 + n_4 + n_9 + n_{10} = S \dots (3)$
12 n7 n8 n9 10 11 12 n7		$n_4 + n_5 + n_{10} + n_{11} = S \dots (4)$
-+---+-+---+-+--		$n_5 + n_6 + n_{11} + n_{12} = S \dots (5)$
18 13 14 15 16 17 18 13		$n_6 + n_1 + n_{12} + n_7 = S \dots (6)$
-+---+-+---+-+--		$n_7 + n_8 + n_{13} + n_{14} = S \dots (7)$
24 19 20 21 22 23 24 19		$n_8 + n_9 + n_{14} + n_{15} = S \dots (8)$
-+---+-+---+-+--		$n_9 + n_{10} + n_{15} + n_{16} = S \dots (9)$
30 25 26 27 28 29 30 25		$n_{10} + n_{11} + n_{16} + n_{17} = S \dots (10)$
-+---+-+---+-+--		$n_{11} + n_{12} + n_{17} + n_{18} = S \dots (11)$
36 31 32 33 34 35 36 31		$n_{12} + n_7 + n_{18} + n_{13} = S \dots (12)$
'-----'		$n_{13} + n_{14} + n_{19} + n_{20} = S \dots (13)$
n6 n1 n2 n3 n4 n5 n6 n1		$n_{14} + n_{15} + n_{20} + n_{21} = S \dots (14)$
<hr/>		
n15+n16+n21+n22 = S ... (15)		n26+n27+n32+n33 = S ... (26)
n16+n17+n22+n23 = S ... (16)		n27+n28+n33+n34 = S ... (27)
n17+n18+n23+n24 = S ... (17)		n28+n29+n34+n35 = S ... (28)
n18+n13+n24+n19 = S ... (18)		n29+n30+n35+n36 = S ... (29)
n19+n20+n25+n26 = S ... (19)		n30+n25+n36+n31 = S ... (30)
n20+n21+n26+n27 = S ... (20)		n31+n32+ n1+ n2 = S ... (31)
n21+n22+n27+n28 = S ... (21)		n32+n33+ n2+ n3 = S ... (32)
n22+n23+n28+n29 = S ... (22)		n33+n34+ n3+ n4 = S ... (33)
n23+n24+n29+n30 = S ... (23)		n34+n35+ n4+ n5 = S ... (34)
n24+n19+n30+n25 = S ... (24)		n35+n36+ n5+ n6 = S ... (35)
n25+n26+n31+n32 = S ... (25)		n36+n31+ n6+ n1 = S ... (36)

#3. What can you find if you take these new Composite Conditions?

Keep taking only these 36 simultaneous equations now, and making some more Algebraic Calculations. What properties can you find for your object, then?

$$(1)+(3)+(5): (n_1 + n_2 + n_3 + n_4 + n_5 + n_6) + (n_7 + n_8 + n_9 + n_{10} + n_{11} + n_{12}) = L_1 + L_7 = 3*S$$

$$(7)+(9)+(11): (n_7 + n_8 + n_9 + n_{10} + n_{11} + n_{12}) + (n_{13} + n_{14} + n_{15} + n_{16} + n_{17} + n_{18}) = L_7 + L_{13} = 3*S$$

$$(13)+(15)+(17): (n_{13} + n_{14} + n_{15} + n_{16} + n_{17} + n_{18}) + (n_{19} + n_{20} + n_{21} + n_{22} + n_{23} + n_{24}) = L_{13} + L_{19} = 3*S$$

$$(19)+(21)+(23): (n_{19} + n_{20} + n_{21} + n_{22} + n_{23} + n_{24}) + (n_{25} + n_{26} + n_{27} + n_{28} + n_{29} + n_{30}) = L_{19} + L_{25} = 3*S$$

$$(25)+(27)+(29): (n_{25} + n_{26} + n_{27} + n_{28} + n_{29} + n_{30}) + (n_{31} + n_{32} + n_{33} + n_{34} + n_{35} + n_{36}) = L_{25} + L_{31} = 3*S$$

$$(31)+(33)+(35): (n_{31} + n_{32} + n_{33} + n_{34} + n_{35} + n_{36}) + (n_1 + n_2 + n_3 + n_4 + n_5 + n_6) = L_{31} + L_1 = 3*S$$

Therefore, $L_1 = L_{13} = L_{25} = LS_1$; $L_7 = L_{19} = L_{31} = LS_2$; and $LS_1 + LS_2 = 3*S = 3 * 74 = 222$;

But you cannot yet say $LS_1 = LS_2 = 111$.

In the same way

$$(1)+(13)+(25): (n_1 + n_7 + n_{13} + n_{19} + n_{25} + n_{31}) + (n_2 + n_8 + n_{14} + n_{20} + n_{26} + n_{32}) = R_1 + R_2 = 3*S$$

$$(2)+(14)+(26): (n_2 + n_8 + n_{14} + n_{20} + n_{26} + n_{32}) + (n_3 + n_9 + n_{15} + n_{21} + n_{27} + n_{33}) = R_2 + R_3 = 3*S$$

$$(3)+(15)+(27): (n_3 + n_9 + n_{15} + n_{21} + n_{27} + n_{33}) + (n_4 + n_{10} + n_{16} + n_{22} + n_{28} + n_{34}) = R_3 + R_4 = 3*S$$

$$(4)+(16)+(28): (n_4 + n_{10} + n_{16} + n_{22} + n_{28} + n_{34}) + (n_5 + n_{11} + n_{17} + n_{23} + n_{29} + n_{35}) = R_4 + R_5 = 3*S$$

Therefore, $R_1 = R_3 = R_5 = RS_1$; $R_2 = R_4 = R_6 = RS_2$; and $RS_1 + RS_2 = 3*S = 222$

But you cannot yet say $RS_1 = RS_2 = 111$.

If you want every row and every column to have the Constant Magic Sum 111, you must explicitly define as $L_1 = R_1 = 111$ at first by yourself.

Only $n_1 + n_2 + n_3 + n_4 + n_5 + n_6 = 111$; $n_1 + n_7 + n_{13} + n_{19} + n_{25} + n_{31} = 111$ are essentially required.

#4. Some Other Interesting Properties

Before studying about pan-diagonals of object, I like to study about some other interesting properties.

(1) Complementary Pairs Again

$$\begin{array}{lll} n1+n2+n7+n8=S; & n7+n8+n13+n14=S; & \rightarrow n1+n2=n13+n14=P1 \\ n7+n8+n13+n14=S; & n13+n14+n19+n20=S; & \rightarrow n7+n8=n19+n20=Q1 \\ n13+n14+n19+n20=S; & n19+n20+n25+n26=S; & \rightarrow n13+n14=n25+n26=P1 \\ n19+n20+n25+n26=S; & n25+n26+n31+n32=S; & \rightarrow n19+n20=n31+n32=Q1 \end{array}$$

Therefore, you will find two groups of complementary pairs such as:

$$n1+n2=n13+n14=n25+n26=P1; \quad n7+n8=n19+n20=n31+n32=Q1; \quad (P1+Q1=S=74)$$

$$\begin{array}{lll} n1+n7+n2+n8=S; & n2+n8+n3+n9=S; & \rightarrow n1+n7=n3+n9=P2 \\ n2+n8+n3+n9=S; & n3+n9+n4+n10=S; & \rightarrow n2+n8=n4+n10=Q2 \\ n3+n9+n4+n10=S; & n4+n10+n5+n11=S; & \rightarrow n3+n9=n5+n11=P2 \\ n4+n10+n5+n11=S; & n5+n11+n6+n12=S; & \rightarrow n4+n10=n6+n12=Q2 \end{array}$$

You will also find another groups of complementary pairs:

$$n1+n7=n3+n9=n5+n11=P2; \quad n2+n8=n4+n10=n6+n12=Q2; \quad (P2+Q2=S)$$

In the same way you may find many other groups of complementary pairs.

$$\begin{array}{lll} n2+n3=n14+n15=n26+n27=P3; & n8+n9=n20+n21=n32+n33=Q3 \\ n7+n13=n9+n15=n11+n17=P4; & n8+n14=n10+n16=n12+n18=Q4 \\ n3+n4=n15+n16=n27+n28=P5; & n9+n10=n21+n22=n33+n34=Q5 \\ n13+n19=n15+n21=n17+n23=P6; & n14+n20=n16+n22=n18+n24=Q6; \\ \dots & \dots & \end{array}$$

$$P3+Q3=S; \quad P4+Q4=S; \quad P5+Q5=S; \quad \dots \rightarrow Pn+Qn=S=74$$

But $Pn=Qn=37$ is not always true. It appears in a few cases.

(2) What relationship can you find among $n1$, $n3$, $n13$ and $n15$?

$$n1+n7=n3+n9=P2; \quad n7+n13=n9+n15=P5; \quad \rightarrow n1+n15=n3+n13$$

This is an example of 'Equality of Cross-sums', isn't it?

$$n8+n14=n10+n16=Q4; \quad n14+n20=n16+n22=Q6; \quad \rightarrow n8+n22=n10+n20$$

What about $n1$, $n4$, $n19$ and $n22$?

$$\begin{array}{ll} n1+n2=P5; & n3+n4=P5; \quad n19+n20=Q1; \quad n21+n22=Q5; \\ n1+n2+n3+n4=P1+P5; & n19+n20+n21+n22=Q1+Q5; \\ n1+n2+n3+n4+n19+n20+n21+n22=(P1+Q1)+(P5+Q5)=S+S=2*S \end{array}$$

But $n2+n3=P3$; $n20+n21=Q3$; $n2+n3+n20+n21=P3+Q3=S$

Therefore $n1+n4+n19+n22=2*S-S=S$

This is an example of 'Composite-sums of Middle-size Block', isn't it?

[Figure 2(Again): Extended Form of Composite Object 6*6]

36	31	32	33	34	35	36	31

n6	n1	n2	n3	n4	n5	n6	n1
-----+-----+-----							
12	n7	n8	n9	10	11	12	n7
-----+-----+-----							
18	13	14	15	16	17	18	13
-----+-----+-----							
24	19	20	21	22	23	24	19
-----+-----+-----							
30	25	26	27	28	29	30-25	
-----+-----+-----+-----							
36	31	32	33	34	35	36	31
'----- -----' -----							
n6 n1 n2 n3 n4 n5-n6-n1							

{n1, n5, n25, n29} are 4 tops of 5×5 block within.

They are also 4 tops of 3×3 block at the corner on the right hand side bottom.

$$n29+n30=n5+n6$$

$$\rightarrow n30+n25=n6+n1$$

$$n29-n25=n5-n1$$

Therefore $n1+n29=n5+n25$

It is an example of 'Equality of Cross-sums,' isn't it?

$n1+n6+n31+n36=S$ is the largest 'composite 4 tops' and is also the smallest 'composite 4 tops'

at the corner on the right hand side bottom at the same time.

Every 4 tops of any block with 2x2, 4x4, and 6x6 entries must add up to the same Constant Sum 74.

Every 4 tops of any block with 3x3 and 5x5 entries must obey the property "the Cross-sums are equal to each other."

#5. Pan-Diagonals of 'Composite' Squares

What can you find about Pan-diagonals if you accept those Composite Conditions?

[Figure 3: Extended Form and the Pan-Diagonal Conditions]

n4 n5 n6 n1 n2 n3 n4 n5 n6 n1 n2 n3	n1+ n8+n15+n22+n29+n36 = K11 n1+n12+n17+n22+n27+n32 = K12
---+---+---+---+---	n2+ n9+n16+n23+n30+n31 = K21 n2+ n7+n18+n23+n28+n33 = K22
10 11 12 n7 n8 n9 10 11 12 n7 n8 n9	n3+n10+n17+n24+n25+n32 = K31 n3+ n8+n13+n24+n29+n34 = K32
---+---+---+---+---	n4+n11+n18+n19+n26+n33 = K41 n4+ n9+n14+n19+n30+n35 = K42
16 17 18 13 14 15 16 17 18 13 14 15	n5+n12+n13+n20+n27+n34 = K51 n5+n10+n15+n20+n25+n36 = K52
---+---+---+---+---	n6+ n7+n14+n21+n28+n35 = K61 n6+n11+n16+n21+n26+n31 = K62
22 23 24 19 20 21 22 23 24 19 20 21	-----'
28 29 30 25 26 27 28 29 30 25 26 27	
34 35 36 31 32 33 34 35 36 31 32 33	

[1] $n_1+n_{15}=n_3+n_{13}$; ('Equality of Cross-sums' of 3x3 blocks.)

Add both sides to each other and add the same (n_8+n_{29}) to the both sides.

$$n_1+n_8+n_{15}+n_{22}+n_{29}+n_{36}=n_3+n_8+n_{13}+n_{24}+n_{29}+n_{34}=K11=K32;$$

[2] $n_3+n_{17}=n_5+n_{15}$; $n_{24}+n_{32}=n_{20}+n_{36}$; (Cross-sums of 3 x 3 blocks)

and add ($n_{10}+n_{25}$) to the both sides.

$$n_3+n_{10}+n_{17}+n_{24}+n_{25}+n_{32}=n_5+n_{10}+n_{15}+n_{20}+n_{25}+n_{36}=K31=K52;$$

[3] $n_5+n_{13}=n_1+n_{17}$; $n_{20}+n_{34}=n_{22}+n_{32}$; Add ($n_{12}+n_{27}$) to both sides.

$$n_5+n_{12}+n_{13}+n_{20}+n_{27}+n_{34}=n_1+n_{12}+n_{17}+n_{22}+n_{27}+n_{32}=K51=K12;$$

[4] $n_1+n_{27}=n_3+n_{25}$ (Cross-sum of 5 x 5 block);

$n_{12}+n_{22}=n_{10}+n_{24}$ (Cross-sum of 3 x 3 block); Add ($n_{17}+n_{32}$) to the both sides.

$$n_1+n_{12}+n_{17}+n_{22}+n_{27}+n_{32}=n_3+n_{10}+n_{17}+n_{24}+n_{25}+n_{32}=K12=K31;$$

[5] $n_1+n_{29}=n_5+n_{25}$ (Cross-sum of 5 x 5 block);

$n_{8}+n_{22}=n_{10}+n_{20}$ (Cross-sum of 3 x 3 block); Add ($n_{15}+n_{36}$) to the both sides.

$$n_1+n_8+n_{15}+n_{22}+n_{29}+n_{36}=n_5+n_{10}+n_{15}+n_{20}+n_{25}+n_{36}=K11=K52;$$

[6] $n_8+n_{36}=n_{12}+n_{31}$ (Cross-sum of 5 x 5 block);

$n_{15}+n_{29}=n_{17}+n_{27}$ (the same of 3 x 3 block); Add (n_1+n_{22}) to the both sides.

$$n_1+n_8+n_{15}+n_{22}+n_{29}+n_{36}=n_1+n_{12}+n_{17}+n_{22}+n_{27}+n_{32}=K11=K12;$$

After all $K11=K12=K31=K32=K51=K52=PD1$

In the same way $K21=K22=K41=K42=K61=K62=PD2$

All pan-diagonals are classified into two groups according to their sums:

(1) Six pandiagonals with n_1 , n_3 or n_5 , and (2) six ones with n_2 , n_4 or n_6 .

Although this conclusion seems almost the same as the usual pan-diagonal type, you cannot yet say $PD1=PD2$. When you want all pandiagonals to be equal, you must explicitly give the definition $K11 (= K62) = 111$ at first by yourself.

Only $n_1+n_8+n_{15}+n_{22}+n_{29}+n_{36}$ ($= n_6+n_{11}+n_{16}+n_{21}+n_{26}+n_{31}$) = 111; is essentially necessary to make our object become 'Pan-diagonal' or 'Complete' type.

#4. Can't we make any Composite Pan-Magic Square of Order 6?

Suppose you add the next 3 equations to our list of Composite Conditions defined.

$$(1) n_1+n_2+n_3+n_4+n_5+n_6 = 111; \quad (2) n_1+n_7+n_{13}+n_{19}+n_{25}+n_{31} = 111;$$

$$(3) n_1+n_8+n_{15}+n_{22}+n_{29}+n_{36} (= n_6+n_{11}+n_{16}+n_{21}+n_{26}+n_{31}) = 111;$$

And then you will necessarily fail in making your object. Your computer must reply

after a hard calculation, "No answers could be found at all!".

You may already know that there exists neither a 'self-complementary' type nor a 'complete' type of magic square of order 6 in general.

Here you can see a sample of logical proof for this impossibility.

Proof by *Reductio ad Absurdum*:

[Figure 4: Four Blocks of Complete Magic Square of Order 6]

Block P	Block Q	Any Complete Magic type of Order 6 has all complementary pairs of 37 only on the pan-diagonals: $n_1+n_{22}=37$; $n_2+n_{23}=37$; $n_3+n_{24}=37$; $n_7+n_{28}=37$; $n_8+n_{29}=37$; $n_9+n_{30}=37$; $n_{13}+n_{34}=37$; $n_{14}+n_{35}=37$; $n_{15}+n_{36}=37$; ...
n1 n2 n3	n4 n5 n6	Therefore, Block P + Block S = $37 \times 9 = 333$
---+---+	---+---+	But Block P + Block Q = $111 \times 3 = 333$
n7 n8 n9	10 11 12	($n_1+n_2+n_3+n_4+n_5+n_6=111$;
---+---+	---+---+	$n_7+n_8+n_9+n_{10}+n_{11}+n_{12}=111$;
13 14 15	16 17 18	$n_{13}+n_{14}+n_{15}+n_{16}+n_{17}+n_{18}=111$)
---+---+	---+---+	Also Block Q + Block S = 333
19 20 21	22 23 24	($n_4+n_{10}+n_{16}+n_{22}+n_{28}+n_{34}=111$;
---+---+	---+---+	$n_5+n_{11}+n_{17}+n_{23}+n_{29}+n_{35}=111$;
25 26 27	28 29 30	$n_6+n_{12}+n_{18}+n_{24}+n_{30}+n_{36}=111$)
---+---+	---+---+	Therefore, Block Q = Block S = $(333 / 2)$
31 32 33	34 35 36	But this quotient is not an integer at all. Sums of both Q and S must be integers.
Block R	Block S	

These conclusions are so contradictory to each other that they should demonstrate that we cannot make any Complete Magic Squares of Order 6.
(This proof was once worked out by Prof. Mutsumi Suzuki and Kanji Setsuda.)

Is it impossible even in the case of 'Composite Squares'?

Impossible. If you take all the conditions mentioned above, you cannot really make any kind of Pan-magic Square of Order 6 at all.

#5. What type of 'Composite Squares' of Order 6 can we make, then?

Let's now have the following experiments to make any 'Composite Squares':

- (1) Simply 'Composite' Squares of Order 6 only under the 'Composite Conditions'
- (2) 'Composite Pandiagonal' Squares of Order 6 under the Composite Conditions and only with $n_1+n_8+n_{15}+n_{22}+n_{29}+n_{36}$ ($=n_6+n_{11}+n_{16}+n_{21}+n_{26}+n_{31}$) = 111.
- (3) 'Composite Semi-pandiagonal' Squares of Order 6 under the Composite Conditions and only with $n_1+n_2+n_3+n_4+n_5+n_6 = 111$; $n_1+n_7+n_{13}+n_{19}+n_{25}+n_{31} = 111$.

5-1. Result List of the Simply 'Composite' Squares of Order 6

The next list demonstrates the recent result of my experiment to make the simply 'Composite' Squares only under the 36 'Composite Conditions' in the list of #2.

** Simply 'Composite' Squares of Order 6: **

[List of Standard Solutions with Check-Sums:(Part): by Kanji Setsuda]

1/ Row PD1\						612289/ Row PD1\						1210945/ Row PD1\											
1	36	3	34	5	32	111	62\	2	36	3	33	5	31	110	57\	3	36	2	33	5	31	110	57\
35	2	33	4	31	6	111	160\	35	1	34	4	32	6	112	165\	34	1	35	4	32	6	112	165\
7	30	9	28	11	26	111	62\	8	30	9	27	11	25	110	57\	9	30	8	27	11	25	110	57\
29	8	27	10	25	12	111	160\	29	7	28	10	26	12	112	165\	28	7	29	10	26	12	112	165\
13	24	15	22	17	20	111	62\	14	24	15	21	17	19	110	57\	15	24	14	21	17	19	110	57\
18	19	16	21	14	23	111	160\	23	13	22	16	20	18	112	165\	22	13	23	16	20	18	112	165\
-----						-----						-----											
103 119 103 119 103 119 Column	111 111 111 111 111 111 Column	111 111 111 111 111 111 Column	/ 62/160/ 62/160/ 62/160/ PD2Sum	/ 57/165/ 57/165/ 57/165/ PD2Sum																			

1795297 / Row PD1\	2339969 / Row PD1\	2867217 / Row PD1\
4 36 2 34 5 31 112 57\	5 36 2 34 4 31 112 57\	6 36 2 34 4 32 114 63\
33 1 35 3 32 6 110 165\	32 1 35 3 33 6 110 165\	31 1 35 3 33 5 108 159\
10 30 8 28 11 25 112 57\	11 30 8 28 10 25 112 57\	12 30 8 28 10 26 114 63\
27 7 29 9 26 12 110 165\	26 7 29 9 27 12 110 165\	25 7 29 9 27 11 108 159\
16 24 14 22 17 19 112 57\	17 24 14 22 16 19 112 57\	18 24 14 22 16 20 114 63\
21 13 23 15 20 18 110 165\	20 13 23 15 21 18 110 165\	13 19 17 21 15 23 108 159\
111 111 111 111 111 111 Column / 57/165/ 57/165/ 57/165/ PD2Sum	111 111 111 111 111 111 Column / 57/165/ 57/165/ 57/165/ PD2Sum	105 117 105 117 105 117 Column / 63/159/ 63/159/ 63/159/ PD2Sum
3359025 / Row PD1\	3770945 / Row PD1\	4138625 / Row PD1\
7 36 2 34 4 32 115 94\	8 36 2 34 4 31 115 96\	9 36 2 34 5 31 117 98\
30 1 35 3 33 5 107 128\	29 1 35 3 33 6 107 126\	28 1 35 3 32 6 105 124\
19 24 14 22 16 20 115 94\	20 24 14 22 16 19 115 96\	21 24 14 22 17 19 117 98\
6 25 11 27 9 29 107 128\	5 25 11 27 9 30 107 126\	4 25 11 27 8 30 105 124\
31 12 26 10 28 8 115 94\	32 12 26 10 28 7 115 96\	33 12 26 10 29 7 117 98\
18 13 23 15 21 17 107 128\	17 13 23 15 21 18 107 126\	16 13 23 15 20 18 105 124\
111 111 111 111 111 111 Column / 94/128/ 94/128/ 94/128/ PD2Sum	111 111 111 111 111 111 Column / 96/126/ 96/126/ 96/126 PD2Sum	111 111 111 111 111 111 Column / 98/124/ 98/124/ 98/124 PD2Sum
4488049 / Row PD1\	4723745 / Row PD1\	4932993 / Row PD1\
10 36 2 34 11 25 118 57\	11 36 2 34 4 25 112 85\	12 36 2 34 4 26 114 85\
27 1 35 3 26 12 104 165\	26 1 35 3 33 12 110 137\	25 1 35 3 33 11 108 137\
13 33 5 31 14 22 118 57\	17 30 8 28 10 19 112 85\	18 30 8 28 10 20 114 85\
24 4 32 6 23 15 104 165\	6 21 15 23 13 32 110 137\	5 21 15 23 13 31 108 137\
16 30 8 28 17 19 118 57\	31 16 22 14 24 5 112 85\	32 16 22 14 24 6 114 85\
21 7 29 9 20 18 104 165\	20 7 29 9 27 18 110 137\	19 7 29 9 27 17 108 137\
111 111 111 111 111 111 Column / 57/165/ 57/165/ 57/165 PD2Sum	111 111 111 111 111 111 Column / 85/137/ 85/137/ 85/137 PD2Sum	111 111 111 111 111 111 Column / 85/137/ 85/137/ 85/137 PD2Sum
5109969 / Row PD1\	5248865 / Row PD1\	5361505 / Row PD1\
13 36 2 34 4 23 112 61\	14 36 2 34 4 24 114 61\	15 36 2 34 4 24 115 62\
24 1 35 3 33 14 110 161\	23 1 35 3 33 13 108 161\	22 1 35 3 33 13 107 160\
17 32 6 30 8 19 112 61\	18 32 6 30 8 20 114 61\	19 32 6 30 8 20 115 62\
20 5 31 7 29 18 110 161\	19 5 31 7 29 17 108 161\	14 9 27 11 25 21 107 160\
21 28 10 26 12 15 112 61\	22 28 10 26 12 16 114 61\	23 28 10 26 12 16 115 62\
16 9 27 11 25 22 110 161\	15 9 27 11 25 21 108 161\	18 5 31 7 29 17 107 160\
111 111 111 111 111 111 Column / 61/161/ 61/161/ 61/161 PD2Sum	111 111 111 111 111 111 Column / 61/161/ 61/161/ 61/161 PD2Sum	111 111 111 111 111 111 Column / 62/160/ 62/160/ 62/160 PD2Sum
5449729 / Row PD1\	5486177 / Row PD1\	
16 36 2 34 4 23 115 64\	17 36 1 35 2 20 111 76\	
21 1 35 3 33 14 107 158\	16 5 32 6 31 21 111 146\	
20 32 6 30 8 19 115 64\	24 29 8 28 9 13 111 76\	
13 9 27 11 25 22 107 158\	10 11 26 12 25 27 111 146\	
24 28 10 26 12 15 115 64\	30 23 14 22 15 7 111 76\	
17 5 31 7 29 18 107 158\	18 3 34 4 33 19 111 146\	
111 111 111 111 111 111 Column / 64/158/ 64/158/ 64/158 PD2Sum	115 107 115 107 115 107 Column / 76/146/ 76/146/ 76/146 PD2Sum	[Count = 5510592] OK!

5-2. Result List of the 'Composite Pan-diagonal' Squares of Order 6

The following list demonstrates the recent result of my experiment to make the 'Composite Pan-diagonal' Squares under the 36 'Composite Conditions' and only with $n_1+n_8+n_{15}+n_{22}+n_{29}+n_{36}(=n_6+n_{11}+n_{16}+n_{21}+n_{26}+n_{31})=111$; for our definition.

** 'Composite Pan-diagonal' Squares of Order 6 with Check-sums: Part **

1 / Row PD1\	21889 / Row PD1\	43777 / Row PD1\
1 36 4 33 7 30 111 111\	2 36 5 31 10 28 112 111\	3 36 7 29 9 26 110 111\
34 3 31 6 28 9 111 111\	33 3 30 8 25 11 110 111\	34 1 30 8 28 11 112 111\
19 18 22 15 25 12 111 111\	14 24 17 19 22 16 112 111\	15 24 19 17 21 14 110 111\
17 20 14 23 11 26 111 111\	21 15 18 20 13 23 110 111\	22 13 18 20 16 23 112 111\
21 16 24 13 27 10 111 111\	26 12 29 7 34 4 112 111\	27 12 31 5 33 2 110 111\
8 29 5 32 2 35 111 111\	9 27 6 32 1 35 110 111\	10 25 6 32 4 35 112 111\
100 122 100 122 100 122 Column /111/111/111/111/111 PD2Sum	105 117 105 117 105 117 Column /111/111/111/111/111 PD2Sum	111 111 111 111 111 111 Column /111/111/111/111/111 PD2Sum

111 111 111 111 111 111 Column /111/111/111/111/111/111 PD2Sum	110 112 110 112 110 112 Column /111/111/111/111/111/111 PD2Sum	110 112 110 112 110 112 Column /111/111/111/111/111/111 PD2Sum
4609/ Row PD1\	4801/ Row PD1\	4961/ Row PD1\
13 36 1 12 25 24 111 111\	14 35 4 9 26 24 112 111\	15 36 3 10 25 21 110 111\
22 3 34 27 10 15 111 111\	23 2 33 28 11 13 110 111\	22 1 34 27 12 16 112 111\
19 30 7 6 31 18 111 111\	18 31 8 5 30 20 112 111\	19 32 7 6 29 17 110 111\
17 8 29 32 5 20 111 111\	19 6 29 32 7 17 110 111\	14 9 26 35 4 24 112 111\
23 26 11 2 35 14 111 111\	22 27 12 1 34 16 112 111\	23 28 11 2 33 13 110 111\
16 9 28 33 4 21 111 111\	15 10 25 36 3 21 110 111\	18 5 30 31 8 20 112 111\
-----	-----	-----
110 112 110 112 110 112 Column /111/111/111/111/111/111 PD2Sum	111 111 111 111 111 111 Column /111/111/111/111/111/111 PD2Sum	111 111 111 111 111 111 Column /111/111/111/111/111/111 PD2Sum
5057/ Row PD1\	5121/ Row PD1\	
16 36 3 10 25 22 112 111\	17 32 5 8 29 20 111 111\	
21 1 34 27 12 15 110 111\	23 2 35 26 11 14 111 111\	
20 32 7 6 29 18 112 111\	16 33 4 9 28 21 111 111\	
13 9 26 35 4 23 110 111\	13 12 25 36 1 24 111 111\	
24 28 11 2 33 14 112 111\	22 27 10 3 34 15 111 111\	
17 5 30 31 8 19 110 111\	19 6 31 30 7 18 111 111\	
-----	-----	
111 111 111 111 111 111 Column /111/111/111/111/111/111 PD2Sum	110 112 110 112 110 112 Column /111/111/111/111/111/111 PD2Sum	[Count = 5184] OK!

[Listed by Kanji Setsuda on August 17, 2010 with MacOSX and Xcode3.1.2.]

I myself was much surprised with this big count of solutions. If you had only a few solutions of that kind, they might be naturally considered to be 'accidental'. But, the existence of so many solutions may hardly be considered to be accidental essentially. They do exist in the way as they should do, to my impression.

You may expect you should very soon have got the real Magic type with both rows and columns adding up to the same constant sum 111.

But, you should not be able to have got any piece of such the real Magic ones, ever never, I am sorry to say.

5-3. Result List of the 'Composite Semi-panmagic' Squares of Order 6

The next list shows the result of my effort to make the 'Composite Semi-panmagic' Squares under the 36 'Composite Conditions' and only with $n_1+n_2+n_3+n_4+n_5+n_6 = 111$; $n_1+n_7+n_{13}+n_{19}+n_{25}+n_{31} = 111$; for our definitions.

This list is certainly new, printed recently, while the calculation itself has often been made for a long time since 2001. The first result was reported by Prof. Mutsumi Suzuki in his page, and I verified it by my own calculation just after that.

* Composite Semi-panmagic Squares of Order 6: Part List with Check-Sums *

1/ Row PD1\	1969/ Row PD1\	3057/ Row PD1\
1 36 4 33 7 30 111 64\	1 35 2 36 3 34 111 60\	1 34 2 36 3 35 111 60\
35 2 32 5 29 8 111 158\	33 5 32 4 31 6 111 162\	33 6 32 4 31 5 111 162\
3 34 6 31 9 28 111 64\	7 29 8 30 9 28 111 60\	7 28 8 30 9 29 111 60\
27 10 24 13 21 16 111 158\	27 11 26 10 25 12 111 162\	27 12 26 10 25 11 111 162\
19 18 22 15 25 12 111 64\	19 17 20 18 21 16 111 60\	19 16 20 18 21 17 111 60\
26 11 23 14 20 17 111 158\	24 14 23 13 22 15 111 162\	24 15 23 13 22 14 111 162\
-----	-----	-----
111 111 111 111 111 111 Column / 64/158/ 64/158/ 64/158 PD2Sum	111 111 111 111 111 111 Column / 60/162/ 60/162/ 60/162 PD2Sum	111 111 111 111 111 111 Column / 60/162/ 60/162/ 60/162 PD2Sum
3537/ Row PD1\	4337/ Row PD1\	4817/ Row PD1\
1 33 3 35 15 24 111 62\	1 32 3 35 7 33 111 94\	1 31 3 35 8 33 111 96\
32 8 30 6 18 17 111 160\	24 17 22 14 18 16 111 128\	24 18 22 14 17 16 111 126\
5 29 7 31 19 20 111 62\	13 20 15 23 19 21 111 94\	13 19 15 23 20 21 111 96\
36 4 34 2 22 13 111 160\	36 5 34 2 30 4 111 128\	36 6 34 2 29 4 111 126\
9 25 11 27 23 16 111 62\	25 8 27 11 31 9 111 94\	25 7 27 11 32 9 111 96\
28 12 26 10 14 21 111 160\	12 29 10 26 6 28 111 128\	12 30 10 26 5 28 111 126\
-----	-----	-----
111 111 111 111 111 111 Column / 62/160/ 62/160/ 62/160 PD2Sum	111 111 111 111 111 111 Column / 94/128/ 94/128/ 94/128 PD2Sum	111 111 111 111 111 111 Column / 96/126/ 96/126/ 96/126 PD2Sum

111 111 111 111 111 111 Column /110/112/110/112/110/112 PD2Sum	3201/ Row PD1\	111 111 111 111 111 111 Column /110/112/110/112/110/112 PD2Sum	3457/ Row PD1\	111 111 111 111 111 111 Column /112/110/112/110/112/110 PD2Sum
7 35 3 32 9 25 111 110\	8 35 3 31 9 25 111 112\	9 36 1 32 5 28 111 110\		
30 2 34 5 28 12 111 112\	29 2 34 6 28 12 111 110\	25 4 33 8 29 12 111 112\		
19 23 15 20 21 13 111 110\	20 23 15 19 21 13 111 112\	23 22 15 18 19 14 111 110\		
6 26 10 29 4 36 111 112\	5 26 10 30 4 36 111 110\	3 26 11 30 7 34 111 112\		
31 11 27 8 33 1 111 110\	32 11 27 7 33 1 111 112\	35 10 27 6 31 2 111 110\		
18 14 22 17 16 24 111 112\	17 14 22 18 16 24 111 110\	16 13 24 17 20 21 111 112\		
111 111 111 111 111 111 Column /110/112/110/112/110/112 PD2Sum	3969/ Row PD1\	111 111 111 111 111 111 Column /112/110/112/110/112/110 PD2Sum	4225/ Row PD1\	111 111 111 111 111 111 Column /110/112/110/112/110/112 PD2Sum
10 36 4 30 5 26 111 110\	11 34 3 30 7 26 111 110\	12 34 2 30 5 28 111 110\		
27 1 33 7 32 11 111 112\	27 2 35 6 31 10 111 112\	25 3 35 7 32 9 111 112\		
22 24 16 18 17 14 111 110\	23 22 15 18 19 14 111 110\	24 22 14 18 17 16 111 110\		
3 25 9 31 8 35 111 112\	1 28 9 32 5 36 111 112\	1 27 11 31 8 33 111 112\		
34 12 28 6 29 2 111 110\	33 12 25 8 29 4 111 110\	36 10 26 6 29 4 111 110\		
15 13 21 19 20 23 111 112\	16 13 24 17 20 21 111 112\	13 15 23 19 20 21 111 112\		
111 111 111 111 111 111 Column /110/112/110/112/110/112 PD2Sum	4609/ Row PD1\	111 111 111 111 111 111 Column /110/112/110/112/110/112 PD2Sum	4801/ Row PD1\	111 111 111 111 111 111 Column /110/112/110/112/110/112 PD2Sum
13 36 4 10 26 22 111 110\	14 36 4 10 26 21 111 112\	15 35 3 9 25 24 111 110\		
24 1 33 27 11 15 111 112\	23 1 33 27 11 16 111 110\	22 2 34 28 12 13 111 112\		
17 32 8 6 30 18 111 110\	18 32 8 6 30 17 111 112\	19 31 7 5 29 20 111 110\		
20 5 29 31 7 19 111 112\	19 5 29 31 7 20 111 110\	14 10 26 36 4 21 111 112\		
21 28 12 2 34 14 111 110\	22 28 12 2 34 13 111 112\	23 27 11 1 33 16 111 110\		
16 9 25 35 3 23 111 112\	15 9 25 35 3 24 111 110\	18 6 30 32 8 17 111 112\		
111 111 111 111 111 111 Column /110/112/110/112/110/112 PD2Sum	5057/ Row PD1\	111 111 111 111 111 111 Column /112/110/112/110/112/110 PD2Sum	5121/ Row PD1\	111 111 111 111 111 111 Column /110/112/110/112/110/110 PD2Sum
16 35 3 9 25 23 111 112\	17 32 5 8 29 20 111 110\			
21 2 34 28 12 14 111 110\	24 1 36 25 12 13 111 112\			
20 31 7 5 29 19 111 112\	16 33 4 9 28 21 111 110\			
13 10 26 36 4 22 111 110\	14 11 26 35 2 23 111 112\			
24 27 11 1 33 15 111 112\	22 27 10 3 34 15 111 110\			
17 6 30 32 8 18 111 110\	18 7 30 31 6 19 111 112\			
111 111 111 111 111 111 Column /112/110/112/110/112/110 PD2Sum	[Count = 5184] OK!			

[Listed by Kanji Setsuda on August 17, 2010 with MacOSX and Xcode3.1.2]

'Almost Panmagic' objects also count so many as 5184, far more pieces than the 'accidental' existence. They do surely exist as they should do.

Their solution counts are the same with those of Almost Magic type of Composite Pandiagonal objects. It does not seem to be a sheer coincidence.

5-3. Fundamental Solutions of 'Composite Semi-panmagic' Squares of Order 6

Let's make the list of Fundamental Solutions of Composite Semi-panmagic Squares of Order 6, under the following guidance of Prof. Mutsumi Suzuki in 2001.

List-forming Inequality Conditions: Solution Counts

n1=1, n2>n7, n2>n4>n6, n3>n5, n13>n25, n7>n19>n31	41 Solutions
n1<n6, n1<n31, n1<n36, n2>n7	53136 = 41x36x36

(Based upon the Table of Prof. M. Suzuki made in 2001)

** Composite Semi-Panmagic Squares of Order 6: Fundamental Solutions **

1/108/114/108/114/108/114 PD2Sum	/ 60/162/ 60/162/ 60/162 PD2Sum	/ 66/156/ 66/156/ 66/156 PD2Sum
22/ Row PD1\		
1 36 2 35 3 34 111 72\	1 36 2 35 3 34 111 90\	1 36 2 35 3 34 111 96\
33 4 32 5 31 6 111 150\	33 4 32 5 31 6 111 132\	33 4 32 5 31 6 111 126\
7 30 8 29 9 28 111 72\	19 18 20 17 21 16 111 90\	19 18 20 17 21 16 111 96\
24 13 23 14 22 15 111 150\	27 10 26 11 25 12 111 132\	24 13 23 14 22 15 111 126\
25 12 26 11 27 10 111 72\	22 15 23 14 24 13 111 90\	25 12 26 11 27 10 111 96\
21 16 20 17 19 18 111 150\	9 28 8 29 7 30 111 132\	9 28 8 29 7 30 111 126\
<hr/>		
111 111 111 111 111 111 Column / 72/150/ 72/150/ 72/150 PD2Sum	111 111 111 111 111 111 Column / 90/132/ 90/132/ 90/132 PD2Sum	111 111 111 111 111 111 Column / 96/126/ 96/126/ 96/126 PD2Sum
25/ Row PD1\		
1 36 2 35 3 34 111 102\	1 36 5 32 9 28 111 86\	1 36 5 32 9 28 111 110\
33 4 32 5 31 6 111 120\	33 4 29 8 25 12 111 136\	33 4 29 8 25 12 111 112\
22 15 23 14 24 13 111 102\	3 34 7 30 11 26 111 86\	15 22 19 18 23 14 111 110\
21 16 20 17 19 18 111 120\	24 13 20 17 16 21 111 136\	24 13 20 17 16 21 111 112\
25 12 26 11 27 10 111 102\	27 10 31 6 35 2 111 86\	27 10 31 6 35 2 111 110\
9 28 8 29 7 30 111 120\	23 14 19 18 15 22 111 136\	11 26 7 30 3 34 111 112\
<hr/>		
111 111 111 111 111 111 Column /102/120/102/120/102/120 PD2Sum	111 111 111 111 111 111 Column / 86/136/ 86/136/ 86/136 PD2Sum	111 111 111 111 111 111 Column /110/112/110/112/110/112 PD2Sum
28/ Row PD1\		
1 36 5 32 9 28 111 112\	1 36 13 24 25 12 111 102\	1 36 13 24 25 12 111 110\
33 4 29 8 25 12 111 110\	33 4 21 16 9 28 111 120\	33 4 21 16 9 28 111 112\
16 21 20 17 24 13 111 112\	3 34 15 22 27 10 111 102\	7 30 19 18 31 6 111 110\
23 14 19 18 15 22 111 110\	32 5 20 17 8 29 111 120\	32 5 20 17 8 29 111 112\
27 10 31 6 35 2 111 112\	11 26 23 14 35 2 111 102\	11 26 23 14 35 2 111 110\
11 26 7 30 3 34 111 110\	31 6 19 18 7 30 111 120\	27 10 15 22 3 34 111 112\
<hr/>		
111 111 111 111 111 111 Column /112/110/112/110/112/110 PD2Sum	111 111 111 111 111 111 Column /102/120/102/120/102/120 PD2Sum	111 111 111 111 111 111 Column /110/112/110/112/110/112 PD2Sum
31/ Row PD1\		
1 36 13 24 25 12 111 112\	1 36 13 24 25 12 111 114\	1 36 2 35 3 34 111 84\
33 4 21 16 9 28 111 110\	32 5 20 17 8 29 111 108\	27 10 26 11 25 12 111 138\
8 29 20 17 32 5 111 112\	9 28 21 16 33 4 111 114\	7 30 8 29 9 28 111 84\
31 6 19 18 7 30 111 110\	31 6 19 18 7 30 111 108\	24 13 23 14 22 15 111 138\
11 26 23 14 35 2 111 112\	11 26 23 14 35 2 111 114\	31 6 32 5 33 4 111 84\
27 10 15 22 3 34 111 110\	27 10 15 22 3 34 111 108\	21 16 20 17 19 18 111 138\
<hr/>		
111 111 111 111 111 111 Column /112/110/112/110/112/110 PD2Sum	111 111 111 111 111 111 Column /114/108/114/108/114/108 PD2Sum	111 111 111 111 111 111 Column / 84/138/ 84/138/ 84/138 PD2Sum
34/ Row PD1\		
1 36 2 35 3 34 111 108\	1 36 2 35 3 34 111 114\	1 36 4 33 7 30 111 84\
27 10 26 11 25 12 111 114\	27 10 26 11 25 12 111 108\	27 10 24 13 21 16 111 138\
19 18 20 17 21 16 111 108\	22 15 23 14 24 13 111 114\	3 34 6 31 9 28 111 84\
24 13 23 14 22 15 111 114\	21 16 20 17 19 18 111 108\	26 11 23 14 20 17 111 138\
31 6 32 5 33 4 111 108\	31 6 32 5 33 4 111 114\	29 8 32 5 35 2 111 84\
9 28 8 29 7 30 111 114\	9 28 8 29 7 30 111 108\	25 12 22 15 19 18 111 138\
<hr/>		
111 111 111 111 111 111 Column /108/114/108/114/108/114 PD2Sum	111 111 111 111 111 111 Column /114/108/114/108/114/108 PD2Sum	111 111 111 111 111 111 Column / 84/138/ 84/138/ 84/138 PD2Sum
37/ Row PD1\		
1 36 4 33 7 30 111 116\	1 36 4 33 7 30 111 118\	1 36 4 33 7 30 111 120\
27 10 24 13 21 16 111 106\	27 10 24 13 21 16 111 104\	26 11 23 14 20 17 111 102\
19 18 22 15 25 12 111 116\	20 17 23 14 26 11 111 118\	21 16 24 13 27 10 111 120\
26 11 23 14 20 17 111 106\	25 12 22 15 19 18 111 104\	25 12 22 15 19 18 111 102\
29 8 32 5 35 2 111 116\	29 8 32 5 35 2 111 118\	29 8 32 5 35 2 111 120\
9 28 6 31 3 34 111 106\	9 28 6 31 3 34 111 104\	9 28 6 31 3 34 111 102\
<hr/>		
111 111 111 111 111 111 Column /116/106/116/106/116/106 PD2Sum	111 111 111 111 111 111 Column /118/104/118/104/118/104 PD2Sum	111 111 111 111 111 111 Column /120/102/120/102/120/102 PD2Sum
40/ Row PD1\		
1 36 2 35 3 34 111 120\	1 36 5 32 9 28 111 130\	
24 13 23 14 22 15 111 102\	24 13 20 17 16 21 111 92\	
25 12 26 11 27 10 111 120\	25 12 29 8 33 4 111 130\	
21 16 20 17 19 18 111 102\	23 14 19 18 15 22 111 92\	
31 6 32 5 33 4 111 120\	27 10 31 6 35 2 111 130\	
9 28 8 29 7 30 111 102\	11 26 7 30 3 34 111 92\	

```
|111|111|111|111|111|111| Column |111|111|111|111|111|111| Column
/120/102/120/102/120/102| PD2Sum /130/ 92/130/ 92/130/ 92| PD2Sum [Count = 41] OK!
```

[Listed by Kanji Setsuda on August 18, 2010 with MacOSX and Xcode3.1.2]

These 41 Fundamental Solutions were also discovered and reported first by our Prof. Mutsumi Suzuki, and I verified them by my own calculation just after that.

I myself found the transformation system to reconstruct all the 53136 Standard Solutions of that type by these 41 Fundamental Solutions, though I like to skip explaining about it here.

(Original Edition Written in English on Jul.29, 2001;
Revised on Jul. 8, 2006; and Aug.23, 2010 with MacOSX & Xcode 3.1.2)

Written by Kanji Setsuda; E-Mail Address: jag12001@nifty.com

[Appendix] Sample List of My Program:

```
/** 'Composite' Semi-Panmagic Squares 6x6: Fundamental Solutions */
/** 'CSPMS6F.c' dictated by Kanji Setsuda */
/** on Jul.29,'01; Jan.16, Aug.18,'10 */
/** Working with MacOSX & Xcode 3.1.2 */
/**/
#include <stdio.h>
/* Global Variables */
short int cnt, cnt3;
short LSM, S4N;
short nm[37], uflg[37];
short mnmm[6][37];
short sm[5][13], pd[5][13];
/* Sub-Routines */
//void stp01(void);
void stp02(void), stp03(void), stp04(void), stp05(void);
void stp06(void), stp07(void), stp08(void), stp09(void);
void stp10(void), stp11(void), stp12(void), stp13(void);
void stp14(void), stp15(void), stp16(void), stp17(void);
void stp18(void), stp19(void), stp20(void), stp21(void);
void stp22(void), stp23(void), stp24(void), stp25(void);
void stp26(void), stp27(void), stp28(void), stp29(void);
void stp30(void), stp31(void), stp32(void), stp33(void);
void stp34(void), stp35(void), stp36(void), stp37(void);
void stp38(void), stp39(void), stp40(void);
void ansprint(void), printsol(short x);
/**/
/* Main Program: Research */
int main(){
short n;
printf("\n* 'Composite Semi-Panmagic' Squares 6x6: List of 41 Fundamental Solutions *\n");
for(n=0;n<37;n++){nm[n]=0; uflg[n]=0;}
LSM=111; S4N=74; cnt=0; cnt3=0;
nm[1]=1; uflg[1]=1; // Set N1=1
stp02(); // Begin the Calculations
if(cnt3>0){printsol(cnt3);}
printf(" [Count = %d] OK!\n",cnt);
printf(" [Listed by Kanji Setsuda on Aug.18,'10 with Mac OSX and Xcode3.1.2]\n");
return 0;
}/*
/**/
```

```

/* Begin the Calculations */
/* Set N2 */
void stp02(){
    short i;
    for(i=36;i>1;i--){
        if(uflg[i]==0){
            nm[2]=i; uflg[i]=1;
            stp03();
            uflg[i]=0;}
    }
}
/* Set N7(<N2) */
void stp03(){
    short i;
    for(i=nm[2]-1;i>1;i--){
        if(uflg[i]==0){
            nm[7]=i; uflg[i]=1;
            stp04();
            uflg[i]=0;}
    }
}
/* Set n8=74-n1-n2-n7 */
void stp04(){
    short i;
    i=S4N-nm[1]-nm[2]-nm[7];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[8]=i; uflg[i]=1;
            stp05();
            uflg[i]=0;}}
}
/* Set N3 */
void stp05(){
    short i;
    for(i=2;i<37;i++){
        if(uflg[i]==0){
            nm[3]=i; uflg[i]=1;
            stp06();
            uflg[i]=0;}}
}
/* Set n9=74-n2-n3-n8 */
void stp06(){
    short i;
    i=S4N-nm[2]-nm[3]-nm[8];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[9]=i; uflg[i]=1;
            stp07();
            uflg[i]=0;}}
}
/* Set N4(<N2) */
void stp07(){
    short i;
    for(i=nm[2]-1;i>1;i--){
        if(uflg[i]==0){
            nm[4]=i; uflg[i]=1;
            stp08();
            uflg[i]=0;}}
}

```

```

        }
    }

/* Set n10=74-n3-n4-n9 */
void stp08(){
    short i;
    i=S4N-nm[3]-nm[4]-nm[9];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[10]=i; uflg[i]=1;
            stp09();
            uflg[i]=0;}}
    }

/* Set N5(>N3) */
void stp09(){
    short i;
    for(i=nm[3]+1;i<37;i++){
        if(uflg[i]==0){
            nm[5]=i; uflg[i]=1;
            stp10();
            uflg[i]=0;}}
    }

/* Set n11=74-n4-n5-n10 */
void stp10(){
    short i;
    i=S4N-nm[4]-nm[5]-nm[10];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[11]=i; uflg[i]=1;
            stp11();
            uflg[i]=0;}}
    }

/* Set N6(<N4) & n1+n2+n3+n4+n5+n6==LSM */
void stp11(){
    short i,sm;
    for(i=nm[4]-1;i>1;i--){
        if(uflg[i]==0){
            nm[6]=i; uflg[i]=1;
            sm=nm[1]+nm[2]+nm[3]+nm[4]+nm[5]+nm[6];
            if(sm==LSM){stp12();}
            uflg[i]=0;}}
    }

/* Set n12=74-n5-n6-n11 */
void stp12(){
    short i,j;
    i=S4N-nm[5]-nm[6]-nm[11];
    j=S4N-nm[6]-nm[1]-nm[7];
    if((1<i)&&(i<37)&&(i==j)){
        if(uflg[i]==0){
            nm[12]=i; uflg[i]=1;
            stp14();
            uflg[i]=0;}}
    }

/**/
/* Search Level 2: */
/* Set N13 */
void stp14(){
    short i;

```

```

for(i=2;i<37;i++){
    if(uflg[i]==0){
        nm[13]=i; uflg[i]=1;
        stp15();
        uflg[i]=0;}
}
/* Set n14=74-n7-n8-n13 */
void stp15(){
short i;
i=S4N-nm[7]-nm[8]-nm[13];
if((1<i)&&(i<37)){
    if(uflg[i]==0){
        nm[14]=i; uflg[i]=1;
        stp16();
        uflg[i]=0;}}
}
/* Set n15=74-n8-n9-n14 */
void stp16(){
short i;
i=S4N-nm[8]-nm[9]-nm[14];
if((1<i)&&(i<37)){
    if(uflg[i]==0){
        nm[15]=i; uflg[i]=1;
        stp17();
        uflg[i]=0;}}
}
/* Set n16=74-n9-n10-n15 */
void stp17(){
short i;
i=S4N-nm[9]-nm[10]-nm[15];
if((1<i)&&(i<37)){
    if(uflg[i]==0){
        nm[16]=i; uflg[i]=1;
        stp18();
        uflg[i]=0;}}
}
/* Set n17=74-n10-n11-n16 */
void stp18(){
short i;
i=S4N-nm[10]-nm[11]-nm[16];
if((1<i)&&(i<37)){
    if(uflg[i]==0){
        nm[17]=i; uflg[i]=1;
        stp19();
        uflg[i]=0;}}
}
/* Set n18=74-n11-n12-n17 */
void stp19(){
short i,j;
i=S4N-nm[11]-nm[12]-nm[17];
j=S4N-nm[7]-nm[12]-nm[13];
if((1<i)&&(i<37)&&(i==j)){
    if(uflg[i]==0){
        nm[18]=i; uflg[i]=1;
        stp21();
        uflg[i]=0;}}
}
/**/

```

```

/* Search Level 3: */
/* Set N19(<N7) */
void stp21(){
    short i;
    for(i=nm[7]-1;i>1;i--){
        if(uflg[i]==0){
            nm[19]=i; uflg[i]=1;
            stp22();
            uflg[i]=0;}
    }
}
/* Set n20=74-n13-n14-n19 */
void stp22(){
    short i;
    i=S4N-nm[13]-nm[14]-nm[19];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[20]=i; uflg[i]=1;
            stp23();
            uflg[i]=0;}}
}
/* Set n21=74-n14-n15-n20 */
void stp23(){
    short i;
    i=S4N-nm[14]-nm[15]-nm[20];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[21]=i; uflg[i]=1;
            stp24();
            uflg[i]=0;}}
}
/* Set n22=74-n15-n16-n21 */
void stp24(){
    short i;
    i=S4N-nm[15]-nm[16]-nm[21];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[22]=i; uflg[i]=1;
            stp26();
            uflg[i]=0;}}
}
/* Set n23=74-n16-n17-n22 */
void stp26(){
    short i;
    i=S4N-nm[16]-nm[17]-nm[22];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[23]=i; uflg[i]=1;
            stp27();
            uflg[i]=0;}}
}
/* Set n24=74-n17-n18-n23 */
void stp27(){
    short i,j;
    i=S4N-nm[17]-nm[18]-nm[23];
    j=S4N-nm[13]-nm[18]-nm[19];
    if((1<i)&&(i<37)&&(i==j)){
        if(uflg[i]==0){
            nm[24]=i; uflg[i]=1;

```

```

        stp28();
        uflg[i]=0;}}
}
/**/
/* Search Level 4: */
/* Set N25(>N13) */
void stp28(){
    short i;
    for(i=nm[13]+1;i<37;i++){
        if(uflg[i]==0){
            nm[25]=i; uflg[i]=1;
            stp29();
            uflg[i]=0;}
    }
}
/* Set n26=74-n19-n20-n25 */
void stp29(){
    short i;
    i=S4N-nm[19]-nm[20]-nm[25];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[26]=i; uflg[i]=1;
            stp30();
            uflg[i]=0;}}
}
/* Set n27=74-n20-n21-n26 */
void stp30(){
    short i;
    i=S4N-nm[20]-nm[21]-nm[26];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[27]=i; uflg[i]=1;
            stp31();
            uflg[i]=0;}}
}
/* Set n28=74-n21-n22-n27 */
void stp31(){
    short i;
    i=S4N-nm[21]-nm[22]-nm[27];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[28]=i; uflg[i]=1;
            stp32();
            uflg[i]=0;}}
}
/* Set n29=74-n22-n23-n28 */
void stp32(){
    short i;
    i=S4N-nm[22]-nm[23]-nm[28];
    if((1<i)&&(i<37)){
        if(uflg[i]==0){
            nm[29]=i; uflg[i]=1;
            stp33();
            uflg[i]=0;}}
}
/* Set n30=74-n23-n24-n29 */
void stp33(){
    short i,j;
    i=S4N-nm[23]-nm[24]-nm[29];

```

```

j=S4N-nm[24]-nm[19]-nm[25];
if((1<i)&&(i<37)&&(i==j)){
    if((uflg[i]==0)){
        nm[30]=i; uflg[i]=1;
        stp34();
        uflg[i]=0;}}
}
/**/
/* Search Level 5: */
/* Set N31(<N19) & n1+n7+n13+n19+n25+n31==LSM */
void stp34(){
    short i,sm;
    for(i=2;i<nm[19];i++){
        if(uflg[i]==0){
            nm[31]=i; uflg[i]=1;
            sm=nm[1]+nm[7]+nm[13]+nm[19]+nm[25]+nm[31];
            if(sm==LSM){stp35();}
            uflg[i]=0;}}
    }
/* Set n32=74-n25-n26-n31 */
void stp35(){
    short i,j;
    i=S4N-nm[25]-nm[26]-nm[31];
    j=S4N-nm[31]-nm[1]-nm[2];
    if((1<i)&&(i<37)&&(i==j)){
        if(uflg[i]==0){
            nm[32]=i; uflg[i]=1;
            stp36();
            uflg[i]=0;}}
    }
/* Set n33=74-n26-n27-n32 */
void stp36(){
    short i,j;
    i=S4N-nm[26]-nm[27]-nm[32];
    j=S4N-nm[32]-nm[2]-nm[3];
    if((1<i)&&(i<37)&&(i==j)){
        if(uflg[i]==0){
            nm[33]=i; uflg[i]=1;
            stp37();
            uflg[i]=0;}}
    }
/* Set n34=74-n27-n28-n33 */
void stp37(){
    short i,j;
    i=S4N-nm[27]-nm[28]-nm[33];
    j=S4N-nm[33]-nm[3]-nm[4];
    if((1<i)&&(i<37)&&(i==j)){
        if(uflg[i]==0){
            nm[34]=i; uflg[i]=1;
            stp38();
            uflg[i]=0;}}
    }
/* Set n35=74-n28-n29-n34 */
void stp38(){
    short i,j;
    i=S4N-nm[28]-nm[29]-nm[34];
    j=S4N-nm[34]-nm[4]-nm[5];
    if((1<i)&&(i<37)&&(i==j)){

```

```

    if(uflg[i]==0){
        nm[35]=i; uflg[i]=1;
        stp39();
        uflg[i]=0;}}
}
/* Set n36=74-n29-n30-n35 & n36>N1 */
void stp39(){
    short i,j;
    i=S4N-nm[29]-nm[30]-nm[35];
    j=S4N-nm[35]-nm[5]-nm[6];
    if((1<i)&&(i<37)&&(i==j)){
        if(uflg[i]==0){
            nm[36]=i; uflg[i]=1;
            stp40();
            uflg[i]=0;}}
}
/**/
/* The Last Checks */
void stp40(){
    short sm1,sm2;
    sm1=nm[30]+nm[25]+nm[36]+nm[31];
    sm2=nm[36]+nm[31]+nm[6]+nm[1];
    if((sm1==S4N)&&(sm2==S4N)){ansprint();}
}
/**/
/* Print The Answers */
void ansprint(){
    short n;
    cnt++;
    mnmm[cnt3][0]=cnt;
    for(n=1;n<37;n++){mnmm[cnt3][n]=nm[n];}
    cnt3++; if(cnt3==4){printsol(cnt3); cnt3=0;}
}
/* Print the List of Answers */
void printsol(short x){
    short l,m,n;
    for(m=0;m<x;m++){
        printf("%17d/",mnmm[m][0]);
        if(m+1<x){printf("  ");}
    }
    printf("\n");
    for(l=0;l<6;l++){
        for(m=0;m<x;m++){
            for(n=1;n<7;n++){printf("%3d",mnmm[m][l*6+n]);}
            if(m+1<x){printf("  ");}
        }
        printf("\n");
    }
    //printf("\n");
}
*/

```