

Part 3: New Algebraic Studies of Magic Squares: **Kanji Setsuda**
 Chapter 4: Algebraic Study of Magic Squares of Order 6
 Section 4: 'Even-Odd Patterns' for Magic Squares 6x6

#0. What do the "Even-Odd Patterns" look like?

Let me introduce you my newest method of studying Magic Squares of order 6.

Right here in this section I would like to demonstrate my idea how to use "Even-Odd Patterns" for our object Magic Squares of order 6.

"Even-Odd Patterns" and "EOP" in mathematical form are shown in the following list.

*** List of Standard Type of Magic Squares of Order 6 ***
 *** Standard Solutions with their 'Even/Odd Patterns' ***

	1/ /Even-Odd	/EOP
1 36 3 35 34 2	0 E 0 0 E E	1 0 1 1 0 0
32 8 23 19 5 24	E E 0 0 0 E	0 0 1 1 1 0
4 20 28 12 22 25	E E E E E 0	0 0 0 0 0 1
31 7 30 21 6 16	0 0 E 0 E E	1 1 0 1 0 0
10 29 14 15 26 17	E 0 E 0 E 0	0 1 0 1 0 1
33 11 13 9 18 27	0 0 0 0 E 0	1 1 1 1 0 1
	3/ /Even-Odd	/EOP
1 36 3 35 34 2	0 E 0 0 E E	1 0 1 1 0 0
32 9 24 18 5 23	E 0 E E 0 0	0 1 0 0 1 1
4 22 27 12 21 25	E E 0 E 0 0	0 0 1 0 1 1
31 8 30 20 6 16	0 E E E E E	1 0 0 0 0 0
10 29 14 15 26 17	E 0 E 0 E 0	0 1 0 1 0 1
33 7 13 11 19 28	0 0 0 0 0 E	1 1 1 1 1 0
.		

Standard forms of MS66 are listed on the left hand side, and made into the 'Even-Odd Patterns' in the center abstracting the only property whether each number is 'even' or 'odd' from the original MS66. The 'EOP' on the right hand side are made by substituting 'Even' with '0' and 'Odd' with '1' to be designed for our mathematical expression. Let's use the last form all the time for our further study.

What do those EOP tables mean mathematically?

- (1) Each table of EOP must have the same counts of '0' and '1' in total: $18+18=36$
 Remember we must use all the serial natural numbers {1, 2, 3, 4, 5, ..., 33, 34, 35, 36} to make our original MS66 taking each strictly once. It is one of the basic promises when we compose our magic things. It means we must take {1, 0, 1, 0, 1, ..., 1, 0, 1, 0} to make the EOP using '0' eighteen times as often as '1'.

- (2) What can we find about any row, any column or any primary diagonal?
 We can only find '1' either once, or three times, or five times in any row, column or primary diagonal. Why?

In the original MS66 six numbers on each row, column and diagonal must add up to 111, the magic constant. But the sum 111 is an odd number. That means in our EOP six on each row, column and diagonal must also add up to the odd sum.

$$0+0+0+0+0+1=1; \quad 0+0+0+1+1+1=3; \quad 0+1+1+1+1+1=5;$$

Only those three combinations are possible, and those sums (1, 3, 5) above mean odd sum '1'. In our EOP we only deal with the remainder 0 and 1 when divided by 2, so that every sum could simulate the magic constant 111 all right.

Watch the next list presenting some example solutions with EOP and check if you could find these properties in them, please.

*** List of Standard Type of Magic Squares of Order 6 ***
 *** Standard Solutions with their 'Even/Odd Patterns' ***

1/						931/																	
/EOP						/EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	8	23	19	5	24	0	0	1	1	1	0	32	8	22	20	5	24	0	0	0	0	1	0
4	20	28	12	22	25	0	0	0	0	0	1	4	19	28	12	25	23	0	1	0	0	1	1
31	7	30	21	6	16	1	1	0	1	0	0	31	7	29	21	6	17	1	1	1	1	0	1
10	29	14	15	26	17	0	1	0	1	0	1	10	30	13	14	26	18	0	0	1	0	0	0
33	11	13	9	18	27	1	1	1	1	0	1	33	11	16	9	15	27	1	1	0	1	1	1
1557/						3912/																	
/EOP						/EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	7	21	20	5	26	0	1	1	0	1	0	32	7	24	18	5	25	0	1	0	0	1	1
4	18	29	13	24	23	0	0	1	1	0	1	4	20	29	13	23	22	0	0	1	1	1	0
31	6	30	22	8	14	1	0	0	0	0	0	31	6	28	21	8	17	1	0	0	1	0	1
10	28	17	12	25	19	0	0	1	0	1	1	10	30	16	9	27	19	0	0	0	1	1	1
33	16	11	9	15	27	1	0	1	1	1	1	33	12	11	15	14	26	1	0	1	1	0	0
5414/						10150/																	
/EOP						/EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	7	25	19	5	23	0	1	1	1	1	1	32	7	26	17	5	24	0	1	0	1	1	0
4	18	29	14	24	22	0	0	1	0	0	0	4	18	30	14	22	23	0	0	0	0	0	1
31	6	30	20	9	15	1	0	0	0	1	1	31	9	29	21	6	15	1	1	1	1	0	1
10	27	16	11	26	21	0	1	0	1	0	1	10	28	12	16	25	20	0	0	0	0	1	0
33	17	8	12	13	28	1	1	0	0	1	0	33	13	11	8	19	27	1	1	1	0	1	1
14223/						17822/																	
/EOP						/EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	6	23	21	5	24	0	0	1	1	1	0	32	6	24	18	5	26	0	0	0	0	1	0
4	17	30	14	20	26	0	1	0	0	0	0	4	20	29	14	21	23	0	0	1	0	1	1
31	8	28	22	9	13	1	0	0	0	1	1	31	7	27	22	8	16	1	1	1	0	0	0
10	29	16	12	25	19	0	1	0	0	1	1	10	30	11	13	28	19	0	0	1	1	0	1
33	15	11	7	18	27	1	1	1	1	0	1	33	12	17	9	15	25	1	0	1	1	1	1
20189/						28205/																	
/EOP						/EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	6	21	22	5	25	0	0	1	0	1	1	32	6	26	19	5	23	0	0	0	1	1	1
4	16	29	15	23	24	0	0	1	1	1	0	4	18	28	15	22	24	0	0	0	1	0	0
31	9	30	20	8	13	1	1	0	0	0	1	31	7	29	21	11	12	1	1	1	1	1	0
10	26	17	12	27	19	0	0	1	0	1	1	10	27	16	13	25	20	0	1	0	1	1	0
33	18	11	7	14	28	1	0	1	1	0	0	33	17	9	8	14	30	1	1	1	0	0	0
36039/						41759/																	
/EOP						/EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	6	26	17	5	25	0	0	0	1	1	1	32	6	27	18	5	23	0	0	1	0	1	1
4	20	30	15	23	19	0	0	0	1	1	1	4	19	29	15	20	24	0	1	1	1	0	0
31	8	27	22	7	16	1	0	1	0	1	0	31	9	26	22	7	16	1	1	0	0	1	0
10	29	14	13	24	21	0	1	0	1	0	1	10	30	14	8	28	21	0	0	0	0	0	1
33	12	11	9	18	28	1	0	1	1	0	0	33	11	12	13	17	25	1	1	0	1	1	1
46553/						58251/																	
/EOP						/EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	6	26	18	5	24	0	0	0	0	1	0	32	6	27	18	5	23	0	0	1	0	1	1
4	19	27	16	22	23	0	1	1	0	0	1	4	19	28	16	20	24	0	1	0	0	0	0
31	8	30	20	9	13	1	0	0	0	1	1	31	7	29	21	8	15	1	1	1	1	0	1
10	25	11	15	29	21	0	1	1	1	1	1	10	26	11	12	30	22	0	0	1	0	0	0
33	17	14	7	12	28	1	1	0	1	0	0	33	17	13	9	14	25	1	1	1	1	0	1

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#1. How many 'Even-Odd Patterns' could we make for MS66 in all?

I guess you may wonder how many different EOP you could make in all for the standard type of magic squares of order 6, when you see those solution examples.

Why don't we actually make all those possible tables and know the total count of them? Let's dictate any program for that and make our computer calculate.

** Basic Form and Basic Equations for Definition of 'EOP' **

n1	n2	n3	n4	n5	n6	$n1+n2+n3+n4+n5+n6=K$ (b1)
n7	n8	n9	n10	n11	n12	$n7+n8+n9+n10+n11+n12=K$ (b2)
n13	n14	n15	n16	n17	n18	$n13+n14+n15+n16+n17+n18=K$ (b3)
n19	n20	n21	n22	n23	n24	$n19+n20+n21+n22+n23+n24=K$ (b4)
n25	n26	n27	n28	n29	n30	$n25+n26+n27+n28+n29+n30=K$ (b5)
n31	n32	n33	n34	n35	n36	$n31+n32+n33+n34+n35+n36=K$ (b6)
n1+n7	n13	n19	n25	n31		$n1+n7+n13+n19+n25+n31=K$ (b7)
n2+n8	n14	n20	n26	n32		$n2+n8+n14+n20+n26+n32=K$ (b8)
n3+n9	n15	n21	n27	n33		$n3+n9+n15+n21+n27+n33=K$ (b9)
n4+n10	n16	n22	n28	n34		$n4+n10+n16+n22+n28+n34=K$ (b10)
n5+n11	n17	n23	n29	n35		$n5+n11+n17+n23+n29+n35=K$ (b11)
n6+n12	n18	n24	n30	n36		$n6+n12+n18+n24+n30+n36=K$ (b12)
n1+n8	n15	n22	n29	n36		$n1+n8+n15+n22+n29+n36=K$ (d1)
n6+n11	n16	n21	n26	n31		$n6+n11+n16+n21+n26+n31=K$ (d2)

- (1) Each variable should take only '0' or '1' for its value.
- (2) Every EOP table should take '0' eighteen times as often as '1' in total.
- (3) Six variables in each of those 14 equations above should add up to K. The sum K means odd number '1', just the remainder when each actual sum is divided by 2.

The next list shows a part of the program I recently dictated for EOP of MS66.

```

/** Simulate MS66 by 'Even-Odd Patterns' **/
/** 'SimlMS66EOP.c' built by Kanji Setsuda **/
/** on Nov.22, '07 with MacOSX & Xcode 2 **/
/**/
#include <stdio.h>
/* Global Variables */
long int cnt, cnt2;
long cntr[5];
short cnt3;
short nm[37],ucnt[2];
short anm[5][37];
/**/
/* Main Program for Research */
int main(){
short n;
printf("\n");
printf("*** Simulate Magic Squares 6x6 by 'Even-Odd Patterns' **\n");
printf(" ** by Kanji Setsuda on Nov.22, '07 with MacOSX **\n");
cnt=0; cnt3=0;
for(n=0;n<37;n++){nm[n]=-1;}; ucnt[0]=0; ucnt[1]=0;
stp01(); /* Begin the Calculations */
if(cnt3>0){pr2ans();}
printf("\n [Count = %d]\n",cnt);
printf(" OK!\n");
return 0;
}
/* Begin the Calculations */

```

```

/* Search Level 1: */
/* Set n1 */
void stp01(){
    short a;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[1]=a; ucnt[a]++;
            stp02();
            ucnt[a]--;}
    }
}
/* Set n2 */
void stp02(){
    short a;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[2]=a; ucnt[a]++;
            stp03();
            ucnt[a]--;}
    }
}
/* Set n3 */
void stp03(){
    short a;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[3]=a; ucnt[a]++;
            stp04();
            ucnt[a]--;}
    }
}
/* Set n4 */
void stp04(){
    short a;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[4]=a; ucnt[a]++;
            stp05();
            ucnt[a]--;}
    }
}
/* Set n5 */
void stp05(){
    short a;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[5]=a; ucnt[a]++;
            stp06();
            ucnt[a]--;}
    }
}
/* Set n6 & Check(n1+n2+n3+n4+n5+n6==K) */
void stp06(){
    short a,b;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[6]=a; ucnt[a]++;
            b=nm[1]+nm[2]+nm[3]+nm[4]+nm[5]+nm[6];
            if((b%2)>0){cnt2=0; stp07();}
        }
    }
}

```

```

        ucnt[a]--;}
    }
}
/* Set n7 */
void stp07(){
    short a;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[7]=a; ucnt[a]++;
            stp08();
            ucnt[a]--;}
        }
}
/* Set n13 */
void stp08(){
    short a;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[13]=a; ucnt[a]++;
            stp09();
            ucnt[a]--;}
        }
}
/* Set n19 */
void stp09(){
    short a;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[19]=a; ucnt[a]++;
            stp10();
            ucnt[a]--;}
        }
}
/* Set n25 */
void stp10(){
    short a;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[25]=a; ucnt[a]++;
            stp11();
            ucnt[a]--;}
        }
}
/* Set n31 & Check(n1+n7+n13+n19+n25+n31==K) */
void stp11(){
    short a,b;
    for(a=0;a<2;a++){
        if(ucnt[a]<18){
            nm[31]=a; ucnt[a]++;
            b=nm[1]+nm[7]+nm[13]+nm[19]+nm[25]+nm[31];
            if((b%2)>0){stp12();}
            ucnt[a]--;}
        }
}
/* Search Level 2: */
/* Set n11 */
. . . .
. . . .
/* Search Level 6: */

```

```

/* Set n23 */
void stp31(){
  short a;
  for(a=0;a<2;a++){
    if(ucnt[a]<18){
      nm[23]=a; ucnt[a]++;
      stp32();
      ucnt[a]--;}
  }
}
/* Set n24 & Check(n19+n20+n21+n22+n23+n24==K) */
void stp32(){
  short a,b;
  for(a=0;a<2;a++){
    if(ucnt[a]<18){
      nm[24]=a; ucnt[a]++;
      b=nm[19]+nm[20]+nm[21]+nm[22]+nm[23]+nm[24];
      if((b%2)>0){stp33();}
      ucnt[a]--;}
  }
}
/* Set n35 & Check(n5+n11+n17+n23+n29+n35==K) */
void stp33(){
  short a,b;
  for(a=0;a<2;a++){
    if(ucnt[a]<18){
      nm[35]=a; ucnt[a]++;
      b=nm[5]+nm[11]+nm[17]+nm[23]+nm[29]+nm[35];
      if((b%2)>0){stp34();}
      ucnt[a]--;}
  }
}
/* Set n34 & Check(n31+n32+n33+n34+n35+n36==K) */
void stp34(){
  short a,b;
  for(a=0;a<2;a++){
    if(ucnt[a]<18){
      nm[34]=a; ucnt[a]++;
      b=nm[31]+nm[32]+nm[33]+nm[34]+nm[35]+nm[36];
      if((b%2)>0){stp35();}
      ucnt[a]--;}
  }
}
/* Set n28 & Check(n4+n10+n16+n22+n28+n34==K) */
void stp35(){
  short a,b;
  for(a=0;a<2;a++){
    if(ucnt[a]<18){
      nm[28]=a; ucnt[a]++;
      b=nm[4]+nm[10]+nm[16]+nm[22]+nm[28]+nm[34];
      if((b%2)>0){stp36();}
      ucnt[a]--;}
  }
}
/* Set n30 & Check(n6+n12+n18+n24+n30+n36==K)&(n25+n26+n27+n28+n29+n30==K) */
void stp36(){
  short a,b,c;
  for(a=0;a<2;a++){
    if(ucnt[a]<18){

```

```

        nm[30]=a; ucnt[a]++;
        b=nm[6]+nm[12]+nm[18]+nm[24]+nm[30]+nm[36];
        c=nm[25]+nm[26]+nm[27]+nm[28]+nm[29]+nm[30];
        if(((b%2)>0)&&((c%2)>0)){ansprint();}
        ucnt[a]--;
    }
}
}
/**/
/* Print the Answers */
void ansprint(){
    short n;
    cnt2++;
    if(cnt2==1){cntr[cnt3]=cnt+1; cntr[cnt3+1]=0;
        for(n=1;n<37;n++){anm[cnt3][n]=nm[n]; anm[cnt3+1][n]=0;}
        cnt3++;
        if(cnt3==4){pr4ans(); cnt3=0;}
    }
    cnt++;
}
/**/
/* Print 4 Answers */
void pr4ans(){
    short l,l6,m,n,d;
    printf("%18d/%19d/%19d/%19d/\n",
        cntr[0],cntr[1],cntr[2],cntr[3]);
    for(l=0;l<6;l++){l6=l*6;
        for(m=0;m<4;m++){printf(" ");
            for(n=1;n<7;n++){d=anm[m][l6+n];
                if(d>=0){printf("%3d",d);}else{printf("  _");}
            }
            printf(" ");
        }
        printf("\n");
    }
}
/**/
/* Print 2 Answers */
void pr2ans(){
    short l,l6,m,n,d;
    printf("%18d/%19d/\n",cntr[0],cntr[1]);
    for(l=0;l<6;l++){l6=l*6;
        for(m=0;m<2;m++){printf(" ");
            for(n=1;n<7;n++){d=anm[m][l6+n];
                if(d>=0){printf("%3d",d);}else{printf("  _");}
            }
            printf(" ");
        }
        printf("\n");
    }
}
/**/

```

It took only one minute for me to know the total count of tables. It was easy, but I was much surprised at the big count of solutions. It was far beyond my imagination.

**** Simulate Magic Squares 6x6 by 'Even-Odd Patterns' ****
**** by Kanji Setsuda on Nov.22, '07 with MacOSX ****

0 0 0 0 0 1	0 0 0 0 1 0	0 0 0 1 0 0	0 0 0 1 1 1
0 0 0 0 0 1	0 0 1 1 0 1	0 0 1 1 0 1	0 0 0 0 0 1
0 0 1 0 1 1	0 0 1 0 1 1	0 0 1 0 1 1	0 0 1 0 1 1
0 1 0 1 0 1	0 1 0 0 1 1	0 0 0 1 1 1	0 0 0 0 1 0
0 1 1 1 1 1	0 0 0 1 1 1	0 0 1 1 0 1	0 1 1 1 1 1
1 1 1 1 1 0	1 0 1 1 1 1	1 1 0 1 1 1	1 0 1 1 1 1
252593/	311385/	387577/	463353/
0 0 1 0 0 0	0 0 1 0 1 1	0 0 1 1 0 1	0 0 1 1 1 0
0 0 1 1 0 1	0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1
0 0 1 0 1 1	0 0 0 0 1 0	0 0 0 0 0 1	0 0 1 0 1 1
0 0 0 1 1 1	0 0 0 1 1 1	0 1 0 0 1 1	0 1 0 0 1 1
0 0 1 1 0 1	0 1 1 1 1 1	0 1 1 1 1 1	0 0 0 1 1 1
1 1 1 0 1 1	1 0 1 1 1 1	1 1 1 1 1 0	1 0 1 1 1 1
539337/	598129/	673905/	750097/
0 1 0 0 0 0	0 1 0 0 1 1	0 1 0 1 0 1	0 1 0 1 1 0
0 0 1 1 0 1	0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1
0 0 1 0 1 1	0 0 0 0 1 0	0 0 1 0 1 1	0 0 1 0 1 1
0 1 0 0 1 1	0 0 0 1 1 1	0 0 0 0 1 0	0 1 0 0 1 1
0 0 0 1 1 1	0 1 1 1 1 1	0 1 1 1 1 1	0 0 0 1 1 1
1 1 1 1 0 1	1 1 0 1 1 1	1 1 1 1 0 1	1 1 0 1 1 1
826081/	902273/	978257/	1054241/
0 1 1 0 0 1	0 1 1 0 1 0	0 1 1 1 0 0	0 1 1 1 1 1
0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1
0 0 0 0 1 0	0 0 1 0 1 1	0 0 1 0 1 1	0 0 0 0 0 1
0 0 0 1 1 1	0 1 0 0 1 1	0 1 0 0 1 1	0 0 0 0 0 1
0 1 1 1 1 1	0 0 0 1 1 1	0 0 0 1 1 1	0 1 1 1 1 1
1 1 1 1 0 1	1 1 1 0 1 1	1 1 1 1 0 1	1 1 1 1 1 0
1113057/	1171873/	1247857/	1323841/
1 0 0 0 0 0	1 0 0 0 1 1	1 0 0 1 0 1	1 0 0 1 1 0
0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1
0 0 1 0 1 1	0 0 1 0 1 1	0 1 1 0 0 1	0 0 1 0 1 1
0 1 0 1 0 1	0 0 0 1 1 1	0 1 0 1 1 0	0 0 0 1 1 1
0 1 1 1 1 1	0 0 1 1 1 0	0 0 1 0 1 1	0 1 1 1 1 1
0 1 1 1 1 1	0 1 1 1 1 1	0 1 1 1 1 1	0 0 1 0 1 1
1400033/	1476017/	1552209/	1627985/
1 0 1 0 0 1	1 0 1 0 1 0	1 0 1 1 0 0	1 0 1 1 1 1
0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1
0 1 0 0 1 1	0 0 1 0 1 1	0 0 0 0 0 1	0 0 1 0 1 1
0 1 0 1 1 0	0 0 0 1 1 1	0 1 0 0 1 1	0 0 0 1 1 1
0 0 1 1 0 1	0 1 1 1 1 1	0 1 1 1 1 1	0 0 0 0 1 0
0 1 1 1 1 1	0 0 0 1 1 1	0 1 1 1 1 1	0 1 1 1 1 1
1686777/	1762761/	1838537/	1914729/
1 1 0 0 0 1	1 1 0 0 1 0	1 1 0 1 0 0	1 1 0 1 1 1
0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1
0 0 1 0 1 1	0 0 1 0 1 1	0 0 1 0 1 1	0 0 1 0 1 1
0 1 0 1 0 1	0 1 0 1 0 1	0 0 0 1 1 1	0 0 0 1 1 1
0 0 1 1 1 0	0 1 1 1 1 1	0 1 1 1 1 1	0 0 1 1 1 0
0 1 1 1 1 1	0 0 1 1 0 1	0 1 1 0 0 1	0 0 1 0 1 1
1973521/	2049713/	2108505/	2167297/
1 1 1 0 0 0	1 1 1 0 1 1	1 1 1 1 0 1	1 1 1 1 1 0
0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1	0 0 0 0 0 1
0 0 1 0 1 1	0 0 1 0 1 1	0 0 0 0 0 1	0 0 0 0 0 1
0 0 0 1 1 1	0 0 0 1 1 1	0 1 0 0 1 1	0 0 0 0 0 1
0 1 1 1 1 1	0 0 1 1 1 0	0 0 1 1 1 0	0 1 1 1 1 1
0 1 0 1 0 1	0 0 0 1 1 1	0 1 1 1 1 1	0 1 1 1 1 1

[Count = 2226112] OK!

#2. How many magic squares 6x6 are there for each EOP?

You may wonder next how many pieces of real MS66 you could make for every EOP.

Why don't we make them actually, based on an EOP at first? Let's define a single EOP and then calculate to reconstruct all the real MS66 corresponding to that table.

As we now know where we have to place 'even' numbers and 'odd' ones according to our EOP, we may well take {1, 3, 5, 7, 9, 11, ..., 31, 33, 35} for odd numbers, and take {2, 4, 6, 8, 10, 12, ..., 30, 32, 34, 36} for even numbers in any step in advance. I wished I could really cut down our steps and save time by half.

Let's skip listing my long program and come quickly to the actual result of our interesting experiment.

**** Reconstruction of Standard Magic Squares of Order 6 ****
**** for a single EOP by Kanji Setsuda on Nov.22, 2007 ****

1/ /EOP						811/ /EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	8	23	19	5	24	0	0	1	1	1	0	32	8	19	21	7	24	0	0	1	1	1	0
4	20	28	12	22	25	0	0	0	0	0	1	4	18	26	12	22	29	0	0	0	0	0	1
31	7	30	21	6	16	1	1	0	1	0	0	31	5	30	23	6	16	1	1	0	1	0	0
10	29	14	15	26	17	0	1	0	1	0	1	10	27	20	11	28	15	0	1	0	1	0	1
33	11	13	9	18	27	1	1	1	1	0	1	33	17	13	9	14	25	1	1	1	1	0	1
1930/ /EOP						3322/ /EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	12	21	11	9	26	0	0	1	1	1	0	32	12	21	9	11	26	0	0	1	1	1	0
4	22	28	8	24	25	0	0	0	0	0	1	4	24	28	8	18	29	0	0	0	0	0	1
31	7	30	23	6	14	1	1	0	1	0	0	31	5	30	25	6	14	1	1	0	1	0	0
10	29	16	19	20	17	0	1	0	1	0	1	10	27	16	19	22	17	0	1	0	1	0	1
33	5	13	15	18	27	1	1	1	1	0	1	33	7	13	15	20	23	1	1	1	1	0	1
4988/ /EOP						6844/ /EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	12	19	11	13	24	0	0	1	1	1	0	32	12	13	17	15	22	0	0	1	1	1	0
4	26	28	8	22	23	0	0	0	0	0	1	4	24	30	6	18	29	0	0	0	0	0	1
31	5	30	21	6	18	1	1	0	1	0	0	31	7	28	23	8	14	1	1	0	1	0	0
10	25	14	27	20	15	0	1	0	1	0	1	10	27	26	9	20	19	0	1	0	1	0	1
33	7	17	9	16	29	1	1	1	1	0	1	33	5	11	21	16	25	1	1	1	1	0	1
8410/ /EOP						9800/ /EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	20	7	13	17	22	0	0	1	1	1	0	32	14	9	13	19	24	0	0	1	1	1	0
4	18	28	6	26	29	0	0	0	0	0	1	4	26	30	6	16	29	0	0	0	0	0	1
31	5	30	21	8	16	1	1	0	1	0	0	31	5	28	17	12	18	1	1	0	1	0	0
10	23	24	25	14	15	0	1	0	1	0	1	10	23	20	25	22	11	0	1	0	1	0	1
33	9	19	11	12	27	1	1	1	1	0	1	33	7	21	15	8	27	1	1	1	1	0	1
10833/ /EOP						11806/ /EOP																	
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	12	9	15	21	22	0	0	1	1	1	0	32	20	13	5	23	18	0	0	1	1	1	0
4	26	30	6	16	29	0	0	0	0	0	1	4	24	26	8	22	27	0	0	0	0	0	1
31	5	24	23	8	20	1	1	0	1	0	0	31	7	30	21	6	16	1	1	0	1	0	0
10	25	28	19	18	11	0	1	0	1	0	1	10	15	28	25	14	19	0	1	0	1	0	1
33	7	17	13	14	27	1	1	1	1	0	1	33	9	11	17	12	29	1	1	1	1	0	1

12463/						/EOP	12857/						/EOP										
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	16	11	9	25	18	0	0	1	1	1	0	32	18	7	5	27	22	0	0	1	1	1	0
4	28	30	8	14	27	0	0	0	0	0	1	4	24	26	12	16	29	0	0	0	0	0	1
31	7	24	15	12	22	1	1	0	1	0	0	31	11	28	21	6	14	1	1	0	1	0	0
10	19	26	23	20	13	0	1	0	1	0	1	10	9	30	23	20	19	0	1	0	1	0	1
33	5	17	21	6	29	1	1	1	1	0	1	33	13	17	15	8	25	1	1	1	1	0	1
13040/						/EOP	13110/						/EOP										
1	36	3	35	34	2	1	0	1	1	0	0	1	36	3	35	34	2	1	0	1	1	0	0
32	16	11	5	29	18	0	0	1	1	1	0	32	10	25	17	5	22	0	0	1	1	1	0
4	28	30	12	14	23	0	0	0	0	0	1	6	20	24	12	26	23	0	0	0	0	0	1
31	7	26	19	6	22	1	1	0	1	0	0	31	7	30	21	4	18	1	1	0	1	0	0
10	9	24	27	20	21	0	1	0	1	0	1	8	29	16	11	28	19	0	1	0	1	0	1
33	15	17	13	8	25	1	1	1	1	0	1	33	9	13	15	14	27	1	1	1	1	0	1
.																							

To my surprise, I knew I would have too many solutions and take too long a time for me to count them up to the last. I could not help giving it up on my way. I could not even imagine how many MS66 I could make in all.

I think we had better make any other type of MS66 with smaller size of solution set. But what else can we make, then? We already know we can hopelessly compose any piece of neither 'Self-complementary' type nor 'Complete' one.

#3. How can you use our 'Even-Odd Patterns' as an effective and fruitful method?

Let me demonstrate next here, how to use this EOP method to prove effectively that we can make nothing of certain special magic squares of order 6.

It is a question from a reader of my homepage that made me develop this new way of solving it and proving that we could compose no piece of that type.

I recently received an interesting mail from an Armenian intellectual, **Dr. Arthur Babayan**, with a serious question if I could make any pieces of such a type as follows.

-	-	-	-	-	-	-	-	-	-	-	-	
-	-	1	11	-	-	Anna Terean's Type #6	-	-	1	1	-	-
-	34	-	-	8	-	of Magic Squares 6x6	-	0	-	-	0	-
-	5	-	-	21	-	-	1	-	-	1	-	
-	-	24	7	-	-	-	-	0	1	-	-	
-	-	-	-	-	-	-	-	-	-	-	-	

(Blank Form)

('EOP' made by K.S.)

Type #6: {n9,n10,n14,n17,n20,n23,n27,n28} = {1, 5, 7, 8, 11, 21, 24, 34}

How can we compose any pieces of Standard Magic Squares 6x6 with those eight numbers {1, 5, 7, 8, 11, 21, 24 and 34} put on such fixed positions as above?

It seemed rather easy for me to solve it and know the total count of solutions.

But I could not really find any answer for it, and I could not come to the end of my calculation spending too long a time, either. I could not help giving it up halfway.

We have truly come up to one of the hardest problems in our magic square world.

I decided to try a new way of studying here instead, yes, our special simulation of MS66 by making the 'Even- Odd Patterns' for that type.

Why don't we make any correct EOP filling it up only with '0' and '1'?

It did not take a long time to do that job all right. My computer gave me no answer at last. It should clearly mean we could make nothing for that type of MS66.

**** Make 'EOP' for Anna Terean's Type #6 of Magic Squares 6x6: ****
[Count = 0]

**** Monitor List: Forced Inspection of Calculations on the Way ****

1/						1663/						3429/						5091/					
0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1
_	0	1	1	0	_	_	0	1	1	0	_	_	0	1	1	0	_	_	0	1	1	0	_
0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0
0	1	0	0	1	1	0	1	1	0	1	0	0	1	0	0	1	1	0	1	1	0	1	0
_	1	0	1	0	_	_	1	0	1	0	_	_	1	0	1	0	_	_	1	0	1	0	_
1	1	0	1	_	1	0	1	1	1	_	1	1	1	0	1	_	1	0	1	1	1	_	1
6857/						8525/						10285/						11779/					
0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1
_	0	1	1	1	_	_	0	1	1	0	_	_	0	1	1	0	_	_	0	1	1	0	_
0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	1	1	0	0	1	0	1
0	1	0	0	1	1	0	1	1	0	1	0	0	1	0	0	1	1	0	1	1	0	1	0
_	1	0	1	0	_	_	1	0	1	0	_	_	1	0	1	0	_	_	1	0	1	0	_
1	1	0	0	_	1	1	1	1	0	_	1	0	1	0	0	_	1	1	1	1	0	_	1
12973/						14733/						16401/						17991/					
0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1
_	0	1	1	0	_	_	0	1	1	1	_	_	0	1	1	0	_	_	0	1	1	0	_
0	0	1	0	0	0	0	0	1	0	0	0	1	0	1	0	0	1	1	0	1	0	0	1
0	1	0	0	1	1	0	1	1	0	1	0	0	1	0	0	1	1	0	1	1	0	1	0
_	1	0	1	0	_	_	1	0	1	0	_	_	1	0	1	0	_	_	1	0	1	0	_
1	1	1	1	_	0	1	1	0	1	_	0	1	1	1	1	_	0	0	1	0	1	_	0
19185/						20753/						21925/						23493/					
0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1
_	0	1	1	0	_	_	0	1	1	0	_	_	0	1	1	0	_	_	0	1	1	0	_
0	0	1	1	0	1	0	0	1	1	0	1	1	0	1	1	0	0	1	0	1	1	0	0
0	1	0	0	1	1	0	1	1	0	1	0	0	1	0	0	1	1	0	1	1	0	1	0
_	1	0	1	0	_	_	1	0	1	0	_	_	1	0	1	0	_	_	1	0	1	0	_
0	1	1	0	_	0	1	1	0	0	_	0	0	1	1	0	_	0	1	1	0	0	_	0

[Count = 24664] (This count means nothing important.)

When I tried to examine calculations of my computer on the way, I found in every case a part of rows or columns should inevitably add up to 'even' sum, or the total count of '0' was inevitably different from '1' in the same EOP table. Because they were all against the first definitions, we could not get any correct table of our EOP.

In the objective MS66 side, it indicates some sets of 6 numbers on rows or columns should fail in adding up to 111, or each objective square should fail in using all the serial natural numbers {1, 2, 3, 4, 5, ..., 33, 34, 35, 36} taking each strictly once. Just because we failed in making any correct EOP, we could not really compose any correct piece of such a special type of MS66 as requested, we believe.

Theoretically speaking, we may well examine the smallest set of EOP at first in place of the biggest set of all the objective MS66 solutions, just in the case we want to know that they are really impossible to be composed. Can't you believe that?

I suppose the fact tells us that our 'EOP' should be such a good abstract from the original magic squares that the former EOP could act as a good simulator for the latter MS66 in some certain aspects.

#4. Let's prove that we could not compose any piece of Pan-diagonal MS66.

We now know in our magic square world we could hardly make any piece of neither 'Self-complementary' type nor 'Complete' type of MS66. I once reported about how to prove their impossibility of those types in my old article.

We also know we could hardly make any piece of 'Pan-diagonal' type of MS66. But, I have not yet proved that it is impossible to be composed.

Now let me challenge this problem, one of the most difficult ones of ours, to prove that we could not compose any piece of Pan-diagonal magic squares of order 6.

I want to try to make our EOP tables and examine them here instead of trying to examine all the imaginary solution set of Pan-diagonal MS66.

I prepared such a diagram and equations as below to define everything for our EOP tables simulating the original Pan-diagonal MS66.

** Basic Form of Pan-Magic Squares of Order 6 **

34	35	36	31	32	33	34	35	36	31	32	33
4	5	6	1	2	3	4	5	6	1	2	3
10	11	12	7	8	9	10	11	12	7	8	9
16	17	18	13	14	15	16	17	18	13	14	15
22	23	24	19	20	21	22	23	24	19	20	21
28	29	30	25	26	27	28	29	30	25	26	27
34	35	36	31	32	33	34	35	36	31	32	33
4	5	6	1	2	3	4	5	6	1	2	3

** Basic Equations: **

$n_1+n_2+n_3+n_4+n_5+n_6=K$ (b1)	$n_1+n_7+n_{13}+n_{19}+n_{25}+n_{31}=K$ (b7)
$n_7+n_8+n_9+n_{10}+n_{11}+n_{12}=K$ (b2)	$n_2+n_8+n_{14}+n_{20}+n_{26}+n_{32}=K$ (b8)
$n_{13}+n_{14}+n_{15}+n_{16}+n_{17}+n_{18}=K$ (b3)	$n_3+n_9+n_{15}+n_{21}+n_{27}+n_{33}=K$ (b9)
$n_{19}+n_{20}+n_{21}+n_{22}+n_{23}+n_{24}=K$ (b4)	$n_4+n_{10}+n_{16}+n_{22}+n_{28}+n_{34}=K$ (b10)
$n_{25}+n_{26}+n_{27}+n_{28}+n_{29}+n_{30}=K$ (b5)	$n_5+n_{11}+n_{17}+n_{23}+n_{29}+n_{35}=K$ (b11)
$n_{31}+n_{32}+n_{33}+n_{34}+n_{35}+n_{36}=K$ (b6)	$n_6+n_{12}+n_{18}+n_{24}+n_{30}+n_{36}=K$ (b12)

** Pan-Diagonal Equations: **

$n_1+n_8+n_{15}+n_{22}+n_{29}+n_{36}=K$ (p1)	$n_1+n_{12}+n_{17}+n_{22}+n_{27}+n_{32}=K$ (p2)
$n_2+n_9+n_{16}+n_{23}+n_{30}+n_{31}=K$ (p3)	$n_2+n_7+n_{18}+n_{23}+n_{28}+n_{33}=K$ (p4)
$n_3+n_{10}+n_{17}+n_{24}+n_{25}+n_{32}=K$ (p5)	$n_3+n_8+n_{13}+n_{24}+n_{29}+n_{34}=K$ (p6)
$n_4+n_{11}+n_{18}+n_{19}+n_{26}+n_{33}=K$ (p7)	$n_4+n_9+n_{14}+n_{19}+n_{30}+n_{35}=K$ (p8)
$n_5+n_{12}+n_{13}+n_{20}+n_{27}+n_{34}=K$ (p9)	$n_5+n_{10}+n_{15}+n_{20}+n_{25}+n_{36}=K$ (p10)
$n_6+n_7+n_{14}+n_{21}+n_{28}+n_{35}=K$ (p11)	$n_6+n_{11}+n_{16}+n_{21}+n_{26}+n_{31}=K$ (p12)

- (1) Each variable should take only '0' or '1' for its value.
- (2) Every EOP table should take '0' eighteen times as often as '1' in total.
- (3) Six variables in each of those 24 equations above should add up to K. The sum K means odd number '1', just the remainder when each actual sum is divided by 2.

Let's skip listing my long program, and come to the result of my experiment.

**** Make the EOP tables for Pan-diagonal MS66 and Examine them ****
[Count = 0]

**** Monitor List: Forced Inspection of Calculations on the Way ****

	1/	1425/	2783/	4141/
0 0 0 0 0 1	0 0 0 0 1 0	0 0 0 1 0 0	0 0 0 1 1 1	
0 1 1 0 0 1	0 1 0 1 0 1	0 1 1 0 0 1	0 1 0 0 0 0	
0 1 0 0 1 1	0 0 1 0 1 1	0 1 0 1 1 0	0 1 0 0 1 1	
0 1 0 1 0 1	0 1 1 1 1 1	0 0 0 0 1 0	0 1 0 0 1 1	
0 1 1 1 1 1	0 1 0 0 1 1	0 1 1 1 1 1	0 1 1 1 1 1	
1 1 1 - - 0	1 0 1 - - 1	1 0 1 - - 1	1 1 0 - - 1	
	5805/	7229/	8899/	10569/
0 0 1 0 0 0	0 0 1 0 1 1	0 0 1 1 0 1	0 0 1 1 1 0	
0 0 1 1 0 1	0 1 0 1 0 1	0 0 0 0 0 1	0 0 0 1 0 0	
0 1 0 0 1 1	0 0 1 0 1 1	0 0 1 0 1 1	0 1 0 0 1 1	
0 1 0 1 0 1	0 1 0 1 1 0	0 1 1 0 0 1	0 1 0 0 1 1	
0 0 1 0 1 1	0 1 0 0 1 1	0 0 1 1 1 0	0 0 1 0 1 1	
1 1 0 - - 1	1 0 1 - - 1	1 0 1 - - 1	1 1 1 - - 0	
	12233/	13587/	15543/	17259/
0 1 0 0 0 0	0 1 0 0 1 1	0 1 0 1 0 1	0 1 0 1 1 0	
0 0 0 0 0 1	0 0 0 0 0 1	0 0 1 1 0 1	0 0 0 0 0 1	
0 1 0 0 1 1	0 0 1 0 1 1	0 0 0 0 1 0	0 1 0 0 1 1	
0 1 1 1 1 1	0 0 0 1 1 1	0 1 0 0 1 1	0 1 1 0 0 1	
0 1 1 1 0 0	0 1 1 1 0 0	0 1 1 0 0 1	0 1 0 1 0 1	
1 1 1 - - 0	1 1 1 - - 1	1 0 1 - - 1	1 1 0 - - 1	
	18969/	20679/	22635/	24351/
0 1 1 0 0 1	0 1 1 0 1 0	0 1 1 1 0 0	0 1 1 1 1 1	
0 0 0 0 0 1	0 0 1 1 0 1	0 1 1 0 0 1	0 0 0 0 0 1	
0 1 0 0 1 1	0 1 0 0 1 1	0 0 1 0 1 1	0 1 0 0 1 1	
0 1 0 1 1 0	0 0 0 1 1 1	0 1 0 0 1 1	0 1 0 0 0 0	
0 1 1 1 0 0	0 0 1 0 1 1	0 0 1 1 0 1	0 1 0 1 0 1	
1 1 1 - - 0	1 1 0 - - 1	1 0 1 - - 1	1 1 0 - - 1	
	25705/	27059/	28775/	30731/
1 0 0 0 0 0	1 0 0 0 1 1	1 0 0 1 0 1	1 0 0 1 1 0	
0 1 0 0 0 0	0 1 1 0 0 1	0 0 1 1 0 1	0 1 1 0 0 1	
0 1 1 1 1 1	0 0 0 0 1 0	0 1 0 0 1 1	0 0 0 0 1 0	
0 1 1 0 0 1	0 1 0 0 1 1	0 0 0 1 1 1	0 1 0 1 1 0	
0 1 0 1 1 0	0 0 1 1 0 1	0 0 1 0 1 1	0 1 1 1 1 1	
0 1 1 - - 1	0 1 1 - - 1	0 0 1 - - 0	0 0 1 - - 1	
	32441/	34151/	35867/	37823/
1 0 1 0 0 1	1 0 1 0 1 0	1 0 1 1 0 0	1 0 1 1 1 1	
0 0 1 1 0 1	0 0 1 1 0 1	0 1 0 1 0 1	0 0 0 0 0 1	
0 1 0 0 1 1	0 0 1 0 1 1	0 1 0 0 1 1	0 0 1 0 1 1	
0 0 1 0 1 1	0 1 0 0 1 1	0 1 0 1 0 1	0 0 0 1 0 0	
0 1 1 0 0 1	0 1 1 0 0 1	0 1 0 0 1 1	0 0 0 1 1 1	
0 1 1 - - 0	0 1 1 - - 1	0 1 0 - - 1	0 1 1 - - 1	
	39177/	40841/	42511/	44181/
1 1 0 0 0 1	1 1 0 0 1 0	1 1 0 1 0 0	1 1 0 1 1 1	
0 0 0 0 0 1	0 1 0 0 0 0	0 1 0 1 0 1	0 0 0 0 0 1	
0 1 1 0 0 1	0 1 1 0 0 1	0 0 0 0 1 0	0 1 0 0 1 1	
0 1 1 0 1 0	0 0 0 0 1 0	0 1 0 1 1 0	0 0 0 1 1 1	
0 1 0 1 0 1	0 1 1 1 1 1	0 1 0 0 1 1	0 0 0 1 1 1	
0 1 1 - - 1	0 1 1 - - 1	0 1 1 - - 1	0 1 1 - - 0	

45605/						47269/						48627/						49985/					
1	1	1	0	0	0	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	0
0	0	0	0	0	1	0	1	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1
0	1	1	0	0	1	0	1	0	0	1	1	0	0	1	0	1	1	0	1	0	0	0	0
0	1	0	0	1	1	0	1	0	0	0	0	0	1	0	1	1	0	0	1	0	1	0	1
0	1	0	1	0	1	0	0	1	1	0	1	0	0	0	1	1	1	0	1	1	1	0	0
0	1	1	-	-	1	0	1	0	-	-	1	0	1	1	-	-	1	0	1	1	-	-	1

[Count = 51408] (This count means nothing important.)

It took only 20-30 seconds to do that job all right. But my computer gave me no answer at all. It should clearly mean we could make nothing of Pan-diagonal MS66.

When I tried to inspect calculations of my computer on the way, I found in every case some rows or columns should inevitably add up to 'even' sum, or the total count of '0' should inevitably be different from '1' in the same EOP table. They were all against the first definitions, so that we could not get any correct table of our EOP.

In the objective PMS66 side, it indicates some sets of 6 numbers on all rows or columns should fail in adding up to 111, or each objective square should fail in using all the serial natural numbers {1, 2, 3, 4, 5, ..., 33, 34, 35, 36} taking each strictly once. The fact we failed in making any correct EOP table means that we should necessarily fail in composing any correct piece of Pan-diagonal MS66. If the former EOP is impossible, then the latter PMS66 is also impossible to be composed.

Can't you believe that? But you should believe that it is logically true.

You don't have to take any more time in examining such the biggest set as of imaginary PMS66 solutions. You may well examine the smallest set of EOP quickly instead, when you want to know just whether they are possible or not.

It might be quite a relief to any researcher who is at a loss to face any hardest problems of magic things, I hope.

(On November 28, 2007 by Kanji Setsuda with MacOSX & Xcode 2)

[Additional Report]

#5. Let's prove some problems of MS66 by *reductio ad absurdum*.

I have been really asked some more interesting questions by Dr. Arthur Babayan about the special types of magic squares 6x6 with such conditions as follows.

He asked me if I could make any correct solutions for the standard type of MS66 under these strict conditions like below.

** Anna Terean's Type of Standard Magic Squares of Order 6 **

1/						2/						3/						4/					
1	-	-	-	-	5	11	-	-	-	-	21	-	11	-	-	-	-	-	1	-	-	-	-
-	-	-	21	-	-	-	-	-	5	-	-	-	-	-	-	21	-	-	-	-	-	-	5
-	11	-	-	-	-	-	1	-	-	-	-	-	-	1	5	-	-	-	-	11	21	-	-
-	-	-	-	24	-	-	-	-	-	7	-	-	-	8	7	-	-	-	-	34	24	-	-
-	-	34	-	-	-	-	-	8	-	-	-	34	-	-	-	-	-	8	-	-	-	-	-
8	-	-	-	-	7	34	-	-	-	-	24	-	-	-	-	24	-	-	-	-	-	7	-

How could I solve them for him using *reductio ad absurdum*? Let me report here, though it has nothing much to do with the 'EOP', our main topic in this section.

These problems once looked to be rather easy for us to solve and get any piece of

answers before long, but they were nothing easy at all in reality.

Any piece of solutions would not come up to us and we had to wait and wait, while we took too long a time for our calculations. We had to give it up on the way before coming to an end. I began to wonder if they might be impossible to be composed.

I started to develop any logical proof for that, using *reductio ad absurdum*.

Finally I found and knew how to prove them.

Let me define the basic diagrams and basic equations at first as follows.

AT1/	AT2/	AT3/	AT4/
1 a b c d 5	11 a b c d 21	a 11 i m - e	a 1 i m - e
e i - 21 m q	e i - 1 m q	- b j n f 21	- b j n f 5
f 11 j n - r	f 8 j n - r	q r 1 5 s t	q r 11 21 s t
g - o k 24 s	g - o k 5 s	u v 8 7 w x	u v 34 24 w x
h p 34 - l t	h p 7 - l t	34 g k o c -	8 g k o c -
8 u v x y 7	34 u v x y 24	h - l p 24 d	h - l p 7 d

AT1/ $1+a+b+c+d+5=111 \rightarrow a+b+c+d=111-1-5=105$
 $1+e+f+g+h+8=111 \rightarrow e+f+g+h=111-1-8=102$
 $1+i+j+k+l+7=111 \rightarrow i+j+k+l=111-1-7=103$
 $5+m+n+o+p+8=111 \rightarrow m+n+o+p=111-5-8=98$
 $5+q+r+s+t+7=111 \rightarrow q+r+s+t=111-5-7=99$
 $8+u+v+w+x+7=111 \rightarrow u+v+w+x=111-8-7=96 (+$

$$a+b+c+d +e+f+g+h +i+j+k+l +m+n+o+p +q+r+s+t +u+v+w+x = 603 \dots Sm1$$

Add both sides of those basic equations and you could know the total sum of all 24 variables {a, b, c, d, e, ... , u, v, w and x} we are using as above.

What value should we take for each variable? Of course, we should take it out of the serial natural numbers {1, 2, 3, 4, 5, 6, 7, ... , 33, 34, 35 and 36}, using each strictly once. As we are making a magic square of order 6, we can neither use the same number twice or more often, and nor un-use any one of them. Therefore, we can not take any of {1, 5, 7 and 8} again for those 24 variables.

What value should we take for each variable, again? The average value of those variables is over 25 ($>603 \div 24$). What combination could we find for them?

Let's try to put them as follows, picking out of all the larger numbers available.

$$36+35+33+32 +31+30+29+28 +27+26+25+23 +22+20+19+18 +17+16+15+14 +13+12+10+9 =$$

$$136 + 118 + 101 + 79 + 62 + 44 = 540$$

Why! These 24 numbers could only add up to the sum 588, smaller than the required value 603.

And it is the most important that we could not make any other larger sum than this. Yes, this 540 is the largest sum we could only get.

$$(a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s+t+u+v+w+x) \leq 540 \dots Sm2$$

Sm1 and Sm2 are contradictory to each other. We could not accept both at the same time. This should surely demonstrate that we could make no correct solution for that type under the first strict condition.

AT2/ $11+a+b+c+d+21=111 \rightarrow a+b+c+d=111-11-21=79$
 $11+e+f+g+h+34=111 \rightarrow e+f+g+h=111-11-34=66$
 $11+i+j+k+l+24=111 \rightarrow i+j+k+l=111-11-24=76$
 $21+m+n+o+p+34=111 \rightarrow m+n+o+p=111-21-34=56$
 $21+q+r+s+t+24=111 \rightarrow q+r+s+t=111-21-24=66$
 $34+u+v+w+x+24=111 \rightarrow u+v+w+x=111-34-24=53 (+$

$$a+b+c+d +e+f+g+h +i+j+k+l +m+n+o+p +q+r+s+t +u+v+w+x = 396 \dots Sm1$$

AT1/					AT2/					AT3/					AT4/								
1	a	b	c	d	5	11	a	b	c	d	21	a	11	i	m	-	e	a	1	i	m	-	e
e	i	-	21	m	q	e	i	-	1	m	q	-	b	j	n	f	21	-	b	j	n	f	5
f	11	j	n	-	r	f	8	j	n	-	r	q	r	1	5	s	t	q	r	11	21	s	t
g	-	o	k	24	s	g	-	o	k	5	s	u	v	8	7	w	x	u	v	34	24	w	x
h	p	34	-	l	t	h	p	7	-	l	t	34	g	k	o	c	-	8	g	k	o	c	-
8	u	v	x	y	7	34	u	v	x	y	24	h	-	l	p	24	d	h	-	l	p	7	d

Add both sides of those basic equations above to know the total sum of all 24 variables {a, b, c, d, e, ... , u, v, w and x} we are using as above.

What value should we take for each variable? Of course, we should take it out of the serial natural numbers {1, 2, 3, 4, 5, 6, 7, ... , 33, 34, 35 and 36}, using each strictly once. As we are composing a magic square of order 6, we can not take any of {1, 5, 7, 8, 11, 21, 24 and 34} again for those 24 variables.

The average value of those variables is under 17 ($396 \div 24 = 16.5$). Let's try to put them as follows, picking out of all the smaller numbers available.

$$2+3+4+6 +9+10+12+13 +14+15+16+17 +18+19+20+22 +23+25+26+27 +28+29+30+31 =$$

$$15 + 44 + 62 + 79 + 101 + 118 = 419$$

Why! These 24 numbers could only add up to the sum 419, larger than the required value 396.

And it is the most important that we could not make any other smaller sum than this. Yes, this 419 is the smallest sum we could only get.

$$(a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s+t+u+v+w+x) \geq 419 \dots Sm2$$

Sm1 and Sm2 are contradictory to each other. We could not accept both at the same time. This should surely demonstrate that we could make no correct solution for that type under the first strict condition.

$$AT3/ \begin{array}{ll} a+b+1+7+c+d=111 & \rightarrow a+b+c+d=111-1-7=103 \\ e+f+5+8+g+h=111 & \rightarrow e+f+g+h=111-5-8=98 \\ i+j+1+8+k+l=111 & \rightarrow i+j+k+l=111-1-8=102 \\ m+n+5+7+o+p=111 & \rightarrow m+n+o+p=111-5-7=99 \\ q+r+1+5+s+t=111 & \rightarrow q+r+s+t=111-1-5=105 \\ u+v+8+7+w+x=111 & \rightarrow u+v+w+x=111-8-7=96 (+) \end{array}$$

$$a+b+c+d +e+f+g+h +i+j+k+l +m+n+o+p +q+r+s+t +u+v+w+x = 603 \dots Sm1$$

Add both sides of those basic equations above to know the total sum of all 24 variables {a, b, c, d, e, ... , u, v, w and x} we are using as above.

What value should we take for each variable? Of course, we should take it out of the serial natural numbers {1, 2, 3, 4, 5, 6, 7, ... , 33, 34, 35 and 36}, using each strictly once. As we are composing a magic square of order 6, we can not take any of {1, 5, 7, and 8} again for those 24 variables.

The average value of those variables is over 25 ($>603 \div 24$). Let's try to put them as follows, picking out of all the larger numbers available.

$$36+35+33+32 +31+30+29+28 +27+26+25+23 +22+20+19+18 +17+16+15+14 +13+12+10+9 =$$

$$136 + 118 + 101 + 79 + 62 + 44 = 540$$

Why! These 24 numbers could only add up to the sum 540, smaller than the required value 603.

And it is the most important that we could not make any other larger sum than this. Yes, this 540 is the largest sum we could only get.

$$(a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s+t+u+v+w+x) \leq 540 \dots Sm2$$

Sm1 and Sm2 are contradictory to each other. We could not accept both at the same time. This should surely demonstrate that we could make no correct solution for that type under the first strict condition.

	AT1/		AT2/		AT3/		AT4/
1	a b c d 5	11	a b c d 21	a 11	i m - e	a 1	i m - e
e	i - 21 m q	e	i - 1 m q	- b	j n f 21	- b	j n f 5
f	11 j n - r	f	8 j n - r	q r	1 5 s t	q r	11 21 s t
g	- o k 24 s	g	- o k 5 s	u v	8 7 w x	u v	34 24 w x
h	p 34 - l t	h	p 7 - l t	34	g k o c -	8	g k o c -
8	u v x y 7	34	u v x y 24	h - l	p 24 d	h - l	p 7 d

AT4/ a+b+11+24+c+d=111 -> a+b+c+d=111-11-24=76
e+f+21+34+g+h=111 -> e+f+g+h=111-21-34=56
i+j+11+34+k+l=111 -> i+j+k+l=111-11-34=66
m+n+21+24+o+p=111 -> m+n+o+p=111-21-24=66
q+r+11+21+s+t=111 -> q+r+s+t=111-11-21=79
u+v+34+24+w+x=111 -> u+v+w+x=111-34-24=53 (+

a+b+c+d +e+f+g+h +i+j+k+l +m+n+o+p +q+r+s+t +u+v+w+x = 396 ... Sm1

Add both sides of those basic equations above to know the total sum of all 24 variables {a, b, c, d, e, ... , u, v, w and x} we are using as above.

What value should we take for each variable? Of course, we should take it out of the serial natural numbers {1, 2, 3, 4, 5, 6, 7, ... , 33, 34, 35 and 36}, using each strictly once. As we are composing a magic square of order 6, we can not take any of {1, 5, 7, 8, 11, 21, 24 and 34} again for those 24 variables.

The average value of those variables is under 17 (396÷24=16.5). Let's try to put them as follows, picking out of all the smaller numbers available.

$$2+3+4+6 +9+10+12+13 +14+15+16+17 +18+19+20+22 +23+25+26+27 +28+29+30+31 = 419$$

Why! These 24 numbers could only add up to the sum 419, larger than the required value 396.

And it is the most important that we could not make any other smaller sum than this. Yes, this 419 is the smallest sum we could only get.

$$(a+b+c+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s+t+u+v+w+x) \geq 419 \dots Sm2$$

Sm1 and Sm2 are contradictory to each other. We could not accept both at the same time. This should surely demonstrate that we could make no correct solution for that type under the first strict condition.

Let's study further about this method in a little more generalized form, shall we?

	AT1/AT2		
n1	a b c d n6	n1+a+b+c+d+n6=111	-> a+b+c+d=111-n1-n6
e	i - - m q	n1+e+f+g+h+n31=111	-> e+f+g+h=111-n1-n31
f	- j n - r	n1+i+j+k+l+n36=111	-> i+j+k+l=111-n1-n36
g	- o k - s	n6+m+n+o+p+n31=111	-> m+n+o+p=111-n6-n31
h	p - - l t	n6+q+r+s+t+n36=111	-> q+r+s+t=111-n6-n36
n31	u v x y n36	n31+u+v+w+x+n36=111	-> u+v+w+x=111-n31-n36 (+

$$a+b+c+d +e+f+g+h +i+j+k+l +m+n+o+p +q+r+s+t +u+v+w+x = 6*111 - 3*(n1+n6+n31+n36) = 666-3*(n1+n6+n31+n36) \dots Sm24$$

But in the present problems 419 < Sm24 < 540; meaning 419 < 666-3*(n1+n6+n31+n36) < 540
Therefore 83 > (n1+n6+n31+n36) > 42 ... Sm4-1

Value combinations of this inequality condition vary according to the actual requirements of that problem of yours.

If you unconditionally adopt the next condition as strict as possible, 300 < Sm24 < 588; between the possible smallest number: 1+2+3+4+5+ ... +22+23+24=300; and the largest number: 36+35+34+33+ ... +16+15+14+13=588; then 300 < 666-3*(n1+n6+n31+n36) < 588

$$\text{Therefore } 122 > (n1+n6+n31+n36) > 26 \dots Sm4-2$$

It shows the widest range of all. You can never imagine anything wider than this.

Suppose you are going out of this range, and you could not absolutely compose any correct solutions of your objective magic squares of order 6.

AT3/AT4

a	-	i	m	-	e	a+b+n15+n22+c+d=111	->	a+b+c+d=111-n15-n22
-	b	j	n	f	-	e+f+n16+n21+g+h=111	->	e+f+g+h=111-n16-n21
q	r	n15	16	s	t	i+j+n15+n21+k+l=111	->	i+j+k+l=111-n15-n21
u	v	n21	22	w	x	m+n+n16+n22+o+p=111	->	m+n+o+p=111-n16-n22
-	g	k	o	c	-	q+r+n15+n16+s+t=111	->	q+r+s+t=111-n15-n16
h	-	l	p	-	d	u+v+n21+n22+w+x=111	->	u+v+w+x=111-n21-n22 (+

$$a+b+c+d + e+f+g+h + i+j+k+l + m+n+o+p + q+r+s+t + u+v+w+x = 6*111-3*(n15+n16+n21+n22) = 666-3*(n15+n16+n21+n22) \dots Sm24'$$

But in our problems $419 < Sm24 < 540$; meaning $419 < 666-3*(n15+n16+n21+n22) < 540$
 Therefore $83 > (n15+n16+n21+n22) > 42 \dots Sm4'-1$

Value combinations vary according to the requirements of that problem of yours.

If you unconditionally adopt the next condition $300 < Sm24 < 588$; between the possible smallest number: $1+2+3+4+5+ \dots +22+23+24=300$; and the largest number: $36+35+34+33+ \dots +16+15+14+13=588$; then $300 < 666-3*(n15+n16+n21+n22) < 588$

Therefore $122 > (n15+n16+n21+n22) > 26 \dots Sm4-2$

It shows the widest range that you could never imagine anything wider.

Suppose you are going out of this range, and you could not absolutely compose any correct solutions of your objective magic squares of order 6.

We have to find out the exact range of some square sums of four variables according to the actual conditions of that problem. If you go out of that range, you should be at a loss and could not make any correct solutions of your object at last.

It is really a disappointing result report to Dr. Arthur Babayan, but our discovery must be such a relief that we could save our energy and time. We could make any challenge to other topics and get our success, I hope.

(Revised on December 10, 2007 by Kanji Setsuda with MacOSX & Xcode 2)

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