

Chapter 6: Fundamental Studies of Multi-Dimensional Extra-Cubic Magic Forms and their Decompositions

Section 2: Study of 4-Dimensional Extra-Cubic Magic Forms of Order 2 and their Decompositions

Section 2-1: Transformations and 'Fundamental Solutions' of Self-Complementary Squares 4 x 4: Kanji Setsuda

#1. In the former section we studied about 4-dimensional extra-cubic magic form of order 2, and finally obtained three 'initial' solutions of (1) self-complementary type and of (2) pan-diagonal (or complete or 'composite and complete') type of magic squares of order 4, by decomposing the original and converting its higher dimension down to lower.

```
**** Four Dimensional Extra-Cubic Forms 2^4 ****
** With 2 Types of Decomposed Magic Squares 4^2 **
** Under the Conditions: n1<n16, n1<n10, n1<n7, n2>n3, n2>n9, n3>n5 **
```

```
0/n1      /n2      0/SC      /PD
n1---n5   n9---13   n1 n2 n9 10   n1 n2 10 n9
 |n2---n6 |10---14   n3 n4 11 12   n3 n4 12 11
n3-|-n7 | 11-|-15 |   n5 n6 13 14   n7 n8 16 15
  n4---n8   12---16   n7 n8 15 16   n5 n6 14 13
[Basic Positions] [Self-Complementary] [Pan-Magic]
```

```
** Three Reperesentative Solutions: **
```

```
1/n1      /n2      1/SC      /PD
 1---12    8---13    1 15  8 10    1 15 10  8
 |15--- 6  |10--- 3    14  4 11  5    14  4  5 11
14-|- 7 | 11-|- 2 |    12  6 13  3    7  9 16  2
  4--- 9    5---16    7  9  2 16    12  6  3 13
```

```
2/n1      /n2      2/SC      /PD
 1--- 8    12---13    1 15 12  6    1 15  6 12
 |15---10  | 6--- 3    14  4  7  9    14  4  9  7
14-|-11 | 7-|- 2 |    8 10 13  3    11  5 16  2
  4--- 5    9---16    11  5  2 16    8 10  3 13
```

```
3/n1      /n2      3/SC      /PD
 1--- 8    14---11    1 15 14  4    1 15  4 14
 |15---10  | 4--- 5    12  6  7  9    12  6  9  7
12-|-13 | 7-|- 2 |    8 10 11  5    13  3 16  2
  6--- 3    9---16    13  3  2 16    8 10  5 11
```

```
[Count = 3]
```

Each solution on the right most in the figure above is transformed from each Self-Complementary one in the center. It is made into Pan-diagonal (or 'Complete' or 'C&C') type by exchanging the whole lines of both 3rd and 4th rows and columns.

#2. Let's study about the Self-Complementary type right here. Are these three solutions really 'fundamental' to all self-complementary magic squares of order 4?

We have to verify that by demonstrating such a clear evidence as a 'transformation method' of making these three into 48 object solutions.

I know there are various ways of making them, but now let me introduce you a typical transformation system for making all 48 self-complementary magic squares out of F3.

The goal objects we want to get to are listed full as below.

```
*** Self-Complementary Magic Squares 4x4 ***
*** Print All 48 Solutions ***
```

```

      1/G1      2/G2      3/G1      4/G3      5/G2      6/G3
1 15 8 10  1 15 12 6  1 15 8 10  1 15 14 4  1 15 12 6  1 15 14 4
14 4 11 5  14 4 7 9  12 6 13 3  12 6 7 9  8 10 13 3  8 10 11 5
12 6 13 3  8 10 13 3  14 4 11 5  8 10 11 5  14 4 7 9  12 6 7 9
7 9 2 16  11 5 2 16  7 9 2 16  13 3 2 16  11 5 2 16  13 3 2 16

      7/G2      8/G3      9/G1      10/G3      11/G1      12/G2
1 14 8 11  1 14 15 4  1 14 12 7  1 14 15 4  1 12 14 7  1 12 15 6
12 7 13 2  12 7 6 9  8 11 13 2  8 11 10 5  8 13 11 2  8 13 10 3
15 4 10 5  8 11 10 5  15 4 6 9  12 7 6 9  15 6 4 9  14 7 4 9
6 9 3 16  13 2 3 16  10 5 3 16  13 2 3 16  10 3 5 16  11 2 5 16

      13/G1      14/G2      15/G1      16/G3      17/G2      18/G3
2 16 7 9  2 16 11 5  2 16 7 9  2 16 13 3  2 16 11 5  2 16 13 3
13 3 12 6  13 3 8 10  11 5 14 4  11 5 8 10  7 9 14 4  7 9 12 6
11 5 14 4  7 9 14 4  13 3 12 6  7 9 12 6  13 3 8 10  11 5 8 10
8 10 1 15  12 6 1 15  8 10 1 15  14 4 1 15  12 6 1 15  14 4 1 15

      19/G2      20/G3      21/G1      22/G3      23/G1      24/G2
2 13 7 12  2 13 16 3  2 13 11 8  2 13 16 3  2 11 13 8  2 11 16 5
11 8 14 1  11 8 5 10  7 12 14 1  7 12 9 6  7 14 12 1  7 14 9 4
16 3 9 6  7 12 9 6  16 3 5 10  11 8 5 10  16 5 3 10  13 8 3 10
5 10 4 15  14 1 4 15  9 6 4 15  14 1 4 15  9 4 6 15  12 1 6 15

      25/G2      26/G1      27/G2      28/G1      29/G1      30/G2
3 16 6 9  3 16 10 5  3 16 6 9  3 16 10 5  3 13 6 12  3 13 10 8
13 2 12 7  13 2 8 11  10 5 15 4  6 9 15 4  10 8 15 1  6 12 15 1
10 5 15 4  6 9 15 4  13 2 12 7  13 2 8 11  16 2 9 7  16 2 5 11
8 11 1 14  12 7 1 14  8 11 1 14  12 7 1 14  5 11 4 14  9 7 4 14

      31/G2      32/G1      33/G2      34/G1      35/G2      36/G1
3 10 13 8  3 10 16 5  4 15 5 10  4 15 9 6  4 15 5 10  4 15 9 6
6 15 12 1  6 15 9 4  14 1 11 8  14 1 7 12  9 6 16 3  5 10 16 3
16 5 2 11  13 8 2 11  9 6 16 3  5 10 16 3  14 1 11 8  14 1 7 12
9 4 7 14  12 1 7 14  7 12 2 13  11 8 2 13  7 12 2 13  11 8 2 13

      37/G1      38/G2      39/G2      40/G1      41/G3      42/G3
4 14 5 11  4 14 9 7  4 9 14 7  4 9 15 6  5 16 4 9  5 16 4 9
9 7 16 2  5 11 16 2  5 16 11 2  5 16 10 3  11 2 14 7  10 3 15 6
15 1 10 8  15 1 6 12  15 6 1 12  14 7 1 12  10 3 15 6  11 2 14 7
6 12 3 13  10 8 3 13  10 3 8 13  11 2 8 13  8 13 1 12  8 13 1 12

      43/G3      44/G3      45/G3      46/G3      47/G3      48/G3
5 11 10 8  5 10 11 8  6 15 3 10  6 15 3 10  6 12 9 7  6 9 12 7
4 14 15 1  4 15 14 1  12 1 13 8  9 4 16 5  3 13 16 2  3 16 13 2
16 2 3 13  16 3 2 13  9 4 16 5  12 1 13 8  15 1 4 14  15 4 1 14
9 7 6 12  9 6 7 12  7 14 2 11  7 14 2 11  10 8 5 11  10 5 8 11
```

('/Gn' on the right shoulder means the group each solution belongs to.)

[Count = 48]

```

** Self-Complementary Magic Squares 4x4: **
*** Print 3 Fundamental Solutions ***

```

1/				2/				4/			
1	15	8	10	1	15	12	6	1	15	14	4
14	4	11	5	14	4	7	9	12	6	7	9
12	6	13	3	8	10	13	3	8	10	11	5
7	9	2	16	11	5	2	16	13	3	2	16

[Count = 3]

We have to make these three fundamental solutions into all those 48 goal objects. But how can we do that?

- (1) Classify all those 48 into three groups each of which contains 16 solutions including the fundamental one. Compare every solution to the F3, and find any similar ones whose rows and columns consist of the same 4 members and collect them into the same group.
- (2) Analyse all in the same group and try to imagine and invent anything like a 'transformation method' making all from each fundamental.
- (3) Any transformation method you make must be simple and common enough to apply to every group.

#3. Let me skip explaining all to my conclusion:

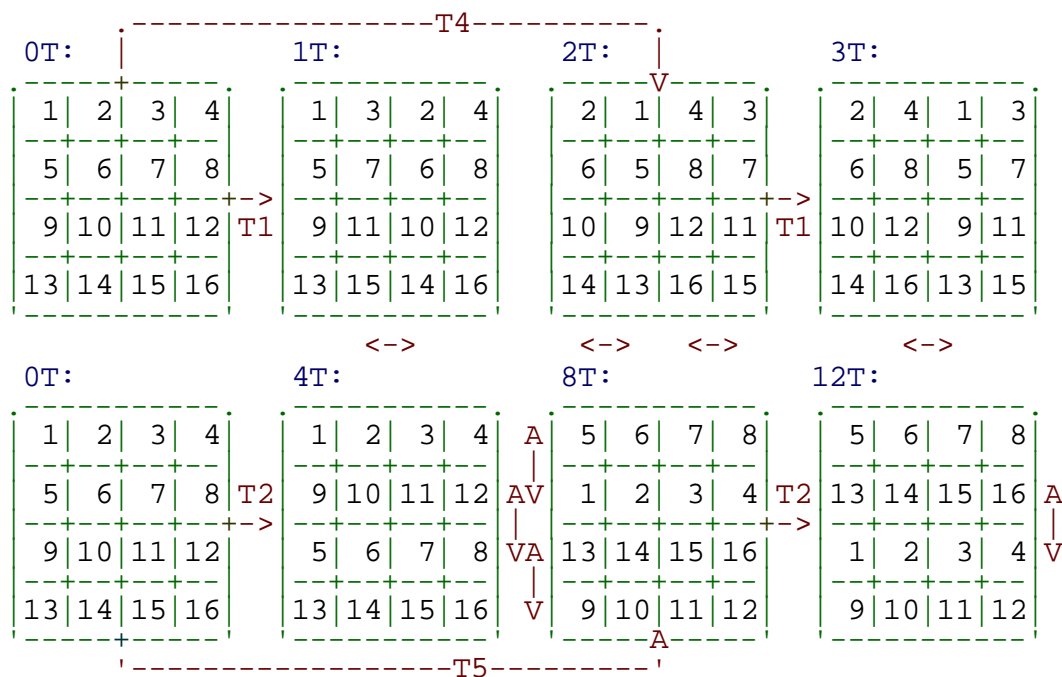
I invented such a powerful transformation system as shown below.

We only use 4 types of transformation method applied in common to every group:

- (1) Type 1 exchanges the 2nd column with the 3rd one.
- (2) Type 4 exchanges the 1st column with the 2nd one and the 4th with the 3rd at the same time.
- (3) Type 2 exchanges the 2nd row with the 3rd one.
- (4) Type 5 exchanges the 1st row with the 2nd and also the 4th with the 3rd.

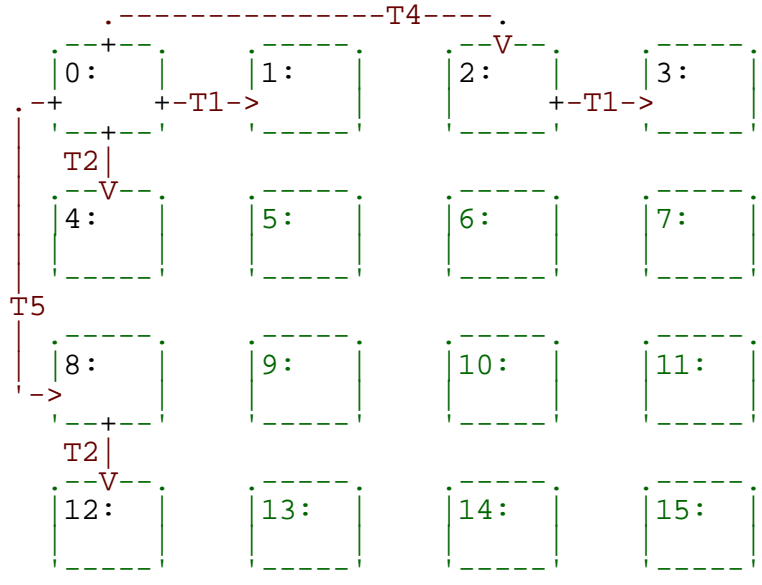
We only do the same thing both in two directions: vertically and horizontally.

'The Process Transition Diagram' illustrates how to use them to make our 15 objects of each fundamental on the left top.



**** Process Transition Diagrams ****

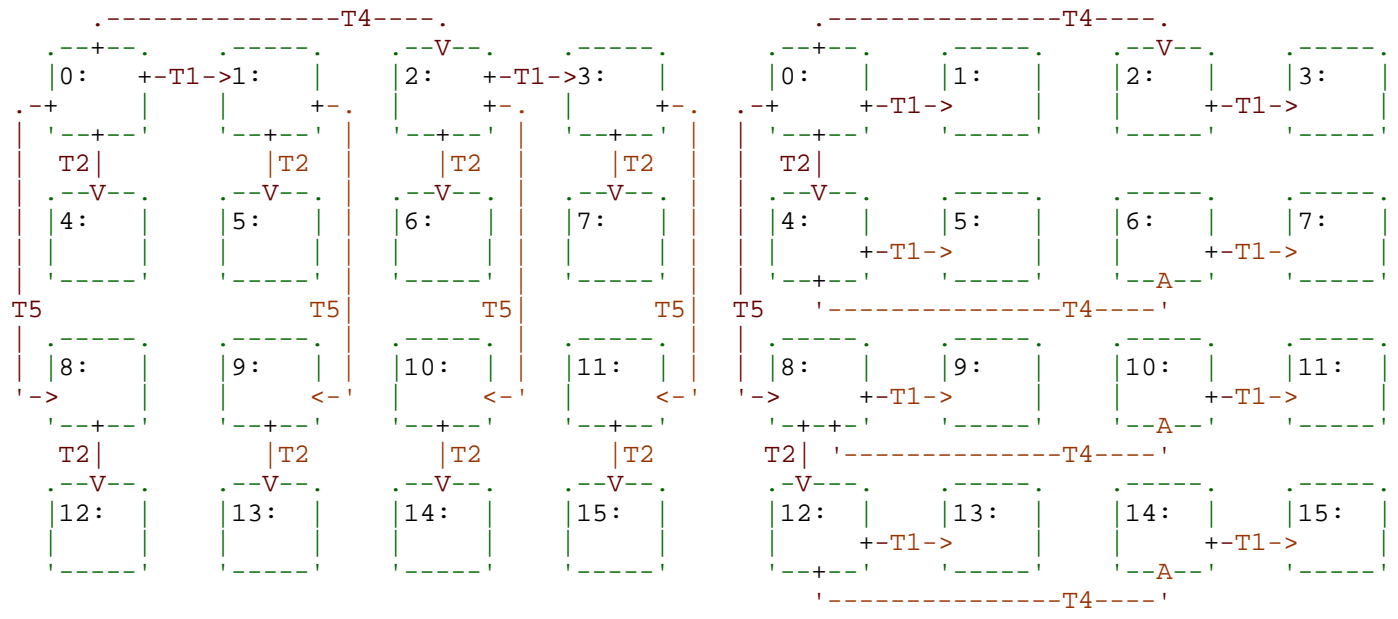
[Step 1:]



Please choose one you like out of the two ways illustrated below for the Step 2 (Either will make the same things.), and make No.5, 6, 7, 9, 10, 11, 13, 14 and 15 solutions.

[Step 2/]

[Step 2]



I dictated our new transformation rules for our program sheet as below:

```

/* Transformation Rules: Type 1~5 */
/**/
void trnsf1(void) {
  short n;
  d[1]=c[1];      d[2]=c[3];      d[3]=c[2];      d[4]=c[4];
  d[5]=c[5];      d[6]=c[7];      d[7]=c[6];      d[8]=c[8];
  d[9]=c[9];      d[10]=c[11];     d[11]=c[10];    d[12]=c[12];
  d[13]=c[13];    d[14]=c[15];     d[15]=c[14];    d[16]=c[16];
  for(n=1;n<17;n++){c[n]=d[n];}
}

```

```

/**/
void trnsf2(void) {
    short n;
    d[1]=c[1];    d[2]=c[2];    d[3]=c[3];    d[4]=c[4];
    d[5]=c[9];    d[6]=c[10];   d[7]=c[11];   d[8]=c[12];
    d[9]=c[5];    d[10]=c[6];   d[11]=c[7];   d[12]=c[8];
    d[13]=c[13];  d[14]=c[14];  d[15]=c[15];  d[16]=c[16];
    for(n=1;n<17;n++) { c[n]=d[n]; }
}
/* We don't use trnsf3(= T1 + T2) */
/**/
void trnsf4(void) {
    short n;
    d[1]=c[2];    d[2]=c[1];    d[3]=c[4];    d[4]=c[3];
    d[5]=c[6];    d[6]=c[5];    d[7]=c[8];    d[8]=c[7];
    d[9]=c[10];   d[10]=c[9];   d[11]=c[12];  d[12]=c[11];
    d[13]=c[14];  d[14]=c[13];  d[15]=c[16];  d[16]=c[15];
    for(n=1;n<17;n++) { c[n]=d[n]; }
}
/**/
void trnsf5(void) {
    short n;
    d[1]=c[5];    d[2]=c[6];    d[3]=c[7];    d[4]=c[8];
    d[5]=c[1];    d[6]=c[2];    d[7]=c[3];    d[8]=c[4];
    d[9]=c[13];   d[10]=c[14];   d[11]=c[15];  d[12]=c[16];
    d[13]=c[9];   d[14]=c[10];   d[15]=c[11];  d[16]=c[12];
    for(n=1;n<17;n++) { c[n]=d[n]; }
}
/**/
/* Normalisation */
void normal s(void) {
    short n;
    for(n=1;n<17;n++) { d[n]=c[n]; }
    if(c[2]<c[5]) {
        d[1]=c[1];    d[2]=c[5];    d[3]=c[9];    d[4]=c[13];
        d[5]=c[2];    d[6]=c[6];    d[7]=c[10];   d[8]=c[14];
        d[9]=c[3];    d[10]=c[7];   d[11]=c[11];  d[12]=c[15];
        d[13]=c[4];   d[14]=c[8];    d[15]=c[12];  d[16]=c[16];
        for(n=1;n<17;n++) { c[n]=d[n]; }
    }
    if(c[1]>c[13]) {
        d[1]=c[13];  d[2]=c[14];   d[3]=c[15];   d[4]=c[16];
        d[5]=c[9];   d[6]=c[10];   d[7]=c[11];   d[8]=c[12];
        d[9]=c[5];   d[10]=c[6];   d[11]=c[7];   d[12]=c[8];
        d[13]=c[1];  d[14]=c[2];   d[15]=c[3];   d[16]=c[4];
        for(n=1;n<17;n++) { c[n]=d[n]; }
    }
    if(c[1]>c[16]) {
        d[1]=c[16];  d[2]=c[12];   d[3]=c[8];    d[4]=c[4];
        d[5]=c[15];  d[6]=c[11];   d[7]=c[7];    d[8]=c[3];
        d[9]=c[14];  d[10]=c[10];  d[11]=c[6];   d[12]=c[2];
        d[13]=c[13]; d[14]=c[9];   d[15]=c[5];   d[16]=c[1];
        for(n=1;n<17;n++) { c[n]=d[n]; }
    }
    if(c[1]>c[4]) {
        d[1]=c[4];   d[2]=c[3];    d[3]=c[2];    d[4]=c[1];

```

```

d[5]=c[8];      d[6]=c[7];      d[7]=c[6];      d[8]=c[5];
d[9]=c[12];     d[10]=c[11];     d[11]=c[10];    d[12]=c[9];
d[13]=c[16];    d[14]=c[15];    d[15]=c[14];    d[16]=c[13];
for(n=1;n<17;n++){c[n]=d[n];}
}
/**/

```

The next list shows the first result of my experiment applying our new transformation system to each fundamental: Every answer is kept pre-normalised.

```

**** Self-Complementary Magic Squares 4x4: ****
** Transform Each Fundamental into 15 Solutions **
** With the Original and Prenoramalized Answers **
[1]
0T:      1/      1T:      2T:      3T:
-----
| 1|15| 8|10| | | 15|10| 1| 8| |
|14| 4|11| 5| | | 4| 5|14|11| |
|12| 6|13| 3| | | 6| 3|12|13| |
| 7| 9| 2|16| | | 9|16| 7| 2| |
-----
4T:      5T:      6T:      7T:
-----
| 1|15| 8|10| | | 15|10| 1| 8| |
|12| 6|13| 3| | | 6| 3|12|13| |
|14| 4|11| 5| | | 4| 5|14|11| |
| 7| 9| 2|16| | | 9|16| 7| 2| |
-----
8T:      9T:      10T:     11T:
-----
|14| 4|11| 5| | | 4| 5|14|11| |
| 1|15| 8|10| | | 15|10| 1| 8| |
| 7| 9| 2|16| | | 9|16| 7| 2| |
|12| 6|13| 3| | | 6| 3|12|13| |
-----
12T:     13T:     14T:     15T:
-----
|14| 4|11| 5| | | 4| 5|14|11| |
| 7| 9| 2|16| | | 9|16| 7| 2| |
| 1|15| 8|10| | | 15|10| 1| 8| |
|12| 6|13| 3| | | 6| 3|12|13| |
-----

```

But we have to rewrite some answers so smart as to meet with the list standard, if they are made against the primary conditions: $n1 < n4$, $n1 < n13$, $n1 < n16$, $n2 > n5$. Otherwise you feel too hard to find their old numbers in the former solution list of the objects.

The next long list gives you the result of my final experiment:

**** Self-Complementary Magic Squares 4x4: ****
 ** Transform Each Fundamental into 15 Solutions **
 *** With Original and Noramlized Version ***

[1]

0T: 1/	1T: 9/	2T: 13/	3T: 21/
$\begin{array}{ c c c c } \hline 1 15 8 10 \\ \hline 14 4 11 5 \\ \hline 12 6 13 3 \\ \hline 7 9 2 16 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 1 14 12 7 \\ \hline 8 11 13 2 \\ \hline 15 4 6 9 \\ \hline 10 5 3 16 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 16 7 9 \\ \hline 13 3 12 6 \\ \hline 11 5 14 4 \\ \hline 8 10 1 15 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 13 11 8 \\ \hline 7 12 14 1 \\ \hline 16 3 5 10 \\ \hline 9 6 4 15 \\ \hline \end{array}$
4T: 3/	5T: 11/	6T: 15/	7T: 23/
$\begin{array}{ c c c c } \hline 1 15 8 10 \\ \hline 12 6 13 3 \\ \hline 14 4 11 5 \\ \hline 7 9 2 16 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 1 12 14 7 \\ \hline 8 13 11 2 \\ \hline 15 6 4 9 \\ \hline 10 3 5 16 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 16 7 9 \\ \hline 11 5 14 4 \\ \hline 13 3 12 6 \\ \hline 8 10 1 15 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 11 13 8 \\ \hline 7 14 12 1 \\ \hline 16 5 3 10 \\ \hline 9 4 6 15 \\ \hline \end{array}$
8T: 26/	9T: 28/	10T: 34/	11T: 36/
$\begin{array}{ c c c c } \hline 3 16 10 5 \\ \hline 13 2 8 11 \\ \hline 6 9 15 4 \\ \hline 12 7 1 14 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 16 10 5 \\ \hline 6 9 15 4 \\ \hline 13 2 8 11 \\ \hline 12 7 1 14 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 15 9 6 \\ \hline 14 1 7 12 \\ \hline 5 10 16 3 \\ \hline 11 8 2 13 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 15 9 6 \\ \hline 5 10 16 3 \\ \hline 14 1 7 12 \\ \hline 11 8 2 13 \\ \hline \end{array}$
12T: 29/	13T: 32/	14T: 37/	15T: 40/
$\begin{array}{ c c c c } \hline 3 13 6 12 \\ \hline 10 8 15 1 \\ \hline 16 2 9 7 \\ \hline 5 11 4 14 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 3 10 16 5 \\ \hline 6 15 9 4 \\ \hline 13 8 2 11 \\ \hline 12 1 7 14 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 14 5 11 \\ \hline 9 7 16 2 \\ \hline 15 1 10 8 \\ \hline 6 12 3 13 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 4 9 15 6 \\ \hline 5 16 10 3 \\ \hline 14 7 1 12 \\ \hline 11 2 8 13 \\ \hline \end{array}$

[2]

0T: 2/	1T: 7/	2T: 14/	3T: 19/
$\begin{array}{ c c c c } \hline 1 15 12 6 \\ \hline 14 4 7 9 \\ \hline 8 10 13 3 \\ \hline 11 5 2 16 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 1 14 8 11 \\ \hline 12 7 13 2 \\ \hline 15 4 10 5 \\ \hline 6 9 3 16 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 16 11 5 \\ \hline 13 3 8 10 \\ \hline 7 9 14 4 \\ \hline 12 6 1 15 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 13 7 12 \\ \hline 11 8 14 1 \\ \hline 16 3 9 6 \\ \hline 5 10 4 15 \\ \hline \end{array}$
4T: 5/	5T: 12/	6T: 17/	7T: 24/
$\begin{array}{ c c c c } \hline 1 15 12 6 \\ \hline 8 10 13 3 \\ \hline 14 4 7 9 \\ \hline 11 5 2 16 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 1 12 15 6 \\ \hline 8 13 10 3 \\ \hline 14 7 4 9 \\ \hline 11 2 5 16 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 16 11 5 \\ \hline 7 9 14 4 \\ \hline 13 3 8 10 \\ \hline 12 6 1 15 \\ \hline \end{array}$	$\begin{array}{ c c c c } \hline 2 11 16 5 \\ \hline 7 14 9 4 \\ \hline 13 8 3 10 \\ \hline 12 1 6 15 \\ \hline \end{array}$

8T: 25/	9T: 27/	10T: 33/	11T: 35/
<pre> +---+ 3 16 6 9 +---+---+ 13 2 12 7 +---+---+ 10 5 15 4 +---+---+ 8 11 1 14 +---+ </pre>	<pre> +---+ 3 16 6 9 +---+---+ 10 5 15 4 +---+---+ 13 2 12 7 +---+---+ 8 11 1 14 +---+ </pre>	<pre> +---+ 4 15 5 10 +---+---+ 14 1 11 8 +---+---+ 9 6 16 3 +---+---+ 7 12 2 13 +---+ </pre>	<pre> +---+ 4 15 5 10 +---+---+ 9 6 16 3 +---+---+ 14 1 11 8 +---+---+ 7 12 2 13 +---+ </pre>

12T: 30/	13T: 31/	14T: 38/	15T: 39/
<pre> +---+ 3 13 10 8 +---+---+ 6 12 15 1 +---+---+ 16 2 5 11 +---+---+ 9 7 4 14 +---+ </pre>	<pre> +---+ 3 10 13 8 +---+---+ 6 15 12 1 +---+---+ 16 5 2 11 +---+---+ 9 4 7 14 +---+ </pre>	<pre> +---+ 4 14 9 7 +---+---+ 5 11 16 2 +---+---+ 15 1 6 12 +---+---+ 10 8 3 13 +---+ </pre>	<pre> +---+ 4 9 14 7 +---+---+ 5 16 11 2 +---+---+ 15 6 1 12 +---+---+ 10 3 8 13 +---+ </pre>

[3]

0T: 4/	1T: 8/	2T: 16/	3T: 20/
<pre> +---+ 1 15 14 4 +---+---+ 12 6 7 9 +---+---+ 8 10 11 5 +---+---+ 13 3 2 16 +---+ </pre>	<pre> +---+ 1 14 15 4 +---+---+ 12 7 6 9 +---+---+ 8 11 10 5 +---+---+ 13 2 3 16 +---+ </pre>	<pre> +---+ 2 16 13 3 +---+---+ 11 5 8 10 +---+---+ 7 9 12 6 +---+---+ 14 4 1 15 +---+ </pre>	<pre> +---+ 2 13 16 3 +---+---+ 11 8 5 10 +---+---+ 7 12 9 6 +---+---+ 14 1 4 15 +---+ </pre>

4T: 6/	5T: 10/	6T: 18/	7T: 22/
<pre> +---+ 1 15 14 4 +---+---+ 8 10 11 5 +---+---+ 12 6 7 9 +---+---+ 13 3 2 16 +---+ </pre>	<pre> +---+ 1 14 15 4 +---+---+ 8 11 10 5 +---+---+ 12 7 6 9 +---+---+ 13 2 3 16 +---+ </pre>	<pre> +---+ 2 16 13 3 +---+---+ 7 9 12 6 +---+---+ 11 5 8 10 +---+---+ 14 4 1 15 +---+ </pre>	<pre> +---+ 2 13 16 3 +---+---+ 7 12 9 6 +---+---+ 11 8 5 10 +---+---+ 14 1 4 15 +---+ </pre>

8T: 41/	9T: 42/	10T: 45/	11T: 46/
<pre> +---+ 5 16 4 9 +---+---+ 11 2 14 7 +---+---+ 10 3 15 6 +---+---+ 8 13 1 12 +---+ </pre>	<pre> +---+ 5 16 4 9 +---+---+ 10 3 15 6 +---+---+ 11 2 14 7 +---+---+ 8 13 1 12 +---+ </pre>	<pre> +---+ 6 15 3 10 +---+---+ 12 1 13 8 +---+---+ 9 4 16 5 +---+---+ 7 14 2 11 +---+ </pre>	<pre> +---+ 6 15 3 10 +---+---+ 9 4 16 5 +---+---+ 12 1 13 8 +---+---+ 7 14 2 11 +---+ </pre>

12T: 43/	13T: 44/	14T: 47/	15T: 48/
<pre> +---+ 5 11 10 8 +---+---+ 4 14 15 1 +---+---+ 16 2 3 13 +---+---+ 9 7 6 12 +---+ </pre>	<pre> +---+ 5 10 11 8 +---+---+ 4 15 14 1 +---+---+ 16 3 2 13 +---+---+ 9 6 7 12 +---+ </pre>	<pre> +---+ 6 12 9 7 +---+---+ 3 13 16 2 +---+---+ 15 1 4 14 +---+---+ 10 8 5 11 +---+ </pre>	<pre> +---+ 6 9 12 7 +---+---+ 3 16 13 2 +---+---+ 15 4 1 14 +---+---+ 10 5 8 11 +---+ </pre>

(Each 'n/' on the right shoulder means the number of the old list.)

[Count = 3 x 16]

I checked how often every old number('n/') was consulted.
No repetition and no drop-off of solution could be found.

```

1:1,    2:1,    3:1,    4:1,    5:1,    6:1,    7:1,    8:1,
9:1,    10:1,   11:1,   12:1,   13:1,   14:1,   15:1,   16:1,
17:1,   18:1,   19:1,   20:1,   21:1,   22:1,   23:1,   24:1,
25:1,   26:1,   27:1,   28:1,   29:1,   30:1,   31:1,   32:1,
33:1,   34:1,   35:1,   36:1,   37:1,   38:1,   39:1,   40:1,
41:1,   42:1,   43:1,   44:1,   45:1,   46:1,   47:1,   48:1

```

Nothing Wrong!

#4. We could really make all 48 object solutions out of the Fundamental Three.
We could demonstrate that F3 are really 'fundamental solutions' for all by this certain evidence, couldn't we?

We have also discovered the origins of all 48 solutions of our self-complementary magic squares 4x4. All of them prove to be the real offsprings of F3, say, of the utmost origin 4-dimensional extra-cubic forms of order 2. I think it is more important. In the 4-dimensional world we cannot help thinking there is essentially only one solution.

Now we have to prove that F3 are really the children of this utmost origin.
But how can we do that?

Find any property showing F3 are similar enough to be considered as 'brothers and sisters' of the same parent.

- (1) Compare each to the others, and find any similar pair half of whose members are the same and common.
- (2) Try to find and make any transformation method for them.

```
** Print All 3 Fundamental Solutions **
```

```

          1/                2/                3/
┌───┬───┬───┬───┐ ┌───┬───┬───┬───┐ ┌───┬───┬───┬───┐
│ 1|15|12| 6 │ │ 1|15| 8|10 │ │ 1|15|14| 4 │
├───┴───┴───┴───┤ ├───┴───┴───┴───┤ ├───┴───┴───┴───┤
│14| 4| 7| 9 │ │14| 4|11| 5 │ │12| 6| 7| 9 │
├───┴───┴───┴───┤ ├───┴───┴───┴───┤ ├───┴───┴───┴───┤
│ 8|10|13| 3 │ │12| 6|13| 3 │ │ 8|10|11| 5 │
├───┴───┴───┴───┤ ├───┴───┴───┴───┤ ├───┴───┴───┴───┤
│11| 5| 2|16 │ │ 7| 9| 2|16 │ │13| 3| 2|16 │
└───┬───┬───┬───┘ └───┬───┬───┬───┘ └───┬───┬───┬───┘

```

```
[Count = 3]
```

```
** Find Any Similar Pairs to Each Other **
```

```

[ 1]          1/                <-- Unmatched -->                2/
  1 15 12  6    * * 12  6    * * 8 10    1 15  8 10
 14  4  7  9    * *  7  9    * * 11  5    14  4 11  5
  8 10 13  3    8 10 * * 12  6 * *    12  6 13  3
 11  5  2 16   11  5 * *  7  9 * *    7  9  2 16

[ 2]          1/                <-- Unmatched -->                3/
  1 15 12  6    * * 12  6    * * 14  4    1 15 14  4
 14  4  7  9    14  4 * * 12  6 * *    12  6  7  9
  8 10 13  3    * * 13  3    * * 11  5    8 10 11  5
 11  5  2 16   11  5 * * 13  3 * *    13  3  2 16

```

```
[ 3]          2/          <-- Unmatched -->          1/
  1 15  8 10    * *  8 10    * * 12  6    1 15 12  6
 14  4 11  5    * * 11  5    * *  7  9    14  4  7  9
 12  6 13  3    12  6 * *    8 10 * *    8 10 13  3
  7  9  2 16    7  9 * *    11  5 * *    11  5  2 16

[ 4]          3/          <-- Unmatched -->          1/
  1 15 14  4    * * 14  4    * * 12  6    1 15 12  6
 12  6  7  9    12  6 * *    14  4 * *    14  4  7  9
  8 10 11  5    * * 11  5    * * 13  3    8 10 13  3
 13  3  2 16    13  3 * *    11  5 * *    11  5  2 16
```

[Count = 4]

```
/**/
/* Transformation Rules: Type 1 & 2 */
void trnsf1(void){
  d[1]=c[1];    d[2]=c[2];    d[3]=c[9];    d[4]=c[10];
  d[5]=c[5];    d[6]=c[6];    d[7]=c[13];   d[8]=c[14];
  d[9]=c[3];    d[10]=c[4];   d[11]=c[11];  d[12]=c[12];
  d[13]=c[7];   d[14]=c[8];   d[15]=c[15];  d[16]=c[16];
}
/**/
void trnsf2(void){
  d[1]=c[1];    d[2]=c[2];    d[3]=c[5];    d[4]=c[6];
  d[5]=c[3];    d[6]=c[4];    d[7]=c[7];    d[8]=c[8];
  d[9]=c[9];    d[10]=c[10];   d[11]=c[13];  d[12]=c[14];
  d[13]=c[11];  d[14]=c[12];   d[15]=c[15];  d[16]=c[16];
}
/**/
```

We found we could make No.1 into both No.2 and 3 by those 'exchange-a-half' transformation methods dictated above.

No.1 of F3 prove to be the 'most fundamental' solution for all!

But the process: No.2 --> No.1 --> No.3 is also possible to be made, and No.3 --> No.1 --> No.2 is also possible to be constructed.

Each one of F3 can be the Most Fundamenta Solution. They are equal.

What does this mean?

It is natural for us to think they are the brothers and sisters of the same parent.

Therefore each of F3 can be nominated for the representative solution of all.

The only parent cannot be but the 4-D EC magic form of order 2.

#5. How about 48 solutions of the pandiagonal magic squares 4x4? Are they also the offsprings of the same origin? They are expected to be so, say, they must be so, because we know there is a notable one-to-one correspondence between the two solution sets by the next way of mutual transformation.

```
          /SC<-->/PD
n1  n2  n3  n4    n1  n2  n4  n3    Exchange 3rd column with 4th.
n5  n6  n7  n8    n5  n6  n8  n7    And then exchange 3rd row with 4th.
n9  n10 n11 n12   n13 n14 n16 n15   Or exchange 3rd row with 4th, and
n13 n14 n15 n16   n9  n10 n12 n11   then exchange 3rd column with 4th.
```

But we have to check if there is such a perfect correspondence as for all 48 solutions.
The next list shows the test result.

**** Conversion Test for SC & CC Magic Squares 4x4 ****
****** Print All 48 Solutions & Converted Ones ******

1/SC	1/Complete	1/Normalised
1 15 8 10	1 15 10 8	1 15 10 8
14 4 11 5	14 4 5 11	14 4 5 11
12 6 13 3	7 9 16 2	7 9 16 2
7 9 2 16	12 6 3 13	12 6 3 13
2/S	2/C	2/N
1 15 12 6	1 15 6 12	1 15 6 12
14 4 7 9	14 4 9 7	14 4 9 7
8 10 13 3	11 5 16 2	11 5 16 2
11 5 2 16	8 10 3 13	8 10 3 13
3/S	3/C	3/N
1 15 8 10	1 15 10 8	1 15 10 8
12 6 13 3	12 6 3 13	12 6 3 13
14 4 11 5	7 9 16 2	7 9 16 2
7 9 2 16	14 4 5 11	14 4 5 11
4/S	4/C	4/N
1 15 14 4	1 15 4 14	1 15 4 14
12 6 7 9	12 6 9 7	12 6 9 7
8 10 11 5	13 3 16 2	13 3 16 2
13 3 2 16	8 10 5 11	8 10 5 11
5/S	5/C	5/N
1 15 12 6	1 15 6 12	1 15 6 12
8 10 13 3	8 10 3 13	8 10 3 13
14 4 7 9	11 5 16 2	11 5 16 2
11 5 2 16	14 4 9 7	14 4 9 7
6/S	6/C	6/N
1 15 14 4	1 15 4 14	1 15 4 14
8 10 11 5	8 10 5 11	8 10 5 11
12 6 7 9	13 3 16 2	13 3 16 2
13 3 2 16	12 6 9 7	12 6 9 7
7/S	7/C	7/N
1 14 8 11	1 14 11 8	1 14 11 8
12 7 13 2	12 7 2 13	12 7 2 13
15 4 10 5	6 9 16 3	6 9 16 3
6 9 3 16	15 4 5 10	15 4 5 10
8/S	8/C	8/N
1 14 15 4	1 14 4 15	1 14 4 15
12 7 6 9	12 7 9 6	12 7 9 6
8 11 10 5	13 2 16 3	13 2 16 3
13 2 3 16	8 11 5 10	8 11 5 10
9/S	9/C	9/N
1 14 12 7	1 14 7 12	1 14 7 12
8 11 13 2	8 11 2 13	8 11 2 13
15 4 6 9	10 5 16 3	10 5 16 3
10 5 3 16	15 4 9 6	15 4 9 6
10/S	10/C	10/N
1 14 15 4	1 14 4 15	1 14 4 15
8 11 10 5	8 11 5 10	8 11 5 10
12 7 6 9	13 2 16 3	13 2 16 3
13 2 3 16	12 7 9 6	12 7 9 6

	11/S	11/C	11/N	
1	12 14 7	1 12 7 14	1 12 7 14	
8	13 11 2	8 13 2 11	8 13 2 11	
15	6 4 9	10 3 16 5	10 3 16 5	
10	3 5 16	15 6 9 4	15 6 9 4	
	12/S	12/C	12/N	
1	12 15 6	1 12 6 15	1 12 6 15	
8	13 10 3	8 13 3 10	8 13 3 10	
14	7 4 9	11 2 16 5	11 2 16 5	
11	2 5 16	14 7 9 4	14 7 9 4	
	13/S	13/C	13/N	
2	16 7 9	2 16 9 7	2 16 9 7	
13	3 12 6	13 3 6 12	13 3 6 12	
11	5 14 4	8 10 15 1	8 10 15 1	
8	10 1 15	11 5 4 14	11 5 4 14	
	• • • • •			
	27/S	27/C	27/N	
3	16 6 9	3 16 9 6	3 16 9 6	
10	5 15 4	10 5 4 15	10 5 4 15	
13	2 12 7	8 11 14 1	8 11 14 1	
8	11 1 14	13 2 7 12	13 2 7 12	
	28/S	28/C	29/N	28/L
3	16 10 5	3 16 5 10	3 16 5 10	3 16 2 13
6	9 15 4	6 9 4 15	6 9 4 15	10 5 11 8
13	2 8 11	12 7 14 1	12 7 14 1	15 4 14 1
12	7 1 14	13 2 11 8	13 2 11 8	6 9 7 12
	29/S	29/C	31/N	29/L
3	13 6 12	3 13 12 6	3 13 12 6	3 16 5 10
10	8 15 1	10 8 1 15	10 8 1 15	6 9 4 15
16	2 9 7	5 11 14 4	5 11 14 4	12 7 14 1
5	11 4 14	16 2 7 9	16 2 7 9	13 2 11 8
	30/S	30/C	33/N	30/L
3	13 10 8	3 13 8 10	3 13 8 10	3 16 2 13
6	12 15 1	6 12 1 15	6 12 1 15	6 9 7 12
16	2 5 11	9 7 14 4	9 7 14 4	15 4 14 1
9	7 4 14	16 2 11 5	16 2 11 5	10 5 11 8
	31/S	31/C	24/N	31/L
3	10 13 8	3 10 8 13	2 11 5 16	3 13 12 6
6	15 12 1	6 15 1 12	7 14 4 9	10 8 1 15
16	5 2 11	9 4 14 7	12 1 15 6	5 11 14 4
9	4 7 14	16 5 11 2	13 8 10 3	16 2 7 9
	32/S	32/C	23/N	32/L
3	10 16 5	3 10 5 16	2 11 8 13	3 13 2 16
6	15 9 4	6 15 4 9	7 14 1 12	10 8 11 5
13	8 2 11	12 1 14 7	9 4 15 6	15 1 14 4
12	1 7 14	13 8 11 2	16 5 10 3	6 12 7 9
	33/S	33/C	35/N	33/L
4	15 5 10	4 15 10 5	4 15 10 5	3 13 8 10
14	1 11 8	14 1 8 11	14 1 8 11	6 12 1 15
9	6 16 3	7 12 13 2	7 12 13 2	9 7 14 4
7	12 2 13	9 6 3 16	9 6 3 16	16 2 11 5
	34/S	34/C	36/N	34/L
4	15 9 6	4 15 6 9	4 15 6 9	3 13 2 16
14	1 7 12	14 1 12 7	14 1 12 7	6 12 7 9
5	10 16 3	11 8 13 2	11 8 13 2	15 1 14 4
11	8 2 13	5 10 3 16	5 10 3 16	10 8 11 5
	35/S	35/C	37/N	35/L
4	15 5 10	4 15 10 5	4 15 10 5	4 15 10 5
9	6 16 3	9 6 3 16	9 6 3 16	14 1 8 11
14	1 11 8	7 12 13 2	7 12 13 2	7 12 13 2
7	12 2 13	14 1 8 11	14 1 8 11	9 6 3 16

	36/S	36/C	39/N	36/L
4	15 9 6	4 15 6 9	4 15 6 9	4 15 6 9
5	10 16 3	5 10 3 16	5 10 3 16	14 1 12 7
14	1 7 12	11 8 13 2	11 8 13 2	11 8 13 2
11	8 2 13	14 1 12 7	14 1 12 7	5 10 3 16
	37/S	37/C	41/N	37/L
4	14 5 11	4 14 11 5	4 14 11 5	4 15 10 5
9	7 16 2	9 7 2 16	9 7 2 16	9 6 3 16
15	1 10 8	6 12 13 3	6 12 13 3	7 12 13 2
6	12 3 13	15 1 8 10	15 1 8 10	14 1 8 11
	38/S	38/C	43/N	38/L
4	14 9 7	4 14 7 9	4 14 7 9	4 15 1 14
5	11 16 2	5 11 2 16	5 11 2 16	9 6 12 7
15	1 6 12	10 8 13 3	10 8 13 3	16 3 13 2
10	8 3 13	15 1 12 6	15 1 12 6	5 10 8 11
	39/S	39/C	12/N	39/L
4	9 14 7	4 9 7 14	1 12 6 15	4 15 6 9
5	16 11 2	5 16 2 11	8 13 3 10	5 10 3 16
15	6 1 12	10 3 13 8	11 2 16 5	11 8 13 2
10	3 8 13	15 6 12 1	14 7 9 4	14 1 12 7
	40/S	40/C	11/N	40/L
4	9 15 6	4 9 6 15	1 12 7 14	4 15 1 14
5	16 10 3	5 16 3 10	8 13 2 11	5 10 8 11
14	7 1 12	11 2 13 8	10 3 16 5	16 3 13 2
11	2 8 13	14 7 12 1	15 6 9 4	9 6 12 7
	41/S	41/C	42/N	41/L
5	16 4 9	5 16 9 4	4 14 1 15	4 14 11 5
11	2 14 7	11 2 7 14	9 7 12 6	9 7 2 16
10	3 15 6	8 13 12 1	16 2 13 3	6 12 13 3
8	13 1 12	10 3 6 15	5 11 8 10	15 1 8 10
	42/S	42/C	38/N	42/L
5	16 4 9	5 16 9 4	4 15 1 14	4 14 1 15
10	3 15 6	10 3 6 15	9 6 12 7	9 7 12 6
11	2 14 7	8 13 12 1	16 3 13 2	16 2 13 3
8	13 1 12	11 2 7 14	5 10 8 11	5 11 8 10
	43/S	43/C	34/N	43/L
5	11 10 8	5 11 8 10	3 13 2 16	4 14 7 9
4	14 15 1	4 14 1 15	6 12 7 9	5 11 2 16
16	2 3 13	9 7 12 6	15 1 14 4	10 8 13 3
9	7 6 12	16 2 13 3	10 8 11 5	15 1 12 6
	44/S	44/C	22/N	44/L
5	10 11 8	5 10 8 11	2 13 3 16	4 14 1 15
4	15 14 1	4 15 1 14	7 12 6 9	5 11 8 10
16	3 2 13	9 6 12 7	14 1 15 4	16 2 13 3
9	6 7 12	16 3 13 2	11 8 10 5	9 7 12 6
	45/S	45/C	32/N	45/L
6	15 3 10	6 15 10 3	3 13 2 16	5 16 3 10
12	1 13 8	12 1 8 13	10 8 11 5	4 9 6 15
9	4 16 5	7 14 11 2	15 1 14 4	14 7 12 1
7	14 2 11	9 4 5 16	6 12 7 9	11 2 13 8
	46/S	46/C	28/N	46/L
6	15 3 10	6 15 10 3	3 16 2 13	5 16 2 11
9	4 16 5	9 4 5 16	10 5 11 8	4 9 7 14
12	1 13 8	7 14 11 2	15 4 14 1	15 6 12 1
7	14 2 11	12 1 8 13	6 9 7 12	10 3 13 8
	47/S	47/C	44/N	47/L
6	12 9 7	6 12 7 9	4 14 1 15	6 15 4 9
3	13 16 2	3 13 2 16	5 11 8 10	3 10 5 16
15	1 4 14	10 8 11 5	16 2 13 3	13 8 11 2
10	8 5 11	15 1 14 4	9 7 12 6	12 1 14 7

48/S				48/C				10/N				48/L			
6	9	12	7	6	9	7	12	1	14	4	15	6	15	1	12
3	16	13	2	3	16	2	13	8	11	5	10	3	10	8	13
15	4	1	14	10	5	11	8	13	2	16	3	16	5	11	2
10	5	8	11	15	4	14	1	12	7	9	6	9	4	14	7

[Count = 48]

**** Print How Often the Old List Number was consulted ****

```

1/1,    2/1,    3/1,    4/1,    5/1,    6/1,    7/1,    8/1,
9/1,    10/2,   11/2,   12/2,   13/1,   14/1,   15/1,   16/1,
17/1,   18/1,   19/1,   20/1,   21/1,   22/2,   23/2,   24/2,
25/1,   26/1,   27/1,   28/1,   29/1,   30/0,   31/1,   32/1,
33/1,   34/1,   35/1,   36/1,   37/1,   38/1,   39/1,   40/0,
41/1,   42/1,   43/1,   44/1,   45/0,   46/0,   47/0,   48/0

```

Several repetitions and drop-offs are found! The one-to-one correspondence is not perfect at all, although it is quite satisfactory during the first 27 solutions.

That means we cannot apply this transformation method to two sets of all 48 solutions between Self-Complementary and Complete types, but we can apply it to the Fundamental Three: No.1, 2 and 4.

We can only make the F3 of Complete type from the Self-Complementary ones by that transformation method. We have to make all the rest from those new F3 by any other transformation system.

(Calculated by MWCW and written on February 16, 2003 by Kanji Setsuda)

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